FURTHER RESULTS FOR CRACK-EDGE MAPPINGS BY RAY METHODS

A.N. Norris and J.D. Achenbach

The Technological Institute
Northwestern University
Evanston, IL 60201

L. Ahlberg and B.R. Tittmann

Rockwell International Science Center
Thousand Oaks, Cal. 91360

ABSTRACT

Methods to map crack edges by the use of arrival times of edge-diffracted signals, which have been presented earlier by the authors, have been extended to the pulse-echo case and to configurations of water immersed specimens and transducers. Applications of the methods to experimental data are presented in a companion paper.

INTRODUCTION

In Refs.[1]-[3] two analytical methods have been developed to map the edge of a crack by the use of data for diffraction of elastic waves by the crack-edge. These methods are based on elastodynamic ray theory and the geometrical theory of diffraction, and they require as input data the arrival times of diffracted ultrasonic signals. The first method maps flash points on the crack edge by a process of triangulation with the source and receiver as given vertices of the triangle. By the use of arrival times for neighboring positions of the source and/or the receiver, the directions of signal propagation, which determine the triangle, can be computed. This inverse mapping is global in the sense that no a-priori knowledge of the location of the crack edge is necessary. The second method is a local edge mapping which determines planes relative to a known point close to the crack edge. Each plane contains a flash point. The envelope of the planes maps an approximation to the crack edge. Mathematical details and a fairly detailed error analysis have been given in Ref.[3]. Applications of the methods to synthetic and experimental data have been presented in Ref.[4]. It is of particular interest that the local mapping technique allows for an iteration procedure whereby the result of a computation suggests an improved choice of the base point which in the subsequent iteration yields a better approximation to the crack edge. Extensions to include anisotropy of the material can be found in Refs.[5]-[6].

In this paper we discuss further extensions of the local edge mapping method to the pulse-echo case and to configurations of water-immersed specimens and transducers. Further tests on experimental data obtained at the Rockwell International Science Center are presented in a companion paper in these Proceedings [7].

REVIEW OF LOCAL EDGE MAPPING IN A HOMOGENEOUS MEDIUM

The local edge mapping method for locating and sizing cracks has been described previously [1]-[6] for the general case of pitch-catch data. In this Section we will consider the simple case of pulse-echo data for review purposes.

A source S is located at the vector position $\underset{\sim}{x}_S$ in a homogeneous, isotropic solid which contains a crack. The measured data is in the form of travel times of signals from the transducer to the crack edge C and back to the transducer. At the crack edge the signal is diffracted according to the Geometrical Theory of Diffraction [8]. The point of diffraction is known as the flash point because it appears to the receiver as a virtual point source. The flash points are points on the edge which make the signal travel time stationary in accordance with Fermat's Principle. Suppose that the incident and observed signals are both waves of longitudinal motion. Then, in most practical examples there are two flash points, one on the "near" crack edge, the other on the "far" crack edge. These correspond to two time-separated received signals, whose arrival times can be measured [4].

It is assumed that the travel time τ of the diffracted longitudinal-longitudinal (L-L) signal has been measured. The local edge mapping method requires that a base point B be chosen which lies relatively close to the crack. Let $\underset{\sim}{x}_B$ and $\underset{\sim}{x}_D$ be the vector positions of the base point and the flash point, respectively. Let $\underset{\sim}{p}_B$ be the unit vector from $\underset{\sim}{x}_S$ to $\underset{\sim}{x}_B$ and let R be the distance between $\underset{\sim}{x}_S$ and $\underset{\sim}{x}_B$. The distance from the source to the flash point is $\frac{1}{2} c_L \tau$, where c_L is the longitudinal wave speed. It follows that

$$\frac{1}{2}c_L\tau - R = (\underset{\sim}{x}_D-\underset{\sim}{x}_B)\cdot\underset{\sim}{p}_B + \frac{1}{2R}\,|(\underset{\sim}{x}_D-\underset{\sim}{x}_B) \wedge \underset{\sim}{p}_B|^2 + --- . \tag{2.1}$$

The third term in eq.(2.1) is small compared with the first two if the distance from the base point to the crack is small compared with R. Under this assumption, we ignore all but the first two terms in eq.(2.1) and we say that the flash point lies on the plane

$$(\underset{\sim}{x} - \underset{\sim}{x}_B)\cdot\underset{\sim}{p}_B - (\frac{1}{2}c_L\tau - R) = 0. \tag{2.2}$$

We note that all the quantities in eq.(2.2) are known except the flash point position. The basic idea of the local edge mapping is to take several pulse-echo measurements, construct the planes in each case according to eq.(2.2), and then find the flash points by intersection of the planes. Any three planes will produce a point, provided that no two of the planes are parallel. However, for any three given source positions, the intersection of the planes may produce a point which is nowhere near the crack. To see this we can imagine three sources situated such that the three corresponding flash points are at quite distinct positions on the crack edge. Obviously, the intersection of the planes will yield a point far from any of the actual flash points. Therefore it is imperative to devise an algorithm that selects from all the triads of planes those which give good approximations to the crack edge. Two possible algorithms will be discussed in the next section.

It should be noted that the local edge mapping is necessarily iterative. At each step in the iteration we select the base point as the spatial mean of the previously determined flash points. In practice, it is found that only two or three iterations are required for convergence.

LOCAL EDGE MAPPING ALGORITHMS

A basic requirement for the intersection process is that the three planes have one normal quite different from the other two. Let the three normals be $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$ and $\underset{\sim}{p}_3$. Then we require that the triple product of the normals be of order unity,

$$|\underset{\sim}{p}_1 \cdot \underset{\sim}{p}_2 \wedge \underset{\sim}{p}_3| = O(1). \tag{3.1}$$

Mathematically, this means that the three simultaneous linear equations for the flash point are not ill-conditioned and stability with respect to source positions is ensured. In geometrical terms eq.(3.1) implies that the three vectors should not be parallel. In terms of source locations, this requires that one of the three sources must be at a distance from the other two which is of the same order as the distance to the crack. The alternative is that the sources are situated close together spatially. Then the edge mapping process becomes equivalent to the global triangulation method of Norris and Achenbach [1]-[3]. In Ref.[4] it is shown that for the same accuracy of data the local edge mapping method discussed here yields better results than the global triangulation method.

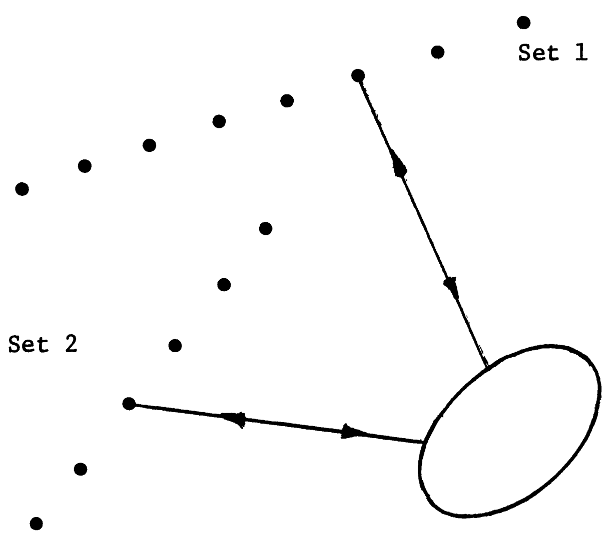

Fig. 1. Configuration of source-receiver positions and crack.

A suitable configuration of source-receiver positions which meets the requirement of eq.(3.1) is illustrated in Fig. 1. Here we have two sets of source -receivers which are sufficiently removed from one another. The idea is to use the planes from two adjacent members of one set with the plane of a single member of the other set. However, we must first be sure that the three planes correspond to proximate flash points. To do this we have developed two alternative algorithms, each requiring not three but five planes.

The first algorithm, which we shall refer to as A_1, has been presented in Ref.[3]. Let P_1, P_2 and P_3 be the planes for three adjacent source-receivers in the first set and let $\overline{P}_1$ and $\overline{P}_2$ be the planes for two adjacent source-receivers in the second set. Define

L_1, L_2 and $\bar{L}$ as the lines formed by the intersections of P_1 with P_2, P_2 with P_3 and $\bar{P}_1$ with $\bar{P}_2$, respectively. If $\bar{L}$ intersects P_2 between L_1 and L_2, the point of intersection is selected as a tentative flash point.

The second algorithm, which we call A_2, uses the same five planes as A_1. The criterion now is that L_1 and L_2 intersect $\bar{P}_1$ on opposite sides of $\bar{L}$. If this happens, the flash point is defined as the point where $\bar{L}$ cuts the line segment between the intersections of L_1 and L_2 with $\bar{P}_1$.

It has been demonstrated by the inversion of experimental data [7] that the two algorithms are not equivalent.

LOCAL EDGE MAPPING THROUGH AN INTERFACE

In practical nondestructive evaluation it is sometimes difficult or impractical to bond transducers directly to the test sample. The alternative is to immerse the test sample in a water bath and to conduct the measurements by the use of immersed transducers. The ray paths are now complicated by the presence of the interface between the two media. In this Section we describe the application of the local edge mapping to this configuration.

The typical configuration is shown in Fig. 2 for pulse-echo measurements. The rays undergo refraction at the water-solid interface. The interface may have arbitrary curvature. We consider for simplicity the case that the edge diffraction is longitudinal to longitudinal. Transverse to transverse or mode converted diffraction presents no extra difficulty other than notational. The angles θ_W and θ_L that a ray makes with the interface normal are related by Snell's law:

$$\frac{1}{c_W}\sin\theta_W = \frac{1}{c_L}\sin\theta_L, \tag{4.1}$$

where c_W is the wave speed in water.

We first select a base point B with vector position $\underset{\sim}{x}_B$ in the solid. Let R equal $c_L\tau_B$, where τ_B is the travel time along a ray from the source-receiver S to B. Thus, R may be computed from the known positions of S and B, the speeds c_W and c_L and the shape of the interface. Referring to Fig. 2, we have

$$R = \frac{c_L}{c_W} R_W + R_L. \tag{4.2}$$

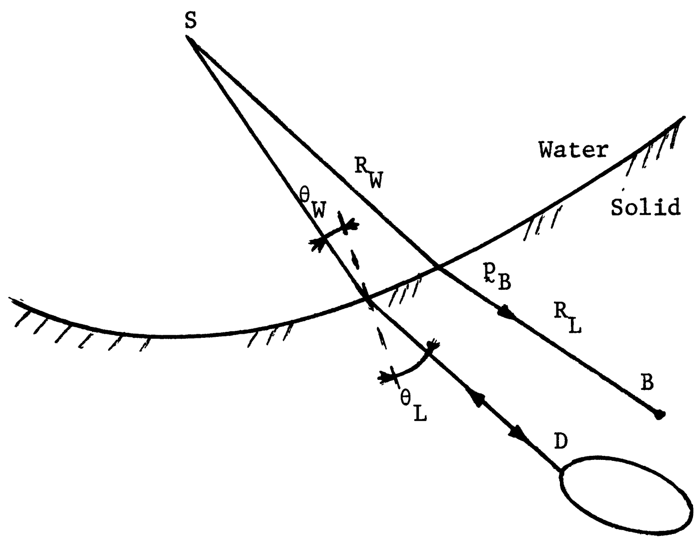

Fig. 2. Water-bath configuration for pulse-echo measurements; S-transducer,B-base point: D-flash point.

Let τ be the pulse-echo travel time from the source-receiver S to the flash point D on the crack edge at vector position $\underset{\sim}{x}_D$.

In order to obtain an equation analogous to eq.(2.1) we must compute $\underset{\sim}{\nabla} R$ and $\underset{\sim}{\nabla}\underset{\sim}{\nabla} R$ where the gradient operator is defined with respect to B. From eq.(4.2)

$$\underset{\sim}{\nabla} R = \frac{c_L}{c_W} \underset{\sim}{\nabla} R_W + \underset{\sim}{\nabla} R_L. \tag{4.3}$$

Since the ray satisfies Fermat's Principle, or equivalently Snell's law, it can be shown that $\underset{\sim}{\nabla} R_W$ is zero and

$$\underset{\sim}{\nabla} R_L = \underset{\sim}{p}_B, \qquad |\underset{\sim}{p}_B| = 1, \tag{4.4}$$

where $\underset{\sim}{p}_B$ is the ray direction in the solid (see Fig.2). Also

$$\underset{\sim}{\nabla}\underset{\sim}{\nabla}R = \frac{1}{\rho_1}\underset{\sim}{n}\underset{\sim}{n} + \frac{1}{\rho_2}\underset{\sim}{m}\underset{\sim}{m}\ , \tag{4.5}$$

where ρ_1 and ρ_2 are the wavefront radii of curvature and $\underset{\sim}{n}$ and $\underset{\sim}{m}$ are the wavefront binormals at B. Thus, if the distance DB is small in comparison with the lesser of ρ_1 and ρ_2, we obtain the result that the flash point lies approximately on the plane defined by

$$(\underset{\sim}{x}-\underset{\sim}{x}_B)\cdot\underset{\sim}{p}_B - (\tfrac{1}{2}c_L\dot{\tau} - R) = 0. \tag{4.6}$$

We note that eq.(4.6) is the generalization of eq.(2.2) to inhomogeneous media. The unit vector $\underset{\sim}{p}_B$ is the local ray direction at the base point. The distance R is $c_L\tau_B$ where τ_B is the travel time from S to B and c_L is the local wave speed at B.

The local edge mapping method is now completely analogous to the one for the homogeneous medium. The same algorithms apply to the intersection process and the choice of the base point is improved by iteration as before.

EDGE MAPPING THROUGH AN INTERFACE USING SYNTHETIC DATA

The interface is taken as a cylindrical surface with the solid on the convex side. In non-dimensional rectangular coordinates the cylindrical water-solid interface is defined by

$$x^2 + z^2 = 36, \quad -\infty < y < \infty. \tag{5.1}$$

The procedure will be illustrated by using synthetic data for a three-dimensional problem. The crack is located in the plane x=0, and it is of elliptical shape. The crack center is (0,0,8) and the major and minor axes, which are pointed in the y and z directions, are of length 2 and 1, respectively. The data is of pulse-echo type, with two sets of 10 source-receivers equally spaced between the points (2,-3,4) and (2,3,4) for one set, and (-1,-3,4) and (-1,3,4) for the other set. Synthetic pulse-echo data was computed by minimizing the travel times from the source to the crack edge and back to the source.

The results of the local edge mapping applied to synthetic data are illustrated in Fig. 3. The initial base point is chosen as (1,1,7). Convergence is obtained after two iterations.

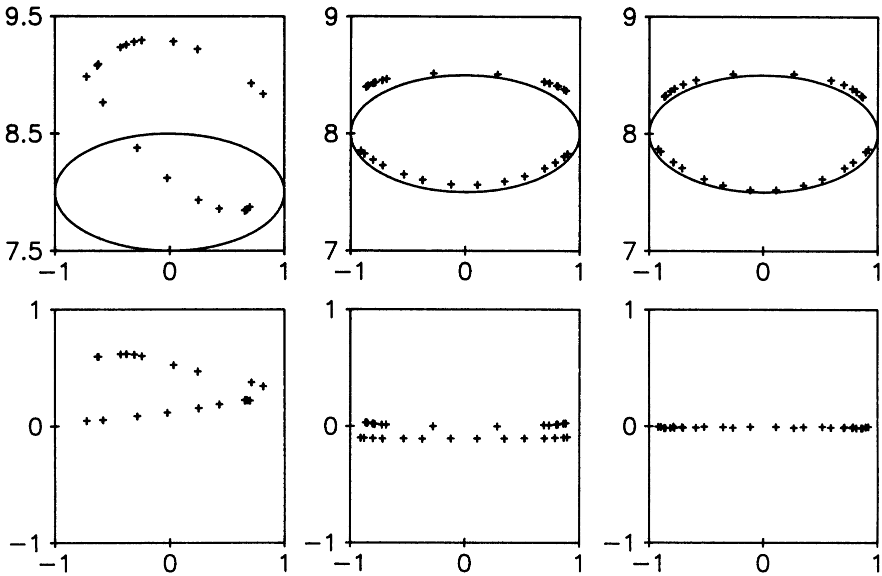

Fig. 3. Mapping of an elliptical crack. Base points (1,1,7); (.3,.05,8.55); and (-.05,0,8.05).

ACKNOWLEDGEMENT

This work was sponsored by the Center for Advanced Nondestructive Evaluation, operated by the Ames Laboratory, USDOE, for the Defense Advanced Research Projects Agency and the Air Force Wright Patterson Aeronautical Laboratories/Materials Laboratory under Contract No. W-7405-ENG-82 with Iowa State University.

REFERENCES

1. J.D. Achenbach and A.N. Norris, "Crack Characterization by the Combined Use of Time-Domain and Frequency-Domain Scattering Data," 1981 AF/DARPA Review of Quantitative NDE, Plenum Publishing Corp.
2. J.D. Achenbach, A.N. Norris and K. Viswanathan, "Mapping of Crack Edges by Seismic Methods," Bull. Seism. Soc. Am. 72, pp. 779-792, 1982.
3. A.N. Norris and J.D. Achenbach, "Mapping of a Crack Edge by Ultrasonic Methods," J. Acoust. Soc. Am. 72, pp. 264-272, 1982.

4. J.D. Achenbach, A.N. Norris, L. Ahlberg and B.R. Tittmann, "Crack Mapping by Ray Methods," 1982 Review of Progress in Quantitative NDE, Plenum Publishing Co.
5. A.N. Norris, "Inverse Ray Tracing in Elastic Solids with Unknown Anisotropy," J. Acoust. Soc. Am. 73, pp. 421-426, 1983.
6. A.N. Norris, "Inverse Ray Tracing in Anisotropic Elastic Solids," 1982 Review of Progress in Quantitative NDE, Plenum Publishing Co.
7. B.R. Tittmann, L.A. Ahlberg, J.D. Achenbach and A.N. Norris, "Experimental Determination of Crack Inversion Parameters," These Proceedings.
8. J.D. Achenbach, A.K. Gautesen and H. McMaken, Ray Methods for Waves in Elastic Solids, Pitman, Boston-London-Melbourne, 1983.

DISCUSSION

J.C. Coffey (CEGB, Manchester, England): Your sources of sound, your receivers, I thought in your talk, they were points. Is that correct?

A.N. Norris: They are considered as points here.

J.C. Coffey: Would you care to comment about how you might have to modify or extend the model to the case where the probes, transducers have a finite size, enough size to be recognized?

A.N. Norris: Well, from the experimental data that we have used up till now, we have considered the center of the transmitting phase as the source point, and that has been fairly adequate. Naturally, there's some error involved in the finite size of the transducer, but I think the best way to consider it would be to have a narrow band at this stage; it produces a narrow bandwidth results. That's one way of looking at it.