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Frequency dependence of hysteresis curves in conducting magnetic materials

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An extension of the hysteresis model has been developed that takes into account the effects on the hysteresis curves of eddy currents in electrically conducting media. In the derivation presented it is assumed that the frequency of the applied field is low enough (or the thickness of the material medium small enough) that the skin effect can be ignored so that the magnetic field penetrates uniformly throughout the material. In this case, the dc hysteresis equation is extended by the addition of a classical eddy-current-loss term depending on (i) the rate of change of magnetization with time, (ii) the resistivity of the material, and (iii) the shape of the specimen; and on an anomalous (or excess) eddy-current-loss term which depends on $(dB/dt)^{3/2}$. In the limit, as the frequency of the magnetic field tends to zero, the frequency-dependent hysteresis curve approaches the dc curve.

I. INTRODUCTION

The hysteresis model equation used in this work has been described previously.[1–3] The hysteresis equation can be used to describe both major (symmetric) loops and minor (asymmetric) loops.[4] Previously, the model equation has been extended to take into account the change in shape of hysteresis loops under a constant applied stress.[5] The earlier work on the development of the hysteresis model was concerned with quasistatic or dc hysteresis loops. The model is based on the concept of the anhysteretic, which provides a global optimum macroscopic state for the magnetization of a material subjected to a magnetic field. The equation for the anhysteretic magnetization was derived, and this resulted in the following expression:

$$M_{\rm an}(H)=M_s\mathscr{L}\left(\frac{H+\alpha M_{\rm an}(H)}{a}\right), \tag{1}$$

where $\mathscr{L}(x)$ is the Langevin function,

$$\mathscr{L}(x)=\coth x-1/x. \tag{2}$$

In Eq. (1), $M_{\rm an}(H)$ is the anhysteretic magnetization under an applied field H, M_s is the saturation magnetization, α is a coupling coefficient which arises from the exchange interaction, and $a=k_BT/\mu_0\langle m\rangle$, where k_B is Boltzmann's constant (1.38×10^{-23} J K^{-1}), T is the temperature in K, μ_0 is the permeability of free space ($4\pi\times10^{-7}$ H m^1), and $\langle m\rangle$ is the effective domain size. This $\langle m\rangle$ is the size of a typical domain in a classical ferromagnetic Ising array with a mean-field interaction which obeys the Langevin equation. The parameter a is therefore proportional to the effective domain density and the absolute temperature.

The equation for hysteresis can be derived from an energy-balance equation in which the magnetic energy supplied, for example, to an initially demagnetized material, can appear either as a change in total magnetization M (magnetostatic energy), or be dissipated due to irreversible changes in magnetization $M_{\rm irr}$ (hysteresis loss). If there is no dissipation (hysteresis) then, by definition, the magnetization must follow the anhysteretic (hysteresis-free) curve. In the presence of hysteresis, the energy-balance equation is

$$\mu_0\int M_{\rm an}\,dH_e=\mu_0\int M\,dH_e+\mu_0k\delta(1-c)\int\left(\frac{dM_{\rm irr}}{dH_e}\right)dH_e, \tag{3}$$

where the first term of the right-hand side is the contribution to the magnetostatic energy, and the second term on the right-hand side is the dissipation loss due to pinning. In this equation $H_e=H+\alpha M$, the coefficient k is the pinning parameter which determines the amount of energy dissipated, and δ is a directional parameter which ensures that energy is always lost through dissipation. $\delta=+1$ when $dH/dt>0$ and $\delta=-1$ when $dH/dt<0$. The coefficient c is a measure of the amount of reversible change in magnetization, as is discussed shortly. Under quasistatic or equilibrium conditions, energy losses arise only from changes in the irreversible magnetization $M_{\rm irr}$. The irreversible magnetization is the component of magnetization that would remain if all domain walls were relaxed back to the planar state and all reversible rotations were reset.

The irreversible and reversible components of magnetization can be separated by differentiating Eq. (3). Since $M=M_{\rm irr}+M_{\rm rev}$,

$$M_{\rm an}=M_{\rm irr}+M_{\rm rev}+k\delta(1-c)\,\frac{dM_{\rm irr}}{dH_e}, \tag{4}$$

and simply by considering that if $c=0$ and $M_{\rm rev}=0$, it follows that

$$M_{\rm irr}=M_{\rm an}-k\delta\,\frac{dM_{\rm irr}}{dH_e} \tag{5}$$

and

$$M_{\rm rev}=c(M_{\rm an}-M_{\rm irr}). \tag{6}$$

Equations (5) and (6) are the basic equations of hysteresis which have been given previously,[6] and the energy equation (3) can be derived directly from them.

The rate of change of magnetization with field dM/dH is then dependent on the displacement of the magnetization from the anhysteretic. In order that the pinning always gives rise to a loss of energy (i.e., is dissipative), this dependence on the displacement $M_{an}(H)-M_{irr}(H)$ applies when the magnetization is below the anhysteretic and the field is increasing ($M_{an}>M_{irr}$ and $dH/dt>0$) and when the magnetization is above the anhysteretic and the field is decreasing ($M_{an}<M_{irr}$ and $dH/dt<0$).

The differential susceptibility dM/dH can be expressed as a sum of the irreversible and reversible components,

$$\frac{dM(H)}{dH}=\frac{dM_{irr}(H)}{dH}+\frac{dM_{rev}(H)}{dH}, \tag{7}$$

where $M_{irr}(H)$ is the irreversible component of magnetization and $M_{rev}(H)$ is the reversible component of magnetization. This leads to the differential equation for hysteresis,[6]

$$\frac{dM(H)}{dH}=\frac{M_{an}(H)-M_{irr}(H)}{k\delta-\alpha[M_{an}(H)-M(H)]}+c\left(\frac{dM_{an}(H)}{dH}-\frac{M_{an}(H)-M_{irr}(H)}{k\delta-\alpha[M_{an}(H)-M(H)]}\right), \tag{8}$$

where the terms on the right-hand side are, respectively, the irreversible and reversible differential susceptibilities. The term c, which determines the amount of reversible magnetization, is derived from the amount of domain-wall bending[7] and is dependent on the domain-wall surface energy.

II. OBJECTIVE OF THE PRESENT WORK

Recently it has been shown how the frequency dependence of hysteresis can be described in "nonconducting" media, such as high-frequency ferrites, through a second-order differential equation.[8] In the present work, the hysteresis equation is extended to describe the frequency dependence of hysteresis in electrically conducting materials. In this extension of the model to account for these effects it is assumed, for the sake of simplicity, that the magnetic field penetrates the entire cross-sectional area of the material uniformly. In most practical cases, the frequency of operation of electrically conducting magnetic media, such as transformer or motor laminations, is much lower than for nonconducting magnetic media, such as high-frequency inductor cores. Therefore, the effects of magnetic relaxation and resonance which have been described in a previous article[8] can be ignored and these phenomena can be entirely separated from the effects of eddy currents. Furthermore, the cross sections of laminations are chosen so that the thickness is comparable, or less than, the depth of penetration of the magnetic field, to ensure high penetration of the field throughout the volume of material.

III. DERIVATION OF THE FREQUENCY-DEPENDENT HYSTERESIS EQUATION

Additional energy dissipation resulting from eddy current losses that arise in electrically conducting media can be added to the right-hand side of Eq. (3) and will thereby lead to a modification of the hysteresis loops with the rate of change of field dH/dt.

The approach taken in this work is to determine the instantaneous power losses due to eddy currents, and then to include these in the energy-balance equation given above. The power losses are therefore separated into hysteresis loss dW_H/dt, classical eddy current loss dW_{EC}/dt, and anomalous (or excess) loss dW_A/dt. The classical eddy current loss is obtained by solving the Maxwell equation $\nabla\times E=-dB/dt$ for a given geometry, assuming that the magnetic field penetrates uniformly throughout the material.

The classical eddy current instantaneous power loss per unit volume is proportional to the square of the rate of change of magnetization as discussed by Chikazumi.[9] This gives

$$\frac{dW_{EC}}{dt}=\frac{d^2}{2\rho\beta}\left(\frac{dB}{dt}\right)^2=\frac{\mu_0^2d^2}{2\rho\beta}\left(\frac{dM}{dt}\right)^2. \tag{9}$$

Where ρ is the resistivity in Ω m, d is the cross-sectional dimension in meters (thickness for laminations, diameter for cylinders and spheres) and β is a geometrical factor which varies from $\beta=6$ in laminations to $\beta=16$ in cylinders and $\beta=20$ in spheres. The approximation $B=\mu_0M$, which is valid for soft magnetic materials, has been made in Eq. (9). Under the restricted condition of sinusoidal variation of B with time, at moderate and low frequencies of magnetic-field excitation, such that the flux penetration is complete, the eddy current power loss per unit volume becomes equal to the well-known expression[10]

$$\frac{dW_{EC}}{dt}=\frac{\pi^2B_{max}^2d^2f^2}{\rho\beta}, \tag{10}$$

where B_{max} is the peak flux density in the cycle and f is the frequency in Hz.

The anomalous loss results from domain-wall motion during changes in the domain configuration, and this has been treated in detail by Bertotti[11–13] and Fiorillo and Novikov.[14] In many cases, this component of the loss can be expressed as

$$\frac{dW_A}{dt}=\left(\frac{GdwH_0}{\rho}\right)^{1/2}\left(\frac{dB}{dt}\right)^{3/2}, \tag{11}$$

where G is a dimensionless constant of value 0.1356, w is the width of laminations, d the thickness, ρ the resistivity, and H_0 is a parameter representing the fluctuating internal potential experienced by domain walls. H_0 has dimensions of A m^{-1} and so is equivalent to a magnetic field.

Incorporating the eddy-current-loss mechanisms into the theory of hysteresis allows perturbations to the hysteresis curve arising from eddy current losses to be modeled. The energy Eq. (3) which forms the starting point of the derivation of the hysteresis equations can be extended to

$$\mu_0 \int M_{an}(H) dH_e$$

$$= \mu_0 \int M(H) dH_e + \mu_0 k \delta (1-c) \int \left(\frac{dM_{irr}}{dH_e}\right) dH_e + \int \frac{\mu_0^2 d^2}{2\rho\beta} \left(\frac{dM}{dt}\right)^2 dt + \int \left(\frac{GdwH_0}{\rho}\right)^{1/2} \left(\frac{\mu_0 dM}{dt}\right)^{3/2} dt, \quad (12)$$

where the final two terms on the right-hand side are the classical eddy current loss and the anomalous loss.

This equation needs to be modified slightly before proceeding further,

$$M_{irr} = \frac{1}{1-c} (M - cM_{an}), \quad (13)$$

and therefore Eq. (12) becomes

$$\mu_0 \int M_{an}(H) dH_e$$

$$= \mu_0 \int M(H) dH_e + \mu_0 k \delta \int \left(\frac{dM}{dH_e}\right) dH_e - \mu_0 k \delta c \int \left(\frac{dM_{an}}{dH_e}\right) dH_e + \frac{\mu_0^2 d^2}{2\rho\beta} \int \left(\frac{dM}{dt}\right)^2 dt + \left(\frac{GdwH_0}{\rho}\right)^{1/2} \mu_0^{3/2} \int \left(\frac{dM}{dt}\right)^{3/2} dt. \quad (14)$$

A. Case 1: Classical loss only

In the case of a material in which the classical loss is the dominant frequency-dependent term, the quasistatic hysteresis model can be extended using a perturbation of the hysteresis equation. In other words, we set H_0 equal to 0 in Eq. (14). The differential equation now needs to be converted into a manageable form.

Therefore, replacing $(dM/dt)^2 dt$ by $(dM/dt)(dM/dH_e) dH_e$, gives

$$\frac{\mu_0^2 d^2}{2\rho\beta} \int \left(\frac{dM}{dt}\right)^2 dt = \frac{\mu_0^2 d^2}{2\rho\beta} \int \left(\frac{dM}{dt}\right) \left(\frac{dM}{dH_e}\right) dH_e. \quad (15)$$

Substituting this back into the hysteresis equation, and dividing by μ_0,

$$\int M_{an}(H) dH_e = \int M(H) dH_e + k\delta \int \left(\frac{dM}{dH_e}\right) dH_e - k\delta c \int \left(\frac{dM_{an}}{dH_e}\right) dH_e + \frac{\mu_0 d^2}{2\rho\beta} \int \left(\frac{dM}{dt}\right) \left(\frac{dM}{dH_e}\right) dH_e. \quad (16)$$

Differentiating with respect to H_e,

$$M_{an}(H) = M(H) + k\delta \frac{dM}{dH_e} - k\delta c \left(\frac{dM_{an}}{dH_e}\right) + \frac{\mu_0 d^2}{2\rho\beta} \left(\frac{dM}{dt}\right) \left(\frac{dM}{dH_e}\right). \quad (17)$$

Equation (17) is equivalent to the quasistatic hysteresis equation with the addition of the final term on the right-hand side. The equation can now be rearranged. Let

$$k' = k\delta + \frac{\mu_0 d^2}{2\rho\beta} \left(\frac{dM}{dt}\right), \quad (18)$$

so that under dc conditions $k' = k\delta$, and the difference between k and k' depends on dM/dt.

$$M_{an}(H) = M(H) + k' \frac{dM}{dH_e} - k\delta c \frac{dM_{an}}{dH_e}. \quad (19)$$

It can then be seen that the effects of the classical eddy currents can be incorporated into a modified energy-loss parameter k' while maintaining the same form of hysteresis equation

$$\frac{dM}{dH_e} = \frac{1}{k'} \left(M_{an}(H) - M(H) + k\delta c \frac{dM_{an}}{dH_e}\right), \quad (20)$$

and substituting $H_e = H + \alpha M$ gives

$$\frac{dM}{dH} = \frac{M_{an}(H) - M(H) + k\delta c (dM_{an}/dH_e)}{k' - \alpha[M_{an}(H) - M(H) + k\delta c (dM_{an}/dH_e)]}, \quad (21)$$

which gives an equation for the differential susceptibility as a result of both hysteresis and classical eddy currents that is analogous to the equation obtained under quasistatic hysteresis conditions. The equivalent expressions to Eqs. (5) and (6) for the irreversible and reversible components of magnetization are

$$M_{irr}(H) = M_{an}(H) - \mu_0 \delta \left(k + \frac{\mu_0 d^2}{2\rho\beta} \frac{dM}{dt}\right) \frac{dM_{irr}}{dH_e}, \quad (22)$$

$$M_{rev}(H) = c[M_{an}(H) - M_{irr}(H)]. \quad (23)$$

In order to determine dM/dH, Eq. (21) can be reduced to a quadratic,

$$\left(\frac{\mu_0 d^2}{2\rho\beta} \frac{dH}{dt}\right) \left(\frac{dM}{dH}\right)^2 + \left[k\delta - \alpha \left(M_{an} - M + k\delta c \frac{dM_{an}}{dH_e}\right)\right] \frac{dM}{dH} - \left(M_{an} - M + k\delta c \frac{dM_{an}}{dH_e}\right) = 0 \quad (24)$$

which can then be solved directly.

In most cases, the external excitation on a material, that is the magnetic field H generated by an electrical current in a solenoid, can be controlled and known, and is often sinusoidal. Therefore, this form of Eq. (24) can be used to calculate the effects on the hysteresis curve of various different, but known, dependencies of magnetic field on time dH/dt, such as sinusoidal or triangular field wave forms. On the other hand the response of the material dB/dt is dependent on the magnetic properties of the material and is rarely sinusoidal, as mentioned by Fiorillo and Novikov.[14] Equation

(24) therefore provides a means of calculating the time dependence of $M(t)$ [and hence $B(t)$] under a given form of external excitation $H(t)$ for situations in which the dominant eddy current power loss term is the classical loss.

B. Case 2: Classical and anomalous losses

In those cases where both types of losses are involved, the presence of the excess loss term depending on $(dM/dt)^{3/2}$ in Eq. (12) makes it impossible to get an expression for dM/dH in the same form as Eq. (24). Therefore, the solution is a little more difficult but can be achieved by numerical methods.

Beginning again from Eq. (12) and making the same substitution for $(dM/dt)^2\,dt$, leads to

$$\int M_{\rm an}(H)dH_e = \int M(H)dH_e + k\delta(1-c)\int\left(\frac{dM_{\rm irr}}{dH_e}\right)dH_e + \frac{\mu_0 d^2}{2\rho\beta}\int\left(\frac{dM}{dt}\right)\left(\frac{dM}{dH_e}\right)dH_e + \left(\frac{Gdw\mu_0H_0}{\rho}\right)^{1/2}\int\left(\frac{dM}{dt}\right)^{1/2}\frac{dM}{dH_e}\,dH_e. \quad (25)$$

Differentiating this equation gives the following:

$$M_{\rm an}(H) = M(H) + k\delta(1-c)\left(\frac{dM_{\rm irr}}{dH_e}\right) + \frac{\mu_0 d^2}{2\rho\beta}\left(\frac{dM}{dt}\right)\left(\frac{dM}{dH_e}\right) + \left(\frac{Gdw\mu_0H_0}{\rho}\right)^{1/2}\left(\frac{dM}{dt}\right)^{1/2}\left(\frac{dM}{dH_e}\right), \quad (26)$$

and using the expression for $M_{\rm irr}$ in Eq. (13),

$$M_{\rm an}(H) = M(H) + k\delta\frac{dM}{dH_e} - k\delta c\frac{dM_{\rm an}}{dH_e} + \frac{\mu_0 d^2}{2\rho\beta}\left(\frac{dM}{dt}\right)\left(\frac{dM}{dH_e}\right) + \left(\frac{Gdw\mu_0H_0}{\rho}\right)^{1/2}\left(\frac{dM}{dt}\right)^{1/2}\left(\frac{dM}{dH_e}\right), \quad (27)$$

which is equivalent in form to the standard quasistatic hysteresis equation

$$M_{\rm an}(H) = M(H) + k''\delta\left(\frac{dM}{dH_e}\right) - k\delta c\frac{dM_{\rm an}}{dH_e}, \quad (28)$$

where the energy-loss parameter k'' is now

$$k'' = k + \frac{\mu_0 d^2}{2\rho\beta}\left(\frac{dM}{dt}\right) + \left(\frac{Gdw\mu_0H_0}{\rho}\right)^{1/2}\left(\frac{dM}{dt}\right)^{1/2}. \quad (29)$$

Equation (27) can then be rearranged in terms of the differential susceptibility dM/dH to give

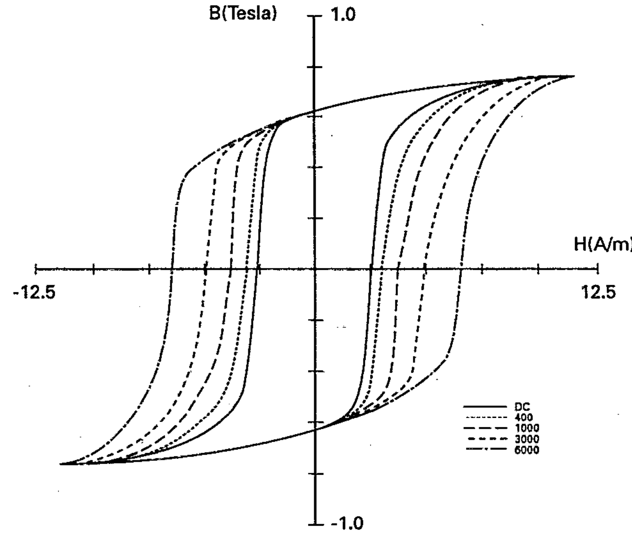

FIG. 1. Measured hysteresis curves of Permalloy 80 at dc, 400, 1000, 3000, and 6000 Hz (see Ref. 15).

$$\left(\frac{\mu_0 d^2}{2\rho\beta}\frac{dH}{dt}\right)\left(\frac{dM}{dH}\right)^2 + \left(\frac{Gdw\mu_0H_0}{\rho}\right)^{1/2}\left(\frac{dH}{dt}\right)^{1/2}\left(\frac{dM}{dH}\right)^{3/2} + \left[k\delta - \alpha\left(M_{\rm an}(H) - M(H) + k\delta c\frac{dM_{\rm an}}{dH_e}\right)\right]\left(\frac{dM}{dH}\right) - \left(M_{\rm an}(H) - M(H) + k\delta c\frac{dM_{\rm an}}{dH_e}\right) = 0. \quad (30)$$

This equation can then be solved for dM/dH using numerical methods, and from the solution the frequency-dependent hysteresis curve can be calculated.

III. RESULTS

Permalloy 80 was chosen as a widely used material for testing the agreement between the model predictions and measured properties. The hysteresis curves of Permalloy 80 measured at various frequencies are given in Fig. 1. From the dc hysteresis curve, the magnetic properties were determined and these are shown in Table I. The hysteresis parameters for the model equation were determined from these measured

TABLE I. Comparison of measured and modeled magnetic properties of square Permalloy 80.

	Measured	Modeled
Saturation magnetic induction	1.05 T	1.0 T
Anhysteretic permeability	···	-0.56×10^6
Initial permeability	26×10^3	24×10^3
Field at loop tip	12.8 A/m	12.5 A/m
Magnetic induction at loop tip	0.8 T	0.8 T
Differential permeability at loop tip	1.9×10^3	8.1×10^3
Magnetic induction at remanence	0.62 T	0.61 T
Differential permeability at remanence	23×10^3	43×10^3
Coercivity	2.4 A/m	2.7 A/m
Differential permeability at coercive point	0.73×10^6	1.2×10^6

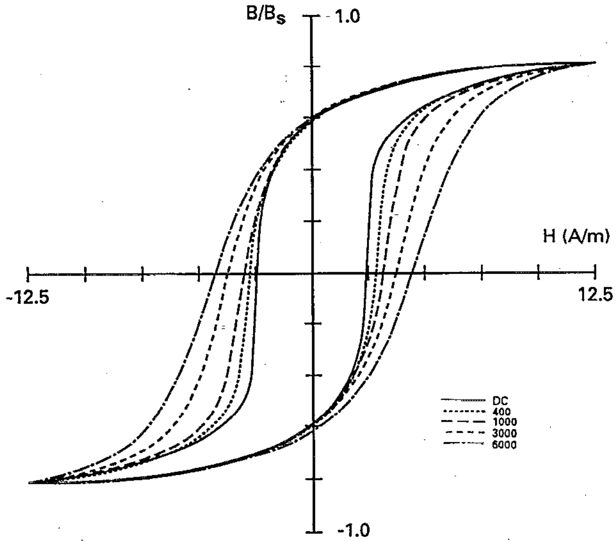

FIG. 2. Modeled hysteresis curves of Permalloy 80 at dc, 400, 1000, 3000, and 6000 Hz calculated by incorporating effects of classical eddy current power losses into the hysteresis equation.

properties using the methods described previously.[6] The model hysteresis loop was then calculated under quasistatic conditions and the values of the modelled magnetic properties of Permalloy 80 obtained from the solution of the equation are also shown in Table I.

The modeled hysteresis curves were calculated first using only the classical eddy current contribution and then using both classical and anomalous (or excess) eddy current contributions. The hysteresis curves obtained by considering only the classical eddy currents are shown for various different frequencies in Fig. 2. These show a progressive widening of the hysteresis loops as the frequency was increased, leading to higher coercivity. The dependence of coercivity on frequency, according to the model calculations, is shown in Table II together with the measured values at the same frequency.

The measured hysteresis curves show two other features of interest. The remanence remains independent of the frequency of excitation, and the loop tips show no evidence of "rounding" as the field is reduced from its maximum excursion. Both of these features also result naturally from the model calculations.

However, one feature of the calculations based solely on the effects of classical eddy currents did not give such good agreement with observation; this was the numerical values of the coercivity at higher frequencies. At 6 kHz the discrepancy was 1.5 A m^{-1} in a coercivity of 5.9 A m^{-1}. Therefore the effects of anomalous (or excess) eddy currents were incorporated into the calculation. The results are shown in Fig. 3. In this case the increase in coercivity with frequency is more accurately modeled, as shown in Table II, while the invariance of remanence with frequency and the sharpness of the recoils at the loop tips are maintained.

TABLE II. Comparison of measured and modeled values of coercivity of square Permalloy 80 at different frequencies. These values were obtained using the following model parameters: $B_s = 1$ T; $a = 3.75$ A m^{-1}; $k = 2.4$ A m^{-1}; $\alpha = 1.5\times10^{-5}$; $c = 0.35$. For classical losses the following additional parameters were used: $\rho = 0.57\times10^{-6}$ Ω m; $d = 15\times10^{-6}$ m; $\beta = 6$. For anomalous (or excess) losses the following additional parameters were used: $w = 0.005$ m; $H_0 = 0.0075$ A m^{-1}.

	H_c (A m^{-1})		
		Modeled	
Frequency (Hz)	Measured	Classical loss only	Classical + excess losses
dc	2.4	2.4	2.5
400	2.9	2.8	3.5
1000	3.6	3.1	4.0
3000	4.6	3.7	4.9
6000	5.9	4.4	5.6

IV. CONCLUSIONS

The work presented here shows how the hysteresis equation can be extended to account for energy losses resulting from the generation of eddy currents in electrically conducting media. The model represents a first approximation in which the skin effect is ignored, or equivalently the field penetration is assumed to be uniform throughout the material. This means that the calculations are really only applicable to thin laminations.

Both classical and excess eddy current losses have been included in the model, and these appear as different terms in the energy-balance equation (25). The results show some interesting features, including the increase in coercivity with frequency, the increase in energy loss per cycle, the invari-

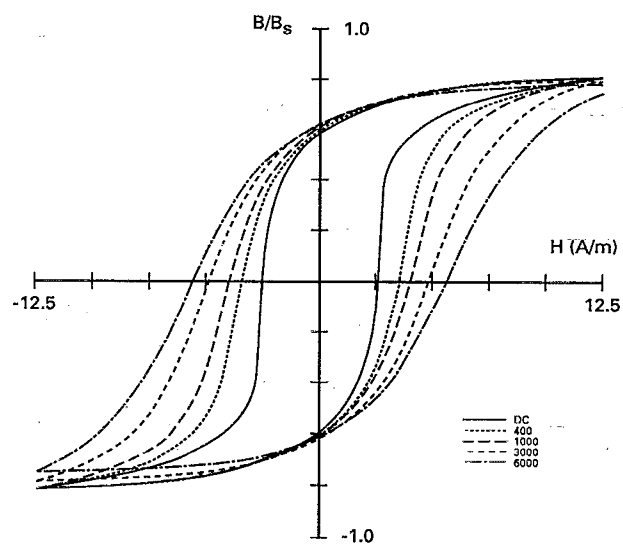

FIG. 3. Modeled hysteresis curves of Permalloy 80 at dc, 400, 1000, 3000, and 6000 Hz calculated by incorporating effects of both classical and anomalous (or excess) eddy current power losses into the hysteresis equation.

ance of remanence with frequency, and the persistence of sharp reversal points at the loop tips, as distinct from the rounding of the hysteresis curves observed in high-frequency ferrites where were modeled via a relaxation mechanism.

The model provides a relatively simple way of predicting the changes in hysteresis curves as a result of eddy currents at different frequencies. The information needed to calculate the hysteresis curves where only classical eddy current losses occur is limited to the dc hysteresis curve of the material, the resistivity, and the shape of the component (e.g., lamination, cylinder), including its cross-sectional width (e.g., lamination thickness or cylinder diameter). In order to calculate hysteresis curves in the presence of excess eddy current losses, additional information is needed, including the width of the laminations and the field parameter representing the fluctuating internal potential experienced by the domain walls.

Future work should include a means of correcting these curves for the effects of the depth of penetration of the magnetic field, or skin depth, in conducting media. This would enable the model to be extended to "thick" components in which the energy dissipation at a given frequency was not uniform over the entire cross section.

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APPENDIX: ENERGY-BALANCE EQUATION FOR QUASISTATIC HYSTERESIS

In the absence of time-dependent effects, the energy balance equation is given by

$$\text{energy supplied} = \text{change in magnetostatic energy} + \text{energy dissipated}. \tag{A1}$$

If there is no hysteresis, then the energy dissipated is zero and consequently the magnetization follows the anhysteretic. Therefore,

$$\int \mu_0 M_{\text{an}}(H)\, dH_e = \int \mu_0 M(H)\, dH_e, \tag{A2}$$

where $H_e = H + \alpha M$ is used instead of H because of the need to include internal coupling energy. If there is hysteresis, then this hysteresis arises from irreversible changes in magnetization M_{irr}, and therefore the most precise form of the energy equation for hysteresis is

$$\int \mu_0 M_{\text{an}}(H)\, dH_e = \int \mu_0 M(H)\, dH_e + k\delta\mu_0 \int dM_{\text{irr}}(H) \tag{A3}$$

using the notation described in the text. The important difference here is that the magnetostatic energy depends on the total magnetization M, whereas the hysteresis loss depends on M_{irr}. As we shall see, compared with earlier derivations, this merely results in a scaling correction to k, which amounts to multiplying k by a constant.

Differentiating the above equation,

$$M_{\text{an}}(H) = M(H) + k\delta \frac{dM_{\text{irr}}(H)}{dH_e}, \tag{A4}$$

and from previous results[6]

$$M(H) = (1-c) M_{\text{irr}}(H) + c M_{\text{an}}(H), \tag{A5}$$

and therefore, rearranging Eq. (A4) with this substitution leads to

$$M_{\text{an}}(H) = M_{\text{irr}}(H) + \frac{k\delta}{(1-c)} \frac{dM_{\text{irr}}(H)}{dH_e}, \tag{A6}$$

which is identical to previous equations except for the $1/(1-c)$ factor, which effectively leads only to a scaling of k. Equation (A6) is an exact derivative of the energy-balance equation (A3). It can be manipulated into an expression for the differential susceptibility $dM_{\text{irr}}(H)/dH$ which is particularly useful for calculating hysteresis curves. Equation (A6) can be shown to be exactly equivalent to

$$\frac{dM_{\text{irr}}(H)}{dH} = \frac{M_{\text{an}}(H) - M_{\text{irr}}(H)}{\frac{k\delta}{(1-c)} - \alpha[M_{\text{an}}(H) - M_{\text{irr}}(H)] \frac{dM}{dM_{\text{irr}}}}. \tag{A7}$$

Now it has also been shown elsewhere[2] that

$$M = M_{\text{irr}} + M_{\text{rev}} \tag{A8}$$

$$= (1-c) M_{\text{irr}} + c M_{\text{an}}, \tag{A9}$$

and therefore for small values of c it follows that $dM/dM_{\text{irr}} \approx 1$, and therefore Eq. (A7) becomes equal to the equation for the irreversible differential susceptibility that has been quoted before,[6] namely

$$\frac{dM_{\text{irr}}(H)}{dH} = \frac{M_{\text{an}}(H) - M_{\text{irr}}(H)}{[k\delta/(1-c)] - \alpha[M_{\text{an}}(H) - M_{\text{irr}}(H)]} \tag{A10}$$

and when summed with the reversible differential susceptibility,

$$\frac{dM_{\text{rev}}}{dH} = c\left(\frac{dM_{\text{an}}(H)}{dH} - \frac{dM_{\text{irr}}(H)}{dH}\right), \tag{A11}$$

so that the total differential susceptibility becomes

$$\frac{dM(H)}{dH} = (1-c) \frac{M_{\text{an}}(H) - M_{\text{irr}}(H)}{\frac{k\delta}{(1-c)} - \alpha[M_{\text{an}}(H) - M_{\text{irr}}(H)]} + c \frac{dM_{\text{an}}(H)}{dH}. \tag{A12}$$

Another possibility that has not been fully explored is what happens when c is not small, so that $dM/dM_{\text{irr}} \neq 1$, and the approximations leading to Eq. (A12) cannot be made. The only known case in this situation is $c=1$, when the solution is simply $M(H) = M_{\text{an}}(H)$.

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