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THEORY OF GRANULAR BED FILTRATION AND CONTACT FLOCCULATION

Iowa State University

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Theory of granular bed filtration and contact flocculation

by

Fathy M. A. Saleh

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

Department: Civil Engineering
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1981
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LIST OF SYMBOLS

$A, A_0$ = total number of pores available for blocking in clogged and clean bed

$A_1' = \frac{V}{C_0} = \text{time coefficient } T^{-1}$

$A_2' = \frac{1}{\gamma s_c} \ln \left( \frac{C_0}{C} - 1 \right) = \text{depth coefficient } X^{-1}$

$A_4, \ldots, A_5$ = coefficients, the exact value of each is presented at first appearance

$A_c$ = cross-sectional area of a capillary cylinder, $\pi r^2$

$A_f$ = filter cross-sectional area

$A_s$ = surface area of a capillary cylinder, $2\pi r L$

$a$ = depth coefficient in the breakthrough equation $= \frac{K_0 \sigma U}{2 V_s}$

$a_o$ = maximum separation distance for removal to occur within a time interval $< 0$

$a, b, c$ = semi-axis lengths

$B = \frac{\epsilon^2 v_s^2 + 4 \alpha_2 \alpha_{27}}{4 \epsilon \alpha_{27}}$

$B_1 = \text{coefficient } = \left( \frac{4 \pi e^2}{DKT_a} \sum \frac{N_i \omega_i}{2} \right)^{1/2}$

$B_2 = \text{head loss coefficient, varies from 84.3 to 12.8 for bed porosity, } \epsilon, 0.26 \text{ to } 0.46 \text{ respectively}$

$B_3 = \text{an empirical constant with values between 6.03 and 10.5 for all sands}$

$B_4, B_4$ = head loss constant at $t = 0, t$ respectively, equals 2.0 for circular cross-sections

$B_5 = \text{Kozeny constant } = B_4 \left( \frac{Le}{L} \right)^2$

$B_6 = \frac{36 B_4}{\psi_s^2 \left( \frac{Le}{L} \right)^2}$
\[ B_7 = \frac{e^{T_1} - 1}{e^{T_1} + 1} \]

- \( B_r \) = break-up constant
- \( b \) = filtration time coefficient in the breakthrough equations = \( 8K_{co} \)
- \( C_o \) = suspension concentration at bed depth, \( x= 0 \) and time \( t \)
- \( C_c \) = the ionic concentration at the colloid's surface
- \( C_D \) = capacity of the diffuse layer
- \( C_i \), \( C_f \) = initial and final filtrate concentrations respectively (at \( t = 0 \), \( t = t_f \))
- \( C_L \) = capacity of the double layer
- \( C_S \) = capacity of the Stern layer
- CMF = completely mixed flow reactor
- \( C' \) = proportionality constant of sedimentation probability, experimentally found equal to 0.1
- \( C_1 \) = coefficient of \( \sigma \) in Iwasaki's equation 63
- \( C_1, C_2 \) = constants
- \( C_2, C_3 \) = coefficients of \( \sigma, \sigma^2 \) in Ives impediment modulus equation 56
- \( C_4 \) = a shear factor dependent on particle shear strength grain size and fluid viscosity
- \( D \) = medium dielectric constant
- \( D_{2}^{N'}(t) \) = variance of the random variable \( N'(t) \)
- \( D_e \) = volume space at which removal occurs (on Lerk and Heertjes model)
- \( D_{ij} \) = diffusion constant for particles in perikinetic transport
- \( d \) = grain diameter
- \( d_{ei} \) = equivalent diameter of capillary inside a clean bed
\[ d_{eo} = \text{equivalent diameter of capillary inside a clogged bed} \]
\[ d_{eq} = \text{pore equivalent diameter} \]
\[ d_o = \text{grain diameter at the inlet surface of the filter} \]
\[ d_p = \text{particle diameter} = 2R_p \]
\[ d_s = \text{grain sieve size} \]
\[ E = \text{streaming potential in mv} \]
\[ E\, N'(t) = \text{the first moment of the random variable } N'(t), \text{ or the expected value of the state at which } n_p \text{ pores are blocked} \]
\[ E_n = \text{denotes the system state at which } n_p \text{ pores are blocked} \]
\[ e = \text{the charge on the electron} \]
\[ erf = \text{error function} \]
\[ F = \text{theoretical filter capacity} \]
\[ F_l = \text{frictional force per unit area} \]
\[ F(l,t) = \text{a generation function} \]
\[ \bar{F}_{t',t} = \frac{e}{L} = \text{porosity factor at } t = 0, t \text{ respectively} \]
\[ F(\sigma) = \text{function of } \sigma \]
\[ G = \text{velocity gradient} \]
\[ \bar{G} = \text{root mean square of velocity gradient} \]
\[ \bar{G}_a = \text{volume weighted root mean square of velocity gradient} \]
\[ \bar{G}_t = \text{root mean square velocity gradient at time, } t \]
\[ \bar{G}_u = \text{ultimate value of the root mean square of velocity gradient} \]
\[ \bar{G}_w = \text{flow weighted root mean square of velocity gradient} \]
\[ G_e, G_s = \text{specific gravity of liquid and particle respectively} \]
\[ G_p = \text{gravitational parameter} = \frac{\gamma}{\gamma_s} \]
G

s

= specific gravity of particle (dimensionless)

G

l

= specific gravity of liquid (dimensionless)

g

= gravity constant

H

= total pressure drop at any time, t

H

m

= Hamaker constant

H

o

= initial pressure drop throughout the entire bed as calculated from Kozeny equation

\hbar

= Planck's constant

\hbar \nu

= ionization energy

I

I, II, III = integrals

i

= diffusion flux

J

= flow hydraulic gradient

j

= the rate of change of grain diameter with depth L

K''

= either K_a, or K_\omega

\bar{K}_a

= K_a \left( \sigma_u - \sigma \right)

K

s

= conductivity in microho/cm

K_F

= a performance parameter characterizing the stirring arrangement

K

G

= geometric coefficient = \frac{2}{\pi} \left( \frac{n_12/3}{n_2} \right)

K

o

= attachment coefficient = k_a \cdot K_{po}

K_{or}

= orthokinetic flocculation constant

K_p, K_{po}

= hydraulic conductivity or coefficient of conductivity for clogged and clean bed respectively

k

= Boltzmann's constant

k_a

= attachment coefficient

k_d

= detachment coefficient
\(k_f\) = a conversion factor \((\text{mg/l})/(\text{vol./vol.})\) or T.U./(vol./vol.)

\(k_r\) = perikinetic flocculation constant

\(L\) = filter total depth

\(\overline{L} = \frac{dP}{d} = \text{interception dimensionless parameter}\)

\(L_e\) = length of sinuous path pursued by an element of the fluid

\(l_e\) = length of a unit cell in Heertjes and Lerk model

\(M\) = inertial parameter

\(m\) = hydraulic radius for granular media = \(\frac{\text{volume of fluid in bed}}{\text{surface presented to fluid}}\)

\(N\) = concentration of primary particles at depth, \(x\), and time, \(t\)

\(N'(t)\) = random variable characterizing the amount of solid particles retained in the porous medium

\(N_a\) = number of atoms per \(\text{cm}^3\)

\(N_c\) = number of capillaries per filter cross-sectional area

\(N_e\) = number of laminar cells per unit bed volume

\(N_F\) = number of flocs

\(N_i\) = concentration of \(i\) particles

\(N_j\) = concentration of \(j\) particles

\(N_l\) = number of laminar cells per bed unit volume at a height of bed \(l_e\)

\(N_o\) = concentration of primary particles present initially

\(N_P\) = number of particles deposited per pore

\(n_1\) = number of grains per unit bed volume

\(n_2\) = number of capillaries per unit bed surface area = \(N_c\)
\[ P = \text{flow pressure} \]
\[ P_n = \text{Pedet number} = \left( \frac{kT_a}{3m_\mu d_p} \right) \]
\[ P_{np} = \text{concentration of blocked pores when their number was } n_p \]
\[ P_{(t)} = \text{concentration of pores blocked by solid particles at time } (t) \]
\[ \text{pH} = \text{hydrogen ion concentration} \]
\[ \Delta P = \text{pressure difference across a depth } \Delta L \]
\[ Q = \text{flow rate} \]
\[ q = \text{flow rate through an elementary area} \]
\[ R = \text{radius of the capillary tube} \]
\[ R_{eq} = \text{equivalent average radius of a capillary } \text{ (mm)} \]
\[ R_F = \text{floc radius} \]
\[ R_{ij} = \text{distance between centers of two particles i and j in contact} \]
\[ R_n = \text{Reynolds number} = \frac{\rho V_s d}{\mu} \]
\[ R_p = \text{radius of primary particles} \]
\[ r = \text{radius of a capillary at which the velocity } u \text{ is calculated} \]
\[ r' = \text{the rate constant for removal by straining} \]
\[ r'' = \text{the rate constant for removal by sedimentation} \]
\[ S = \text{grain total surface area per unit bed volume} \]
\[ S_v = \text{a shape factor} = \frac{6}{\psi_d \text{eq}} \]
\[ s = \text{a factor that relates grain sieve size to the equivalent volume sphere grain size} \]
\[ s^* = \text{constant used in head loss used proposed by Willson, et al.} \]
\[ T = \text{total force exerted by fluid shear stresses on a fluid lamina} \]
\[ T_1 = \frac{2Vc_0 t^{(a_{13} - a_{14})}}{d_0 (1 - \rho_s)} \]

- \( T_a \) = absolute temperature
- \( t \) = time
- \( t_b \) = bed residence time
- \( t_r \) = run time or time to breakthrough
- \( u \) = flow velocity inside a capillary tube at radius, \( r \)
- \( u, v, w \) = fluid element velocities in the directions \( x, y, z \) respectively
- \( \bar{u} \) = mean velocity in the capillary tube
- \( \bar{u}^2 \) = the mean square velocity of fluctuation = \( K_G \)
- \( U_{\max} \) = max flow velocity at the center of the capillary tube
- \( U_{(r)} \) = Lennard-Jones potential energy function.
- \( V \) = volume of liquid passing through a unit area
- \( V_a \) = energy of attraction between two slabs
- \( V_i \) = flow interstitial velocity
- \( V_m \) = volume of total solid mass suspended per volume of fluid
- \( V_p \) = particle settling velocity
- \( V_s \) = superficial flow velocity
- \( \Delta V \) = change in grain volume due to deposition
- \( W \) = total work per unit of time in the capillary between sections 1 and 2
- \( X, x \) = filter bed depth
- \( \bar{X} \) = particle translation due to Brownian diffusion
- \( X_{\text{CHP}, \text{OHP}} \) = the outer Helmoltze plane thickness and potential respectively
y = a substitute variable = \( \frac{\sigma_0}{\sigma_u} \) or = \( \frac{\sigma}{\sigma_u} \)

y, \( \omega \) = empirical indices

z = intermolecular distance

\( \alpha \) = atoms polarizability

\( \alpha_1 \) to \( \alpha_{30} \) = constants the value of each is presented at their first appearance

\( \alpha_1, \alpha_j \) = constants

\( \alpha_p, \alpha_o \) = dimensionless perikinetic or orthokinetic transport

\( \beta \) = grain packing coefficient

\( \beta_1, \beta_2 \) = constants in Lennard-Jones potential energy function

\( \nabla^2 \) = the Laplace operator \( \left( \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2} \right) \)

\( \varepsilon, \varepsilon_o \) = clogged and clean bed porosities

\( \xi \) = media zeta potential

\( \theta \) = cell residence time

\( \delta, \bar{\delta} \) = Mints and Ives scour coefficients respectively

\( \lambda, \lambda_o \) = filter impedance modulus at time, \( t \), zero respectively \( L^{-1} \)

\( \lambda_D \) = the Debye length

\( \mu \) = fluid absolute viscosity

\( \nu \) = fluid kinematic viscosity

\( \nu_o \) = an empirically derived frequency for the molecules, probably closely related to the frequency of fundamental electronic oscillations within the molecules

\( \rho_c \) = the charge density

\( \rho_s \) = deposit self porosity
\( \rho_s, \rho \) = mass densities of the particle and fluid
\( \sigma \) = specific deposit content vol/vol
\( \sigma_c \) = surface charge density
\( \sigma_s, \sigma_d \) = the Stern and diffuse layers charge respectively
\( \sigma_u \) = ultimate value of specific deposit
\( \psi \) = the electric potential at any point away from the surface of a colloid
\( \psi_j(x) \) = the electric potential for the \( j \)th kind of ions at a distance \( x \) from the colloid surface
\( \psi_o \) = the electric potential at the surface of a colloid
\( \psi_s \) = a surface factor unit for a sphere
\( \omega \) = the valence of the ion
\( \delta \) = electrolyte dielectric constant
\( \delta W \) = work done by the fluid per unit volume per unit time
\( \phi \) = total work done by shear per unit of volume per unit of time in all three planes \( x, y, z \) on an element of fluid
\( \phi_0, \phi_1 \) = grain equivalent radius at time, \( t = 0, t \) respectively
\( \phi_s \) = grain sieve radius, or one half of the mean of adjacent sieve sizes that pass and retains the grain
\( \eta \) = the fraction of collisions resulting in aggregation
\( \tau \) = shear stresses
INTRODUCTION AND SPECIFIC OBJECTIVES

Rapid sand filters have been commonly used for clarifying dilute liquid suspensions in municipal water supply for over 80 years. Throughout the earlier years of their use, the development and improvement of rapid sand filters had been evolved by trial-and-error innovations without a basic knowledge of the mechanisms involved in filtration by a porous sand bed. Many studies made during recent years have shown that a rapid sand filter involves many dynamic, complex mechanisms in which the removal of particulate matter from water is a depth and time related phenomenon.

As a result of the advancement in the coagulation and sedimentation processes, water treatment plants have been able to reduce appreciably the particulate load in the filter influent. In many well-operated rapid sand filter plants, it is quite common to have a filter influent turbidity of one unit or less. When one considers the maximum contaminent level of turbidity (one NTU) for public water supplies, it is obvious that such filters are merely functioning as a final polishing treatment for the water. The benefit to cost ratio for this final polishing is proportionately low since the filters, which are expensive to construct and operate, are doing the least work.

Conversely, many treatment facilities are required to treat waters that are diverse in their characteristics due to the complexity of contamination from industrial and agricultural sources. Nevertheless,
they must meet stringent treatment specifications. Old conventional treatments are not necessarily adequate to handle economically and/or effectively the burden of removal of those newly introduced contaminants.

In recent years, more and more attention is being turned towards filtration research. The use of high rate and declining rate filtration, direct filtration, diatomaceous earth or precoat filtration, the preconditioning of filter media, multimedia filtration and polymeric filtration aids are examples of the recent improvements on the process of filtration.

Extensive investigations and an enormous amount of effort were carried out to develop a clear understanding of the process and the feasibility of the contemporary techniques. These investigations are necessary to help the design engineer cope with increasing variation in raw water characteristics and understand the new filtration technology and equipment.

The last four decades have witnessed very intensive efforts to develop the basics of the theory of filtration. Research workers including Ives, Lerk and Heertjes in England, Mints and Shekhtman in Russia, Mackrle and Mackrle in Czechoslovakia, Deb and Sakthivadivel in India, and Camp, O'Melia, Cleasby, Baumann and Stumm in the United States, are among those who elaborated on the filtration theory and have presented process mathematical models. However, the theorists have disagreed among themselves to the extent that the reliability or practicality of the models is still questionable.
One fairly recent development in the field of porous media filtration is the utilization of the bed for the purpose of coagulation, flocculation, and floc removal. The new process is called "contact flocculation" due to the occurrence of flocculation of particles while they are in contact with the bed grain surfaces or while flowing inside the bed. The process combines the characteristics of both flocculation and filtration. Therefore, any representative model should include the contribution of each of the two processes.

In the literature, there seems to be as many theories of filtration or flocculation as there are filtration research workers. However, according to the current state of knowledge there is no theoretical work cited anywhere concerning bed flocculation mechanisms or a mathematical model representing them. It is almost impossible, at least for the time being, to develop a theory of this process without prior gathering of specially designed experimental observations under controlled laboratory conditions.

Therefore, the objectives of the current study include:

1. To conduct an experimental investigation of the new process.
2. To establish a new theory of granular media simultaneous contact flocculation and filtration.
3. To develop mathematical formulas describing the process pressure drop and the root mean square of velocity gradient in the bed.

The introductory part of the study includes a literature review which will highlight some of the theoretical and practical aspects of
the processes. The literature covers the following sections:

1. Process mechanisms
   a. Filtration mechanisms
   b. Contact flocculation mechanisms
2. Hydraulics of flow through granular media flocculator
3. Process theoretical literature
   a. Theoretical aspects of filtration
   b. Theoretical aspects of granular media contact flocculation.

The literature review presented in the introductory section covers such subjects that are directly or indirectly related to the specific objectives of this thesis. The main reason for such a comprehensive coverage is to provide the reader with a complete summary of the past achievements and to show the current status beyond which the contributions of the current research begins.
Filtration Mechanisms

The phenomenon of particle removal occurs when a suspension is filtered through a granular bed. This results from at least two different groups of mechanisms, acting simultaneously and independently. These are: transport mechanisms and attachment mechanisms. Basically, the process of removal occurs via two steps: a transport step and an attachment step.

Particle transport is a physical hydraulic process and is affected by parameters which govern mass transfer. Particle attachment is basically a chemical process, and is influenced by both physical and chemical parameters.

In spite of the complexity of the process of flow through porous media, it is widely accepted that within the practical limits of physical operating variables the flow is laminar and is of the Poiseuille type. Therefore, the fluid velocity at grain surfaces is zero, and is maximum at the center of pore spaces between the grains.

The role of a transport mechanism is to bring the particles away from their streamlines, and into the vicinity of the grain surface where the flow effect is minimum and attachment mechanisms are active. These transport mechanisms can be classified as:

1. Straining
2. Sedimentation
3. Inertial impingement and centrifugal collection
4. Diffusion and Brownian movements
5. Hydrodynamic
6. Interception and chance contact.

**Straining**

For a long time it has been believed that filtration is a pure surface phenomenon and that straining at the entering face of the media should be as negligible as possible for successful operation (82, 90, 98). If filters are preceded by adequate pre-treatment, only small particles will reach the filter, thus providing no chance for straining. The result of straining is development of a surface mat which causes head loss to rise up rather rapidly, resulting in short filter runs. The rate of head loss progress for surface straining is described by Boucher's law (57), which is a power law function of time.

Regardless of its disadvantages, particle removal by straining mechanisms will always occur as long as there are particles too large to enter the pores of a filter, or large enough to span crevices in the pore structure (7, 35, 57). Hall postulated an interstitial straining theory stating that

\[ r' = \left( \frac{d_p}{d} \right)^{3/2} \]

where \( r' \) is the rate constant for removal by straining and \( d_p, d \) are the particle and grain diameters, respectively (35).
Sedimentation

Hazen (36) was the first investigator to postulate the similarity between void spaces in the filter and settling basins. He was concerned with slow sand filtration, where particles as small as 0.0003 mm in diameter are removed by the effect of their settling velocity. The proposed sedimentation mechanism by Hazen was later supported by other investigators (19, 28, 35, 49, 98, 99).

Hall (35) proposed a gravitational-sedimentation theory based on the relative ratio of fluid velocity \( V_g \), and particle settling velocity due to the effect of gravity \( V_p \). According to Stoke's law of sedimentation in which \( V_p \) is proportional to \( \frac{dp^2}{V_s} \), Hall proposed that the probability of removal due to sedimentation, \( r'' \) follows the equation:

\[
r'' = C' \ m \ \frac{dp^2}{V_s}
\]

where \( m \) is the coefficient of \( \frac{dp^2}{V_s} \) and \( C' \) is experimentally found to be 0.1.

In water filtration, most of the suspended particles are of higher density than water. Consequently, they are subjected to the effect of sedimentation. Even those particles that are less dense than water can settle inside the voids if they are close enough to the grain surface, where the fluid velocity is zero. Solving the equations governing the sedimentation of particles influenced by gravity in a laminar flow, Ives (59) proposed a dimensionless group, called the gravitational parameter
Inserting Stoke's Law for $V_p$, the following is obtained for $G_p$

$$G_p = \frac{V_p}{V_s}$$

$$G_p = \frac{(\rho_s - \rho)}{18 \mu V_s} dp^2$$

where $\rho_s$, $\rho$ are the mass densities of the particle and the fluid respectively, and $V_s$ is the approach velocity of filtration. Ives (59) proposed that for a constant Reynolds number, $R_n$, the efficiency of removal is found to vary with $G_p$ to the power of 1.3.

**Inertial impingement and centrifugal collection**

In laminar flow through granular media, particles are carried by the stream lines in a sinuous pathway around the spherical grains. Because of inertia, particles with sufficient momentum tend to practice downward movement and impinge on the sand grains. As presented by Ives (54), the inertial parameter, $M$, is:

$$M = \rho_s \frac{dp^2}{9 \mu} \frac{V_s}{d}$$

Due to the fluid viscosity, small particle sizes, and low flow rates, the inertial mechanism is of negligible effect in water filtration (68). Herzig et al. (42) demonstrated that the centrifugal force exerted on the particles in a stream of fluid flowing around a grain is 1/100 to 1/10,000 of the gravitational force, for typical flow rates and grain sizes.
Diffusion and Brownian movement

In 1963, Hunter and Alexander (47) studied the diffusion of small particles into the surface of media where the flow velocity is diminished. They proposed a mechanism for diffusion which is due to the concentration gradient initially present between the center of the capillary, and the pore boundary. Diffusion was observed regardless of the repulsive forces between the negatively charged particles and grains. However, in water filtration several investigators have found that the effect of diffusion on the transport of large particles is negligible (52, 80, 98).

The effect of Brownian movement as a transport mechanism has been studied by several investigators (53, 56, 62). In water filtration, the atmosphere of the dipole water molecules causes small suspended particles to move randomly into close proximity with grain surfaces. Particle translation due to Brownian diffusion (10) is equated as follows:

\[ \bar{x} = \frac{2k T a t}{3\mu dp} 1/2 \]

in which \( \bar{x} \) is the mean translation that a particle of diameter \( dp \) would practice in time \( t \), \( k \) is Boltzmann's constant, and \( T_a \) is the absolute temperature. In water filtration, Ives (57) proposed that the Brownian diffusion mechanism is characterized by the dimensionless Peclet Number, \( P_n \) where
The efficiency of removal due to Brownian motion was experimentally investigated (49), and was found to be inversely proportional to the flow rate and also to the square of fluid viscosity. In general, it is agreed that only particles less than 1 μm in size will be transported by this mechanism. Within the range of water viscosity, flow rates, and suspended particle sizes encountered in typical filtration applications, the effect of Brownian motion is practically insignificant (42, 52, 62, 98).

**Hydrodynamics**

In laminar flow, the velocity distribution at any section is parabolic. Particles that are carried by the flow streamlines are subjected to rotational and lateral drifts due to the difference in shear forces acting on their boundaries. If a particle is not spherical or continuously changes its shape in a shear field that is not uniform or stationary, the effect of the lateral movement is further accentuated. The net result of these hydrodynamic forces is a random helical motion, which in some instances will bring the particle to the media surface.

The effect of this hydrodynamic mechanism is characterized by the dimensionless Reynolds Number, $R_n = \frac{\rho v S d}{\mu}$, and was experimentally found (49) to vary as $R_n^{-2.7}$.

**Interception and chance contact**

There are additional removal chances for those particles that are flowing in a stream line at a separation distance $\leq \frac{dp}{2}$ away from the
grain surface. The mechanism of removal is purely interception, and is characterized by the dimensionless parameter $\bar{L} = \frac{dp}{d}$. Ison and Ives (48) postulated that if the other parameters are unchanged the effect of the interception parameter on removal efficiency is proportional to $(\bar{L})^{2.3}$.

Stein (99) studied the mechanisms of particle removal at constrictions in a tube because of the close similarity to a sand bed. The author proposed that chance contact due to the convergence of fluid streamlines is proportional to $\frac{dp^2}{d^3}$.

**Attachment mechanisms**

Particles frequently present in raw water to be filtered exhibit surface charges. The existence of these charges is due to one or more of the following reasons (80):

1. Ionization of molecules at the particle surface.
2. Imperfections or isomorphic substitution in the crystal lattice, with the result that the surface charge density is fixed and constant.
3. Direct chemical reactions with specific ions in solution, (chemisorption).
4. Physical adsorption of ions from solution (an example is hydrogen bonding).

Filter media commonly used in water filtration exhibit negative surface charges (40, 47, 80). The net interaction between the surface charges influences particle attachment to a great extent.
As discussed earlier, particle transport is controlled by the flow, suspension, and media physical variables. In contrast, during the attachment step the process is predominantly controlled by the surface properties of the materials involved. The surface interactions encountered are often described as the net result of attraction produced by hydrogen bonding or van der Waals forces, and electrostatic repulsion of the coulombic type. The simplest colloid-chemical model used to describe the interactions between suspended particles and the surfaces of the filter grains is based upon the theory of electrical double-layer interaction.

Aside from physical and chemical adsorption of particles on the grain surfaces, additional removal mechanisms may simply be due to surface tension (92), or friction between the irregular rough-texture grain surface and particles in suspension.

Evidence of electrokinetic phenomena in water filtration is cited elsewhere. Ruth (88) concluded that particles of several orders of magnitude smaller than the media pore size can only be removed by adhesion to the grain surface if the electrokinetic attraction forces overcome the repelling electrostatic forces. Smith (96), Hunter and Alexander (47) studied the effects of media zeta potential ($\xi$) and particle surface charges on the failure or success of filtration. When both particle and grain surface charges were the same, removal was impaired due to the energy barrier. Media charge reversal increased the efficiency of filtration. In their study, the value of ($\xi$) is
evaluated based on the streaming potential effect. The relationship between electrokinetic potential ($\xi$) in mV, and streaming potential, $E$, in mV as proposed by Ives is as follows:

$$\xi = \frac{E}{\Delta p} \cdot \frac{4\pi \mu K}{\delta}$$  \hspace{1cm} (1)

where, $\Delta p$ is the pressure difference across the column of grains in cm of water, $\mu$ is the viscosity of the electrolyte, $\delta$ is the dielectric constant of the electrolyte, and $K$ is the conductivity in micromho/cm.

Stanley (98) investigated iron floe penetration inside the filter bed and the effect of pH on floe filterability. The study indicated that particles are most easily removed when their electrokinetic repelling forces are at a minimum in the pH region encompassing their isoelectric point. He also postulated the effect of other ions present in water on floe filterability.

Cleasby and Baumann (19) observed that ferric floe particles coat sand media uniformly without any pronounced preference, with respect to positions on the grains. The study indicated that the electrokinetic forces were primarily responsible for the removal of the hydrous ferric oxide particles. It was also concluded that particle size had little effect on the removal efficiency.

Oulman and Baumann (83) and Oulman et al. (84), reported on streaming potentials in diatomite filtration of water, and the effect of electropositive materials used for coating diatomite on the zeta
potential of coated filter media. They demonstrated that the originally negatively charged diatomaceous earth reverses to a positive charge upon the addition of iron or aluminum salts, or upon the addition of cationic polyelectrolytes. The effect of coating the diatomaceous earth with polyelectrolytes is a remarkable increase in the media capacity for turbidity, bacteria, and color removal, at a low head loss.

To describe the process of particle attachment in a mathematical way, or to explain it in a theoretical model it is necessary to study the nature of the molecular and electrostatic forces that are active. In the following section, discussion of each of the attachment mechanisms is briefly presented.

van der Waals forces

The earliest attempts to develop a model of the liquid state grew out of observations of the continuity of the liquid and gaseous states, and the existence of a critical point. An early effort was that of van der Waals, in which he accounted for the molecule's volume, as well as its decrease in pressure due to intermolecular forces. In 1938, Lennard-Jones as cited from Rosenberg text (87), presented a mathematical model for molecular interaction, known as the Lennard-Jones potential energy function \( U(r) \);

\[
U(r) = \frac{-\beta_1}{r^6} + \frac{\beta_2}{r^{12}}
\]  

(2)
where \( z \) is the intermolecular distance, and \( \beta_1, \beta_2 \) are constants. In his model the attractive term \( \frac{-\beta_1}{z^6} \) is a starting point to calculate the net attraction exerted by a given surface, considered as an infinite-sized slab, and a colloid suspension, by the summation of overall atom–atom interactions, (if simple additivity of forces can be assumed). The constant \( \beta_1 \) is taken to be \( \left(\frac{3}{4}\right) \hbar \nu_o \alpha^2 \) where \( \alpha \) is the atoms polarizability, and has units of volume in the cgs system, \( \hbar \nu_o \) is approximately the ionization energy. Since ionization potentials are about 1.0 ev or \( 10^{-12} \) erg, and polarizabilities, about \( 10^{-24} \) cm\(^3\), \( \beta_1 \) will have a value of about \( 10^{-60} \) erg cm\(^6\)/atom. Now if one desires the attractive energy between two slabs, \( V_a \), a further integration over the size of the colloid particle is needed. This energy will now vary as \( z^{-2} \),

\[
V_a = \frac{-H_m dp}{6z}
\]  

(3)

where

- \( V_a \) = attractive energy between two slabs (the surface of a colloid particle considered to be a slab compared to the size of an individual molecule).
- \( H_m \) = the Hamaker constant = \( \frac{3}{4} \pi N_a^2 \hbar \nu_o \alpha^2 \)
- \( N_a \) = the number of atoms per cubic centimeter
- \( \hbar \) = Planck's constant, and
- \( \nu_o \) = an empirically derived frequency for the molecules, probably closely related to the frequency of fundamental electronic oscillations within the molecules.
It might be noted that $H_m$ contains the quantity $(N_a \alpha)^2$. For most elements, this quantity is about 0.1, so the Hamaker constant is of the order of $10^{-13}$ erg. It is also necessary to realize that equation 3 is only applicable for a very close approach, such that $z \ll \frac{dp}{2}$.

Application of Van der Waals intermolecular forces to water filtration models was first proposed by Mackrle and Mackrle (71). The authors contended that adhesion between suspended particles and grain media is solely due to attraction resulting from Van der Waals forces. Heertjes and Lerk (38, 39) proposed a mathematical filtration model based on the equilibrium between molecular forces of adhesion (arising at the surfaces of the filter grains) and hydrodynamic forces acting on the particles in suspension. Ives (52), Stein (99), Camp (14) and many other investigators postulate that adsorption of suspended particles to the surface of media is a primary factor in rapid sand filtration.

Theory of double layer interaction

An important group of electrical phenomena has to do with the nature of the distribution of ions in solution in the presence of an electric field. To begin with, consider a plane surface bearing a uniform charge density in contact with a solution phase containing both positive and negative ions. To be specific, the surface is assumed to be negatively charged. The electrical potential at the surface is taken to be $\psi_0$, and it decreases as one proceeds into the solution in a manner yet to be determined. At any point, the potential
\( \psi \) determines the potential energy, \( \omega e \psi \), of the ion in the electric field, where \( \omega \) is the valence of the ion and \( e \) is the charge on the electron. The ion distribution with distance away from the plane surface assumes Boltzmann's distribution:

\[
\rho_c = \omega e \left[ \frac{C_c e}{k T_a} - \frac{e}{k T_a} \right]
\]

where \( \rho_c \) is the charge density, \( k \) is Boltzmann's constant, \( T_a \) is the absolute temperature, and \( C_c \) is the ionic concentration at the colloid's surface.

In the case of a neutral system, the net charge should be zero. At a long enough separation distance from the surface, the positive and negative charges are of the same density. The integration of \( \rho_c \) out to infinity gives the total excess charge in the solution, per unit area, and is equal in magnitude but opposite in sign to the surface charge density \( \sigma_c \).

The mathematics are complicated by one additional theorem, given by Poisson's equation. This relates the divergence of the gradient of the electrical potential at a given point to the charge density at that point:

\[
\nabla^2 \psi = -4 \frac{\rho_c}{\varepsilon D} \pi
\]

where \( \nabla^2 \) is the Laplace operator \( \left( \delta^2 / \delta x^2 + \delta^2 / \delta y^2 + \delta^2 / \delta z^2 \right) \) and \( D \) is the dielectric constant of the medium.
Various forms of the solutions to equations 4 and 5, have been studied by Gouy (34), Debye and Huckel (24), and a summary of the theory of colloid stability has been presented by Verwey and Overbeek (106). In their theory, the potential \( \psi \) falls off exponentially with distance \((x)\), for example, for the jth kind of ion:

\[
\psi_j(x) = \frac{4\pi a}{\omega_j e} e^{-\frac{xB}{\omega_j}}
\]

(6)

de here,

\[
B_j = \left( \frac{4\pi e^2}{kT} \sum n_j \omega_j^2 \right)^{1/2}
\]

(7)

where \( n_j \) is the concentration of ions of valency \( \omega_j \).

The quantity \( B_j \) is associated with the size of the ion atmosphere around each ion, and \( \frac{1}{B_j} \) is commonly called the ion atmosphere radius, or the Debye characteristic length, \( \lambda_D \). Equation 6 if restated after substituting for \( \psi_o \) (the surface potential) and \( \lambda_D \) (the Debye length) takes the form of

\[
\psi_x = \psi_o e^{-\frac{x}{\lambda_D}}
\]

Here the definition of the characteristic Debye length, \( \lambda_D \), is more understandable, which is the length required for the potential to fall off by an order of magnitude. In other words, \( \lambda_D \) denotes the thickness of the double layer as if resembled by a two plate condenser. In such cases, the capacity of the double layer, \( C_L \), is equivalent to:
\[ C_L = C_s + C_D \] (8)

where

\[ C_s = \text{capacity of the Stern layer} \]

\[ = \frac{D}{4\pi \lambda_{OHP}} = \frac{\sigma_s}{(\psi_o - \psi_{OHP})} \]

and \( C_D = \text{capacity of the diffuse layer} \)

\[ = \frac{D}{4\pi \lambda_D} = \frac{\sigma_D}{\lambda_D} \]

\( \sigma_s, \sigma_D \) are the Stern and diffuse layers charge, respectively, and \( X_{OHP}, \psi_{OHP} \) are the outer Helmoltz plane thickness and potential, respectively.

Based on equations 6 to 8, it is possible to explain the effect of such chemical parameters as ion concentration, pH, temperature, type of ions in dispersion, and the dielectric constant of the system. It is more useful to study the effects of \( \psi_{OHP}, \lambda_D \) on the potential for repulsion, and hence the system's stability. For good flocculation or particle attachment to a filter grain, it is generally necessary to decrease \( \psi_{OHP} \) and decrease \( \lambda_D \). The potential \( \psi_{OHP} \) could be decreased by controlling the concentration of the system's potential-determining ions, and the Debye length is reduced by increasing the system's ion concentration and valancy, and reducing its temperature.
In water filtration it is plausible to consider the existence of double layer interactions between the media and medium ions. As previously stated, most filter media exhibit negative charges, whereas the suspension particles include a variety of charges that are mostly negative too. In order for attachment to occur, addition of positively charged electrolytes is often recommended to compact the double layers, thereby reducing substantially the repulsion region and allowing particles of like charges to come close enough to the grain surface to become attached due to the van der Waals attractive forces previously discussed. The effect of pH changes affects the grain and particle zeta potential, since in water systems hydrogen is usually the potential determining ion.

The treatment of the attachment step in light of the double layer theory and the molecular interactions forces seems to have wide acceptance among investigators in water filtration. Mackrle and Mackrle (71), Heertjes and Lerk (38), Ives and Gregory (60), O'Melia and Stumm (81), O'Melia and Crapps (80), Wenk (110), and many others have postulated adsorption of particles on the grain surface as the primary removal mechanism, taking into account the applicability of the double layer theory and the molecular forces in the process of particulate removal. The general form of interaction energy versus separation for two like charges shows predominance of attraction at very small and very large separation distances, and predominance of repulsion in between (110). These areas are denoted (110) as the primary minimum and the secondary minimum in
attraction energy, and maximum of repulsion or an energy barrier, respectively. Flocculation or attachment of the two charged surfaces is almost possible if their separation distance coincides with either the primary or secondary minimum. In the second case, the two particles will attach with a liquid film between them. In the first case, attachment would occur if the particles have enough potential or thermal energy to overcome the energy barrier (100).

Ives and Gregory (59) discussed the basic concepts of filtration, primarily the effect of surface force mechanisms. They concluded that electrokinetic phenomena are significant in water filtration. In another paper, the same authors (60) plotted the attraction and repulsion energy curves between polyvinylchloride microspheres of ~ 1 micron size and sand, anthracite and ballotini grain surfaces in various electrolytes. (Also the same curves for the interaction energy between ferric floc particles and ferric floc coated sand grains surfaces.) In both cases, there was a linear inverse correlation between height of energy barrier and filtration efficiency. The authors observed that substantial decrease of the energy barrier resulted from increasing the electrolyte concentration and valency. According to their experimental data, the energy barriers are at separations of about 100 to 200 Å, and in general, the whole range of the surface forces is less than 200 - 300 Å for most natural waters containing moderate amounts of dissolved salts. This indicates that the effect of such surface forces is limited to the very close proximity of the grain surfaces.
Hydrogen bonding

Hydrogen bonding is a result of an interaction between a proton donor group A–H and a proton acceptor group B to form a bond of the type A–H——B, where the hydrogen atom acts as a bridge between the two electronegative atoms A and B. A particularly strong dipole-dipole attraction occurs when hydrogen is covalently bound to a very small electronegative element such as fluorine, oxygen, or nitrogen. In these instances, very polar molecules result in which the extremely small hydrogen atom carries a substantial positive charge. Because the positive end of this dipole can approach close to the negative end of a neighboring dipole, the force of attraction between the two is quite large. The positive end of the instantaneous dipole will attract electrons in a neighboring atom, thereby inducing another instantaneous dipole. These dipoles attract one another briefly before they disappear. While attractions between the instantaneous dipoles may be fairly strong, their duration is very short (59). The strength of hydrogen bonds is on the order of 5.0 K cal/mole, which is in the neighborhood of physical adsorption bonds.

Wenk (110) discussed the evidences of hydrogen bond existence in water flocculation and filtration reported by other investigators. The effect of pH on polymer adsorption was considered as a crucial factor in explaining Michaels and Morelos (75) observation regarding the flocculation ability of sodium polyacrylate and polyacrylamide for colloidal kaolinite suspensions. Retarded flocculation was reported
(75) for both low and high values of pH, while at intermediate pH values the conditions for flocculation were favorable. Wenk pointed out that even at pH = 6, since hydrogen atoms are attracted to the negatively charged particle surface, it is possible to find unionized acid groups within the double layer even though in the bulk they are ionized.

Adsorption by hydrogen bond occurs only over a few molecular layers or a separation range of 10 – 20 Å. For this reason and others, particle adhesion by hydration is postulated to be questionable (59). It is the writer's understanding that hydration referred to hydrogen bonding.

Due to the complexity of the process, it is almost impossible to be certain of which mechanisms have caused the removal of an individual particle. Moreover, it is frequently reported that particle removal in experimental filter units was observed to occur under conditions for which theories predict no removal. According to theories, negatively charged particles a few angstroms in size should not attach to negatively charged grain surfaces. However, a certain portion does. Cationic or anionic polyelectrolytes do not cause agglomeration of like charged particles in light of the double layer theory. However, experimental evidence to the contrary has been reported. Still, substantial research efforts are required to break down the process complexity.
Contact Flocculation Mechanisms

Contact flocculation is a process whereby a water containing a dilute suspension of suspended solids is passed through a granular medium or fixed matrix in such a manner that the particles are brought together, or flocculated, to form larger particles that are either more settleable or more filterable than in the original suspension. A frequently used form of contact flocculation is in the direct filtration of low turbidity waters on filters containing relatively coarse media. Coagulent aids are added to the filter influent with little or no detention time, simply to destabilize the charge on the suspended solids. Upon passing through the filter, the combined action of the tortuous flow pathway and surface contact between the coagulated suspension and the surface of filter media grains causes the dispersed particles to agglomerate into larger particles. These larger particles are then removed within the filter bed.

When using large media, high flow-through rates and low coagulent doses, the filter bed may become, in the purest sense, a contact medium that simply causes flocculation of the suspension into larger particles that pass through the bed for subsequent removal by filtration or sedimentation. In this latter, ideal case, the contact bed would not become plugged with entrapped solids but would continue to promote flocculation indefinitely.

It generally is accepted that within the normal range of media size and filtration flow rates, the flow through filters is laminar.
There is great similarity between particles moving in sinuous streamlines in granular beds and flow through capillary tubes. However, in the former case the streamlines are not always parallel but frequently converge and diverge as the flow channel changes shape and size throughout the bed depth. At some locations the cross-sectional area of the flow path might approximate a circular shape, in others it might be elliptical or triangular.

Also, it is hypothesized that as a heterogeneous mixture of particles moves around the spherical grains or inside the voids, chances of their random contact with each other or with the grain surfaces are always possible due to the field velocity gradient, the random Brownian motion, and the contribution of other transport mechanisms such as hydrodynamics, sedimentation, and interception.

As previously discussed, electrokinetic effects exist simultaneously between the filter bed and the particles themselves. A change in these effects may thus influence both particle-bed and particle-particle interactions. For instance, Smith (97) found that only particle-bed interactions take place when particles were filtered without any coagulant. On the other hand, he noticed that interactions between particles (flocculation) play a great part when coagulants or flocculants were added. In this case, the particle-bed interactions were found to have no effect upon bed efficiency and the particle diameter was observed to increase. Ghosh (32, 33), and Borchardt-O'Melia (12) concluded that adding certain ions causes flocculation and formation of larger particles.
which can be subjected to sedimentation or to straining between grains, whereas single particles are not. Stanley (98), however, claimed that flocculation does not occur inside the bed owing to the small particle size, low residence time, and small suspension concentrations. However, Camp (13), analyzing the results of Eliassen (26), considered that flocculation is probably an important mechanism. Finally, O'Melia-Crapps (79) proposed that bed contact flocculation occurs due to bed-particle interactions between themselves.

Although the concept of in-bed flocculation is fairly new and has not been well-defined and modeled, there is ample evidence to support its occurrence. For example, particles that are invisible or of a very minute size before reaching a filter show up as large, well-defined flocs in the backwash waste water. Such an observation would not happen if the filter media did not exhibit a reasonable flocculation potential. Also, large mean size particles emerging in the effluent from a filter that receives smaller size particles, as discussed below, would indicate that small particles are agglomerated into large flocs while moving through the bed.

Stein (99) elaborately investigated the theoretical aspects of water filtration and proposed that bed flocculation can occur inside the media voids and along the sinuous capillary flow channels. The principal factors promoting flocculation were the flow velocity gradient and the effect of curvature of streamlines on the particles moving around the spherical grains.
Perhaps the most significant evidence for bed flocculation is that reported by both Smith (96) and by an anonymous paper (109). Smith (96) presented a paper about the electrokinetic phenomena in particulate removal by rapid sand filtration. The author, without any prior awareness of bed flocculation, used the Coulter Counter technique as a tool to verify the mechanisms of particulate removal. However, during the particle size distribution study, he observed a significant increase in the mean particle sizes in the effluent. Therefore, it was concluded that particles do flocculate inside the bed. Smith reported that when conditions were favorable, the bed acted as a "super coagulator."

In Smith's (96) filtration work, it is possible to assume that small particles were removed by attachment to the grain surface. It is also possible that large particles were removed by straining or sedimentation inside the media voids. However, the author concluded that particles do flocculate while flowing inside the bed capillaries. This conclusion was based on the appearance of larger sized particles in the effluent that weren't present in the influent.

In the period from 1958 to 1970, a controversy (109) grew between Ives in London, and Mints in Moscow, concerning the occurrence of particle detachment in deep bed filtration. An anonymous summary (109) of the controversy referred to Mints, and to Raga and Kalsi's observations to support the mechanism of detachment. The independent experimental work of Mints and Raga and Kalsi, under Ives supervision, included an interesting observation regarding the break-up of large, well-defined,
flocs in the filter effluent towards the end of a run. However, the authors disagreed in their interpretation of the reason for this observation. At the time, Mints' interpretation suggested the existence of continuous dynamic scouring of the old deposit simultaneously with continuous attachment of the new ones. Ives followed a different line of reasoning, and suggested the occurrence of in-bed flocculation of new particles only after the filter reached its ultimate removal capacity. Ives commented that particles which were able to be attached under equilibrium with the fluid shearing forces would not detach again as long as the existing equilibrium was maintained.

It is interesting to note that similar observations to those of Smith (96) and Mints (109), and Raga and Kalsi (109) have been reported by Shea, et al. (93) and were observed in the writer's filtration work at the Ames and Iowa City water treatment plants. However, it is still unknown whether these flocs are formed in the liquid phase prior to their deposition or grew on the media surface and subsequently were detached.

Variables Affecting Contact Flocculation

As in conventional flocculation and filtration, contact flocculation is controlled by the process physical and chemical variables. The physical parameters are:

1. Media size, shape, and depth
2. Concentration of particles in suspension
3. Flow rate

The chemical variables are:

1. Type of coagulant
2. Coagulant dose
3. Ion concentration.

Physical Variables

Media size, shape, and depth

One of the most important floc characteristics is size. In conventional flocculation it is necessary to produce large settleable flocs for the removal of the bulk turbidity in sedimentation basins. However, in direct filtration, sedimentation is omitted and consequently floc filterability is more important than settleability. Therefore, smaller unisize flocs are optimum for bed filtration, while large ones are counterproductive (104).

Hudson (46) commented on the shortcoming of conventional mechanical flocculation producing a wide range of floc sizes. Large flocs are removed on top of filter media and fill the voids rapidly, thereby causing short filter runs. Small flocs break through rather quickly and short runs can also occur due to breakthrough. Therefore, a uniform flow of an optimum floc size is the goal.

In contact flocculation, the final floc size is limited by the average pore size of the media. Therefore, no extremely large flocs are produced. Also, the substantial appearance of flocs in the effluent
begins when the storage capacity of the entire bed nears exhaustion. At this point there are possible mechanisms which could contribute to that appearance. First, it is possible that the released flocs are those that were previously retained by the filter and grew in size and weight to the point where the retaining mechanisms are no longer capable of holding them. The released flocs then should be relatively uniform in size as long as the in-bed retaining forces and the scouring forces are the same throughout the bed.

On the other hand, another possibility is that essentially no scouring may take place. In this case, the processed flocs would be formed from the fresh suspended solids that enter the bed and have no chance for removal past the point of ultimate bed holding capacity. According to the foregoing assumption, it is possible to assume uniform floc size as long as the bed volume for flocculation, flow-through time, and influent suspended solids are the same.

The effect of media shape on the process of bed flocculation has not been investigated before. Intuitively, it is more likely to expect higher storage capacity for the same size of coal than that of sand since the former has the greater bed porosity. However, the effect of media shape factor and pore size on the rate of particle aggregation requires investigation.

As for single-versus multi-media contact flocculators, Shea et al. (93) discussed the possible bed configurations, and concluded that fine on top of coarse media is inadequate to produce long run lengths. On
the other hand, coarse on top of fine media can act as a complete unit process serving the dual function of flocculation and floc removal by filtration. Accordingly, it is the writer's opinion that in the design of a contact flocculator for the sole purpose of flocculation, a single type of large media is recommended.

Influent suspended solids concentration

For direct filtration or contact flocculation, the influent suspended solids concentration, $C_0$, is a significant variable. For high influent concentrations, coarse grained media beds might exceed the performance of the fine-grained ones. Better performance is characterized by providing greater in-bed storage capacity and longer run lengths. On the other hand, if the influent concentration is low, the use of smaller size media would help avoid fast breakthrough. Unfortunately, there are no good criteria for selecting the best media size for a particular water and pilot plant testing is almost always necessary.

One of the unique advantages of contact flocculators is the ability to handle the fluctuations in influent concentration of particulates. Conventional flocculators are sensitive to the inlet concentration. Low concentrations may require excessive amounts of mixing energy or mixing time to bring the dispersed particles into contact. In the case of contact flocculation of dilute suspensions, the bed brings together particles that reach the bed at two different times during the run length. Therefore, flocculation can continue to occur between the deposited particles and the incoming particles, even if the concentration
of the incoming particles gets very low.

Flow rate

The filter hydraulic loading determines the rate of headloss build-up as well as the mixing intensity in the void spaces. The higher the flow rate, the higher the value of velocity gradient, G, inside the bed capillaries, but the lower the flow-through time. Therefore, changes in the critical flocculation parameters tend to offset each other as the flow-through rate increases. One then has to resort to actual experimental observations to determine the optimum operating conditions.

On the other hand, increasing the flow rate reduces the bed retaining capacity and increases the floc break-up term. Consequently, high flow-through rates might be the optimum mode for continuous operation primarily as a flocculation unit preceding filters to avoid frequent backwashing.

Chemical Variables

Type of coagulant

The storage capacity of a contact flocculator is limited to a certain percent of the bed volume. Consequently, for optimum utilization it is necessary to reduce the floc water content as much as possible. For example, reducing the floc water content from 95%, as is normal, to 90% doubles the bed storage capacity.

Floc compaction in contact flocculation occurs as a result of the continuous reduction of its water content. To introduce the same effect
in conventional flocculation processes, high energy is required. Marrow and Rausch (74) and Adin and Rebhun (2) proposed that polymers are better than alum for use as a coagulant in flocculation since lower polymer doses are required which produce strong and dense flocs with low water content. Shea, et al. (93), also reported that 1.0 mg/l of cat-floc polymer gives better performance than 20 mg/l of alum. The suspension used was a mixture of kaolinite and bentonite clay.

**Coagulant dose**

Adin and Rebhun (2) studied the effects of polymer dose and concluded that the optimum dose required for contact flocculation has a broader effective range than indicated by jar testing. It was also concluded by Adin and Rebhun (2) and by Shea et al. (93) that the polymer dose seems to be proportional to the influent solids concentration and in all cases was much less than that required in mechanical flocculation.

**Ion concentration**

The cation concentration of the liquid phase influences particle destabilization by helping to counteract and reduce the repulsive effect of the double layers of the counter-ions around each particle. The double layer theory has been elaborately investigated and experimentally verified for water flocculation and filtration (60). This same effect occurs in contact flocculation. Accordingly, the presence of high concentration of counter-ions enhances floc formation and attachment to the grains of the contact bed.
HYDRAULICS OF WATER FILTRATION

In 1856, D'Arcy presented the fundamental equation of permeability based on measurements of water flow through porous media, which may be represented as

\[ V_i = k_p \frac{\Delta P}{L} \]  \hspace{1cm} (9)

where \( \frac{\Delta P}{L} \) is dimensionless pressure drop gradient in length of water column per unit length of bed, \( V_i \) the interstitial velocity = \( V_s / \varepsilon \), and \( k_p \) is the coefficient of permeability, expressed as the rate of flow of water across a unit cube of media at unit pressure head. The law is closely analogous to Poiseuille's law for the flow of viscous fluid through a circular capillary, presented as follows by Slichter (94):

\[ V_i = \frac{d_{eq}}{32 \nu} \frac{\Delta P}{L} \]  \hspace{1cm} (10)

where \( d_{eq} \) is the diameter of equivalent channel, in cm and equal to 4m for non-circular channels, \( m \) is the hydraulic radius. However, Slichter mentioned poise units for \( \nu \), but the equation is only dimensionally correct if the usual units of kinematic viscosity are used (\( \nu = L^2/T \)).

The first modification of the simple D'Arcy law was made by Slichter (94), whose treatment represents the first real attempt to
derive expressions for the equivalent channels from the geometry of a bed of equally-sized spheres. Slichter assumed the average cross-section to be triangular, and deduced expressions for the cross-sectional area in terms of particle size and bed porosity. Introducing a correction factor of Poiseuille's law, he came out with the equation:

$$\frac{d}{L} = 10.2 \left( \frac{\Delta P}{\mu} \right)$$

where $d_2$ is a function of bed porosity, $\epsilon$, varying from 84.3 for $\epsilon = 0.26$ to 12.8 for $\epsilon = 0.46$. Smith (95), has summarized Slichter's treatment and revised it to give better agreement with experiment.

A modification of Slichter's constant $B_2$ was introduced by Terzaghi, (102) who deduced a relationship between porosity and permeability in close agreement with experiment:

$$V_s = B_3 \left( \frac{\epsilon - 0.13}{3} \right)^2 \frac{d^2}{\mu} \frac{\Delta P}{L}$$

where $B_3$ is an empirical constant with values between 6.03 and 10.5 for all sands. However, up to that time, neither Terzaghi's mathematical treatment, nor Slichter's geometrical treatment of an idealized bed of spheres established the analogy between Poiseuille's law and D'Arcy's law.

In 1927, Kozeny (64) presented the first theoretical equation for the hydraulic gradient, $i$, of flow through granular media. He pointed out that a unique plot is obtained if the dimensionless groups, $\frac{F_1}{\rho V_i^2}$
\[
\frac{\rho V_i d_{eq}}{\mu} = Re, \text{ are plotted against one another, where } F_i = \text{frictional force per unit area, and } \frac{\rho V_i d_{eq}}{\mu} \text{ is called the Reynold's number. The correlation is still close if } \frac{F_i}{\rho V_i^2} \text{ is plotted against } \frac{\rho V_i m}{\mu}.
\]

The factor \( m = \frac{\text{cross-sectional area normal to flow}}{\text{perimeter presented to fluid}} \) and is termed the hydraulic radius. For a bed of granular material, Kozeny provided that \( m = \frac{\text{Volume of fluid in bed}}{\text{Surface presented to fluid}} \).

\[
m = \frac{\varepsilon}{S}
\]

where \( S \) is the surface area of the bed grains per unit bed volume. The general law of streamline flow through granular media (generally known as Kozeny's equation) is:

\[
V_S = \frac{\varepsilon^3}{B_4 \mu S^2} \frac{\Delta P}{L} \cdot g \tag{13}
\]

Equation 13 can be converted to equation 10 by substituting \( \frac{\varepsilon}{S} = \frac{d_{eq}}{4} \), and \( B_4 = 2 \) into the latter. It is interesting to note that \( B_4 = 2 \) does not necessarily denote a circular cross section, nor even a shape resembling a circle. This probably explains much of the success in applying Poiseuille's law for circular channels to granular beds.

In equation 13, the interstitial velocity \( V_i \) was assumed equal to \( \frac{V_S}{\varepsilon} \) which still should be modified such that \( V_i = \frac{V_S}{\varepsilon} \cdot \frac{L_e}{L} \), where \( L_e \) is the length of the sinuous path pursued by an element of the fluid. Therefore, equation 13 becomes,
where $B_5 = B_4 \left(\frac{L_e}{L}\right)^2$, which was found by both Kozeny, Fair and Hatch, to be around 5.0.

In dealing with beds of spheres,

$$S = \frac{6}{d}\left(1 - \epsilon\right)$$

(16)

and equation 15 becomes

$$V_s = \frac{d^2}{36B_5\mu} \frac{\epsilon^3}{(1 - \epsilon)^2} \frac{\Delta P}{L} g$$

(17)

For non-spherical particles, a similar type of transformation may be made in which

$$S = \frac{6(1 - \epsilon)}{\psi_s d}$$

(18)

In this, $\psi_s$ is a shape factor, which is unity for a sphere, and, since a sphere has minimum specific surface, is less than unity for all non-spherical shapes. Hence, for non-spheres

$$V_s = \frac{d^2}{36B_5\left(\frac{\psi_s}{2}\right)^2 \mu} \frac{\epsilon^3}{(1 - \epsilon)^2} \frac{\Delta P}{L} g$$

(19)
In order to experimentally determine the value of the amount \( \frac{36B_5}{\psi_s^2} = \frac{36B_4}{\psi_s^2} \left( \frac{L_e}{L} \right)^2 = \beta_6 \) it is necessary to experimentally determine the separate values of \( \beta_4, \left( \frac{L_e}{L} \right) \) and \( \psi_s \) individually which was thought \((14)\) to increase the risk of accumulated errors. Camp \((14)\) suggested experimentally evaluating \( \beta_6 \) as an over-all characteristic of a filter material to reduce the number of expected errors. He further suggested that \( \beta_6 \) does not stay constant for a certain granular material during the filtration process since the deposit material would change the particle shape and the value of \( \frac{L_e}{L} \) obtained for a clean bed.

Values of head loss obtained using Kozeny's equation would represent the viscous energy loss alone, and therefore are correct only under laminar flow conditions characterized by low Reynold's number, \( R_n \), less than 12 \((14)\), where

\[
R_n = \frac{V_s d}{\nu \sqrt{\varepsilon (1 - \varepsilon)}} \tag{20}
\]

after Camp \((14)\). At higher values of \( R_n \) the flow is not laminar and falls into the transition region, and substantial error would result if Kozeny's equation was mistakenly used, \((14)\). For turbulent flow the energy losses are assumed to follow the formula

\[
\Delta P = \alpha_1 V_s + \alpha_2 V_s^{\alpha_3} \tag{21}
\]

in which \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) \((1.7 < \alpha_3 < 2.0)\), are constants characteristic of the bed, and the first and the second terms denote streamline flow and turbulent flow, respectively. Ergun \((27)\) presented a new head
loss equation to account for both viscous and turbulent energy losses. The first term of it is very similar to Kozeny's equation (17, 20) except for the constant. Ergum proposed that

\[
\frac{\Delta P}{L} = 150 \frac{\mu}{g} \frac{(1 - \varepsilon)^2}{\varepsilon^3} \left(\frac{s_v}{6}\right)^2 \frac{v_s}{s} + 1.75 \frac{(1 - \varepsilon)}{\varepsilon^3}
\]

where \( s_v = \frac{6}{\psi_s d_{eq}} \)

In general, workers in the domain of water filtration have agreed that flow through granular clean filter media under normal filtration rates, media sizes, and water temperatures is laminar and can be described by the previously discussed Kozeny equation (17, 20). During filtration of dilute suspensions the initial values of the variables in Kozeny's equation change considerably, causing a rise in head loss. To be able to predict the head loss values at any time and depth during a filter run it would be necessary to know the new values of the parameters in the Kozeny equation that will vary with filtration. The foremost variables that are worth investigating are clogged bed porosity, surface area, tortuosity factor \((L_e/L)^2\), and the shape factor \(B_4\). Investigators, who have studied head loss changes during filtration, disagree on the way these parameters vary, and consequently different models are cited. Exact calculation of the individual changes in each parameter is extremely difficult and has been accomplished only
approximately by assuming simplified mathematical models for the mode of deposition.

The factor $B_5$ has very little changes (1.5 - 2) during filtration, as has been shown by Carman (17) and Wyllie (112) for wide differences in shape. The change in tortuosity factor $(L_e/L)^2$ due to bed clogging is not possible to predict either analytically or experimentally. However, it is possible to determine its upper and lower limits. If one assumes that the stream lines in a clean bed follow the shape of the spherical grains, then the maximum value of tortuosity factor is $\frac{\pi}{2}$ being the ratio between half a circle circumference to the diameter of the circle. On the contrary, for a bed that had reached its ultimate retaining capacity, it might be possible to assume that the stream lines follow straight lines parallel to the direction of flow, and the corresponding value of $\frac{L_e}{L}$ could approach one.

There are essentially three models for clogged media surface areas. The first model assumes that the bed is represented by an assembly of equal spheres and the deposit uniformly coats the grains. The mathematical expression for surface area according to that model is (57):

$$S = S_o \left(1 + \beta \frac{d}{E_o}\right)^{2/3}$$

(23)

where $\beta$ is grain packing coefficient.

The second is the capillary model which assumes the bed is represented by a bundle of sinuous capillaries and the particles are
uniformly deposited inside the channels. The surface area is expressed as (57):

$$ S = S_0 \left(1 - \frac{\sigma}{\epsilon_0}\right)^{1/2} $$

(24)

The third model (72) takes into consideration the common effect of the previous two models and expresses the surface area as:

$$ S = S_0 \left(1 + \beta \frac{\sigma}{\epsilon_0}\right)^4 \left(1 - \frac{\sigma}{\epsilon_0}\right)^5 $$

(25)

and bed porosity as:

$$ \varepsilon = \varepsilon_0 \left(1 - \frac{\sigma}{\epsilon_0}\right) $$

(25a)

Ives, (57) proposed a head loss model considering the special case when \( \alpha_4 = \alpha_5 = 1 \). Substituting the values of \( S \) and \( \varepsilon \) from equations 25, and 25a into equation 15, the ratio of pressure drop at any time \( (\Delta P) \) to the initial pressure drop \( (\Delta P_0) \) is as follows:

$$ \frac{\Delta P}{\Delta P_0} = \frac{(1 + \beta \frac{\sigma}{\epsilon_0})^2}{(1 - \frac{\sigma}{\epsilon_0})} $$

(26)

and by expanding the right-hand side:

$$ \frac{\Delta P}{\Delta P_0} = 1 + (2\beta + 1) \frac{\sigma}{\epsilon_0} + (\beta + 1)^2 \left(\frac{\sigma}{\epsilon_0}\right)^2 + $$

$$ (\beta + 1)^3 \left(\frac{\sigma}{\epsilon_0}\right)^3 + $$

(27)
For low values of \( \frac{\sigma}{\varepsilon_0} \), the equation could be approximated by the form:

\[
\frac{\Delta P}{\Delta P_0} = 1 + (2\beta + 1) \frac{\sigma}{\varepsilon_0} \tag{28}
\]

which represents a linear relationship between head loss and amount of deposit or time of filtration. Equation (28) is more frequently presented as (14, 48, 99, 102).

\[
H = H_0 + \alpha L \int_0^L \sigma \, dL \tag{29}
\]

where \( \alpha \) is a constant characteristic of filtration conditions, \( H_0 \) is the initial pressure drop throughout the entire bed as calculated from Kozeny equation, and \( H \) is the total pressure drop at any time, \( t \).

Solution of the filtration mass balance equation determines the functional relationship between specific deposit, \( \sigma \), at any time at a certain depth and accordingly equation 29 can be evaluated analytically or numerically. If the effluent concentration \( C \) is small as compared to influent concentration \( C_0 \), an approximate solution (49) is

\[
H = H_0 + \alpha_6 \frac{V_0 C_0 t}{(1 - \varepsilon_0)} \tag{30}
\]

Mohanka (78) followed a similar approach and proposed that for filtrate concentrations less than 10% of the inlet concentration
\[
H = H_0 + \alpha_7 C_0 V_s t \quad (31)
\]
or otherwise
\[
H = H_0 + \alpha_7 (C_0 - C) V_s t \quad (32)
\]
where \( \alpha_7 \) was determined empirically as follows:
\[
\alpha_7 = \alpha_8 V_s^{0.4} S^{0.9}
\]
where \( \alpha_8 \) is a proportionality constant. The author's (78) extended Mackrle and Mackrle's (71) model for surface area, equation 25, and explicitly assumed that \( B_4 \) is constant during filtration simply because it is not possible to monitor its value either experimentally or theoretically. Furthermore, the power coefficients \( \alpha_4, \alpha_5 \) were equated to 1 without any rational reasons, but just for simplicity.

Diaper and Ives (25) simplified equation 28 assuming constant value for the constant \( B_5 \) and proposed that:
\[
\frac{\delta H}{\delta L} = \frac{\alpha_9}{d_s^2} \frac{\alpha_{10}}{d_s} \sigma \quad (33)
\]
where \( \alpha_9 \), in \( \text{cm}^2 \) is a constant for given flow conditions, and porosity in Kozeny equation \( \alpha_{10} \) is a constant for head loss in \( \text{cm} \), and \( d_s \) is the sieve size of grains in \( \text{cm} \). The authors analytically solve equation 33 for size graded media filters where
\[
d_s = d_o + jL
\]
in which \( d_o \) is the grain diameter at the inlet surface of the filter and \( j \) is the gradient or rate of change of diameter with depth \( L \).

The solution proposed was:

\[
H = \frac{\alpha_9 L}{d_o (d_o + jL)} + \alpha_{11} \left\{ \frac{\left( B_7^{-2} - 1 \right) \left( \frac{d_o + jL}{d_o} \right)^{\alpha_{12}} + 1}{\left( B_7^{-2} - 1 \right) \left( \frac{d_o + jL}{d_o} \right)^{1/2} + 1} \right\}^{1/2} \tag{34}
\]

where

\[
B_7 = \frac{e^{T_1} - 1}{e^{T_1} + 1}
\]

\[
T_1 = \frac{2V_s C_o t}{d_o (1 - \rho_s)} \frac{\alpha_{13} \alpha_{14}}{\alpha_{13} \alpha_{14}}
\]

\[
\alpha_{11} = \frac{\alpha_{10}}{2(\alpha_{13} + jL)} \frac{\alpha_{13}}{\alpha_{14}}
\]

\[
\alpha_{12} = \frac{2(\alpha_{13} + j)}{j}
\]

and \( \alpha_{13}, \alpha_{14} \) are dimensionless constants to be determined experimentally.

In case of deep filtration where \( \frac{C}{C_o} < 0.05 \), equation 34 was simplified to

\[
\]
\[ H = \frac{\alpha L}{d_0(d_0 + jL)} + \frac{\alpha_1 \alpha_3}{(\alpha_3 + j)} \frac{V \, C_0 \, t}{d_0(1 - \rho S)} \]  

(35)

Utilization of equation 34 is only practical with the computer assistance. However, the simplified equation 35 would suggest that head loss at any time is equal to the initial head loss plus a constant in head loss proportional to the applied load \( V \, C_0 \, t \).

A number of additional papers have also tried to relate the change in pressure drop to the accumulation of deposited material in the pores, as this latter causes a change in permeability. These are summarized below in their mathematical forms, where \( \frac{\Delta P}{\Delta P_0} \) is the ratio of the pressure drop due to accumulated specific deposit, \( \sigma \), to the pressure drop of the clean filter, when \( \sigma = 0 \). Common symbols are used in the following equations wherever possible. New symbols are defined at the end of the series of equations.

1. Stein (99) (1940).

\[ \frac{\Delta P}{\Delta P_0} = \frac{[1 + \frac{\beta \sigma}{(1 - \epsilon_0)}]^2}{(1 - \frac{\beta \sigma}{\epsilon_0})^3[1 + 2 \frac{\beta \sigma}{\alpha_15}(1 - \epsilon_0)]^2} \]  

(36)


\[ \frac{\Delta P}{\Delta P_0} = \frac{1}{(1 - \frac{\beta \sigma}{\epsilon_0})^3} \]  

(37)

\[
\frac{\Delta P}{\Delta P_0} = \frac{1}{(1 - \alpha_{16} N_p)_{17}}
\]  
(38)


\[
\frac{\Delta P}{\Delta P_0} = 1 + \frac{\alpha_{18}}{\Delta P_0} \left\{ (\alpha_{19} + \alpha_{20} \varepsilon_0)\sigma + \frac{\alpha_{19} + \alpha_{20}}{2} \sigma^2 
\right. 
+ \alpha_{20} \varepsilon_0^2 \ln\left(\frac{\varepsilon_0 - \sigma}{\sigma_0}\right) \right\}
\]  
(39)


\[
\frac{\Delta P}{\Delta P_0} = \left[ 1 + \varepsilon \sigma / \alpha_{21} (1 - \varepsilon_0) \right]^{4/3} 
(1 - \beta \sigma / \varepsilon_0)^3
\]  
(40)


\[
\frac{\Delta P}{\Delta P_0} = \alpha_{22} \frac{t}{d^{0.5}} \frac{L}{L}
\]  
(41)

\[
\Delta P = \Delta P_o L + \alpha_{23} \varepsilon_o L - \frac{\alpha_{23}}{\gamma_6} L \ln \left\{ 1 + e^{\gamma_6 \varepsilon_o L} \left( e^{\gamma_6 \varepsilon_o L} - 1 \right) \right\}
\]

(42)

\[
\gamma_6 = \frac{\gamma_4 (1 - \varepsilon_o) \mu_d}{\mu \varepsilon \gamma_s}
\]

\[\gamma_4, \alpha_{21} \text{ are proportionally constants.}\]


\[
\frac{\Delta P}{\Delta P_o} = [1 + \alpha_{25} (1 - 10^{-\alpha_{24}^\sigma})] \left( \frac{\varepsilon_o}{\varepsilon_o - \sigma} \right)^3
\]

(43)


\[
\frac{\Delta P}{\Delta P_o} = \left( \frac{B}{B_o} \frac{F^2}{F_t^4} \right) \left( \frac{\varepsilon_o - \sigma}{(1 - \varepsilon_o + \sigma)} \right)^3
\]

(44)


\[
\frac{\Delta P}{\Delta P_o} = (1 - \varepsilon_o + \sigma)^2 \left( \frac{\varepsilon_o}{\varepsilon_o - \sigma} \right)^3 \left( \frac{1}{\sqrt{3(1 - \varepsilon_o)^3 + 1}} + \frac{1}{3(1 - \varepsilon_o)} + \frac{1}{3(1 - \varepsilon_o)} + \frac{1}{3(1 - \varepsilon_o)} \right)
\]

(45)
where
\[ \gamma_4, \alpha_{15}, \ldots, \alpha_{25} \] are dimensionless constants,
\[ N_p = \text{number of particles deposited per pore}, \]
\[ \gamma = \frac{\gamma_4 (1 - \varepsilon_0)}{\mu d^3} \]
\[ \bar{B}_4, B_4 = \text{head loss constant at } t = 0, t \text{ respectively}. \]
\[ \alpha_{13} \text{ to } \alpha_{23} \] are experimentally determined constants, and \( \gamma_6 \) is an attachment coefficient.

The equations of Stein (99), Mints (76), Maroudas (73), are all based on alterations of the porosity and specific surface terms in the Kozeny equation, although Mints (76) thought the change in specific surface could be neglected. They have all assumed that the tortuosity remains unchanged during deposition, and have made various attempts to define the internal geometry of the porous media when containing deposited solids. Both Heertjes' equation 38 and Moroudas' equation 40 are based on complete blocking of some flowpaths by particles, thus diverting flow at increased velocity through the remaining open pores. Ives' equation 39 is semi-empirical, based on the internal geometry and flow conditions in the pores containing deposits.

Most of these equations reduce to the form
\[
\frac{\Delta P}{\Delta P_0} = \frac{1}{(1 - \alpha_1 \phi \alpha_j)} + F(\phi) \quad (46)
\]
where \( \alpha_i, \alpha_j \) are constants, and \( F(\sigma) \) is a function of \( \sigma \), or constant or zero. The second term in equation 48 is generally included to make allowances for inaccuracies in the theory (i.e. the values of \( \alpha_i, \alpha_j \)) or to fit particular experimental data. In the regime of laminar flow, most investigators omit the second term and the equation becomes:

\[
\frac{\Delta P}{\Delta P_0} = \frac{1}{(1 - \alpha_i \sigma)^\alpha_j}
\]  

(47)
THEORY OF WATER FILTRATION

The first attempt at a theoretical study on sand filtration started when Iwasaki (63) presented his article "Notes on Sand Filtration." In his paper Iwasaki introduced mathematical expressions for the mass balance and the kinetic equations for the first time in the history of the process. As pointed out by the author:

1. The process is dynamic and varies with filter depth and time.
2. Particle removal at any lamina and a given time is proportional to suspension concentration existing at that time and location.
3. The rate constant of removal is a function of time and depth, and is denoted as the "impediment modulus."
4. For continuity the amount of removal must equal the rate of increase in deposit accumulation.

Iwasaki's foregoing statements are translated to the following mathematical formula:

\[-\frac{\delta C}{\delta X} = \lambda C\]  \hspace{1cm} (48)

\[-\frac{\delta C}{\delta X} = \frac{1}{Q} \frac{\delta C}{\delta t}\]  \hspace{1cm} (49)

where \(\lambda\) is the filter impediment modulus, and \(C(X, t)\) is the suspension concentration at depth, \(X\), and time, \(t\).

The kinetic equation 48 is a widely accepted proposition and can be regarded as having statistical bases (69). The equation is a
partial differential because the concentration not only varies with depth but also with time. At the initial run conditions for \( t = 0 \) the equation may be integrated to yield:

\[
C = C_0 e^{-\lambda X}
\]  

(50)

where, \( \lambda_0 \) is the initial value for the impediment modulus \( \lambda \), and \( C_0 \) is the inflow concentration at the top of the filter. However, as filtration proceeds the internal geometry of the porous medium changes because of the continuous deposition of particles and consequently the filter coefficient \( \lambda \) changes, giving rise to Iwasaki's third equation:

\[
\lambda = \lambda_0 + C_1 \sigma
\]  

(51)

Although the kinetic part in Iwasaki's model is widely accepted, there is a great debate about the functional relationship that describes the variation of filter impediment modulus \( \lambda \) and specific deposit \( \sigma \). Equation 51 which was first formulated by Iwasaki and was used subsequently by Stein (99) shows that \( \lambda \) is linearly proportional to \( \sigma \). That is, particle removal increases linearly with deposit accumulation without any definite limits, which is obviously unrealistic. In actual filtration, efficiency of removal is observed to steadily increase up to a stage at which equilibrium occurs between the shearing forces impairing particle removal and mechanisms of removal. Past that stage, removal slows down and practically stops after
an ultimate deposit value is reached. Ives (51) explained such phenomena in light of the following assumptions: a) removal characteristic of a filter depends on the surface area available on the grains, on the tortuosity of flow within the pores, and on the interstitial velocity, b) the principal force operating to remove suspended particles from the flow stream lines is gravitational.

Ives and Gregory (59) conducted several filtration experiments using pilot filters and through microscopic observations were able to study the mode of deposition around the media grains. Based on the microscopic observations the authors proposed the following expression for the filter coefficient \( \lambda \):

\[
\lambda = \lambda_0 + C_2 \sigma - C_3 \left( \frac{\sigma^2}{\varepsilon - \sigma} \right)
\]  

(52)

The third term in equation 52 explicitly accounts for the deterioration in filter performance due to clogging and continuous decrease in the filter specific area available for removal.

Ives (54) postulated a detachment mechanism encompassing the shearing forces on deposited particles. The shearing forces would either break the bond between the deposited particles, the deposited particles and the granular medium, or ruptured the particles themselves. Ives (54) presented a mathematical development and arrived at the shear term.
53

\[
\text{shear term} = \frac{c_4 \sigma (1 - e_o)^{1/3}}{(e_o - \sigma) (1 - e_o + \sigma)^{1/3}}
\]

(53)

in which \( c_4 \) is a shear factor dependent on particle shear strength, grain size, and fluid viscosity.

This shear effect requires modification of Equation 48 to

\[
\frac{\delta c}{\delta x} = - \lambda_o + c_2 \sigma - c_3 \frac{\sigma^2}{(e_o - \sigma)} c + c_4 \frac{\sigma c}{(e_o - \sigma)} \frac{1 - e_o}{1 - e_o + \sigma}^{1/3}
\]

(54)

However, in subsequent papers, the shear term was dropped by the authors.

Later on, in another paper (49) by the author, the following propositions were listed:

1. During the initial part of the run, the spherical media grains are uniformly coated with deposit particles without changing the shape of the grains appreciably.

2. With the progress of filtration, the side spaces are filled in causing the internal media geometry to change to approximately cylindrical flowpaths which keep narrowing with further deposition.

3. Fluid interstitial velocity, \( V_i = \frac{V_s}{e} \), increases proportionately to the decrease in bed porosity, \( e \), up to a limiting value beyond which deposition is inhibited.

From considerations of the bed geometries, the above assumptions, and others Ives (49) presented, the following model for the functional dependence of the filter coefficient \( \lambda \) and the specific deposit \( \sigma \):
\[ \frac{\lambda}{\lambda_o} = (1 + \frac{\sigma}{\varepsilon})^{\alpha_4} (1 - \frac{\sigma}{\varepsilon})^{\alpha_5} (1 - \frac{\sigma}{\sigma_u}) \]  

(55)

where \( \sigma_u \) is the ultimate specific deposit, \( \alpha_4, \alpha_5 \) are empirical indices and for most cases it has been found that:

\[ \alpha_4 = \alpha_5 = 1, \text{ therefore,} \]

\[ \frac{\lambda}{\lambda_o} = 1 + \text{constant #1} \sigma - \text{constant #2} \sigma^2 + \text{constant #3} \sigma^3 \]

(56)

Equation 56 is further modified by omitting the third term for simplicity.

Although Ives' equation 52 was more adequate than the previous one by Iwasaki, and was experimentally verified for the filtration of algae and polyvinyl particles (59), yet Fox and Cleasby (31) questioned its validity for filtration of other suspended solids. The authors investigated filtration of hydrous ferric oxide flocs under controlled laboratory conditions. Through the early stages of filtration, initial filtrate quality improvement was observed giving support to the exactness of the linear relation between \( \lambda \) and \( \sigma \) as denoted by the first two terms in equation 52. However, the authors (31) reported deviation of experimental observations from Ives' model. The major source of error (31) was due to:

1. Ives' model assumed certain hypotheses that are not necessarily valid under conditions normally met in filtration of flocculated suspension. These hypotheses included that particles are discrete
and unisize, and floc water content is unchanging.

2. Adding the shear term, Eq. 53, previously suggested by the author (54) didn't materially improve the model fitness to experimental observations.

3. The power in the third term of equation 52 was overestimated for hydrous ferric oxide flocs filtration and close agreement between theory and practice was obtained by replacing \( \sigma^{1.25} \) for \( \sigma^2 \). However, that adjustment was not recommended for general applicability as a substitute to Ives' model.

One of the most serious problems Fox and Cleasby ran into while drawing the filter modulus, \( \lambda \), vs. specific deposit, \( \sigma \), curves was the conversion of deposit accumulation in weight per unit volume to volume per unit volume. The authors pointed out their disagreement with Ives' hypothesis concerning the deposit water content. An iron floc that is 98% water and 2% solids loses about 5 to 10% of its water content due to the action of deposition of layers on top of each other and the effect of fluid pressure. However, Fox and Cleasby did not present an exact way to estimate the true deposit volume per unit weight at any time and depth. A compromise suggested by Camp (14) involved the average percentage solids weight to solve the dispute.

It is interesting to know that five years later in 1970, Charles R. Ott, and Richard H. Bogan (82) approached the same subject. The authors used Camp's (14) conclusion to support Ives hypothesis about the unchanging deposit volume per unit weight before and after filtration. A more acceptable way for calculating suspension volume per
unit weight is by using coulter counter data for influent and effluent suspensions, as those conducted by Smith (97).

Several forms of filtration kinetic and mass balance equations, have been proposed by different investigators. Numerical (25, 39, 71) and analytical (22, 43, 51) solutions for the filtration equations were also cited elsewhere. A summary of the progress of filtration model studies is presented in the next section to help in understanding the role of the process variables and hopefully to lead to a combined theory for both contact flocculation and filtration in one model.

Bodziony and Litwiniszyn (11, 10) were among the prominent workers who investigated the process of colmatage (pore blockage) in bed filtration as having statistical or stochastic nature. The authors assumed unisize spherical suspension particles would randomly block individual pores. Therefore, at any instant, the rate of pore blocking according to the foregoing hypothesis is proportional to the number of empty pores available at that instant. In other words:

\[
\frac{\delta P(t)}{\delta(t)} = \alpha_{26} \cdot C \left[ A_0 - P(t) \right]
\]

where

- \( P(t) \) = concentration of pores blocked by solid particles at time \( t \),
- \( A_0 \) = total number of pores available for blocking in the clean bed,
\[ \frac{\delta \sigma}{\delta t} = K_a \left[ e_o - \sigma \right] \]  

(58)

where

\[ K_a \] is the attachment coefficient.

In 1963, Litwiniszyn (70) proposed another model considering the analogy between the process of colmatage and a particle process of "births" discussed by Feller (30). If \( E_n \) denotes the system state at which \( n_p \) pores are blocked, then the expected value of that state, \( E \left[ N'(t) \right] \), may be defined as the first moment of the random variable \( N'(t) \) from the formula

\[ E \left[ N'(t) \right] = \sum_{n_p} A_0 n_p P_{n_p} (t) \]

where \( N'_t \) characterizes the amount of solid particles retained in the porous medium.

Consequently, the expected value of the random variable, \( N'(t) \), calculated by utilizing the concept of the generation function,

\[ F(1,t) = \sum_{n_p} A_0 e^n P_{n_p} (t) \]

is as follows:
The mean value of the random variable \( N'(t) \) characterizing the amount of solid particles retained in the porous medium, tends with time, \( t \), to reach the value \( A_0 \). Also, the variance of the process of colmatage, as shown in equation 60,\
\[
D^2 \{ N'(t) \} = A_0 e^{-\gamma_1 t} (1 - e^{-\gamma_1 t})
\]
where \( \gamma_1 \) is the coefficient of colmatage characterizing the run of the process.

As shown in equation 59, the mean value of the random variable \( N'(t) \) characterizing the amount of solid particles retained in the porous medium, tends with time, \( t \), to reach the value \( A_0 \). Also, the variance of the process of colmatage, as shown in equation 60, first follows an ascending trend, from zero to the maximum value \( \frac{A_0}{4} \), and then a descending one down to zero.

Litwiniszyn, three years later, presented a different model (70) of colmatage - scouring kinetics in the light of stochastic "birth-death" process. In the new model, the author contended that the probability of transition \( E_n \rightarrow E_{n+1} \) is proportional to the number of free places susceptible to be filled in the volume unit of the porous medium. Also, due to scouring process, the system may transform from the state \( E_n \) into the state \( E_{n-1} \). The probability of transition \( E_n \rightarrow E_{n-1} \) within the time interval \( \Delta t \) is \( \gamma_2 n_p \Delta t \), \( \gamma_2 \) being a constant. That is, the probability of moving from the state \( E_n \rightarrow E_{n-1} \) is proportional to the amount of particles trapped per unit volume of the porous media.
According to the foregoing assumptions, the probability of changing the system from the state $E_n + E_{n-1}$ is equal to

$$1 - \gamma_1 (A_o - n_p) \Delta t - \gamma_2 n_p \Delta t.$$ 

Using the same methodology as in the previous model (22), Litwiniszyn (70) pointed out that the new distribution of probability of the random variable $N'_c(t)$ has an average value.

$$E \{ N'_c(t) \} = y_1 A_o \left[ \frac{1 - e^{-(y_1 + y_2)t}}{(y_1 + y_2)} \right]$$

and a variance

$$D^2 \{ N'_c(t) \} = \frac{y_1 A_o}{y_1 + y_2} \left[ \frac{1 - e^{-(y_1 + y_2)t}}{1 - e^{-(y_1 + y_2)t}} \right] \frac{1 - y_1 (1 - e^{-(y_1 + y_2)t})}{(y_1 + y_2)}$$

In Poland, and particularly among the scientific workers in Cracow, it was the publication of a paper by Bodziony and Litwiniszyn (10) that gave rise to the interest in colmatage problems. In the last named paper, a certain mathematical model reproducing the process of colmatage was advanced and the phenomenon itself was described by two equations:
\[
\frac{\delta N'(x,t)}{\delta t} + V_i \frac{\delta N'(x,t)}{\delta x} + N'(x,t) \frac{\delta V_i}{\delta x} = -\frac{\delta P(x,t)}{\delta (t)} \tag{63}
\]

\[
\frac{\delta P(x,t)}{\delta (t)} = \gamma_1 \left[ A_0 - P(x,t) \right] N'(x,t) \tag{64}
\]

where \( N'(x,t) \) denotes the volumetric (dimensionless) concentration of the suspension in an immobile porous medium at the point \( x \) and the moment \( t \) after the process has been started, and \( P(x,t) \) means the concentration of pores of the medium which already underwent colmatation.

Equation 63 may be regarded as a balance of particles carried by the fluid flowing through a porous medium. The second equation 64 describes the kinetics of the process of colmatation.

The system of equations 63 and 64 was solved with respect to the functions \( N(x,t) \) and \( P(x,t) \) the following boundary conditions being assumed

\[
N'(0,t) = n_p = \text{constant}
\]

\[
P(x,t) = 0 \text{ for } x - \int_0^t V_i(x,t) \, dt \geq 0
\]

\[
V_i(x,t) = \text{constant}.
\]

(In the final solution, the author replaced this with a constant superficial velocity.)
The solutions obtained are of the following form:

\[
P(x,t) = \begin{cases} 
0 & \text{for } x-V_s t > 0, \\
\left( -\frac{n_p \gamma l (X - V_s t)}{e V_s} - 1 \right) & \text{for } x-V_s t < 0
\end{cases}
\]

\[
N'(x,t) = \begin{cases} 
0 & \text{for } x-V_s t > 0, \\
\left( -\frac{n_p \gamma l (X - V_s t)}{e V_s} + \frac{A_o \gamma l X}{e V_s - 1} \right) & \text{for } x-V_s t < 0
\end{cases}
\]

Trzaska (104), presented the experimental verifications of the equations 65 and 66 with reasonable agreement. Also, one year later, he proposed a very similar solution of his own (103).

Litwiniszyn (67) followed a different approach to describe the phenomenon of colmatage due to a diffusion mechanism under the assumption that the intensity of diffusion flux, \( i \), is proportional to the gradient concentration, \( N \). The system resembles a one dimensional
process occurring in a tube with rectilinear x-axis and with a constant circular cross section. The author assumed the same kinetic equation as in equation 63, but a different continuity equation of the form:

$$\varepsilon \frac{\delta N'}{\delta x} + \varepsilon \frac{\delta N'}{\delta t} - \alpha_{27} \frac{\delta^2 N'}{\delta x^2} + \frac{\delta P}{\delta t} = 0$$  \hspace{1cm} (67)

where $\alpha_{27}$ is a constant.

The solution of equations 63 and 67 is a rather complicated one, of very little utility for practical calculations, and hard to verify experimentally (66).

However, in 1966, the same author considered the case of a continuous and dynamic diffusion and suffusion occurring at the same time. The proposed equation is

$$\frac{\delta P(x,t)}{\delta t} = \alpha_{28} N'(x,t) \left[ A_0 - P(x,t) \right] - \alpha_{28} P(x,t)$$  \hspace{1cm} (68)

where $\alpha_{28}$ is a constant and the term $\alpha_{28}P(x,t)$ has been introduced to account for the phenomenon of suffusion.

The author claimed the possibility of getting a certain solution of Cauchy's problem, related to the phenomenon of colmatage, in a form more convenient for physical interpretation than that of the solution of initial boundary problems. Unfortunately, the complexity of the mathematical solution (66) was not resolved.
Bodziony and Kraj (9) discussed the phenomenon of colmatage-and-suffusion as a simultaneous process. The authors solved the system of the kinetic equation 68 and a new continuity equation of the form:

\[ \varepsilon \frac{\delta N'(x,t)}{\delta t} + \varepsilon V_s \frac{\delta N'(x,t)}{\delta x} = \frac{-\delta P(x,t)}{\delta t} \]  

(69)

Kraj (65) also presented a simplified model of colmatage accompanied by diffusion alone as described by equation 64. The author used the continuity equation:

\[ \varepsilon V_s \frac{\delta N'(x,t)}{\delta t} + \frac{\delta N'(x,t)}{\delta t} - \alpha_{27} \frac{\delta^2 N(x,t)}{\delta x^2} + \frac{\delta P(x,t)}{\delta t} = 0 \]

Kraj was able to solve equations 64, 68 simultaneously using Laplace transformation. He came out with the following function for \( P(x,t) \):

For \( 0 \leq t \leq t_1 \):

\[ P(x,t) = \alpha N_0 e^{-\frac{V_s}{2\alpha_{27}}} X x \int_0^t \left[ 1 + B(t - \tau)e^{-B\tau} \right] \frac{1}{\sqrt{\alpha_{27}}} \frac{e}{2\sqrt{\tau}} \text{erf} \left( \frac{x}{2\sqrt{\tau}} \right) d\tau \]

For \( t > t_1 \):
\[ P(x,t) = \alpha N_0 e^{2\alpha_27} \int_{t_1}^{t} \frac{e^{2\alpha_27}}{e^{2 \alpha_27} + 4 \alpha_26 \alpha_27} \left[ 1 + B(t - \tau) \right] e^{-B\tau} \]

\[ \text{erf} \frac{X \sqrt{\frac{\varepsilon}{\alpha_27}}}{2 \sqrt{\tau}} \, d\tau \]  

where \( \text{erf} \) is an error function, \( \tau \) is now a parameter such that \( \tau = (t - t_1) (X - X_1) \) and the constant \( B \) is defined from the equation

\[ B = \frac{\varepsilon^2 \alpha_2 + 4 \alpha_26 \alpha_27}{4 \varepsilon \alpha_27}, \]

and

\[ N_0 = N(x,t) \text{ for } x = 0, 0 < t < t_1 \]

It should be mentioned that in the foregoing models \( (9, 30, 66, 67, 70) \) the authors didn't support their theories with experimental evidence to verify the soundness and true representation to the process of suspension filtration. Moreover, some of the authors proposed more than one model with different assumptions concerning the questionable step of detachment. Such discrepancies, besides the complexity of the mathematical solutions, render such theoretical filtration theories of very little practical utility, and defeat the purpose for which they were intended.
In the last two decades, several investigators have attempted to combine the physical and chemical aspects of the theory of filtration in a mathematical model that accommodates such variables affecting particulate removal inside a filter bed. Ives, Lerk and Heertjes in London, Litwiniszyn and Bodzioiny in Poland; Mints and Schektman in Moscow; Deb in India, and O'Melia and Strumm in U.S.A.; Mackrle and Mackrle in Czechoslovakia are among workers whose names are cited in the literature.

Heertjes and Lerk (39), for example, presented a model for filtration of flocculated suspensions of particles of 100 to 1000 orders of magnitude less than the pore size of the filter media and with a density close to the water. They concluded that gravitational and centrifugal effects could be ignored in considering the mechanisms of removal for such particles. Furthermore, they also concluded that London-van der Waal forces are the only possible interaction by which such particles can be caught in the filter bed.

According to the foregoing assumption the authors discussed the resemblance between bed geometry and an array of alternating mixing and laminar cells. In the laminar regions, particles falling in the field of the molecular attractive forces are assumed to be subsequently attached to the grain surface. The space available for removal, inside the laminar cell, is that through which an equilibrium exists between attractive forces and Stokes drag forces. Denoting $a_0$ as the maximum separation distance for removal to occur within a time interval
equal or less than the cell residence time, \( \theta = \frac{e \cdot y_3 \cdot e}{v_s} \), where \( e \) is the length of each unit cell. The authors conclude that

\[
a_o = \left( \frac{H_m}{12 \pi \mu} \right)^{1/3}
\]

(71)

The volume space within which removal occurs, denoted as the relative adhesion volume \( D_e \), is equivalent to \( N \left( a_o \right)^3 \) where \( N \) is the number of laminar cells per bed unit volume:

\[
N = \frac{(1 - e_o) \cdot t}{\pi d^3}
\]

Therefore,

\[
D_e = \frac{\gamma_3 \cdot \frac{e \cdot y_3 \cdot H_m}{2 \pi \mu v_1} \cdot (1 - e_o) \cdot \frac{e}{d^3}}{\gamma_4 c}
\]

where \( \gamma_3 \) and \( \gamma_4 \) are proportionality constants.

Using the concept of relative adhesion volume \( D_e \), Heertjes and Lerk proposed the following mass balance over a unit cell:

\[
\frac{- \delta C}{\delta x} = \left( \frac{D_e}{\lambda e} \right) C
\]

which leads to the equation:
The system of equations 49 and 72 were solved for time and depth in filter and the result is:

\[
C(x,t) = \frac{C_0}{1 + e^{\frac{-V_s C \gamma_5 X}{e^{s e_0} \gamma_5 X}}}
\]

(73)

and

\[
\sigma(x,t) = \frac{e^{s e_0} X}{1 + (e^{\frac{V_s C \gamma_6 X}{e^{s e_0} \gamma_6 X}})}
\]

(74)

Utilizing both equations 73 and 74 the author provided an interesting conclusion regarding the relationship between bed depth, L, and time to breakthrough, \( t_r \), as follows:

\[
t_r = \frac{1}{V_s C \gamma_5} \ln \left\{ \frac{e^{s e_0} L}{e^{s e_0} - 1} \right\}
\]

(75)

Expansion of equation 75 results in

\[
y_5 e_0 L + V_s y_5 C_0 t_r + \ln \left( \frac{C_0}{C} - 1 \right)
\]

For the special case
\[ V_s \gamma s \varepsilon_o t_r + \ln \left( \frac{C_0}{C} - 1 \right) >> 1 \]

then,

\[ L \approx \frac{V_s}{\varepsilon_o} C_0 t_r + \frac{1}{\gamma s \varepsilon_o} \ln \left( \frac{C_0}{C} - 1 \right) \]

and for a particular value of \( C_0/C \),

\[ \approx A'_1 t_r + A'_2 \]  

(76)

where

\[ A'_1 = \frac{V_s C_0}{\varepsilon_o}, \quad A'_2 = \frac{1}{\gamma s \varepsilon_o} \ln \left( \frac{C_0}{C} - 1 \right) \]

Combining equations 49 and 72 leads to the kinetic equation:

\[ \frac{\delta \sigma}{\delta t} = \gamma s V_s (\varepsilon_o - \sigma) C \]  

(77)

The right hand side of equation 77 is the attachment term in the author's model. The rate of particle deposition inside a filter lamina at a given time is proportional to approach velocity \( V_s \), suspension concentration \( C \), and net space available for deposition \( (\varepsilon_o - \sigma) \) at that time and depth. As shown from equation 77, a filter bed would virtually continue to remove incoming particles at a declining rate, until all bed porosity is filled in and there is no space for stream line flow. Such a situation is impossible to meet in normal
operation and in the writer's opinion the following correction is suggested to overcome that weakness of Hertjees and Lerk model:

\[
\frac{\delta \sigma}{\delta t} = \gamma \frac{V_s}{5} (\sigma_u - \sigma)C \tag{78}
\]

where \( \sigma_u \) is an ultimate deposit value past which a bed would no longer remove particles. The value of \( \sigma_u \) is always less than \( \varepsilon_o \) and represents a practical saturation limit, a function of type and concentration of suspension, flow approach velocity, type of chemical pretreatment, and bed porosity and surface characteristics.

Herzig et al. (42) proposed the kinetic equation:

\[
\frac{\delta \sigma}{\delta t} = k_a C (\varepsilon_o - \frac{\sigma}{\rho_s}) - k_d \sigma \tag{79}
\]

where \( \rho_s \) is the deposit self-porosity, \( k_a \) and \( k_d \) are the attachment and detachment constants. This equation is similar to a Langmuir adsorption equation:

\[
\frac{\delta \sigma}{\delta t} = k_a C (F - \sigma) - k_d \sigma \tag{80}
\]

and in the usual adsorption terminology, \( k_a \) and \( k_d \) are adsorption and desorption constants respectively, and \( F \) is adsorber ultimate capacity.

Although equation 79 includes the same error as in 77, it can falsely fit actual observations particularly towards the end of a filter
run. The reason Hertzig's model resembles a real breakthrough curve is that it includes two different terms. The first one, \( k_a C \left( \epsilon_o - \frac{\sigma}{\rho_s} \right) \), is an attachment term which is very close to Heertjes' and Lerk's equation 77 and includes the same error pointed out before. The magnitude of that term decreases during the progress of a filtration run as the value of \( \sigma \) continually increases. The second term is a detachment term and is proportional to the existing deposit content. The combined effect of the two terms, one is descending and the other is ascending, is a false fit to the real data which could occur if the values assigned for \( k_a \), \( k_d \) are adequately chosen for this curve fitting technique.

Saatci (89), unaware of Hertjees' and Lerk's work, developed an analytical solution to equation 79. The author pointed out that the detachment term is insignificant and may be ignored. For this particular case, the solution was:

\[
\frac{C}{C_o} = \left[ \frac{k_a \gamma_s}{\rho_s} \frac{\gamma_o \gamma_{st}}{v_s} \right] + e^{\frac{k_a \gamma_s}{\rho_s}} + e^{\frac{k_d \gamma_o \gamma_{st}}{v_s}}
\]

(81)

The solution is very similar to equation 73, presented by Heertjes and Lerk. For convenience, equation 73 has been rearranged to show the similarity as follows:

\[
\frac{C}{C_o} = \frac{v_s \gamma_o \gamma_{st}}{e^{v_s \gamma_o \gamma_{st}} + e^{e^{\gamma_{st} \gamma_o} \gamma_{s0} X - 1}}
\]

(73 rearranged)
The principal difference is that Saatci's solution does not include the minus 1 in the denominator. Saatci presented the following linear transformation to the breakthrough equation 81:

\[ t = \frac{F}{C_0 V_s} L - \frac{1}{k a C_0} \ln \left( \frac{C}{C - 1} \right) \quad (82) \]

The analytical solution presented by Saatci and given in equation 81 was found to include three deficiencies. The first two are the models' lack of fulfillment of two principal boundary conditions:

1. \( C = C_0 \) at \( x = 0 \) and \( t > 0 \).
2. \( C = C_e e^{-k \frac{C}{a C_0} x/V_s} \) at \( t = 0 \) and \( x \leq L \).

where \( L \) is total bed depth.

Applying the first boundary condition to equation 81 yields:

\[ C = \frac{C_0 e^{k a C_0 t/\rho_s}}{e^{k a C_0 t/\rho_s} + 1} \neq C_0 \]

which does not satisfy the first boundary condition. Also, at \( t = 0 \) equation 81 relates \( C = f (C_0, x) \) as follows:

\[ C = \frac{C_0}{1 + e^{k a C_0 x/\rho_s}} \neq \frac{-k \frac{C_e}{a C_0} x}{C_0 e^{-\frac{k a C_0 x}{V_s}}} \]

which does not satisfy the second boundary condition. The third deficiency is similar to that found in Adin's model. Rewriting the kinetic equation 79 as used by Saatci,
\[
\frac{\delta \sigma}{\delta t} = k_a \left( e_o - \frac{1}{\rho_s} \sigma \right) C - k_d \sigma
\]

at \( C = 0 \) (i.e. if the influent is clear water after a period of suspension filtration).

\[\frac{\delta \sigma}{\delta t} = -k_d \sigma\]

i.e. \( \ln \sigma = -k_d t + \text{constant} \). \( (83) \)

If we assume that at the time \( t_1 \) when the influent was free from particles, \( \sigma = \sigma_1 \).

\[\ln \sigma_1 = -k_d t_1 + \text{constant}\]

\[\therefore \text{constant} = \ln \sigma_1 + k_d t_1\]

Accordingly equation 83 may be expressed as

\[\ln \frac{\sigma}{\sigma_1} = k_d (t_1 - t) = k_d (t - t_1)\]

\[\therefore \sigma = \sigma_1 e^{-k_d (t - t_1)} \] \( (84) \)

Equation 84 demonstrates a continuous decay of the amount of deposit inside the bed with respect to time when no solids are being fed to the filter. This is, however, contrary to the observed zero rate of decay during Mackrle's experiments (109).
Adin (1) proposed the following mass balance and kinetic equations:

\[ V \frac{\delta C}{\delta x} + \frac{\delta \sigma}{\delta t} = 0 \]  

(85)

and

\[ \frac{\delta \sigma}{\delta V} = k_a C \left[ F - \sigma \right] - \frac{k_d}{k_p} \sigma \]  

(86)

where,

- \( V = V_s t \) = volume of liquid passing through a unit area in time, \( t \).
- \( K_p = \frac{V_s}{J} \) = hydraulic conductivity or coefficient of permeability,
- \( J \) = the flow hydraulic gradient.

The coefficient of permeability, \( K_p \) is a modified form of the Shekhtman's formula as quoted by Adin (1):

\[ K_p = K_{po} \left( 1 - \sqrt{\frac{\sigma}{F}} \right)^3 \]

Since, in Adin's model the system of the mass balance and kinetic equations do not have an analytical solution, the author resorted to the computer technique and presented a stepwise numerical solution.

Saatci pointed out that making the substitution, \( J = 1 \), renders Adin's model analytically solvable. The approximation presented by Saatci didn't significantly change the predicted breakthrough curves drawn for different values of \( k_a, k_d \) particularly during the initial part of the run before breakthrough.

However, Saleh and Cleasby (91), in a discussion of Adin's paper, pointed out the weakness of the model in handling all possible filtration
conditions. For example, when \( C = C_o = 0 \), Adin's model would also
indicate a continuous decay of the previous deposit as discussed in
the third deficiency of Saatci's model. Such a case, therefore, indicates
that the author's equation 86 is inadequate to handle all the possible
filtration conditions appropriately. Ives (57) pointed out the same
weakness of the scour term in Mints' continuity equation (10):

\[
- \frac{\delta c}{\delta L} = \lambda_o \frac{\delta \sigma}{V_s} \tag{86a}
\]

where \( \tilde{\sigma} \) is Mints' scour coefficient.

Ives followed a different line of reasoning that led to the
impossible situation that \( \sigma \) could, under some circumstances, exceed
the value of the initial porosity of the clean bed. Therefore, Ives
suggested the following form of Mints' equation to circumvent that
impossible situation:

\[
- \frac{\delta c}{\delta L} = (\lambda_o - \frac{\tilde{\sigma}}{V_s}) C \tag{86b}
\]

where \( \bar{\sigma} \) is a new scour coefficient.

Mints' cited in (109) explained the difference between his model and
Mackrle's observation stating that the model is dynamic, and in order
for detachment to occur, the suspension momentum energy is required
to detach the old deposit. Therefore, clear water can not cause
detachment. Accordingly, a detachment term, if proven to exist,
should reflect these principles.

Furthermore, Mints' model of filtration cited in (109) does not take into
consideration the initial improvement of filtrate due to the increase in the bed surface area. This improvement period was previously demonstrated by Ives (55) and by Fox and Cleasby (31).

As a conclusion, it can be stated that current filtration theories, whether simple or complex, provide an interpretation of observed facts concerning the particular filters and suspensions studied by each investigator. It must also be stated, however, that none of these theories can predict the performance of a filter treating a given suspension without extensive laboratory testing. None of these theories is capable of predicting the performance of the same or another filter treating another suspension.

Ives and Sholji (62) compared several filtration theories by evaluating the relationships which these theories predict between the filter coefficient, $\lambda$, and the physical variables, sand size, $d$, filtration velocity, $V_s$, and water viscosity. The results are remarkable in their disagreements. The authors presented the following results: the filter coefficient is reported to vary with $d^{-1}$ to $d^{-3}$, with $V_s^0$ to $V_s^{-1.56}$, and with $\mu^{-1}$ to $\mu^{0.5}$.

It doesn't appear unfair to state that current filtration theories are either too simple to be flexible enough to apply in practice, or too complex (i.e., require an excessive amount of experimentation) to be useful. In either case, they disagree to such an extent that general applicability be questioned.
Modeling Contact Flocculation

A contact flocculator exhibits characteristics of both in-bed filtration as well as in-bed flocculation. Therefore, an acceptable model must include the contribution of both of these two processes.

Conventional flocculation often is described by the kinetic equation:

\[ - \frac{dN}{dt} = \text{(aggregation term)} - \text{(break-up term)} \]  
(87)

where \(N\) = number of particles per unit volume at time \(t\).

The aggregation term takes into consideration particle contact due to the effect of artificially introduced mechanical agitation and particle contact due to molecular random motion. In other words, there are basically two mechanisms of flocculation, perikinetic (molecular) and orthokinetic (mechanical). The relative significance of each is a function of particle size, mixing power and water temperature. Design recommendations for mixing and flocculation tanks suggest that it is plausible to assume the predominance of the orthokinetic mechanism for most conventional flocculation processes (101).

The equations of Smoluchowski and the contributions of numerous authors, listed in Huck and Murphy (45), have enabled the coagulation and flocculation phenomena to be expressed in the following mathematical form for the union of two particles \(i\) and \(j\):

Perikinetic

\[ - \frac{dN_i}{dt} = \alpha_p 4\pi \delta_{ij} N_i N_j \]  
(88)
Orthokinetic

\[
-\frac{dN_i}{dt} = \alpha \frac{4}{3} \pi (R_{ij})^3 N_i N_j \frac{du}{dx} \tag{89}
\]

where

- \( -\frac{dN_i}{dt} \) = the time rate of reduction in concentration of \( i \)-particles (number/ml sec)

- \( \alpha_p \) & \( \alpha_o \) = dimensionless perikinetic or orthokinetic collision-efficiency factors

- \( R_{ij} \) = the distance between centers of two particles \( i \) and \( j \) in contact

- \( D_{ij} \) = diffusion constant for the particles in perikinetic transport

- \( \frac{du}{dx} \) = velocity gradient of the fluid medium in orthokinetic transport.

Perikinetic transport resulting from Brownian motion is represented in equation 88 by the diffusion constant, \( D_{ij} \), as defined, for this system, by the Stokes-Einstein equation:

\[
D_{ij} = \frac{kT \alpha_i}{3\mu R_{ij}}
\]

where

- \( k \) = Boltzmann's constant.

Orthokinetic transport is dependent on the flow velocity gradient, \( \frac{du}{dx} \). In normal flocculation units, the velocity gradient is obtained by exerting mechanical agitation. However, in porous media, fluid
flow is laminar and the velocity is distributed according to laminar flow through capillary tubes according to the model presented by Camp and Stein (15):

\[ u = U_{\text{max}} - 2\tilde{u} \frac{r^2}{R^2} \]

where

\[ U_{\text{max}} \] = max. flow velocity at the center of a capillary channel
\[ \tilde{u} \] = mean velocity in the capillary channel
\[ R \] = radius of the capillary channel.

i.e. \[ u = U_{\text{max}} \left( \frac{R^2 - r^2}{R^2} \right) \]

In a three dimensional flow, a fluid element moves under an equilibrium between the shear stresses acting above its surfaces and the difference in pressure components. So that, in general, the total work done by shear per unit of volume per unit of time in all three planes is (15):

\[ \phi = \mu G^2 = \mu \left[ \left( \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \right)^2 + \left( \frac{\delta u}{\delta z} + \frac{\delta w}{\delta x} \right)^2 + \left( \frac{\delta v}{\delta z} + \frac{\delta w}{\delta y} \right)^2 \right] \]  

(90)

where

\[ G \] = is the absolute velocity gradient at a point.
\[ u, v, w \] = fluid element velocities in the directions, x, y, z respectively.
\( \bar{G} \) is the root mean square velocity gradient and generally is equal to the square root of the input power, \( p \), per unit volume, divided by the fluid viscosity.

\[
\bar{G} = \sqrt{\frac{p}{\mu V}}
\]

Camp and Stein (15) generalized Smoluchowski's equation to include turbulent flow conditions by introducing the variable, \( \bar{G} \), to replace the velocity gradient in laminar flow. For unisize spherical particles, the rate of disappearance is (15):

\[
\frac{dN}{dt} = -\frac{2}{3} \eta d_p^3 N_o N \bar{G}
\]

in which \( N \) = the number of primary particles present at time \( t \); \( N_o \) = the number of primary particles present initially; \( \eta \) = the fraction of collisions resulting in aggregation; \( d_p \) = the particle diameter.

Integrating equation 91 for the boundary conditions \( N = N_o \) at \( t = 0 \) and \( N = N \) at \( t = t \), gives the relationship:

\[
N = N_o e^{-\left(K_1 N_o \bar{G} t\right)}
\]

where

\[
K_1 = \left(\frac{2}{3}\right) \eta d_p^3
\]

Argaman and Kaufman (6) noted that the application of Smoluchowski's theories to turbulent flocculation required that the particles be much smaller than the smallest scale of turbulence, an assumption that was
not justified in the flocculation systems encountered in water treatment practice. The smallest scale of turbulence is defined as the average size of the turbulent eddies or the size of the packet of fluid all of which has the same velocity (6). To account for the length and time scale over which turbulent velocity fluctuations occur, the authors developed a diffusion coefficient for particles and flocs based on the turbulence energy spectrum. Their rate equation for the aggregation of primary particles was:

\[
\frac{dN}{dt} = -4\pi\eta K_2 R_F^3 N_F N \bar{U}^2
\]  \hspace{1cm} (93)

in which \( K_2 \) is a proportionality constant expressing the effect of the turbulence energy spectrum on the particle diffusion coefficient (for a particular turbulence field and particle size, \( K_2 \) is constant);
\( R_F = \) floc radius; \( N_F = \) number of flocs; \( N, \eta \) are the same as before; and \( \bar{U}^2 = \) the mean square velocity of fluctuation = \( K_F G \). Where \( K_F \) is a performance parameter characterizing the stirring arrangement.

Floc concentration may be expressed as a function of the initial particle concentration, \( N_0 \), and floc radius as a function of the number of particles remaining in suspension, according to the formula:

\[
R_F^3 N_F = R_p^3 (N_0 - N)
\]  \hspace{1cm} (94)

where

\( R_p = \) the radius of primary particles.
Equation 93 is therefore transformed to:

\[
\frac{dN}{dt} = -4\pi \eta K_j R_j^2 N F N K_F G
\]

\[
= \frac{-1}{K_3} N (N_o - N) G \tag{95}
\]

in which \( K_3 = 1/(4\pi \eta K_2 K_F R_p^3) \).

Separation of variable to render this equation exact imposes the restriction \( N \neq N_o \). Since \( N_o > 1 \), the lower limit of integration can be set at \( N_o - 1 \) with negligible effect. Integration of this equation for the boundary conditions \( N = N_o - 1 \) at \( t = 0 \) and \( N = N \) at \( t = t \) gives (6):

\[
N = \frac{N_o (N_o - 1) \exp (-K_3 N_o Gt)}{1 + (N_o - 1) \exp (-K_3 N_o Gt)} \tag{96}
\]

When the floe break-up terms are included in the Argaman-Kaufman model, the boundary conditions for integration are the same as for the corresponding aggregation model. By this model, the rate at which primary particles are released depends on the surface shear, the floe size, and the size of the primary particles. As the shearing stress depends on \( U^2 \) or \( K_F G \), a possible expression for the rate reformation of primary particles due to floe break-up was proposed by Argaman and Kaufman (6):

\[
\frac{dN}{dt} = B \frac{N_F}{R_F^2} N F K_F G \tag{97}
\]
in which $B_r$ = the break-up constant. The term $R_p^2 N_F$ may be replaced by another term incorporating both $R_p$ and $N$ by simple manipulation to equation 94 as follows:

$$R_p^2 N_F = R_p^2 (N_0 - N)$$

$$R_p^2 N_F = R_p^2 (N_0 - N)^{2/3} N_F^{1/3}$$

However, Argaman assumed that the number of flocs is proportional to the number of primary particles:

$$N_F = K_4 N_o$$

where $K_4$ is the proportionality constant.

Accordingly,

$$N_F^{1/3} = K_4^{1/3} N_o^{1/3}$$

Equation 97, therefore, becomes:

$$\frac{dN}{dt} = B \frac{K}{R_p^2} (N_0 - N)^{2/3} K_{4^{1/3}} N_o^{1/3} \frac{K_F G}{R_p^2}$$

$$= B \frac{K}{R_p^2} K_{4^{1/3}} N_o^{1/3} (N_0 - N)^{2/3} G$$

$$= K_5 N_o^{1/3} (N_0 - N)^{2/3} G$$

(98)
The net rate of primary particles disappearance is obtained by summation of equations 95 and 98.

\[
\frac{dN}{dt} = \frac{-1}{K_3} N (N_0 - N) G + K_5 N_0^{1/3} (N_0 - N)^{2/3} G 
\]  

(99)

in which \( K_5 = B \frac{K_4^{1/3}}{r} K_F \).

Since \( N_0 \gg N \), equation 99 is further simplified to the form:

\[
\frac{dN}{dt} = -K_6 N_0^2 G N + K_5 N_0 G 
\]

(100)

in which \( K_6 = 1/K_3 \).

Equation 100 can be integrated within the limits \( N = N_0 \) at \( t = 0 \) and \( N = N \) at \( t = t \), to give

\[
N = (N_0 - K_7 e^{-K_6 N_0 G t} + K_F
\]

(101)

where

\[
K_7 = \frac{K_5}{K_6}.
\]

During the process of flocculation, if the mixing power is too intense or is overtimed the previously formed flocs might break up again. For this reason, according to the current state of flocculation technology it is necessary to consider the individual values of \( G \) and \( t \) rather than merely the product of \( G \cdot t \). The values of both \( C \) and \( t \) are confined to certain limits that provides optimum flocculation efficiency.
The mean square root of velocity gradient, $\bar{G}$, for any mixing process is a function of fluid viscosity and mixing volume and energy. In the special case of mixing due to flow velocity gradient inside a granular media reactor, the mixing volume and energy are proportional to bed porosity, $\varepsilon$, and pressure loss per unit bed depth, $\frac{Ap}{AL}$, respectively. At any bed depth, $x$ and time of filtration, $t$, both $\varepsilon$ and $\frac{Ap}{AL}$ are functions of deposit content $\sigma(x,t)$. Consequently, for the same point $(x,t)$, the root mean square of velocity gradient, $\bar{G}$, is a dependent variable of the parameter $\sigma(x,t)$.

Bhole and Mhaisalkar (8) investigated various parameters of the sand flocculator such as bed depth and media size. A combination of three different bed depths, 40, 50, 60 cm, and four media sizes 1-2, 2-3, 3-4, 4-5 mm were included in the study. A constant upflow rate of 4 m/hr of 100 JTU influent suspension was used in each run. The study concluded that among the various depths and sizes of sand, the 40 cm depth and 4-5 mm size gave a velocity gradient of 70 sec$^{-1}$ which was in the range prescribed for flocculation. Other combinations resulted in higher velocity gradients.

Bhole and Mhaisalkar calculated the above flow velocity gradients using headloss measurements and the following equation:

$$\bar{G} = \sqrt{gh_{t}/\nu_{b}}$$  \hspace{1cm} (102)

where $h_{t}$ is the headloss across the sand bed. However, the authors didn't seem to fully comprehend the use of equation 102 for two reasons. First, an upflow granular media bed which is partially clogged
with solids exhibits a descending velocity gradient, \( \bar{G} \), the highest value being at the bottom and the lowest at the top. The authors calculated a single \( \bar{G} \) for the entire bed. Second, the bed residence time, \( t_b \), is not constant during the whole run and decreases with the progress of solids removal. However, the authors used a constant value for the detention time in the bed to calculate \( \bar{G} \).

Ives and Aldibouni (58) used a similar equation to calculate the velocity gradient inside a downflow granular media bed. The authors (58) stated that the velocity gradient (\( \bar{G} \), being a space-averaged mean of the field velocity gradient \( \frac{dV}{dz} \)) in a clean fixed bed can be calculated from the power dissipated in head loss as follows:

\[
G = \sqrt{\frac{P}{V \mu}} = \sqrt{\frac{g \rho V_s}{\varepsilon_o L \mu}}
\]

(103)

where \( \frac{P}{V} \) is the power dissipated per unit liquid volume, and \( \frac{h}{L} \) is the hydraulic gradient.

Using Kozeny's equation for the hydraulic gradient with a constant of 5, equation 103 becomes the following for spherical grains:

\[
G = 13.4 \frac{V_s (1 - \varepsilon_o)}{\varepsilon_o^2 \frac{d}{d}}
\]

(104)

The only shortcoming of the Ives and Aldibouni equation 104, is its limited application to clean beds. The authors (58) used equation 104 to calculate \( V_s \) to provide a \( G \) of 10 sec\(^{-1}\) for flocculation experiments. The experiments were abandoned because particle removal in the
bed interfered with the flocculation observations. Again, it should be recognized that equation 104 can only be used for the clean bed.

Perhaps the most significant development in the area of contact flocculation was recently presented by Willson et al., (111). The authors derived theoretical relationships for the rate of flocculation, mean residence time, $t_b$, and the product $\bar{G}t_b$ from fundamental relationships based on the capillary model for flow through a granular filter. The derived relationship between the average velocity gradient $\bar{G}_t$ at time, $t$, the interstitial velocity, $V_i$, and the average capillary diameter, $d_{eq}$, was as follows:

$$\frac{\bar{G}_t}{d_{eq}} = \frac{4.266V_i}{4.266V_i}$$

(105)

The Hagen-Poiseuille formula for the head loss, $h$, in a capillary is

$$h = \frac{32\mu L V_i}{gd^2_{eq}}$$

(106)

Also, assuming the number of capillaries, $N_c$, remains constant, the porosity, $\varepsilon$, is

$$\varepsilon = \frac{\pi d^2_{eq} N_c}{4 A_f}$$

(107)

where $A_f$ is the cross-sectional area of filter. Combining equations 106 and 107, the average capillary diameter, $d_{eq}$, at any time $t$ is
The combination of equations 105, 107 and 108 results in:

\[ S = \frac{4}{K_8 V_s h} \]  

where

\[ K_8 = \frac{128\mu L A_f}{\pi \rho g N_c} \]

The combination of equations 105, 107 and 108 results in:

\[ G_c = K_9 \sqrt{h^3 V_s} \]  

where

\[ K_9 = \frac{4.266 \rho g}{32 \mu L} \sqrt{\frac{128\mu L A_f}{\pi \rho g N_c}} \]

\[ = 0.343 \left(\frac{\rho g}{\mu L}\right)^{0.75} \left(\frac{A_f}{N_c}\right)^{0.25} \]

The bed residence time \( t_b \) was presented as a function of \( h \) and \( V_s \) as follows:

\[ t_b = \frac{L}{V_s} \frac{d_{eq}^2 \eta N_c}{4 A_f} = \frac{L \rho N_c}{4 A_f} \sqrt{\frac{128 \mu L A_f}{\pi \rho g N_c}} \frac{1}{h V_s} = K_{10} \frac{1}{h V_s} \]

where

\[ K_{10} = \left[ \frac{8 L^{3} \rho \mu N_c}{\rho g A_f} \right]^{1/2} \]

Combining equations 109 and 110, the product \( G_c \cdot t_b \) becomes

\[ G_c \cdot t_b = K_{11} \sqrt{\frac{h}{V_s}} \]  

(111)
Finally, Willson et al., (111) proposed the following relationship for headloss, \( h \), with filtration time \( t \):

\[
h = h_0 + st
\]  

(112)

where \( s \) is the slope of the linear head loss development curve. The functions for \( \bar{G}_r \), \( t_b \), \( G_{t_b} \) were first derived by substituting in equation 109, 110 and 111 for \( h \) as given in equation 112. The derived functions were then integrated and divided by run length, \( t_r \), to get \( \bar{G}_r \), \( \bar{t}_b \), and \( \bar{G}_{t_b} \), thus giving an average value for the parameters during the filter run.

\[
\frac{\bar{G}_r}{K_9} = \left[ h - h_0 \right] \frac{v_s 0.25}{1.75 st_r}
\]  

(113)

\[
\frac{\bar{t}_b}{K_{10}} = \left[ h - h_0 \right] \frac{v_s 0.25}{0.5 st_r}
\]  

(114)

\[
\frac{\bar{G}_{t_b}}{K_{11}} = \left[ h - h_0 \right] \frac{v_s 0.25}{1.25 st_r}
\]  

(115)

Willson et al., (111) did not provide a method for calculating the parameter \( N_c \), and therefore it wasn't possible to explicitly determine the value of \( \bar{G}_r \), \( \bar{t}_b \), and \( \bar{G}_{t_b} \). In order to circumvent this
problem, the authors used the terms $\frac{G}{K_9}$, $\frac{t_b}{K_{10}}$, $\frac{G.t_b}{K_{11}}$ instead.

The work of Willson et al., (Ill) is of interest because it is related to the objectives of this thesis. However, it suffers from several deficiencies.

1. Explicit values of $G$, $t_b$, or $G.t_b$ cannot be calculated from equations 109 through 115 because the constants include the number of capillaries, $N_c$, and no method of determining $N_c$ was presented. Only the related values of $\bar{G}$, $t_b$, and $\bar{G}.t_b$ including the appropriate constant was presented.

2. The authors only applied the equations to the entire bed head loss where in fact, $G$, $t_b$ and $G.t_b$ are functions of both bed depth and time of filtration.

3. In deriving equations 113 through 115, a linear head loss equation was assumed which is not appropriate to all filtration situations as shown in the prior chapter on Hydraulics of Water Filtration. Furthermore, it is not applicable to a coarse media without flocculation at high flow rates or high solids loading where the primary goal is flocculation at steady state rather than filtration.

The dual media consisted of 1.513 mm average grain size anthracite on top of 0.675 mm average grain size sand, the flow rates were 0.1, 0.2, 0.3, 0.4 cm/sec, and influent suspension concentration was 4.0 mg/l. As a result of the above filtration conditions, particle removal predominated rather than contact flocculation and the head loss followed
a straight line relationship with respect to time of filtration as proposed in equation 112. Unfortunately, the authors neither tried larger size media and higher influent concentrations nor extended their runs beyond 8.0 hours to allow for steady state to occur. The occurrence of steady state beyond break-through is proposed by the writer and was experimentally proven by Bhole and Mhaisalkar (8).
ANALYTICAL SOLUTION TO THE FILTRATION EQUATIONS

In the section of literature review of theory of water filtration, several analytical and numerical solutions were presented for the two mass balance and kinetic equation. Two major comments are worth mentioning. First, the discrepancies among filtration models are great. Second, the controversial issue concerning particle dynamic detachment is still unresolved.

In the following section, a new kinetic equation is proposed. Also, an analytical solution of the filtration equations is presented.

It is the writer's opinion that rate of particle deposition is enhanced by the amount of space available for removal and retarded by the increase in fluid shearing forces designated by the value of the hydraulic gradient, J. The space available for particle deposition is the difference between an ultimate deposit content value, $\sigma_u$, characteristic of each filtration condition, and the existing amount of deposition $\sigma$. The new proposed form of kinetic equation is:

$$\frac{\delta \sigma}{\delta t} = k_a \frac{a_s}{J} (\sigma_u - \sigma) \mathcal{C}$$  \hspace{1cm} (1)

where

$$k_a = \text{attachment constant}$$

$$J = \frac{V_S}{K_p}$$

$$= \left[ \frac{k_{po}}{V_S} \left(1 - \sqrt{\frac{\sigma}{\sigma_u}} \right)^3 \right]^{-1}$$ \hspace{1cm} (1a)
92

where \( K_{po} \) = permeability coefficient for the clean bed and the change of \( K_p \) with \( \sigma \) is from Adin (1).

\[
\frac{\delta \sigma}{\delta t} = \frac{k_a s K_{po}}{v_s} (1 - \sqrt[3]{\frac{\sigma}{\sigma_u}}) \left( \sigma_u - \sigma \right) C
\]

\[
= k_a K_{po} (1 - \sqrt[3]{\frac{\sigma}{\sigma_u}}) \left( \sigma_u - \sigma \right) C
\]

\[
= K_0 f_1(\sigma) C
\]

where

\[ K_0 = k_a K_{po} \]

The mass balance equation is similar to that used in any fixed bed process involving the transport of matters from the mobile to the stationary phase. For an axial flow filter with a constant cross-sectional area, this is written as

\[
V \left( \frac{\delta C}{\delta x} \right)_t + \left[ \frac{\delta (C + \varepsilon C)}{\delta t} \right]_x = 0
\]

The other assumptions involved are one-dimensional plug flow and negligible axial dispersion of particulate matters. The porosity (or void fraction) of the bed \( \varepsilon \) changes with time as particle accumulation within the bed increases. If the deposited matters form relatively smooth coatings outside the filter grains, \( \varepsilon \) and \( \sigma \) can be related by the simple expression
\[ \varepsilon = \varepsilon_o - \frac{\sigma}{1 - \rho_s} \]

where

\[ \rho_s = \text{deposit porosity.} \]

However, for all practical purposes the change in concentration of particles in the pores with time is very small compared to specific deposit \( \sigma \) except in the beginning of the filter run. Equation 2 could be written approximately as

\[ -V_s \frac{\delta C}{\delta x} = \frac{\delta \sigma}{\delta t} \quad (3) \]

Combining equations 1b, 2 yields

\[ - \frac{\delta C}{\delta x} = \frac{1}{V_s} k_a K_o \left( 1 - \frac{\sigma}{\sigma_u} \right)^3 \left( \sigma_u - \sigma \right) C \quad (3a) \]

Equations 1 and 3 give a macroscopic description of the dynamic behavior of deep bed filtration with the following initial and boundary conditions:

\[ C = C_o \text{ at } X = 0, \ t > 0 \]
\[ \sigma = \sigma_o \text{ at } X = 0, \ t > 0 \]
\[ \sigma = 0 \text{ at } t = 0, \ X > 0 \]

The solution of these equations yields values of \( C, \sigma \) as functions of time and position. Specifically, one can obtain histories of filtrate concentration and deposit accumulation across the bed, which in turn, can be used as a basis of design and optimization.
The simultaneous solution of the mass balance equation and the kinetic equation perhaps deserves some comment. These equations are of the same type as those found in a variety of engineering applications such as fixed bed adsorption, cross flow heat exchanger, heat generators, etc. If the rate expression is of a certain kind, analytical solutions may be possible.

Consider a porous bed of depth L and unit cross-sectional area. At any instant t during the filtration run the mass balance at any section at depth, x, and for the part of the run elapsed may be written in the following form:

\[
\text{Total amount of solids applied to filter at } x = 0 \text{ for the period } (0 \rightarrow t) = \text{Total amount of solids passed through filter layer at depth } x \text{ for } (0 \rightarrow t) + \text{Total amount of solids stored in the bed layer from the surface up to section } x \text{ at the time } t.
\]

i.e. \( V_s \int_0^t C_0 \, dt = V_s \int_0^t C \, dt + \int_0^x \sigma \, dx \) \hspace{1cm} (4)

If the suspension concentration is substituted by the expression

\[
C = \frac{\frac{d\sigma}{dt}}{K_{f,1}(\sigma)} \frac{1}{K_o}\]

as in equation 1, where

\[
K_o = K_p K_0
\]
\[ f_1(\sigma) = (1 - \sqrt{\frac{\sigma}{\sigma_u}})^3 (\sigma_u - \sigma) \]
\[ = (1 - \sqrt{\frac{\sigma}{\sigma_u}})^3 \sigma_u (1 - \sqrt{\frac{\sigma}{\sigma_u}})(1 + \sqrt{\frac{\sigma}{\sigma_u}}) \]
\[ = \sigma_u (1 - \sqrt{\frac{\sigma}{\sigma_u}})^4 (1 + \sqrt{\frac{\sigma}{\sigma_u}}) \quad (4a) \]

Equation 4 becomes
\[ V_s \int_\sigma^t C_0 \, dt = V_s \int_{\sigma_0}^\sigma \frac{1}{K_0 f_1(\sigma)} \cdot \frac{d\sigma}{dt} \, dt + \int_0^X \sigma \, d\chi \quad (5) \]

Differentiate equation 5 with respect to \( x \)
\[ \therefore 0 = \frac{V_s}{K_0 f_1(\sigma)} \frac{d\sigma}{d\chi} + \sigma \]
and
\[ \frac{d\sigma}{d\chi} = -\frac{K_0}{V_s} \sigma f_1(\sigma) \quad (5a) \]

i.e.
\[ \int_{\sigma_0}^\sigma \frac{d\sigma}{\sigma f_1(\sigma)} = \int_0^X -\frac{K_0}{V_s} \, d\chi \quad (6) \]

In order to carry out this integration the value of the lower integration limit \( \sigma_0 \), the deposit content at \( x = 0 \) and time \( t \) must be known. For this purpose, it is useful to utilize equation 1 by employing a known value of suspension concentration at \( (x = 0, t = t) \).

At depth \( x = 0 \) equation 1 becomes
\[ \frac{d\sigma}{dt} = K_0 f_1(\sigma) C_0 \]

\[ \sigma_0 \int_{\sigma_0}^{\sigma} \frac{d\sigma}{f_1(\sigma)} = \int_{t_0}^{t} K_0 C_0 \, dt \]  

Upon integration of equation 7 the relationship between the deposit content at \( x = 0 \) and time \( t \) is attainable, i.e. \( \sigma_0 \). By feeding back the value of \( \sigma_0 = f_2(t, C_0) \) into equation 6, it is possible to carry out the integration part in the left hand of equation 6. The constant of this integration is generally a function of time \( T(t) \). The functional relationship \( \sigma = f_3[\sigma_0 = f_2(t, C_0), (x, V_s)] \) can therefore be determined.

Combining equation 1, 3 and 5a:

\[ \frac{\delta \sigma}{\delta t} = K_0 f_1(\sigma) C \]  

\[ -V_s \frac{\delta C}{\delta \chi} = \frac{\delta \sigma}{\delta t} \]  

\[ \frac{\delta \sigma}{\delta \chi} = -\frac{K_0}{V_s} \sigma f_1(\sigma) \]  

a useful relationship \( C = f_4(\sigma_0, \sigma, C_0) \) may arise. From equations 1 and 3

\[ -V_s \frac{\delta C}{\delta \chi} = K_0 f_1(\sigma) C \]
also, from equation 5a:

\[ K_0 f_1(x) = -\frac{V_s}{c} \frac{\partial c}{\partial x} \]  

\[ \text{(8)} \]

Equating the R.H.S. of both equations 8 and 9:

\[ \frac{-V_s}{c} \frac{\partial c}{\partial x} = \frac{-V_s}{\sigma} \frac{\partial \sigma}{\partial x} \]

\[ \therefore \int_{C_o}^{C} \frac{\delta c}{c} = \int_{\sigma_o}^{\sigma} \frac{\delta \sigma}{\sigma} \]

\[ \ln \frac{C}{C_o} = \ln \frac{\sigma}{\sigma_o} \]

\[ \therefore \frac{C}{C_o} = \frac{\sigma}{\sigma_o} \]

\[ \therefore C = C_o \frac{\sigma}{\sigma_o} \]  

\[ \text{(10a)} \]

\[ \text{i.e., } C = f_5(C_o, \sigma_o, t, \chi) \]  

\[ \text{(10b)} \]

The system of equations 1 to 10 provides a very useful and general step-wise analytical solution to the filtration equations regardless of the form of \( f(\sigma) \). In the following pages, the new
kinetic equation 1 and the mass balance equation 3 are solved simultaneously in order to develop the relation of specific deposit and suspension concentration with depth and filtration time.

Develop Relationship for \( \sigma_0 \) as Function of Time

First, solve for \( \sigma_0 \) utilizing equation 7

\[
\int_{0}^{\sigma_0} \frac{d\sigma_0}{f_1(\sigma_0)} = \int_{0}^{t} K_0 C_0 dt
\]

The left hand side will be denoted by \( I_1 \) and will be integrated as follows:

\[
I_1 = \int_{0}^{\sigma_0} \frac{d\sigma_0}{f_1(\sigma_0)}
\]

\[
= \int_{0}^{\sigma_0} \frac{d\sigma_0}{(1 - \sqrt{\frac{\sigma_0}{\sigma_u}})^3 (\sigma_u - \sigma_0)}
\]

\[
= \int_{0}^{\sigma_0} \frac{d\sigma_0}{\sigma_u \left[ 1 - \sqrt{\frac{\sigma_0}{\sigma_u}} \right]^3 \left[ 1 - \sqrt{\frac{\sigma_0}{\sigma_u}} \right] \left[ 1 + \frac{\sigma_0}{\sigma_u} \right]}
\]
In order to obtain \( d\sigma \) in terms of \( y \), take the partial differential of both sides of the preceding equation (11a):

\[
\therefore \quad 2ydy = \frac{1}{\sigma_u} \, d\sigma_o
\]

\[
\therefore \quad d\sigma_o = 2\sigma_u ydy \quad (12)
\]

Therefore, equation (11) becomes

\[
I_L = \int_0^\sigma \frac{\sigma o \, 2ydy}{\sigma_u [1 - y]^{\frac{u}{4}} [1 + y]}
\]
However, the quantity \( \frac{1}{(1 - y)^4(1 + y)} \) may be expanded as follows:

\[
\frac{1}{(1 - y)^4(1 + y)} = \frac{A_1}{1 + y} + \frac{A_2}{(1 - y)} + \frac{A_3}{(1 - y)^2} + \frac{A_4}{(1 - y)^3} + \frac{A_5}{(1 - y)^4}
\]

such that

\[
1 = A_1 (1 - y)^4 + A_2 (1 + y)(1 - y)^3 + A_3 (1 + y)(1 - y)^2 + A_4 (1 + y)(1 - y) + A_5 (1 + y)
\]  

\[1 = A_1 (1 - 4y + 6y^2 - 4y^3 + y^4) + A_2 (1 - 2y + 0 + 2y^3 - y^4) + A_3 (1 - y - y^2 + y^3 + 0) + A_4 (1 + 0 - y^2 + 0 + 0) + A_5 (1 + y + 0 + 0 + 0)
\]
In order to solve for the coefficients $A_1, \ldots, A_5$, equate the coefficients of $y^0, y, \ldots, y^4$ in both sides of equation 15. Therefore,

\begin{align*}
A_1 + A_2 + A_3 + A_4 + A_5 &= 1 \\ (-4)A_1 - 2A_2 - A_3 + 0 + A_5 &= 0 \\ 6A_1 + 0 - A_3 - A_4 + 0 &= 0 \\ (-4)A_1 + 2A_2 + A_3 + 0 + 0 &= 0 \\ A_1 - A_2 + 0 + 0 + 0 &= 0
\end{align*}

From equation 20
\[ A_1 = A_2 \]

Solving equations 21 and 19 yields
\[ -4A_2 + 2A_2 + A_3 = 0 \]
\[ A_3 = 2A_2 \]

From equations 22, 21 and 18
\[ 6A_2 - 2A_2 - A_4 = 0 \]
\[ A_4 = 4A_2 \]

Combining equations 22, 21 and 17
\[ -4A_2 - A_2 - 2A_2 + A_5 = 0 \]
\[ A_5 = 8A_2 \]
Substituting for $A_1, A_3, A_4$ and $A_5$ from equations 21, 22, 23 and 24 into equation 16

$$A_2 + A_2 + 2A_2 + 4A_2 + 8A_2 = 1$$

$$A_2 = \frac{1}{16}$$

therefore,

$$A_1 = A_2 = \frac{1}{16}$$

$$A_3 = 2A_2 = \frac{1}{8}$$

$$A_4 = 4A_2 = \frac{1}{4}$$

$$A_5 = 8A_2 = \frac{1}{2}$$

and

$$I_1 = 2 \int \frac{y \, dy}{(1 - y)^4 (1 + y)}$$

$$= \frac{2}{16} \left[ \int \frac{y \, dy}{(1 + y)} + \int \frac{y \, dy}{(1 - y)} + 2 \int \frac{y \, dy}{(1 - y)^2} + 4 \int \frac{y \, dy}{(1 - y)^3} \right.$$

$$+ 8 \int \frac{y \, dy}{(1 - y)^4} \left. \right]$$

$$= \frac{1}{8} \left[ (1 + y - \log^* (1 + y)) + \left\{ 1 - y - \log (1 - y) \right\} \right.$$

$$+ 2 \left\{ \log (1 - y) + \frac{1}{1 - y} \right\} + 4 \left\{ \frac{-1}{1 - y} \right\} +$$

*Log throughout the thesis refers to natural logs base e.*
\[
\frac{1}{2(1 - y)^2} + 8 \left\{ \frac{-1}{2(1 - y)^2} + \frac{1}{3(1 - y)^3} \right\}
\]

\[
I_I = \frac{1}{8} \left[ \log \left( \frac{1 - y}{1 + y} \right) + 2 \right. - \left. \frac{2(3y - 2/3)}{(1 - y)^3} \right]
\]

The right hand side of the above equation includes an exponential term, \( G_1 (\sigma) = \log \left( \frac{1 - y}{1 + y} \right) \), and the term, \( G_2 (\sigma) = -2 \frac{\sigma y - 3y + 2/3}{(1 - y)^3} \).

For values of \( y < 0.5 \) both terms are approximately equal in magnitude and opposite in sign. The sum of both terms is another function of \( \sigma \) that could be regarded as proportional to either term as follows.

\[
G(\sigma) = G_1 (\sigma) - G_2 (\sigma) = \text{constant} \quad G_1 (\sigma)
\]

The value of the proportionality constant is not known and therefore is assumed equal to unity by introducing the proper changes in the value of the attachment constant \( k_0 \).

\[
I_I = \frac{1}{8} \left[ \log \left( \frac{1 - y}{1 + y} \right) + 2 \right] = \frac{1}{8} \left[ \log \left( \frac{1 - \sqrt{\frac{\sigma}{\sigma_u}}}{\sqrt{\frac{\sigma}{\sigma_u}}} \right) + 2 \right]
\]

Substituting into equation 7

\[
I_I = \int_0^{\sigma} \frac{d\sigma}{f(\sigma)} = \int_0^t K_0 C_0 dt
\]

\[
\left. \left[ \log \left( \frac{\sigma}{\sqrt{\sigma_u}} \right) \right]_0^\sigma \right. = K_0 C_0 t
\]

\[
\left. \frac{1}{8} \left[ \log \left( \frac{1 - \sqrt{\sigma/u}}{\sqrt{\sigma/u}} \right) + 2 \right] \right. = K_0 C_0 t
\]
\[ \frac{1}{\theta} \left\{ \log \left( \frac{1 - \sqrt{\sigma_0}}{1 + \sqrt{\sigma_0}} \right) + 2 \right\} - \left\{ \log \left( \frac{1 - \sigma}{1 + \sigma} \right) + 2 \right\} = K_0 C_0 t \]

\[ \therefore \frac{1}{\theta} \log \left( \frac{1 - \sqrt{\sigma_0}}{1 + \sqrt{\sigma_0}} \right) = K_0 C_0 t \]

\[ \frac{1 - \sqrt{\sigma_0}}{1 + \sqrt{\sigma_0}} = e^{K_0 C_0 t} \quad (24a) \]

\[ \frac{\sqrt{\sigma_u} - \sqrt{\sigma_o}}{\sqrt{\sigma_u} + \sqrt{\sigma_o}} = e^{8K_0 C_0 t} \]

\[ -\sqrt{\sigma_o} \left[ 1 + e^{8K_0 C_0 t} \right] = \sqrt{\sigma_u} \left[ e^{8K_0 C_0 t} - 1 \right] \]

\[ \therefore \sqrt{\sigma_o} = \sqrt{\sigma_u} \begin{bmatrix} \frac{8K_0 C_0 t}{1 - e^{8K_0 C_0 t}} \\ \frac{1 - e^{8K_0 C_0 t}}{1 + e^{8K_0 C_0 t}} \end{bmatrix} \]

Upon squaring both sides of this equation, there are two possible solutions

\[ \therefore \sigma_o = \sigma_u \left[ \pm \frac{1 - e^{8K_0 C_0 t}}{1 + e^{8K_0 C_0 t}} \right]^2 \]
Since \( \sigma_0 \) must increase with time, reject the + solution and rearrange the equation as follows:

\[
\sigma_0 = \sigma_u \left[ \frac{\epsilon^{\frac{8K_C t}{\sigma_0}} - 1}{\epsilon^{\frac{8K_C t}{\sigma_0}} + 1} \right]^2
\]

(25)

Develop Relationship for Specific Deposit (\( \sigma \)) as Function of Depth (x) and Time (t)

Second, by substituting the above value of \( \sigma_0 = f_2(t, \sigma_0) \), equation 25, into equation 6 it is possible to determine the function \( \sigma = f_3 \)

\[
[\sigma_0 = f_2(t, \sigma_0), x, v_s]
\]

as follows:

\[
\int_{\sigma_0}^{\sigma} \frac{d\sigma}{\sigma f_1(\sigma)} = -\int \frac{K_0}{V_s} \, dx
\]

(5a)

Call the integral of the left hand side \( I_{II} \) and substitute the expression for \( f_1 \) given in equation 4a

\[
I_{II} = \int_{\sigma_0}^{\sigma} \frac{d\sigma}{f(\sigma)} = \int_{\sigma_0}^{\sigma} \frac{d\sigma}{\sigma_u \sigma [1 - (\sigma/\sigma_u)^2] [1 - (\sqrt[3]{\sigma/\sigma_u})]}
\]

\[
= \int_{\sigma_0}^{\sigma} d\sigma \sigma_u \sigma (1 + \sqrt[3]{\sigma/\sigma_u}) (1 - \sqrt[3]{\sigma/\sigma_u})^4
\]

Assume the substitution \( \nu = \sqrt[3]{\sigma/\sigma_u} \).
\[
\begin{align*}
\therefore \, d\sigma &= 2\sigma_u \, y \, dy \\
\text{and} \\
I_{II} &= \int_{y_0}^{y} \frac{2\sigma_y \, dy}{2(1+y)(1-y)^4} \\
&= \frac{2}{\sigma_u} \int_{y_0}^{y} \frac{dy}{y(1+y)(1-y)^4} \\
&= \frac{2}{16\sigma_u} \left[ \int_{y_0}^{y} \frac{dy}{y(1+y)} + \int_{y_0}^{y} \frac{dy}{y(1-y)} + 2 \int_{y_0}^{y} \frac{dy}{y(1-y)^2} \right. \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + 4 \int_{y_0}^{y} \frac{dy}{y(1-y)^3} + 8 \int_{y_0}^{y} \frac{dy}{y(1-y)^4} \] \\
&= \frac{1}{8\sigma_u} \left[ \int_{y_0}^{y} \frac{2dy}{y(1-y)^2} + 2 \int_{y_0}^{y} \frac{dy}{y(1-y)} + 4 \int_{y_0}^{y} \frac{dy}{y(1-y)^3} \right. \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + 8 \int_{y_0}^{y} \frac{dy}{y(1-y)^3} + 8 \int_{y_0}^{y} \frac{dy}{(1-y)^4} \] \\
&= \frac{1}{8\sigma_u} \left[ \log \left( \frac{y^2}{1-y} \right) + 2 \left\{ \frac{1}{1-y} - \log \left( \frac{1-y}{y} \right) \right\} \right. \\
&\quad \quad \quad \quad \quad \quad \quad \quad + 12 \left\{ \frac{1}{2} \left( \frac{2-y}{1-y} \right)^2 + \log \left( \frac{y}{1-y} \right) \right\} + 8 \left\{ \frac{1}{3} \frac{1}{(1-y)^3} \right\} \right] \\
\end{align*}
\]
In equation 26, the value of the linear term, \[ \frac{-6y^3 + 32y^2 - 52y + 28y}{(1 - y)^3} \]
is negligible compared to that of the logarithmic term, \( \log \left( \frac{y}{1 - y} \right) \left( \frac{1 - y}{1 + y} \right) \), for small values of \( y \), and if ignored it is possible to rewrite equation 26 as:

\[
I_{II} = \frac{1}{8\sigma_u} \left[ \log \left( \frac{y}{1 - y} \right) \left( \frac{1 - y}{1 + y} \right) \right]
\]

\[
= \frac{16}{8\sigma_u} \left[ \log \left( \frac{y}{1 - y} \right) \left( \frac{1 - y}{1 + y} \right)^{1/16} \right]
\]

\[
= \frac{2}{\sigma_u} \left[ \log \left( \frac{1 - y}{y} \right) + \log \left( \frac{1 - y}{1 + y} \right)^{1/16} \right] \quad (27)
\]

The term, \( \log \left( \frac{1 - y}{y} \right)^{1/16} \), has very insignificant effect on the net values of the whole term \( \log \left( \frac{y}{1 - y} \right) \left( \frac{1 - y}{1 + y} \right)^{1/16} \) for all values of \( y \) from zero to one, and if ignored the approximate value of the
integral $I_{II}$ becomes:

$$I_{II} = \frac{2}{\sigma_u} \log \left( \frac{y}{1 - y} \right)$$  \hspace{1cm} (28)$$

Substituting for $I_{II}$ from equation 28 into equation 6 yields

$$I_{II} = \frac{2}{\sigma_u} \log \left( \frac{y}{1 - y} \right) = \frac{-K_o X}{V_s} + \text{integration constant}$$  \hspace{1cm} (29)$$

To determine the value of the integration constant substitute for

$y = y_o = \sqrt{\frac{\sigma_o}{\sigma_u}}$ at $x = 0$ into equation 29

$$\therefore \frac{2}{\sigma_u} \log \left( \frac{\sqrt{\sigma_o}}{\sqrt{\sigma_u}} \right) = \text{constant}$$

However, from equation 25

$$\sqrt{\sigma_o} = \frac{8K_o C_t - 1}{e^{8K_o C_t}}$$

$$\therefore \frac{2}{\sigma_u} \log \left( \frac{8K_o C_t - 1}{e^{8K_o C_t}} \right) = \frac{2}{\sigma_u} \log \left( \frac{e^{8K_o C_o t - 1}}{2} \right) = \text{constant}$$

Inserting the constant into equation 29

$$\therefore I_{II} = \frac{2}{\sigma_u} \log \left( \frac{\sqrt{\sigma_o}}{\sqrt{\sigma_u}} \right) = \frac{-K_o X}{V_s} + \left( \frac{2}{\sigma_u} \right) \log \left( \frac{e^{8K_o C_t - 1}}{2} \right)$$
\[ \frac{2}{u} \log \left( \frac{\sqrt{\sigma}}{\sqrt{\sigma} - \sqrt{\sigma}} \right) \left( \frac{8K_oC_0t}{2} - 1 \right)^{-1} = \frac{-K_oX}{V_s} \]

\[ \left( \frac{\sqrt{\sigma}}{\sqrt{\sigma} - \sqrt{\sigma}} \right) \left( \frac{8K_oC_0t}{2} - 1 \right)^{-1} e^{-\frac{K_oX}{2V_s}} \]

\[ K_o \sigma u X \]

\[ 2e^{-\frac{K_oX}{2V_s}} \sqrt{\sigma} = \left( \sqrt{\sigma} - \sqrt{\sigma} \right) (e^{8K_oC_0t-1}) \]

\[ \sqrt{\sigma} \begin{bmatrix} \frac{K_o \sigma u X}{2V_s} \\ + (e^{8K_oC_0t-1}) \end{bmatrix} = \frac{\sigma_u}{V_u} (e^{8K_oC_0t-1}) \]

\[ \sigma = \sigma_u \left( \frac{(e^{8K_oC_0t-1})}{K_o \sigma u X} \right) ^2 \]

\[ \sigma = \sigma_u \left[ \frac{ae^{bt-1}}{2e^{at} + e^{bt} - 1} \right] ^2 \]

Equation 30 can also be rearranged to the following:

\[ \sigma = \sigma_u \left[ \frac{K_o \sigma u X}{2e^{at} + 8K_oC_0t + 1} \right] ^2 \]
Equation 30 is the relation between specific deposit and filtration times and different depths in the filter.

Development of Suspension Concentration (C) as Function of Depth (X) and Time (t)

The suspension concentration, C, at different bed depths and times of filtration may be concluded utilizing the relationship given in equations 10a, 25 and 30b.

\[
C = \frac{c}{c_o} \sigma \left[ \frac{\sigma}{\frac{k \sigma X}{o u}} \right]^2 + \left[ \frac{1}{\sigma^2} \left( \frac{8k c t + 1}{e^{8k c o t}} \right) \frac{e^{8k c o t} + 1}{e^{8k c o t} - 1} \right]^2
\]

\[
\frac{C}{C_o} = \left[ \frac{e^{8k c o t} + 1}{k \sigma X o u} + \frac{8k c o t}{2e^{2V_s} + e^{8k c o t} - 1} \right] \left[ \frac{e^{bt} + 1}{2e^{ax} + e^{bt} - 1} \right]^2
\]

In order to confirm the correctness of equations 30 and 31, the previously mentioned filtration boundary conditions are used and the calculated values of \( \sigma \) and C are matched with their known values. The expected boundary conditions are:
Applying the first boundary condition, equation 32, to equations 30 and 31 yields:

\[
\sigma = \left[ \frac{\sigma_u}{2e^o + \frac{8Kc_0 t}{e^o + 8Kc_0 t - 1}} \right]^2 = \alpha_u \left[ \frac{8Kc_0 t}{(e^o + 8Kc_0 t - 1)} \right]^2 = \sigma_0 \tag{34}
\]

as in equation 25. Also,

\[
C = C_0 \left[ \frac{8Kc_0 t + 1}{2e^o + \frac{8Kc_0 t}{e^o + 8Kc_0 t - 1}} \right]^2 = C_0 \tag{35}
\]

Applying the second boundary condition, equation 33, to both equations 30 and 31 yields:

\[
\sigma = \sigma_u \left[ \frac{1}{2e \frac{u}{2v_s} + 1} \right]^2 = 0 \tag{36}
\]

and

\[
C = C_0 \left[ \frac{e^o + 1}{2e \frac{u}{2v_s} + e^o - 1} \right]^2
\]
As shown in equations 34, 35, 36, and 37 the analytical solution presented in equations 30 and 31 fulfills the boundary conditions 32 and 33.

One of the advantages of the theoretical model presented in equation 31 is its usefulness in the optimization of filter design for specific treatment objectives. For example, if the coefficients $a$, $b$ are known for a particular filter and specific conditions of flow rate, $V_s$, and influent concentration, $C_o$, it is possible to predict the filter performance under different values of $V_s$ and $C_o$. For the purpose of illustration, suppose a filter bed exhibits a breakthrough curve with coefficients $a_1$, and $b_1$, when the flow rate and influent concentrations were $V_{s1}$, and $C_{01}$. If the same filter is operating under a different flow rate $V_{s2}$ and influent concentration $C_{02}$, the new coefficients $a_2$, $b_2$ may be predicted as follows by considering the components in $a$ and $b$ presented after equation 30.

$$a_2 = a_1 \left( \frac{V_{s1}}{V_{s2}} \right)$$  \hspace{1cm} (38)

and

$$b_2 = b_1 \left( \frac{C_{02}}{C_{01}} \right)$$  \hspace{1cm} (39)
Knowing the new values $a_2$, $b_2$, it is possible to draw the breakthrough curve for the same filter for the new values $V_{s2}$, $C_{02}$.

The values of the coefficients $a$, $b$ determine the shape of the $\sigma/\sigma_u$ or $C/C_0$ versus time curves (by equation 30 or 31). The coefficient $a$ is predominant during the early stage of a filter run, and coefficient $b$ predominates during the later stages.

If $C_i$, $C_f$ are the initial and final effluent concentration at $t = 0$, $t = t_f$, it is possible to estimate the value of both $a$, $b$ as follows:

Rearrangement of equation 37 yields

$$-2ax = \ln \frac{C_i}{C_0}$$

$$a = \frac{\ln \frac{C_0}{C_i}}{x}$$

(40)

Substitute the value of $a$ given in equation 40 into equation 31 and consider the point of the filtration curve where $t = t_f$, $C = C_f$.

$$\frac{C_f}{C_0} = \left[ \frac{e^{bt_f} + 1}{2e^{\ln \frac{C_0}{C_i} + bt_f} - 1} \right]^2$$

(41)

Upon expansion of equation 41 and rearrangement

$$\sqrt{\frac{C_0}{C_f}} (e^{bt_f} + 1) - e^{bt_f} + 1 = 2 \sqrt{\frac{C_0}{C_i}}$$
As shown in equation 40, the higher the value of coefficient \( a \) the lower the effluent concentration \( C_i \) at the beginning of the run. If a particular filter is required to treat a certain suspension at a specific flow rate, the only way to increase the value of coefficient \( a \), in order to prevent an early breakthrough, is to increase the value of \( k_o \) by adjusting the polymer or alum dose.

Also, as shown in equation 42, the higher the value of coefficient \( b \), the shorter the time to breakthrough, \( t^*_f \). This explains why filtration runs are shorter for higher influent concentration.

Rearranging equations 30 and 31 and assuming certain approximations, it is possible to prove the linear relationship between the time of front movement and bed depth. The linear relationship has been previously proven for similar breakthrough equations by Mints and Krishtul (77), Heertjes and Lerk (37) and Saatci (89), and was explicitly presented by Bohart and Adams in their model of adsorption.

Rearranging equation 30a, and considering the negative square root of \( \sigma/\sigma_u \), we get:

\[
e^{bt^*_f} = \frac{\left[ 2\frac{C_o}{\sqrt{C_i}} - (1 + \frac{C_o}{\sqrt{C_f}}) \right]}{(\frac{C_o}{\sqrt{C_f}} - 1)}
\]

\[
ln\left( \frac{2\frac{C_o}{\sqrt{C_i}} - (1 + \frac{C_o}{\sqrt{C_f}})}{(\frac{C_o}{\sqrt{C_f}} - 1)} \right)
\]

\[
b = \frac{t^*_f}{114}
\]
\[
\sqrt{\frac{\sigma_u}{\sigma}} (e^{bt} - 1) = 2e^{ax} + e^{bt} - 1 \quad (43)
\]

\[
\sqrt{\frac{\sigma_u}{\sigma}} - e^{-bt} \left( \sqrt{\frac{\sigma_u}{\sigma}} - 1 \right) = 2e^{ax} - bt + 1
\]

Drop the \( e^{-bt} (\frac{\sigma_u}{\sigma} - 1) \) term for its negligible value. Therefore,

\[
\sqrt{\frac{\sigma}{\sigma_u}} = 2e^{ax} - bt + 1
\]

If \( 2e^{ax} - bt \gg 1.0 \) particularly during the initial stages of the run

\[
\ln \left( \frac{1}{2} \sqrt{\frac{\sigma}{\sigma_u}} \right) = ax - bt
\]

i.e.

\[
t = \frac{a}{b} X - \frac{1}{b} \ln \left( \sqrt{\frac{\sigma}{\sigma_u}} \right)
\]

(44)

Equation 44 is recommended for use only in determining the values of coefficients \( a \) and \( b \). Also, because the net value of \( \ln \left( \frac{\sigma}{\sigma_u} \right) \) varies from negative value, zero and positive value for values of \( \frac{\sigma}{\sigma_u} > 0.5, 0.5 \) and \( < 0.5 \) respectively, and because \( a \) and \( b \) are always positive the use of equation 44 should be restricted to values of \( \frac{\sigma}{\sigma_u} < 0.5 \) only. For values of \( \frac{\sigma}{\sigma_u} \geq 0.5 \) the writer suggests the following modification

\[
t = \frac{a}{b} X - \frac{1}{b} \ln \left( \sqrt{\frac{\sigma}{\sigma_u}} + 1 \right)
\]

(44a)

Equation 44 is a straight line relationship between \( t \), the time to reach a certain value of \( \frac{\sigma}{\sigma_u} \), and bed depth \( X \). As shown in Figure 1, the slope of this linear equation is \( \frac{a}{b} = \frac{\sigma u}{16 \sigma_o \sqrt{\gamma_s}} \) and the intercept is
\[ \frac{-1}{b} \ln \left( \frac{\sqrt{u} + 1}{\sqrt{v}} \right) = \frac{1}{8k_c C_o} \ln \frac{\sqrt{u} + 1}{2} \]

Similarly, equation 31 may be rearranged as follows.

\[ \sqrt{\frac{C_o}{C}} (e^{bt} + 1) = 2e^{2x} + e^{bt} - 1 \]

\[ e^{bt} \left( \sqrt{\frac{C_o}{C}} - 1 \right) + \sqrt{\frac{C_o}{C}} + 1 = 2e^{ax} \]

\[ (\sqrt{\frac{C_o}{C}} - 1) + e^{-bt} (\sqrt{\frac{C_o}{C}} + 1) = 2e^{ax} - bt \]

However,

\[ e^{-bt} \left[ \sqrt{\frac{C_o}{C}} + 1 \right] = 2.0 \text{ for time } t \text{ up to } \]

\[ t = \frac{1}{b} \ln \left( \frac{\sqrt{C_o} + 1}{2} \right) \]

If \( b << 1.0 \) the corresponding value of \( t \) is large enough to cover most of the initial stages of the run. Therefore;

\[ \sqrt{\frac{C_o}{C}} + 1 = 2e^{ax} - bt \]
Fig. 1. Linear transformation of specific deposit data

\[ \frac{1}{b} \left[ \ln \left( \frac{a \sigma_y}{\sigma} + 1 \right) \right] \]

BED DEPTH, X

TIME TO BREAKTHROUGH, \( t_r \)

Fig. 2. Linear transformation of turbidity data

\[ \frac{1}{b} \left[ \ln \left( \frac{\sqrt{C_r/C} + 1}{2} \right) \right] \]

BED DEPTH, X

TIME TO BREAKTHROUGH, \( t_r \)
The slope of equation 45 is \( \frac{a}{b} = \frac{\sigma u}{16v_c} \) and the intercept is

\[
\begin{align*}
- \frac{1}{b} \ln \left( \frac{C_0}{C} + \frac{1}{2} \right) &= - \frac{1}{8K} \ln \left( \frac{C_0}{C} + \frac{1}{2} \right) \\
&= \frac{1}{8K} \ln \left( \frac{C_0}{C} + \frac{1}{2} \right)
\end{align*}
\]

as shown in Figure 2.

The linear transformation of equations 30a and 31 as shown in equations 44 and 45 provide two of the methods used in the experimental verification of the filtration model as will be presented later.

Theory of Bed Contact Flocculation

In contact flocculation, the transport mechanisms are not only orthokinetic and perikinetic, but also include the contribution of filtration mechanisms (transport and attachment). Particles that flocculate and grow in size are then subsequently removed to some extent within the bed. Accordingly, a new mass balance equation is proposed:

\[
- \frac{dN}{dt} = (\text{Aggregation term}) - (\text{breakup term}) + (\text{Removal Term})
\]

While the aggregation term is a function of particle in-bed flocculation and transport mechanisms, the break-up term is a function
of bed flow characteristics such as the interstitial velocity, the amount of accumulated deposit, and other variables affecting the hypothesized detachment mechanism. The removal term is simply the same as in conventional filtration.

In order to be able to design a model for bed flocculation, one must simplify the complexity of the process by assuming its analogy to certain flow conditions that can be described mathematically. Three cases are proposed because of their similarity to flow through porous media. These are:

1. Consider laminar flow through capillary tubes of diameter equivalent to the average pore size and length equal to the sinuous flow pathline inside the granular bed.
2. Consider flow velocity distribution around spherical objects resembling granular media particles.
3. Consider a bed described by an array of mixing cells and laminar cells in sequence within which the overall mixing energy is a function of the flow rate, head loss and fluid density.

The first two models were proposed by Stein (99) in order to study the effect of shearing forces on the removed particles, and to derive an expression for flow velocity gradient inside the bed voids. However, the author didn't present any mathematical model for flocculation.

Camp and Stein (15) studied the velocity gradients and internal work in fluid motions. The authors presented a case of uniform steady
flow through a capillary tube of circular cross section. The case presented is of great interest to the current research for its great similarity to porous media flow as will be demonstrated in a later section.

The third model was presented by Heertjes and Lerk (39) to describe bed filtration. In their model, each individual void space entrapped between media grains resembles a mixing reactor. Mixing reactors are joined together through laminar flow cells. The model (39) closely approximates a number of completely mixed flow (CMF) reactors in series.

The foregoing three fluid flow models bear different degrees of analogy to the case of fluid flow through porous media as in bed flocculation. For the sake of argument, let us consider the model of fluid flowing through circular capillary tubes. As previously mentioned in the first section about flow hydraulics in water filtration, a filter bed may approximate a group of similar parallel capillaries of diameter equal to the equivalent pore opening. The average velocity inside each capillary is \( \frac{V_s}{\varepsilon_o} \), where \( \varepsilon_o \) is the clean bed porosity and \( V_s \) is the superficial flow velocity. In order to deduce the velocity distribution profile across any section perpendicular to flow, let us study the equilibrium between fluid pressure, \( P \), and shear stresses, \( \tau \), in the length \( L \) between sections 1 and 2 on the lamina of thickness \( dr \), Figure 3. Shear stresses are assumed uniformly distributed for a given fluid cylindrical lamina of radius \( r \) and increase in the radial
direction. The total force exerted by fluid shear stresses on the example lamina is:

\[ \tau_s = 2\pi r \mu \left( \frac{\delta v}{\delta r} \right) \]

Since there is no acceleration and the flow is steady, the shear force is equal to the difference in the fluid pressure between the two sections 1 and 2. Therefore,

\[ 2\pi r L \mu (-\frac{\delta v}{\delta r}) = \pi r^2 \rho g h \]

from which

\[ \frac{\delta v}{\delta r} = \frac{-\rho g h}{2\mu L} r \]  

By separating the two variables and integrating within the limits \( v = v \) at \( r = r \) and \( v = 0 \) at \( r = R \), where \( R \) is the tube radius,

\[ \int_v^0 \delta v = -\frac{\rho g h}{2\mu L} \int_r^R r \, dr \]

or,

\[ 0 - v = -\frac{\rho g h}{2\mu L} \left[ \frac{R^2 - r^2}{2} \right] \]

\[ \therefore v = \frac{\rho g h}{4\mu L} \left[ \frac{R^2 - r^2}{2} \right] \]  

The total flow per capillary is calculated by integrating the flow rate through the lamina \( 2\pi r \, dr \) for the same limits.
\[
\int_0^Q dq = \int_0^R v dA_c
\]

where \( dA_c = 2\pi r \, dr \)

\[
\therefore \int_0^Q dq = \int_0^R \frac{\rho gh}{4uL} \left[ r^2 - r^2 \right] 2\pi r \, dr
\]

\[
\therefore Q = \frac{\pi \rho gh}{2uL} \left[ \frac{R^2}{2} - \frac{r^2}{4} \right]_0^R
\]

\[
= \frac{\rho gh}{8uL} \pi R^4
\]

(49)

According to the model of laminar flow through capillaries, the velocity is assumed the same for all the points that are at the same distance from the center line axis. The only change in velocity is in the radial direction. The mean square root of velocity gradient, \( \bar{G} \), therefore, may be expressed as:

\[
\bar{G} = \sqrt{\frac{\delta W}{\mu}}
\]

where \( \delta W \) = work done by the fluid per unit volume per unit time. Also,

\[
\bar{G} = \left| \frac{\delta v}{\delta r} \right|
\]

therefore,

\[
W = \left( \frac{\delta v}{\delta r} \right)^2 \mu
\]
However, the total work exerted per unit time on the fluid lamina between sections 1 and 2 is:

\[ \delta W = \left[ 2\pi r \, dr \, L \right] \]

\[ = \left( \frac{\delta v}{\delta r} \right)^2 \mu \left[ 2\pi r \, dr \, L \right] \]

Substituting for the value of \( \frac{\delta v}{\delta r} \) from equation 47

\[ \therefore W = \left( \frac{\rho g h}{2\mu L} \right)^2 r^2 \mu 2\pi r \, L \, dr \]

\[ = \frac{(\rho gh)^2}{2\mu L} \pi r^3 \, dr \]

The total work per unit of time in the capillary between sections 1 and 2 is then

\[ W = \int_0^R \delta W = \left( \frac{\rho gh}{2\mu L} \right)^2 \pi \int_0^R r^3 \, dr \]

\[ = \frac{(\rho gh)^2 R^4}{8\mu L} \]

As shown in equation 56 the flow velocity gradient varies in the radial direction with the distance from the capillary axis. The mean value of the velocity gradient for the flow inside a capillary may be obtained by integration of \( \frac{\delta v}{\delta r} \) for different values of \( r \). However, since the flow through each lamina per unit time is not the same, it
is necessary to consider a weighted mean of the velocity gradient. The assigned weights for different values of velocity gradient at different values of \( r \) may be related either to the volume of fluid inside each lamina, \( \bar{\delta}_a \), or to the flow through each lamina per unit time, \( \bar{\delta}_w \). In the following section, values of \( \bar{\delta}_a \), \( \bar{\delta}_w \) for steady viscous flow through a capillary are presented as developed in a prior paper (86).

\[
\bar{\delta}_a = \int_0^V \frac{(\delta v)}{\delta r} \, dV
\]

where

\( V = \text{volume of fluid inside the capillary between sections 1 and 2.} \)

\[
\bar{\delta}_a = \int_0^R \frac{\rho gh}{2\mu L} \cdot \frac{r'^2}{2\pi r. dr} \int \frac{2\pi r. dr}{\pi R^2 L}
\]

\[
= \frac{\rho gh}{\mu L R^2} \frac{R^3}{3}
\]

\[
\bar{\delta}_a = \frac{\rho gh}{3\mu L} R \tag{51}
\]

Also,

\[
\bar{\delta}_w = \int_0^Q \frac{\delta v}{\delta r} \frac{\delta q}{Q}
\]
The ratio of the volume weighted to the flow weighted root mean square of velocity gradient, \( \frac{\overline{G_a}}{\overline{G_w}} \), is

\[
\frac{\overline{G_a}}{\overline{G_w}} = \frac{\frac{4}{15} \frac{\rho g h}{\mu L}}{R} = 1.25
\]

\[
\overline{G_w} = \frac{4}{15} \frac{\rho g h}{\mu L} \cdot R
\]
To apply the foregoing model to bed flocculation, values of h, L and R for granular media have to be determined as functions of suspension, media, and flow variables. Such functions may be obtained by assuming the similarity between granular media geometry and a capillary tubes model. For the purpose of this argument, let us consider a cubical unit bed volume of unit surface area and unit side length. Also, assume the following:

\[ n_1 = \text{number of grains per unit bed volume} \]
\[ n_2 = \text{number of capillaries per unit bed surface area} \]
\[ R_{eq} = \text{equivalent average radius of a capillary (mm)} \]
\[ \phi_s = \text{grain sieve radius, or one half of the mean of adjacent sieve sizes that pass and retain the grain (mm)} \]
\[ \phi_s = S\phi_s = \text{is the radius of equivalent volume sphere} \]
\[ s = \text{a factor that relates grain sieve size to the equivalent sphere grain size. The equivalent sphere is that which has the same volume as the grain volume. The value of s is usually } > 1.0 \text{ for sand media and could reach values } < 1.0 \text{ for angular coal media.} \]
\[ \frac{L_e}{L} = \text{tortosity factor, } F_t, \text{ which is the ratio between the longest and the shortest paths the flow streamlines might follow inside the bed. The longest pathline is that following the boundary of the spherical grains } = \frac{\pi}{2} (L). \text{ Here } L \text{ is bed depth or shortest pathline. The ratio} \]
\[ \frac{L_e}{L} = \frac{\text{half sphere circumference}}{\text{sphere diameter } d} \]
The constant, $S$, presented below, may be estimated for granular media by assuming each grain particle resembling the shape of either a prolate spheroid or an ellipsoid. For each of the above cases the constant $S$ is derived as follows:

Case (1): The prolate spheroid shape:

Assume that the equivalent sphere diameter is $d_{eq} = 2\phi$

Equivalent sphere volume $= \frac{4}{3} \pi \phi^3$ and

Prolate spheroid volume $= \frac{4}{3} \pi \bar{a} \bar{b}^2$ where $\bar{a}$, $\bar{b}$ are the major and minor semiaxes. Also, assume that the minor semiaxis $\bar{b}$ is the controlling dimension and is the same as half the sieve opening.

i.e. $\bar{b} = \phi_s$ and $\bar{a} = c\phi_s$

where $c$ is a constant more than one. Therefore,

Prolate spheroid volume $= \frac{4}{3} \pi \bar{c} \phi_s (\phi_s)^2$

Since the equivalent sphere volume is the same as the prolate spheroid volume:

$$\frac{4}{3} \pi \phi^3 = \frac{4}{3} \pi \bar{c} \phi_s \phi_s^3$$

$$\phi = (\bar{c})^{1/3} \phi_s$$

$$= S\phi_s$$
Case (2): The ellipsoid shape.

Volume of ellipsoid = $\frac{4}{3} \pi \bar{a} \bar{b} \bar{c}$

where $\bar{a}$, $\bar{b}$, $\bar{c}$ are the lengths of the semiaxes. Also, $\bar{b} = \phi_s$, $\bar{a} = \bar{c}_1 \phi_s$, $\bar{c} = \bar{c}_2 \phi_s$, where $\bar{c}_1$, $\bar{c}_2$ are constants.

Since volume of ellipsoid = volume of equivalent sphere

\[
\therefore \frac{4}{3} \pi \bar{a} \bar{b} \bar{c} = \frac{4}{3} \pi \phi^3
\]

\[
\frac{4}{4} \pi (\bar{c}_1 \phi_s)(\phi_s)(\bar{c}_2 \phi_s) = \frac{4}{3} \pi \phi^3
\]

or

\[
\phi = (\bar{c}_1 \bar{c}_2)^{1/3} \phi_s
\]

\[
\phi = S \phi_s
\]

The value of $S$ is generally $> 1.0$ for granular sand media and could be $< 1.0$ for angular coal media.

As shown in Figure 4, the volume of capillaries per unit bed volume = $\varepsilon$.

\[
\varepsilon = n_2 \pi R^2_{eq} L_e
\]

\[
= n_2 \pi R^2_{eq} \left[ (n_1)^{1/3} 2 \pi S \phi_s \right]
\]

where

$\phi_s$ = clean grain sieve radius and $(n_1)^{1/3}$ = number of grains per unit side length.
Also, volume of grains/unit bed volume

\[ = 1 - \varepsilon \]

\[ = n_1 \frac{4}{3} \pi s^3 \phi_s \]

(53)

therefore,

\[ \frac{\varepsilon}{1 - \varepsilon} = \frac{n_2 (n_1)}{n_1} \frac{1}{3} \frac{2}{3} \pi s \phi_s \frac{(\pi R_{eq}^2)}{3} \]

\[ = \frac{1}{2} n_2 2/3 \left( \frac{\pi}{2} \right) \frac{R_{eq}^2}{\phi_s} \]

or,

\[ R_{eq} = \sqrt{\frac{2}{\pi} n_1 \frac{2}{3}} \frac{2}{\phi_s} \frac{(S^2)}{\varepsilon} \frac{R_{eq}^2}{1 - \varepsilon} \]

\[ = \sqrt{\frac{K}{g} \frac{\varepsilon}{1 - \varepsilon}} = \phi \sqrt{\frac{K}{g} \frac{\varepsilon}{1 - \varepsilon}} \]

where \( K \) = a geometric constant.

The ratio \( \frac{n_1}{n_2} \) may be estimated considering the case of one particular packing shown in Figure 4. This tetrahedral packing is considered most representative of a typical granular filter. Assume a unit bed volume of side length \( L = 1.0 \) (the ratio of \( \frac{n_1}{n_2} \) should stay constant regardless
of the size of the volume element).

Number of grains/cross sectional area

\[= 5.0 \text{ grains} \]

\[\therefore \text{number of grains/side of example cube} \]

\[= \sqrt{5} \text{ grains} \]

consequently

Number of grains/example cube

\[= (\sqrt{5})^3 \]

\[= 11.18 \text{ grains.} \]

Also, there are 7 complete pores and 6 half pores in the cross-sectional area shown in Figure 4. Therefore, the total number of pores (or capillaries) per cross-sectional area is 10.

\[\therefore \frac{(n_1)^{2/3}}{n_2} = \frac{(11.18)^{2/3}}{10} \approx 0.5 \]

\[\therefore k = \frac{2}{\pi} \frac{(n_1)^{2/3}}{n_2} = \frac{2}{\pi} \frac{(11.18)^{2/3}}{10} = 0.3183 \]

Accordingly,

\[\text{Req} = \phi \sqrt{k \frac{\varepsilon}{1 - \varepsilon}} = 0.564\phi \sqrt{\frac{\varepsilon}{1 - \varepsilon}} \quad (54)\]
Although the tortuosity factor, $F_t$, varies during a filter run, there is no reliable and accurate way of measuring its value with time of filtration. Therefore, for simplicity, it is assumed constant and equal to 1.08 as shown in Fig. 4.

Equation 54 is very similar to Camp's formula for the equivalent pore diameter presented as follows:

$$R_{eq} = \sqrt{\frac{\varepsilon}{1 - \varepsilon}}$$

but Camp's formula provides 43.6% higher values of $R_{eq}$ than given by equation 54.

Furthermore, Deb (21) pointed out that the limits for the equivalent pore radius for the two extreme cases of packing, rhombohedron and cubical, are

$$R_{eq} = 0.378 \text{ to } 0.788\phi$$

Notice that equation 54 falls within Deb's range of values and Camp's equation does not.

According to the capillary tubes model, the granular media bed maintains its characteristic geometry regardless of the degree of bed clogging with the process of particle removal. The capillary channels between the granular media continue to narrow as removal continues. For a clean bed with tetrahedral packing, the average pore size is designated $d_{eo}$, and after any filtration time $t$ it is designated $d_{el}$ such that:
Fig. 4. Horizontal cross-section in a unit bed volume

\[ n_1 = (\sqrt{5})^3 = 11.18, \quad L = 5\phi\cos 30 = 4.33\phi, \]

\[ L_e = (n_1)^{1/3}(120/360)2\pi\phi = 4.683\phi, \quad F_c = \frac{L_e}{L} = 1.08 \]
\[ d_{eo} = 2\phi_o (0.564) \sqrt{\frac{\varepsilon_o}{1 - \varepsilon_o}} \]
\[ d_{e1} = 2\phi_1 (0.564) \sqrt{\frac{\varepsilon}{1 - \varepsilon}} \]
\[ = 2 \left( \phi_o + \Delta\phi \right) (0.564) \sqrt{\frac{\varepsilon_o - \sigma}{1 - \varepsilon_o + \sigma}} \quad (55) \]

where \( \phi_o, \phi_1 \) = grain equivalent radius at time, \( t = 0, t \) respectively
\[ \Delta\phi = \phi_1 - \phi_o \]

\( \sigma \) = amount of deposit per unit bed volume (Vol/Vol).

Assuming that particle deposition is uniformly coating the granular media, the change in each grain volume, \( \Delta V \), due to deposits at any time \( t \) is
\[ \Delta V = \frac{4}{3} \pi \left[ \phi_1^3 - \phi_o^3 \right] \quad (56) \]

Also,
\[ \Delta V = \frac{\sigma}{n_1} \]

but, from equation 53
\[ n_1 = \frac{1 - \varepsilon_o}{\frac{4}{3} \pi \phi_o^3 \phi_s} = \frac{3}{4} \left( \frac{1 - \varepsilon_o}{\pi \phi_o^3} \right) \]
Equating equations 56 and 57

\[ \frac{4}{3} \left[ \phi_i^3 - \phi_o^3 \right] = \frac{4}{3} \pi \phi_o^3 \left( \frac{\sigma}{1 - \varepsilon_o} \right) \]

\[ \therefore \phi_i = \phi_o \left[ \frac{1 - \varepsilon_o + \sigma}{1 - \varepsilon_o} \right]^{1/3} \tag{58} \]

Substituting for \( \phi_i \) in equation 54 results in

\[ R_{eq} = 0.564 \phi_i \sqrt{\frac{\varepsilon}{1 - \varepsilon}} \]

\[ = 0.564 \phi_o \left[ \frac{1 - \varepsilon_o + \sigma}{1 - \varepsilon_o} \right]^{1/3} \sqrt{\frac{(\varepsilon_o - \sigma)}{(1 - \varepsilon_o + \sigma)}} \]

\[ R_{eq} = 0.564 \phi_o \sqrt{\frac{(\varepsilon_o - \sigma)}{(1 - \varepsilon_o)^{2/3}(1 - \varepsilon_o + \sigma)^{1/3}}} \tag{59} \]

In order to develop the relationship between \( \frac{h}{L_e} \) and the filtration variables \( V, \varepsilon_o, \sigma, \phi_{so}, K_g \) it is useful to start with equation 49 for the flow, \( Q \), through a capillary tube

\[ Q = \frac{\rho g h_f}{8 \mu L} \pi R^4 \tag{49} \]
For a granular bed \( L = L_e, \) \( R = R_{\text{eq}} \)

\[
\frac{h}{L_e} = \frac{8\mu Q}{\rho g \pi R_{\text{eq}}} \frac{4}{h} = \frac{8\mu \left(\pi R_{\text{eq}}^2\right) V_i}{\rho g \pi R_{\text{eq}}^4}
\]

where \( V_i \) is the average velocity through a capillary, or the interstitial velocity.

\[
\frac{h}{L_e} = \frac{8\mu}{\rho g} \frac{V_s}{\varepsilon} \cdot \frac{1}{R_{\text{eq}}} \frac{1}{2}
\]

\[
\frac{h}{L_e} = \frac{8\mu}{\rho g} \frac{V_s}{\varepsilon} \left(\varepsilon_o - \sigma\right) \left[ \frac{1 - \varepsilon_o}{0.564\phi_o} \left(1 - \varepsilon_o\right)^{2/3} \left(1 - \varepsilon_o + \sigma\right)^{1/3} \right]
\]

\[
\frac{h}{L_e} = \frac{25.15\mu}{\rho g} \frac{V_s}{\phi_o^2} \frac{(1 - \varepsilon_o)^{2/3} (1 - \varepsilon_o + \sigma)^{1/3}}{(\varepsilon_o - \sigma)^2}
\]

\[
\frac{h}{h_o} = \left(\frac{1 - \varepsilon + \sigma}{1 - \varepsilon_o}\right)^{1/3} \left(\frac{\varepsilon_o}{\varepsilon_o - \sigma}\right)^2
\]

(60a)
Combining equations 51, 59 and 60

\[
\bar{G}_a = \frac{\rho gh}{3\mu L_e} R_{eq}
\]

\[
= \frac{\rho g}{3\mu} \left[ \frac{25.15\mu}{g} \frac{v_s}{\phi^2} \frac{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}{(\varepsilon_0 - \sigma)^2} \right]
\]

\[
= 0.564\phi_o \sqrt{\frac{(\varepsilon_0 - \sigma)}{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}}
\]

\[
= 4.73 \frac{v_s}{\phi_o} \sqrt{\frac{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}{(\varepsilon_0 - \sigma)^3}}
\]

\[
= 4.73 \frac{v_s}{S\phi_{so}} \sqrt{\frac{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}{(\varepsilon_0 - \sigma)^3}} \quad (61a)
\]

Similarly

\[
\bar{G}_w = \frac{1}{1.25} \bar{G}_a
\]

\[
= 3.784 \frac{v_s}{S\phi_{so}} \sqrt{\frac{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}{(\varepsilon_0 - \sigma)^3}} \quad (61b)
\]
In order to develop some feeling for the range of flow velocity gradient inside a granular media bed, according to the capillary flow model, Table 1 includes calculations of $\bar{G}_a$, $\bar{G}_w$, $\bar{G}_a t$, $\bar{G}_w t$ based on equations 61a and 61b, example media grain sizes of 1.0, 2.0 mm E.S., 0.45 clean bed porosity, four feet bed depth, and $S = 1.10$, for superficial flow velocities, 4.0 and 8.0 gpm/ft$^2$, for the initial clean bed conditions and for a value of $\sigma = 0.3$. The value of $S = 1.10$ is arbitrarily selected based on the average value of $\frac{d_{eq}}{d_s}$ for four different sand media included in a previous study (111). Here $d_{eq}$ is the diameter of the equivalent sphere, $d_s$ is the average sieve opening.

For the purpose of illustration, a sample example is brought to demonstrate the step-wise calculations followed in preparation of Table 1.

**Example:** Calculations of $\bar{G}_a$, $\bar{G}_w$, $\bar{G}_a t$, $\bar{G}_w t$ for a four foot clean and clogged bed ($\sigma_u = 0.3$) of 1.0 mm sand media ($S = 1.10$). Assume that:

Case (1):

\[ \varepsilon_0 = 0.45 \]
\[ \sigma = 0 \]
\[ V_s = 4 \text{ gpm/ft}^2 = 2.721 \text{ mm/sec} \]

\[ \bar{G}_a = 4.73 \frac{V_s}{d_{eq}s} \sqrt{\frac{(1 - \varepsilon_0)^{2/3}(1 - \varepsilon_0 + \sigma)^{1/3}}{(1 - \varepsilon_0 + \sigma)^{2/3}}} \]

\[ \bar{G}_a = 4.73 \frac{(2.721 \text{ mm/sec})}{1.10 \ (0.5 \text{ mm})} \sqrt{\frac{(1 - 0.45)^{2/3}(1 - 0.45 + 0)^{1/3}}{(0.45 - 0.0)^3}} \]
\[ t = \text{bed residence time} \]

\[ t = \frac{\text{bed depth}}{V_i} = \frac{L(\varepsilon_o)}{(V_s)} \]

\[ = 4.0 \text{ ft} \times 30.45 \text{ cm/ft} \times 0.45 \]

\[ 2.721 \text{ mm/sec} \times \frac{1}{10} \text{ cm/mm} \]

\[ = 201.4 \text{ sec} \]

\[ \bar{\varepsilon}_a t = 57.5 \times 201.4 \]

\[ = 11,580 \]

\[ \bar{\varepsilon}_w, \bar{\varepsilon}_w t \text{ are } \frac{1}{1.25} \text{ of } \bar{\varepsilon}_a, \bar{\varepsilon}_a t \text{ respectively.} \]

Therefore,

\[ \bar{\varepsilon}_w = 46 \text{ sec}^{-1} \text{ and } \bar{\varepsilon}_w t = 9,264. \]

Case (2):

\[ \varepsilon_o = 0.45 \]

\[ \sigma = \sigma_u = 0.30 \]

\[ V_s = 4 \text{ gpm/ft}^2 = 2.721 \text{ mm/sec} \]

\[ \bar{\varepsilon}_a = 4.73 \frac{2.721 \text{ mm/sec}}{1.10 (0.5 \text{ mm})} \sqrt[3]{(1 - 0.45)^{2/3} (1 - 0.45 + 0.3)^{1/3}} \]

\[ \frac{(0.45 - 0.3)^{1/3}}{(0.45 - 0.3)^3} \]
\[ t = \frac{4.0 \text{ ft}(304.5 \text{ mm/ft})(0.45 - 0.3)}{2.721 \text{ mm/sec}} \]

\[ t = 67.14 \text{ sec} \]

\[ \bar{G}_a \cdot t = 21,566 \]

\[ \bar{G}_w = 257 \text{ sec}^{-1} \]

\[ \bar{G}_w \cdot t = 17253. \]

The relationship between both \(G, G_t\) and bed porosity, \(\varepsilon\), may be drawn utilizing equations 61a and 61b.

\[ \bar{G}_a = 4.73 \left( \frac{V_s}{S\Phi_{so}} \right) \sqrt{\frac{(1 - \varepsilon_o)^{2/3}(1 - \varepsilon_o + \sigma)^{1/3}}{(\varepsilon_o - \sigma)^3}} \quad (61a) \]

\[ \bar{G}_w = K_w \sqrt{\frac{(1 - \varepsilon_o)^{1/3}}{\varepsilon^3}} \]

where \(K_a = 4.73 \left( \frac{V_s}{S\Phi_{so}} \right) (1 - \varepsilon_o)^{1/3} \)

similarly,

\[ \bar{G}_w = K_w \sqrt{\frac{(1 - \varepsilon_o)^{1/3}}{\varepsilon^3}} \]

where \(K_w = 3.784 \left( \frac{V_s}{S\Phi_{so}} \right) (1 - \varepsilon_o)^{1/3} \)
<table>
<thead>
<tr>
<th>Case Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^2 <em>so</em> - E.S. in mm</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Flow rate gpm/ft^2</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Clean bed porosity, ( \varepsilon_o )</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Deposit content, (( \sigma ))</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Bed depth in ft.</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Bed residence time sec.</td>
<td>200</td>
<td>70</td>
<td>100</td>
<td>35</td>
<td>200</td>
<td>70</td>
<td>100</td>
<td>35</td>
</tr>
<tr>
<td>( \bar{G} a ) sec^{-1}</td>
<td>60</td>
<td>320</td>
<td>120</td>
<td>640</td>
<td>30</td>
<td>160</td>
<td>60</td>
<td>320</td>
</tr>
<tr>
<td>( \bar{G} w ) sec^{-1}</td>
<td>45</td>
<td>260</td>
<td>90</td>
<td>510</td>
<td>225</td>
<td>130</td>
<td>45</td>
<td>260</td>
</tr>
<tr>
<td>t.( \bar{G} a )</td>
<td>12,000</td>
<td>22,400</td>
<td>12,000</td>
<td>22,400</td>
<td>6,000</td>
<td>11,200</td>
<td>6,000</td>
<td>11,200</td>
</tr>
<tr>
<td>t.( \bar{G} w )</td>
<td>9,000</td>
<td>18,200</td>
<td>9,000</td>
<td>18,200</td>
<td>4,500</td>
<td>9,100</td>
<td>4,500</td>
<td>9,100</td>
</tr>
</tbody>
</table>
or generally,

\[ \bar{c} = K'' \sqrt{\frac{(1 - \varepsilon)^{1/3}}{\varepsilon^{3/2}}} \]  \hspace{1cm} (62) \]

\( K'' \) = either \( K_a \) or \( K_w \)

The derivative of equation 62 is

\[ \frac{d\bar{c}}{d\varepsilon} = \frac{-K''}{6} \left[ \frac{1}{\varepsilon^{1.5}(1 - \varepsilon)^{5/16}} + \frac{9(1 - \varepsilon)^{1/6}}{\varepsilon^{2.5}} \right] \]

\[ = \frac{-K''}{6} \left[ \frac{(9 - 8\varepsilon)}{(1 - \varepsilon)^{5/6}\varepsilon^{2.5}} \right] \]  \hspace{1cm} (63)

Applying the boundary conditions

1. at \( t = \infty \) \( \sigma = \varepsilon_0 \) and \( \varepsilon = 0 \)

2. at \( t = 0 \) \( \sigma = 0 \) and \( \varepsilon = \varepsilon_0 \)

The limits of the derivative \( \frac{d\bar{c}}{d\varepsilon} \) become

at \( t = \infty \) \( \frac{d\bar{c}}{d\varepsilon} = -\infty \)

at \( \varepsilon = \varepsilon_0 \) \( \frac{d\bar{c}}{d\varepsilon} = \frac{-K''}{6} \left[ \frac{(9 - 8\varepsilon_0)}{(1 - \varepsilon_0)^{5/6}\varepsilon_0^{2.5}} \right] \)

The product \( \bar{c}t_b \) where \( t_b = \frac{\varepsilon L}{V_s} \) is the bed residence time, is

\[ \bar{c}t_b = \frac{\varepsilon L}{V_s} \sqrt{\frac{(1 - \varepsilon)^{1/3}}{\varepsilon}} \]
and the derivative of the product $\bar{G}_t \rho$ is

$$\frac{d(\bar{G}_t \rho)}{d\varepsilon} = \frac{-KL}{6V} \left[ \frac{3 - 2\varepsilon}{\frac{1.5}{\varepsilon} (1 - \varepsilon)^{5/6}} \right]$$

(64)

The limits of the derivative $\frac{d(\bar{G}_t \rho)}{d\varepsilon}$ at $\varepsilon = 0$ and $\varepsilon_o$ are as follows:

at $\varepsilon = 0$ \hspace{1cm} $\frac{d(\bar{G}_t \rho)}{d\varepsilon} = -\alpha$

at $\varepsilon = \varepsilon_o$ \hspace{1cm} $\frac{d(\bar{G}_t \rho)}{d\varepsilon} = \frac{-KL}{6V} \left[ \frac{3 - 2\varepsilon_o}{\frac{1.5}{\varepsilon_o} (1 - \varepsilon_o)^{5/6}} \right]$

Establishing the relationship between the filtration process variables and the velocity gradient inside the filter bed is a forward step towards the development of in-bed contact flocculation theory. The next step towards the completion of the theoretical study includes a presentation of the rate of particle agglomeration and break-up as a result of flocculation and floc break-up mechanisms.

A particle size distribution study is found very helpful in most of the conventional flocculation processes. However, in the case of bed contact flocculation another dimension is added to the problem of model development. The additional term of floc break-up as a result of particle detachment might cause further complications and uncertainties to the proposed model. In order to avoid such obstacles, it is, in the writer's opinion, more feasible to develop a model for the disappearance
if primary particles during the process of contact flocculation as an indication of the extent of the process efficiency.

As previously mentioned, particle disappearance during bed contact flocculation is a result of the contribution of three main terms:

1. Aggregation term
2. Break-up term
3. Removal term

The aggregation term involves particle flocculation due to Brownian motion as well as flow velocity gradient. The rate of primary particle disappearance due to Brownian motion based on equation 88 in the literature section is:

\[- \frac{dN}{dt} = \alpha_p 4\pi D_{ij} R_{ij} \frac{N_i N_j}{N} \]  

(Lit. 88)

In the particular case, where \( R_i = R_j = R_p \) (primary particle radius)

\[ D = \frac{kT}{3\mu \pi (2R_p)} \]

\[ = \frac{kT}{6\mu \pi R_p} \]

and

\[- \frac{dN}{dt} = \alpha_p 4\pi \left( \frac{kT}{6\mu \pi R_p} \right) 2R_p N^2 \]

\[ = \frac{4}{3} \frac{\alpha_p kT}{\mu} N^2 \]  

(65)
where \( k_r \) is the constant for perikinetic flocculation.

The rate of decrease in primary particle concentration due to velocity gradient, \( \frac{du}{dz} \), is described by equation 89 in the literature section which for particular case when \( R_i = R_j = R_p \) becomes:

\[
\frac{-dN}{dt} = \alpha_0 \frac{4}{3} \pi \left( \frac{2R_p}{p} \right)^3 N^2 \frac{dv}{dz}
\]

\( (\text{Lit. 89}) \)

\[
= 8\alpha_0 \frac{4}{3} \pi R_p^3 N \frac{dv}{dz}
\]

\[
= K_{or} v_m \frac{dv}{dz} N
\]

(67)

where, \( v_m \) = volume of total solid mass suspended per volume of fluid and \( K_{or} \) is a constant for orthokinetic flocculation. In conventional flocculation utilizing mechanical energy for mixing, the velocity gradient, \( \frac{du}{dz} \), is constant and independent of time as long as the speed of mixing and other flow and mixing basin physical variables are the same. However, in bed flocculation, the velocity gradient inside the sinuous capillaries varies with time and amount of particle removal. Therefore, the efficiency of bed flocculation varies with time.

Equation 67 is a general form and applies to any type of orthokinetic flocculation model including bed contact flocculation as will be demonstrated in a later section. The only difference in the kinetic equations for orthokinetic flocculation from one model to another
exists in the orthokinetic flocculation constant $K_{or}$ and the expressions for velocity gradients, $\frac{du}{dz}$. For bed contact flocculation capillary tube model, the value of $\frac{du}{dz}$ is given in equation 47. To determine the constant $K_{or}$ it is necessary to follow a similar line of thinking as used in the development of the general flocculation theory. However, this time the theoretical study is confined to the particular case of the rate of disappearance of primary particles due to velocity gradient in a viscous steady laminar flow inside a capillary tube.

The rate of primary particles disappeared due to orthokinetic flocculation inside a capillary channel may be presented as a function rate of collision of particles due to the velocity gradients existing in the capillary. The rate of collision of other particles with a primary particle, carried by a streamline at a certain velocity, is proportional to the number of particles flowing into a sphere of radius, $2R_p$, with its center coinciding with the primary particle center. The rate of primary particles flow across the foregoing sphere is, $qN$, where $q$ is the flow rate across the sphere.

The total rate of disappearance of all the primary particles is then:

$$\frac{-dN}{dt} = \alpha(qN)N$$  \hspace{1cm} (67a)

However, the flow rate, $q$, for a flow stream with radius $2R_p$ is expressed as
Also, the root mean square of velocity gradient of flow through the given sphere, arbitrarily using the volume weighted equation for $G$, is as follows:

$$\bar{G}_a = \frac{\rho gh}{3 \mu L} (2R_p)$$  \hspace{1cm} (51)

The choice of $G_a$ over $G_w$ is strictly arbitrary and has no affect on the following development. Dividing equation 49 by equation 51 results in:

$$q = \frac{3\pi}{8} \bar{G}_a (R_p)^3$$

Therefore, inserting this equation into equation 67a,

$$\frac{-dN}{dt} = \alpha \left[ \frac{3\pi}{8} \bar{G}_a (2R_p)^3 \right] N^2$$

$$\frac{-dN}{dt} = \alpha \left[ \frac{4}{3} \frac{\pi R^3 N}{p} \right] \left[ \frac{8}{4} \bar{G}_a N \right]$$

$$= \alpha \cdot 2.25 \sqrt{V_m} \bar{G}_a N$$  \hspace{1cm} (68)

Combining equations 65 and 68 the rate of primary particles disappearance due to the aggregation term becomes:

$$\frac{-dN}{dt} \text{ (aggregation)} = \frac{4}{3} \alpha_p \frac{kT}{\mu} N^2 + 2.25 \alpha \sqrt{V_m} \bar{G}_a N$$  \hspace{1cm} (69)
Floc break-up is a function of the flow shear stresses as well as the mean value of velocity gradient inside the bed capillaries. However, assuming that break-up of large floc particles would not result in the generation of primary particles, the break-up term in equation 46 may be dropped out.

The removal term in equation 46 is assumed proportional to volume available for removal, concentration of primary particles, and inversely to flow hydraulic gradient, J. Therefore, the rate of removal of primary particles is:

\[
\frac{-dN}{dt} \text{ (removal)} = K_a \frac{(u - \sigma)}{J} N
\]  

(70)

Combining equations 69 and 70 the overall rate of primary particles disappearance is:

\[
\frac{-dN}{dt} = \frac{4}{3} a_p \frac{k}{\mu} \frac{N_0}{2} + 2.25 a_o \frac{V_m}{m} g_a N + K_a N \frac{(u - \sigma)}{J}
\]  

(71)

Equation 71 describes the rate of disappearance of primary particles due to flocculation or filtration during the flow through the filter bed. Therefore, it must be clear that the time, \(dt\), in the left hand side of equation 71 is that required for suspension to flow through an increment of depth, \(dx\). Accordingly, equation 71 may be rewritten as follows for the case of a granular media flocculation:
Application of the Proposed Filtration Model to the Theory of Bed Contact Flocculation

The rate expression for the disappearance of primary particles presented in equation 71a may be rewritten as follows after inserting the expression for $J$ from equation 1a:

$$\frac{-dN}{dx} = \frac{4}{3} \alpha_p \frac{kT}{\mu} N^2 + 2.25 \alpha_o V_m \bar{G}_a N + \kappa_a N \frac{(\sigma - \sigma)}{J}$$

(71a)

where

$$K_1 = \frac{4}{3} \alpha_p \frac{kT}{\mu}$$

$$K_2 = 2.25 \alpha_o V_m$$

$$K_3 = \kappa_a \frac{K_{oo}}{V_s}$$

Following the argument presented by Van de Ven and Mason (105), in which the authors had challenged the concept of additivity of perikinetic and orthokinetic flocculation, the perikinetic flocculation term may be omitted in equation 72 as long as the value of $G$ is higher than zero. In fact, for typical particle sizes encountered in water treatment, and under normal filtration conditions of flow rate, media size, shape and
porosity, the calculated values of \( \bar{c}_a, \bar{c}_w \) shown in Table 1 are high enough to assume the predominance of orthokinetic flocculation inside the bed. Accordingly, a simple form of equation 72 may be presented as follows:

\[
-\frac{dN}{dx} = N \left[ K_2 \bar{c}_a + K_j (\sigma_u - \sigma)(1 - \sqrt[3]{\frac{\sigma}{\sigma_u}})^3 \right]
\]

Inserting equation 61a for \( \bar{c}_a \) and the rearranging of the second term shown in equation 4a:

\[
-\frac{dN}{dx} = N \left[ K_2 \frac{(4.73)(V_s)}{S \phi_{so}} \frac{\sqrt{(1 - \varepsilon_o)^{2/3}(1 - \varepsilon_o + \sigma)^{1/3}}}{(\varepsilon_o - \sigma)^3} \right]
\]

\[
+ K_3 \sigma_u \left( 1 + \frac{\sigma}{\sigma_u} \right) \left( 1 - \frac{\sigma}{\sigma_u} \right)^4
\]

\[
= N \left[ K_4 \frac{(1 - \varepsilon_o + \sigma)^{1/6}}{(\varepsilon_o - \sigma)^{3/2}} + K_3 \sigma_u \left( 1 + \frac{\sigma}{\sigma_u} \right) \left( 1 - \frac{\sigma}{\sigma_u} \right)^4 \right] \quad (73)
\]

where

\[
K_4 = \frac{K_2^1 (4.73)V_s (1 - \varepsilon_o)^{1/3}}{S \phi_{so}}
\]

\[
= \frac{(2.25)(4.73)(a_v V_s m_s) (1 - \varepsilon_o)^{1/3}}{S \phi_{so}}
\]
In the foregoing section of theory of filtration, equation 30 was developed based on the proposed kinetic equation 1. The use of the ultimate deposit content, \( \sigma_u \), in the filtration equations in the prior section is supported by the experimental observations and agrees with the general behavior of a filter bed. However, for the sake of simplicity in solving equation 73 for bed flocculation, the ultimate deposit term \( \sigma_u \) is replaced by bed initial porosity, \( \varepsilon_0 \). Accordingly, equation 73 may be rewritten as:

\[
-dN \over dx = N \left\{ K_4 \frac{(1 - \varepsilon_0 + \sigma)^{1/6}}{(\varepsilon_0 - \sigma)^{3/2}} + K_3 \varepsilon_0 (1 + \sqrt{\frac{\sigma}{\varepsilon_0}}(1 - \sqrt{\frac{\sigma}{\varepsilon_0}})^4 \right\} (73a)
\]

Equation 73a represents the total disappearance of primary particles due to two mechanisms. The first term represents the disappearance due to orthokinetic flocculation and second term, the disappearance due to filtration. Each of the two terms in the right hand side of the above equation will be integrated separately and added together in the final step.

Substituting the value of \( \sigma \) given in the modified equation 30 (with \( \varepsilon_0 \) substituted for \( \sigma_u \)) into the first term of equation 73a results in

\[
-dN \over dx = N \left\{ K_4 \left[ (1 - \varepsilon_0) + \frac{\varepsilon_0 (e^{8K\varepsilon_0 t} - 1)^2}{2e^{2V_s t}} \right]^{1/6} \right\}
\]

\[
(73b)
\]
At a particular time during a filter run, the rate of primary particle disappearance due to orthokinetic flocculating is only a function of depth and can be represented as follows:

\[
\frac{-dN}{dx} = N K_4 \left( \frac{1 - \varepsilon_o}{(2e^{ax} + \alpha)^2} \right)^{1/6} - \frac{\varepsilon_o \alpha^2}{(2e^{ax} + \alpha)^2} \left( \frac{1 - \varepsilon_o}{(2e^{ax} + \alpha)^2} \right)^{3/2} \]

where

\[\alpha = (e^{bt} - 1).\]
In the case the value of coefficients $a, b$ are $<< 1.0$ it is possible to assume that $y << e^{(e^{a} + a)}$ and the term $e^{a/2} (e^{a} + a)$ may be set equal to zero in order to facilitate carrying out the solution to the above equation.

Also, make the substitution

$$y = e^{ax} \quad \therefore dy = ae^{ax} \, dx = ay \, dx$$

i.e. $dx = \frac{1}{ay} \, dy$

$$\therefore - \frac{dN}{N} = K_5 \left[ \frac{y (y + \alpha)}{(y)^{3/2}} \right]^{1/6} \left[ \frac{2y + \alpha}{y + \alpha} \right]^{8/3} \frac{dy}{y}$$

where

$$K_5 = K_4 \frac{4(1 - \varepsilon_0)^{1/6}}{a(4 \varepsilon_0)^{3/2}}$$
Expanding the binomial term and retaining only terms to the second power yields:

\[ \int - \frac{dN}{N} = K_7 \int \left[ 1 + \frac{4}{3} \cdot \frac{\alpha^2}{4y(y + \alpha)} + \frac{4}{3} \cdot \frac{1}{3} \cdot \left( \frac{\alpha^2}{4y(y + \alpha)} \right)^2 \cdot \frac{1}{2} \right] \frac{dy}{y} \]

where

\[ K_7 = K_5 \alpha^{4/3} \]

\[ \int - \frac{dN}{N} = K_7 \left[ \int \frac{dy}{y} + \frac{\alpha^2}{3} \int \frac{dy}{y^2(y + \alpha)} + \frac{1}{72} \int \frac{\alpha^4}{y^3(y + \alpha)^2} \right] \]
\[ 154 \]

\[ = K_7 \left[ \log y + \frac{\alpha^2}{3} \left\{ -\frac{1}{\alpha y} + \frac{1}{\alpha^2} \log \frac{y + \alpha}{y} \right\} \right. \]

\[ + \frac{\alpha^4}{72} \left\{ -\frac{1}{(3-1)\alpha^2 y^2} - \frac{(2 + 3 - 3)}{(3 - 1)\alpha^2} \int \frac{dy}{y(y + \alpha)^2} \right\} \]

\[ - 2\alpha \left( \frac{1 + 3 - 2}{(3 - 1)\alpha^2} \right) \int \frac{dy}{y^2(y + \alpha)^2} \right\} \]

\[ = K_7 \left[ \log y - \frac{\alpha}{3y} + \log \left( \frac{y + \alpha}{y} \right)^{1/3} + \frac{4}{72} \left\{ -\frac{1}{2\alpha^2 y} \right\} \right. \]

\[ - \frac{1}{\alpha^2} \left( \frac{1}{\alpha(y + \alpha)} \right) - \frac{1}{\alpha^2} \log \left( \frac{y + \alpha}{y} \right) - \frac{2}{\alpha} \left( \frac{\alpha + 2y}{\alpha^2 y(y + \alpha)} \right) \]

\[ + \frac{2}{\alpha^2} \log \left( \frac{y + \alpha}{y} \right) \left. \right\} \]

\[ = K_7 \left[ \log y - \frac{\alpha}{3y} + \log \left( \frac{y + \alpha}{y} \right)^{1/3} - \frac{\alpha^2}{144 y^2} - \frac{\alpha}{72(y + \alpha)} \right. \]

\[ + \frac{1}{72} \log \left( \frac{y + \alpha}{y} \right) + \frac{\alpha(\alpha + 2y)}{36y(y + \alpha)} - \frac{1}{18} \log \left( \frac{y + \alpha}{y} \right) \]

\[ = K_7 \left[ \log (y)^{0.7} (y + \alpha)^{0.3} + ((-48\alpha y)(y + \alpha)) - \alpha^2(y + \alpha) \right. \]
Thus, the integrated form prior to determination of the integration constant will be:

\[-\log \frac{N}{N_0} = K_7 \left[ \log \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.7} \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.3} - \frac{\alpha(42y^2 + 45ay + \alpha^2)}{144y^2(y + \alpha)} \right] + c_1 \]

Use the following boundary conditions to determine the integration constant.

at \( X = 0 \rightarrow y = 1, N = N_0 \)

\[0 = K_7 \left[ \log \left( \frac{1 + \alpha}{1 + \alpha} \right)^{0.3} - \frac{\alpha(42 + 45\alpha + \alpha^2)}{144(1 + \alpha)} \right] + c_1\]

\[c_1 = -K_7 \left[ \log \left( \frac{1 + \alpha}{1 + \alpha} \right)^{0.3} - \frac{\alpha(42 + 45\alpha^2 + \alpha^3)}{144(1 + \alpha)} \right] \]

\[-\log \frac{N}{N_0} = K_7 \left[ \log \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.7} \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.3} - \frac{\alpha(42y^2 + 45ay + \alpha^2)}{144y^2(y + \alpha)} \right] + \frac{\alpha(42 + 45\alpha + \alpha^2)}{144(1 + \alpha)}\]

\[= K_7 \left[ \log \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.7} \left( \frac{y + \alpha}{1 + \alpha} \right)^{0.3} \right] + \left\{ -\alpha (1 + \alpha) \right\} \left( 42y^2 + 45ay + \alpha^2 \right)\]
\[ \begin{align*}
+ \alpha(42 + 45\alpha + \alpha^2) y^2 (y + \alpha) \\
\div 144 (1 + \alpha)(y + \alpha) y^2
\end{align*} \]

For convenience, this equation will be written as follows:

\[ -\log \frac{N}{N_0} = K_7 \left[ \log \left( e^{0.7ax} \right) \left[ \frac{e^{ax} + e^{bt}}{e^{bt}} \right]^{0.3} + f(x,t) \right] \] (76)

where

\[ f(x,t) = \frac{-\alpha(1 + \alpha)(42y^2 + 45\alpha y + \alpha^2) + \alpha(y^2)(y + \alpha)(42 + 45\alpha + \alpha^2)}{144(1 + \alpha)(y + \alpha)y^2} \]

\[ = \begin{cases} 
& y^3 \left[ 42 + 45\alpha + \alpha^2 \right] + y^2 \left[ -42 + 45\alpha^2 + \alpha^3 \right] \\
& + y \left[ -45\alpha (1 + \alpha) - \alpha^2 \left[ 1 + \alpha \right] \right] \div 144 (1 + \alpha)(y + \alpha)y^2
\end{cases} \]

Introducing the definitions of \( y = e^{ax} \) and \( \alpha = (e^{bt} - 1) \) and rearranging leads to:

\[ f(x,t) = \begin{cases} 
& e^{3ax} \left[ e^{2bt} + 43e^{bt} - 2 \right] + e^{2ax} \left[ 3^{bt} + 42e^{2bt} \\
& - 87e^{bt} + 2
\end{cases} \]
Solution of equation (76) may be written as follows:

\[ N = N_0 \left\{ e^{0.7ax} \left[ \frac{e^{ax} + e^{bt} - 1}{e^{bt}} \right]^{0.3} - K_7 e^{-K_7 \xi(x,t)} \right\} \quad (78) \]

The second term (i.e., the filtration term) in the right hand side of equation 73a may be rearranged as follows:

\[ K_3 \varepsilon_0 \left( 1 + \frac{\sigma}{\varepsilon_0} \right) \left( 1 - \frac{\sigma}{\varepsilon_0} \right)^4 = \frac{1}{\sqrt{s}} K a K po \left( 1 - \frac{\sigma}{\varepsilon_0} \right)^3 (\varepsilon_0 - \sigma) \]

and

\[ - \frac{\delta N}{\delta x} = \frac{1}{\sqrt{s}} K a K po \left[ 1 + \sqrt{\frac{\sigma}{\varepsilon_0}} \right]^{3} \left( \varepsilon_0 - \sigma \right) N \]

The detailed integration of the expression can be eliminated by noting the similarity of the above equation to equation 3a which is repeated below for convenience:

\[ - \frac{\delta c}{\delta x} = \frac{1}{\sqrt{s}} K a K po \left[ 1 - \sqrt{\frac{\sigma}{\varepsilon_0}} \right]^{3} \left( \varepsilon_0 - \sigma \right) C \quad (3a) \]
Because of their similarity, the solution of equation 73a will be identical to the solution of 3a which was equation 31. Therefore, the solution to equation 73c is as follows:

\[ N = N_o \left\{ \frac{e^{bt} + 1}{2e^{ax} + e^{bt} - 1} \right\}^2 \]  

(79)

This also leads to the conclusion that the extent of removal of primary particles due to filtration, denoted by \( \left( \frac{N}{N_o} \right)_f \) is equivalent to the overall extent of solids removal, \( \left( \frac{C}{C_o} \right) \), i.e. It is possible to assume that

\[ \left( \frac{N}{N_o} \right)_f = \frac{C}{C_o} \]  

(79a)

Combining equations 10a and 79a yields

\[ \frac{C}{C_o} = \frac{\sigma}{\sigma_u} = \left( \frac{N}{N_o} \right)_f \]  

(79b)

Combining equations 78 and 79 the net number of primary particles at time t, and depth x, may be presented as follows:

\[ N = N_o \left\{ e^{0.7ax} \left( \frac{e^{ax} + e^{bt}}{e^{bt} - 1} \right)^{0.3} \right\}^{-K_7} e^{-K_7 f(x,t)} \]

\[ \left\{ \frac{e^{bt} + 1}{2e^{ax} + e^{bt} - 1} \right\}^2 \]  

(80)
where

\( K_f \) will be given the new symbol \( K_r \), and called the flocculation constant.

\[
K_r = (67.58) \frac{\alpha v \mu s}{K_0 S_p s_o} \frac{(1 - \varepsilon_o)^{1/2}}{\varepsilon_o^{5/2}}
\]

However, at the initial stage of filtration, i.e., at \( t = 0 \) the following boundary condition exists:

at \( t = 0, x = L \)

\( C = C_1, N = N_1, f(x, t) = 0 \)

Substituting in equation 80 results in

\[
\frac{N_1}{N_0} = \left\{ e^{-0.7ax - 0.3ax} \right\}^{K_f - 2ax} - \left( (0.7 + 0.2) K_f - 2 \right) e^{-ax}
\]

or

\[
\frac{N_r}{N_1} = e^{2 + K_f}
\]

\[
\ln \frac{N_r}{N_1} = ax \left[ 2 + K_f \right]
\]

\[
K_f = \frac{\ln \frac{N_0}{N_1}}{ax} - 2
\]
Substitute $x = \ln \frac{C_0}{C_1}$ from equation 40.

$$K_7 = \frac{\ln \frac{N_0}{N_1}}{\ln \frac{\sqrt{C_0}}{\sqrt{C_1}}} - 2$$

$$= \ln \left( \frac{N_0}{N_1} - \frac{\sqrt{C_0}}{\sqrt{C_1}} \right) - 2$$  \hfill (81)

As previously mentioned, the process of bed contact flocculation involves three main stages. These are the early breakthrough stages, the active working stage, and finally, the steady state stage. During the steady state stage, the bed ceases to remove any more particles and the rate of disappearance of primary particles would only depend on the orthokinetic flocculation mechanism. Assuming that the deposit content per unit bed volume during the steady state stage reaches an ultimate value, $\sigma_u$, the rate of primary particle disappearance, equation 73, becomes:

$$\frac{-dN}{dx} = N \left[ K_4 \frac{(1 - \varepsilon_o + \sigma_u)^{1/6}}{(\varepsilon_o - \sigma_u)^{3/2}} + 0 \right]$$

$$= N K_2' \tilde{G}_u$$  \hfill (82)

where $\tilde{G}_u$ is the ultimate value of the root mean square of velocity gradient. Solution of equation 82 is:
\[ \log \frac{N}{N_0} = -K_2' \bar{G}_u x \]

i.e. \[ N = N_0 e^{-K_2' \bar{G}_u x} \] (83)

For a given total bed depth, \( L \), the ratio of effluent to influent primary particle concentration becomes:

\[ \frac{N}{N_0} = e^{-K_2' \bar{G}_u L} \] = constant (84)

Equation 84 depicts the steady state stage of the primary particle disappearance model. The extent of contact flocculation increases with \( \bar{G}_u \). Thus, by reviewing equation 61a for \( \bar{G} \), it is apparent that the extent of contact flocculation increases with an increase of bed depth, \( L \), approach velocity, \( V_s \), and ultimate deposit content, \( \sigma_u \), and the decrease in media grain size, \( \phi_s \). However, there is an upper limit to the approach velocity, \( V_s \), and a lower limit to the media size, \( \phi_s \), beyond which floc break-up becomes a significant factor.
The general purpose of the experimental work was to provide exper­
imental verification of both the filtration and contact flocculation 
models. Experimental proof of the filtration model was developed by 
comparing the experimental and the theoretical filtration breakthrough 
curves; whereas, the verification of the contact flocculation model was 
attempted by comparing the measured and predicted changes in the 
number of smallest detectable particles in suspension. In general, the 
experimental runs included in the current study involved filtration 
runs through a deep bed of coarse sand bed at constant flow rate and 
inlet suspension concentration. Flow approach velocity, inlet suspen­sion concentration and media size were generally higher than their 
common values in water filtration for two reasons: First, to force 
breakthrough to occur shortly after the start of the filter run, which 
would hopefully be followed by the proposed steady state breakthrough 
condition. Second, to prove the occurrence of bed flocculation under 
those extreme operation conditions.

The parameters measured during each run included turbidity 
measurements, pressure readings and particle size count with reference 
to bed depth and filtration time. Pressure readings were recorded at 
small increments of bed depth, while samples for turbidity and particle 
count were collected at the inlet and outlet of the filter. In some 
filter runs, several filters of different depth were operated in paral­
lel to obtain filtrate quality as a function of depth. Both flow rate
and influent concentration were frequently monitored and maintained constant.

Pilot Plant Apparatus

The laboratory pilot plant which was used in the filtration and bed contact flocculation study consisted of two 2700 liter suspension storage tanks with recirculation pump and feed pump, a chemical feed pump, a 70 liter polymer solution container and a bank of five filters equipped with necessary valves, sampling taps and flow meters. A layout diagram of the pilot plant is given in Figure 5.

**Raw water tank**

The two 2700 steel suspension tanks used during the study were manufactured by Gabel Tank Company. To keep the suspension homogeneous, a propeller type lighting mixer, Model No. ND2V was used in each tank. The mixers were manufactured by Mixing Equipment Inc., Rochester, New York. Each mixer had a rated HP of 0.5 and speed of 1800 rpm.

**Recirculation pump**

The pump used to recirculate the raw water through the constant head chamber was a submersible pump Model No. SE12N manufactured by the Little Giant Pump Company of Oklahoma City, Oklahoma.

**Chemical feed pump**

A positive displacement chemical feed pump was used in this study. Polymer solution was fed with a chem-feed, Model No. C-614P, manufactured by Presto-Tek Division, Devon Products Corporation of Los Angeles, California. The polymer feeder operates under a maximum pressure of 100
Fig. 5. Schematic diagram of experimental filtration system  
(P = Pump; ∅ = Valve)
psi and maximum feed rate is 0.6 GPH.

**Cartridge filter**

The cartridge filter used was a stainless steel, single shell housing, filter Model No. BRX10 3/4SD manufactured by Fulflo Filters, DIV Carborundum, Lebanon, Indiana 46052. Disposable cotton honeycomb tubes were used inside the filter housing to remove particles down to sub-micron range. The maximum design operating pressure and flow rate of this unit were 150 psi and 10 GPM.

**Filter columns**

The granular bed filter housings were five 3.0 inch I.D. plexiglass columns. A 3.0 inch diameter 50 mesh stainless screen was used to support the media in each column. The five filters denoted A, B, C, D, E, contained 24, 20, 16, 12, 8 inches of identical media respectively. A free board, 20 inches high, was maintained in each filter to allow for bed expansion during backwashing.

The sand medium used in the five filters was that which was retained on a U.S. standard sieve No. 8 and which passed through U.S. standard sieve No. 7. The average media size thus calculated was 2.595 mm.

Tygon tubing piezometers were installed at the top and bottom section of each filter. Additional piezometers on filter A were spaced at 1.5 inches vertical intervals for the full depth of the media and descended on a spiral curve surrounding the filter boundary.
Equipment

Turbidimeter

The turbidimeter used was the Hach 2100 A Model manufactured by Hach Chemical Company of Ames, Iowa. Very low turbidities were measured after standardizing the instrument with a standard of 0.61 NTU and at higher turbidities 10,100 NTU standards were used, all obtained from Hach Chemical Company.

pH meter

Measurements of water pH were made using a Beckman Expandomatic IV pH meter manufactured by Beckman Instruments, Inc., of Irvine, California. The meter was calibrated using pH 4.0 and 7.0 standards at frequent intervals.

Particle dispersion unit

A high-speed commercial colloid dispersing unit was used to disperse and mix the kaolin or minusil materials in water. The unit used was a SD45N Super Dispax with G454 generator manufactured by Tekmar Company of Cincinnati, Ohio. This unit has a maximum speed of 10,000 rpm and a suction head exerting high hydrodynamic shear forces.

The unit is applicable for mixing different suspensions and organic solvents. It is capable of providing a reproducible dispersion of particulate materials.

Jar test apparatus

The jar test machine used to determine the polymer type and dose was a Phipps-Bird 6-place flocculator manufactured by Phipps & Bird,
Inc. of Richmond, Virginia. The maximum speed of the flocculator paddles is 100 rpm. A floc illuminator base was used for easier viewing of floc size. Glass battery jars with a capacity of approximately 2 liters were used in the tests.

**Particle size distribution analyzer**

The particle counter used to determine the particle size distribution of kaolin and minusil materials is a Hiac Model PC-320 particle size analyzer with an automatic digital printer for recording particle size data, and a 1.0 to 60 μm particle sensor. The counter is manufactured by Pacific Scientific Company of Montclair, California.

The particle count analyzer depends on the light blockage techniques and the results are displayed in 12 separate channels. With the aperture used in this research, the size range in μm in each of these twelve channels is as follows: 1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-9, 9-13, 13-17, 17-20, 20-25, 25-60. Particles in fluid suspension flow through a channel and past a window whose area is known accurately. A collimated light beam is directed through the fluid at right angles to the direction of fluid flow. The light passes through the fluid to fall on a photodiode. The output of the photodiode goes to a preamplifier in the sensor and then to the main particle counter. There, a peak level detector generates a signal which controls a servo loop and maintains the sensor preamplifier output to a constant reference voltage when there is no particle in the beam.

When a particle passes across the beam, it partially blocks the light and reduces the area through which light falls on the photodiode.
The pulse generated by the change in the amount of light recorded on the diode is proportional to the projected area of the particle, and the particle size is specified in terms of equivalent spherical diameter, a sphere with the same projected area as the particle. Electronic circuiting is then used to count the number of particles within given size ranges that are present on the counter.

Experimental Procedure

Suspension preparation

Two different suspensions were used for this study. One, mostly a kaolinite clay, was Kentucky No. 5 Ball Clay, a product of Old Hickory Clay Co., Paducah, Kentucky. The second, a crushed quartz, was Berkeley 30 Micron Minusil, produced by Pennsylvania Glass Sand Corporation, Pittsburgh, Pennsylvania. Tables 2 and 3 show the particle characteristics of kaolinite clay and minusil, and the particle count data of the minusil is presented in Figure 6.

Initially, in the experimental investigation, it was mistakenly assumed that the smallest size particles in suspension were 1-2 µm. It was the intention to monitor the disappearance of the 1-2 µm particles to verify the flocculation model. However, in looking at the particle counting results for the early filtration runs it appeared that the filter effluent had a larger number of 1-2 µm size particles than the influent. Thus, the ratio of effluent to influent was greater than 1. Logically, with filtration there should be a decrease in the smallest-size particles. Therefore, it was hypothesized that there must be large
Fig. 6. Minusil particle size distribution
number of sub-micron (≤ 1 μm) particles which flocculated to produce the increase in 1-2 μm particles. This hypothesis was supported by the results of a TEM (Transmission Electronic Microscope) enumeration of the influent and effluent samples. Because these submicron particles were undetectable by the HIAC, the particle counting data for the experimental runs was not used to verify the flocculation model during the initial stages of the filter run. However, the collected data were useful in demonstrating the occurrence of the steady state stage of contact flocculation. Only the preparation procedures and particle count analysis of minusil are presented in the next section. The clay data are not presented because the TEM evaluation revealed to large a number of sub-micron particles.

Routinely before each run, the two large suspension tanks were carefully cleaned to avoid contamination by any extraneous particles. The tanks were then filled with distilled water which had been purified further by cartridge filtration to protect against occasional high particulates in the distilled water supply. Each time a new filter cartridge was used, the distilled water filtrate obtained in the first 30 minutes was discarded because of its high particle count, compared to that of distilled water. The results of particle count of the filtered distilled water are presented in Appendix III, Table 1.

Batches of minusil, each weighing 500 gm were soaked in water and vigorously dispersed in two-liter plastic jars using the SD45N super Dispax unit. The speed of mixing was increased from zero to 6000 rpm within 10 seconds, and kept constant for another two minutes. The
minusil suspension was then transferred to the elutriation column to separate particles one micron or larger. An upflow of 0.42 cm/minute was maintained throughout the elutriation system. Sodium hexametaphosphate was fed to the inlet water at a rate of 0.1 mg/liter to prevent particle agglomeration. Details of minusil elutriation are presented in Appendix IV.

Each elutriation run lasted two hours, after which the contents of the middle third of the elutriation column was transferred to the large suspension tank and the propeller mixer was kept running at 60% of its maximum speed. Several elutriation runs were performed and the elutriate was added to the suspension tank until the average turbidity in the tank was about 50 NTU.

**Preparation of polymer solution**

The polymer used in the filtration of minusil was a highly cationic LT-22 powdered polymer manufactured by Allied Colloids, Inc. The polymer stock solution was prepared fresh on each day a filter run was being conducted. The amount of dry powdered polymer, in grams, required per day was calculated as follows:

\[
\text{weight of polymer/day} = AD V_s \left(3.785 \frac{\text{gal}}{\text{min}}\right)\left(1440 \frac{\text{min}}{\text{day}}\right)\left(10^{-3} \frac{\text{gm}}{\text{mg}}\right) = W_p \text{ gm/day}
\]

where

\[
V_s = \text{filtration flow rate in gpm/ft}^2.
\]

\[
A = \text{filter cross sectional area in ft}^2.
\]
\[ D_s = \text{polymer dose mg/l.} \]
\[ W_p = \text{weight of polymer, in grams, per day.} \]

The amount of dry polymer required per day was weighed and very slowly sprinkled into the vortex of 1000 ml of distilled water which was being magnetically stirred. The polymer addition took place over a period of approximately 30 minutes to prevent the formation of hard-to-dissolve gelatinous "fish eyes". The solution was continuously mixed over night until all the dry powder had been dissolved. The polymer stock solution was then transferred to a 70 liter polymer tank and distilled water was added until the 40 liter mark was reached. The polymer feed pump was subsequently adjusted to maintain a feed rate of 1.67 l/hr or 40 l/day.

**Influent and effluent sampling**

Before the start of each run, a stock of 300 ml glass sampling bottles was soaked in a chromic acid bath for at least one hour to prevent any particle or bacterial contamination. The sampling bottles were then washed with distilled water and kept dry with Saran Wrap underneath the lids. The bottles were marked to indicate the Run No., Sample No., and Filter No.

Samples of influent and effluent were collected every half-hour to one-hour intervals for both turbidity and particle counting measurements. The influent samples were obtained from a sampling cock located in the influent pipe of each filter. The effluent samples were obtained from the effluent pipe of each filter downstream from the rotameters.
Table 2. Kaolinite clay particle characterization

<table>
<thead>
<tr>
<th>Typical Chemical Analysis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon dioxide</td>
<td>55.86%</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>30.38%</td>
</tr>
<tr>
<td>Iron oxide</td>
<td>0.99%</td>
</tr>
<tr>
<td>Titanium dioxide</td>
<td>1.40%</td>
</tr>
<tr>
<td>Calcium oxide</td>
<td>0.11%</td>
</tr>
<tr>
<td>Magnesium oxide</td>
<td>0.05%</td>
</tr>
<tr>
<td>Sodium oxide</td>
<td>0.08%</td>
</tr>
<tr>
<td>Potassium oxide</td>
<td>0.17%</td>
</tr>
<tr>
<td>Ignition loss</td>
<td>10.57%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Typical Particle Size Analysis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>% minus 20 microns</td>
<td>98.1</td>
</tr>
<tr>
<td>% minus 10 microns</td>
<td>93.8</td>
</tr>
<tr>
<td>% minus 5 microns</td>
<td>85.0</td>
</tr>
<tr>
<td>% minus 2 microns</td>
<td>67.4</td>
</tr>
<tr>
<td>% minus 1 microns</td>
<td>57.5</td>
</tr>
<tr>
<td>% minus 0.5 microns</td>
<td>47.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raw Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude color</td>
<td>white</td>
</tr>
<tr>
<td>pH (28%)</td>
<td>5.7</td>
</tr>
<tr>
<td>Filtration rate (ml)</td>
<td>26</td>
</tr>
<tr>
<td>Soluble sulfate (ppm)</td>
<td>114</td>
</tr>
</tbody>
</table>

The above data were provided by the Old Hickory Clay Company, Paducah, Kentucky.
Table 3. Minusil particle characteristics

<table>
<thead>
<tr>
<th>Chemical name and synonyms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon dioxide</td>
</tr>
<tr>
<td>Crystalline silica</td>
</tr>
<tr>
<td>Quartz</td>
</tr>
<tr>
<td>Silica sand</td>
</tr>
</tbody>
</table>
| Trade Name                | Berkeley 30 Micron Min-U-Sil  
| Formula                   |  
| Silicon dioxide (SiO₂)    | 99.700% free silica  
| Iron oxide (Fe₂O₃)        | 0.023%  
| Aluminum oxide (Al₂O₃)    | 0.101%  
| Titanium oxide (TiO₂)     | 0.019%  
| Calcium oxide (CaO)       | Trace  
| Magnesium oxide (MgO)     | Trace  
| Specific Gravity          | 2.650  
| Boiling Point             | 4,046° F.  
| Refractive Index          | 1.547  
| pH                        | 7.0  
| Porosity                  | Non-porous  
| Bulking Value             | 22.144 lb/gal  
| Surface Area              | 0.04522 gal/lb  
| Average Particle Size     | 0.54 m²/gm  
| Apparent Bulk Density     | 8.8 microns  
| Moisture Content          | 63 lb/ft³  
|                           | 0.05% maximum  

The minusil data were provided by the Pennsylvania Glass Sand Corporation, Pittsburgh, Pennsylvania.
Application of the Filtration Model to the Experimental Data

The experimental verification of the filtration model was obtained by using both the author's experimental data and other investigators' data. Depending on the type of data collected, there are five different methods of calculating the filtration coefficients, \( a, b \) and hence the solution of the theoretical breakthrough curve equation 31.

The next section includes a brief discussion of each method. Further details on the use of each method as well as solved examples are presented in Appendix I.

First method: trial and error

Determine the values of the ratios \( \frac{C_i}{C_o} \) and \( \frac{C_f}{C_o} \) from the experimental filtration curve at the time \( t = 0, t_f \) respectively. Substitute the values of \( \frac{C_i}{C_o}, \frac{C_f}{C_o} \) in equations 40 and 42 to calculate the coefficients \( a, b \) respectively. The calculated values of \( a, b \) may be considered for an initial trial to conceive equation 31. If the difference between the theoretical and experimental breakthrough curves is too great, introduce incremental changes in \( a \) and/or \( b \) until an acceptable similarity between the two curves is reached.

This method was used to verify the experimental data of Peterson's (85) filtration runs No. 13, 14, 18 and 20.
Second method: filtrate quality data

If the quality of filtrate at several bed depths was monitored during a filtration run, it is possible to use the linear transformation approach presented in equation 45 to determine the coefficients $a, b$. The procedure involves intercepting the experimental filtration curves drawn at different bed depths, with a horizontal line at an arbitrary break-through level $\frac{C}{C_0}$. The points of interception represent the run length up to the breakthrough for each bed depth. Plot the relationship between bed depth $x$ and time to breakthrough $t$ which is approximately a straight line. The slope and intercept of this straight line are used to determine both $a, b$ based on equation 45 as follows:

$$b = \frac{-\ln\left(\frac{C}{C_0} + 1\right)}{2\text{Intercept}}$$

$$a = b \cdot \text{slope}.$$ 

The linear approximation approach was used in the author's run 7, Eliassen's (26) run 6, and Saatci's (89) run 31.

Third method: head loss data

If the head loss readings were recorded at several small increments of bed depth, it is possible to use equation 60a to solve for the specific deposit content, $\sigma$. The solution is rather tedious and time consuming; therefore, Computer Program No. 1 is recommended (Appendix II).

The entry data are piezometer readings and depths, clean bed porosity, $\varepsilon_0$, and flow rate, $V_s$. The output includes values of $\sigma$, $\sigma_u$, 


G, and total specific deposit content inside the entire bed for all values of filtration time used in the input. Plot the curve that relates the ratio $\frac{G}{G_u}$ vs. time of filtration for several values of bed depth. Intercept this group of curves at any level of $\frac{G}{G_u}$ with a horizontal line and determine the time, $t$, required to reach the value $\frac{G}{G_u}$ for each bed depth, $x$. The relationship between $x$, $t$ is approximated by a straight line. The values of coefficients $a$, $b$ may be calculated from the slope and intercept of the above linear relationship using equation 44 as follows:

$$b = \frac{-\ln \left( \frac{\sqrt{\frac{G}{G_u}} + 1}{2} \right)}{\text{Intercept}}$$

$$a = b \cdot \text{slope}$$

The only filtration data available that could be analyzed using this method were from the writer's runs 10 and 11.

Fourth method: extrapolating the data of other runs

If the data of one filtration run at a specific flow rate, $V_s$, and influent concentration, $C_o$, are known, it is possible to predict filtration curves of other runs at different values of $V_s$, $C_o$ and bed depth, using the same media size and filtration chemical pretreatment. This method involves determining the coefficients $a$ and $b$ for the known run using any of the above methods. The coefficients $a$ and $b$ for the other runs are calculated using equations 38 and 39 as follows:
This approach was used in Peterson's (85) runs 20-24 and runs 34a and 38 of Saatci (89).

Fifth method: extrapolating the data collected for one filter depth

Sometimes, if it is not possible to intercept more than two breakthrough curves with a horizontal line, the second method cannot be applied. In this case, it is better to determine the values of coefficients a and b using one curve for a specific depth. The same coefficients are used to produce filtration curves at any other depth using equation 31.

This approach was used for the analysis of Hegg's runs A-13 and 22, B-2, 4 and 15, Cleasby's (18) data presented in his Figures 3 and 6 and Hsiung's (43) data for run 40.

Saleh's runs 7, 10 and 11

Runs 7, 10 and 11 were particularly designed to investigate and verify both models of filtration and bed contact flocculation. In all runs the influent was a concentrated suspension of minusil mixed with a continuous dose of cationic polymer, Percol LT -22. Other filtration conditions were: flow rate, $V_s = 10 \text{ gpm/ft}^2$, sand media size, $\phi_{so} = 2.56$ mm, bed depths 24, 20, 16, 12, 8 inches for run 7 and 24 inch in runs 10, 11

\[
a_2 = a_1 \left( \frac{v_{s1}}{v_{s2}} \right)
\]

\[
b_2 = b_1 \left( \frac{c_{o2}}{c_{o1}} \right)
\]
<table>
<thead>
<tr>
<th>Investigator's Name and Year Published</th>
<th>Run No.</th>
<th>Filter Size (mm)</th>
<th>Sand Media Bed Depth (inch)</th>
<th>Influent Concentration (mg/l)</th>
<th>Suspension Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saleh, 1981</td>
<td>10</td>
<td>2.596</td>
<td>24</td>
<td>70.5 FTU</td>
<td>Minusil</td>
</tr>
<tr>
<td>Saleh, 1981</td>
<td>11</td>
<td>2.596</td>
<td>21</td>
<td>160</td>
<td>Minusil</td>
</tr>
<tr>
<td>Cleasby (18), 1969</td>
<td>Fig. 3</td>
<td>0.649</td>
<td>3, 5, 9, 24</td>
<td>20</td>
<td>Clay (calcium montmorillonite)</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>20</td>
<td>0.649</td>
<td>52</td>
<td>50</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>21</td>
<td>0.649</td>
<td>52</td>
<td>25</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>22</td>
<td>0.649</td>
<td>52</td>
<td>25</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>23</td>
<td>0.649</td>
<td>52</td>
<td>50</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>24</td>
<td>0.649</td>
<td>52</td>
<td>25</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>13</td>
<td>0.649</td>
<td>52</td>
<td>100</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>14</td>
<td>0.649</td>
<td>52</td>
<td>300</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Peterson (85) 1981</td>
<td>18</td>
<td>0.649</td>
<td>52</td>
<td>200</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>Hegg (41), 1968 A-13</td>
<td></td>
<td>0.649</td>
<td>1.5, 9</td>
<td>7.0</td>
<td>Iron floc from ferrous sulfate</td>
</tr>
<tr>
<td>Hegg (41), 1968 A-22</td>
<td></td>
<td>0.649</td>
<td>1.5, 9</td>
<td>7.0</td>
<td>Iron floc from ferrous sulfate</td>
</tr>
<tr>
<td>Hegg (41), 1968 B-2</td>
<td></td>
<td>0.649</td>
<td>1.5, 9</td>
<td>7.1</td>
<td>Iron floc from ferrous sulfate</td>
</tr>
<tr>
<td>Hegg (41), 1968 B-4</td>
<td></td>
<td>0.649</td>
<td>5, 9</td>
<td>3.5</td>
<td>Iron floc from ferrous sulfate</td>
</tr>
<tr>
<td>Hegg (41), 1968 B-15</td>
<td></td>
<td>0.649</td>
<td>1.5, 9</td>
<td>6.96</td>
<td>Iron floc from ferrous sulfate</td>
</tr>
<tr>
<td>Dose (mg/l)</td>
<td>Type</td>
<td>Flow Rate (gpm/ft²)</td>
<td>Chemical Pretreatment</td>
<td>Slope Intercept (hr/cept inch) (hr.)</td>
<td>Comments</td>
</tr>
<tr>
<td>------------</td>
<td>------</td>
<td>------------------</td>
<td>----------------------</td>
<td>-------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>1.0</td>
<td>Percol LT-22</td>
<td>10</td>
<td>$\frac{C}{C_0}$ = 0.4</td>
<td>0.5121 1.0011 0.1308 0.2554</td>
<td>3rd method</td>
</tr>
<tr>
<td>0.5</td>
<td>Percol LT-22</td>
<td>10</td>
<td>$\frac{C}{C_0}$ = 0.55</td>
<td>0.3197 0.0268 0.0534 0.167</td>
<td>3rd method</td>
</tr>
<tr>
<td>0.5</td>
<td>Percol LT-22</td>
<td>10</td>
<td>$\frac{C}{C_0}$ = 0.6</td>
<td>0.45 2.20</td>
<td>0.162 0.249</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>6.0</td>
<td>None</td>
<td>0.0425 0.1960</td>
<td>5th method</td>
</tr>
<tr>
<td>10</td>
<td>Alum</td>
<td>6.0</td>
<td></td>
<td>0.1</td>
<td>5th method</td>
</tr>
<tr>
<td>0.05</td>
<td>Percol LT-20 &amp; alum</td>
<td>5.0</td>
<td>$\frac{C}{C_0}$</td>
<td>0.1013 0.0684 0.2026</td>
<td>1st method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0</td>
<td>None</td>
<td>0.0684 0.1013</td>
<td>4th method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.5</td>
<td>None</td>
<td>0.0456 0.2026</td>
<td>Run 20 data</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.5</td>
<td>None</td>
<td>0.0456 0.2026</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>None</td>
<td>0.0342 0.1013</td>
<td>&quot;</td>
</tr>
<tr>
<td>15</td>
<td>Alum &amp; Percol LT-20</td>
<td>5.0</td>
<td>$\frac{C}{C_0}$</td>
<td>0.2549 0.0443</td>
<td>1st method</td>
</tr>
<tr>
<td>0.05</td>
<td>Percol LT-20</td>
<td>5.0</td>
<td>$\frac{C}{C_0}$</td>
<td>0.5945 0.0110</td>
<td>1st method</td>
</tr>
<tr>
<td>15</td>
<td>Alum &amp; Percol LT-20</td>
<td>5.0</td>
<td>$\frac{C}{C_0}$</td>
<td>0.5046 0.0205</td>
<td>1st method</td>
</tr>
<tr>
<td>15</td>
<td>Alum &amp; Percol LT-20</td>
<td>5.0</td>
<td>$\frac{C}{C_0}$</td>
<td>0.280 0.165</td>
<td>5th method</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>6.0</td>
<td>None</td>
<td>0.1515 0.1196</td>
<td>5th method</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>6.0</td>
<td>None</td>
<td>0.4298 0.075</td>
<td>5th method</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>6.0</td>
<td>None</td>
<td>0.3991 0.1134</td>
<td>5th method</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>3.0</td>
<td>None</td>
<td>0.5973 0.1561</td>
<td>5th method</td>
</tr>
</tbody>
</table>
Table 4. continued

<table>
<thead>
<tr>
<th>Investigator's Name and Year Published</th>
<th>Run No.</th>
<th>Filter Size (mm)</th>
<th>Sand Media Bed Depth (inch)</th>
<th>Influent Suspension Concentration (mg/l)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saatci (89), 1978</td>
<td>31</td>
<td>0.415</td>
<td>19.49, 17.32, 15.62, 13.58, 9.65</td>
<td>47.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>34a</td>
<td>0.415</td>
<td>19.88, 17.32, 15.28, 13.70, 9.84</td>
<td>77.4</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>38</td>
<td>0.415</td>
<td>19.69, 17.32, 15.75, 13.27, 9.84</td>
<td>81.4</td>
</tr>
<tr>
<td>Agrawal (3), 1966</td>
<td>-</td>
<td>0.5</td>
<td>2.5, 10, 20, 30</td>
<td>50</td>
<td>Algae</td>
</tr>
<tr>
<td>Eliassen (26), 1941</td>
<td>6</td>
<td>0.60</td>
<td>1.68, 4.68, 10.68, 16.68, 23.52</td>
<td>0.6 ppm Iron floe (hydrous ferric oxide)</td>
<td></td>
</tr>
<tr>
<td>Hsiung (43), 1967</td>
<td>40</td>
<td>0.649</td>
<td>1.5, 9</td>
<td>5.75</td>
<td>Iron flocs</td>
</tr>
<tr>
<td>Chemical Pretreatment</td>
<td>Flow Rate gpm</td>
<td>C\textsubscript{cor}</td>
<td>(\frac{\sigma}{\sigma_0})</td>
<td>Slope (hr/in)</td>
<td>Intercept (hr)</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------</td>
<td>----------------</td>
<td>-------------------------</td>
<td>-------------</td>
<td>----------------</td>
</tr>
<tr>
<td>1 ml (Cat Floc T polymer diluted with tap water to 2(\frac{1}{2}) ml)</td>
<td>10.21</td>
<td>(\frac{C}{C_0}=0.21)</td>
<td>0.0963</td>
<td>0.587</td>
<td>0.0698</td>
</tr>
<tr>
<td></td>
<td>10.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ferric sulfate and lime</td>
<td>2.0</td>
<td>(\frac{C}{C_0}=0.34)</td>
<td>4.2589</td>
<td>6.156</td>
<td>0.5130</td>
</tr>
<tr>
<td>None</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
influent concentration, \( C_0 = 200 \text{ mg/1}, 100 \text{ mg/1}, 70.5 \text{ NTU}, \) and polymer doses 0.5, 0.5, 1.0 mg/1 respectively.

In Run 7, filtrate samples were collected from all five filters for both turbidity and particle counting analysis. The results of this run were analyzed following the filtrate data linear transformation approach presented in the second method. The slope, \( b_1 \), and intercept, \( b_0 \), of the linear relationship, at breakthrough level, \( \frac{C}{C_0} = 0.6 \), were 0.463 hr/inch and 2.20 hr respectively. Consequently, the coefficients \( a, b \) of this run were determined as follows:

\[
\frac{\ln \left( \frac{1}{0.6} + 1 \right)}{2.2} = 0.0617 \text{ hr}^{-1}
\]

and

\[
a = b.b_1 = 0.0617 (0.463) = 0.0286 \text{ inch}^{-1}.
\]

The results of the data analysis for run 7 are presented in Table 5 and both the theoretical and experimental breakthrough curves are shown in Figures 7 and 8.

In both filtration runs 10, 11 the head loss measurements were used to determine the coefficients \( a, b \) following the third method of data analysis. Such information as the media equivalent diameter \( S\phi_{50} \), flow rate, \( V_e \), clean bed porosity \( \varepsilon_0 \), and piezometer readings were fed into the second computer program and the output included all values of \( \sigma, \bar{G}_a \) at each piezometer location, calculated according to the filtration model equations 51a and 61a.
Fig. 7. Experimental data of Saleh's run No. 7
Fig. 8. Comparison between the experimental and the theoretical breakthrough curves for Saleh's run 7.
Table 5. Regression analysis of run 7 to determine slope and intercept

<table>
<thead>
<tr>
<th>Time To Breakthrough (hr)</th>
<th>Bed Depth (inch)</th>
<th>x.t&lt;sub&gt;r&lt;/sub&gt;</th>
<th>Slope&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Intercept&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6</td>
<td>12</td>
<td>91.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.8</td>
<td>16</td>
<td>140.8</td>
<td>0.463</td>
<td>-2.20</td>
</tr>
<tr>
<td>11.5</td>
<td>20</td>
<td>230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.2</td>
<td>24</td>
<td>316.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma t_r = 42.1 \quad \Sigma x = 72 \quad \Sigma x.t_r = 778.8 \]
\[ \Sigma x^2 = 1376 \]

\[ a_{\text{slope}} = \frac{778.8 - \frac{(42.1)(72)}{4}}{1367 - \frac{(72)^2}{4}} = 0.463 \, \text{hr/inch} \]

\[ b_{\text{intercept}} = \frac{42.1 - 0.463(72)}{4} = -2.20 \, \text{hr} \]

In order to use the linear transformation approach presented in the third method of data analysis, curves of specific deposit ratio, \( \frac{\sigma}{\sigma_u} \), with respect to filtration time, \( t \), were calculated for five different bed depths 2.5, 7.0, 11.5, 16.0, 20.5 inches and are presented in Table 6.

Curves of \( \frac{\sigma}{\sigma_u} \) versus time were prepared from the data of Table 6, and a horizontal line at a breakthrough level of \( \frac{\sigma}{\sigma_u} = 0.4 \) was drawn as shown in Figure 9.

The breakthrough coefficient \( a, b \) were then calculated using method 3, and the theoretical and experimental curves of \( C/C_0 \) vs time
Fig. 9. Calculated curves of $\sigma/\sigma_u$ vs. bed depth for Salch's run No. 10
Table 6. Values of $\frac{\sigma}{\sigma_u}$ for different bed depths and filtration time as given in Saleh's run No. 10

<table>
<thead>
<tr>
<th>Time hr.</th>
<th>Bed Depth, Inches ($\sigma/\sigma_u$)</th>
<th>2.5</th>
<th>7</th>
<th>11.5</th>
<th>16</th>
<th>20.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>0.551</td>
<td>0.365</td>
<td>0.223</td>
<td>0.132</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.750</td>
<td>0.584</td>
<td>0.291</td>
<td>0.291</td>
<td>0.259</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.716</td>
<td>0.651</td>
<td>0.551</td>
<td>0.450</td>
<td>0.318</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.722</td>
<td>0.601</td>
<td>0.610</td>
<td>0.436</td>
<td>0.404</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.738</td>
<td>0.640</td>
<td>0.578</td>
<td>0.526</td>
<td>0.365</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.742</td>
<td>0.606</td>
<td>0.640</td>
<td>0.551</td>
<td>0.365</td>
<td></td>
</tr>
<tr>
<td>18.5</td>
<td>0.760</td>
<td>0.637</td>
<td>0.600</td>
<td>0.565</td>
<td>0.436</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.760</td>
<td>0.600</td>
<td>0.600</td>
<td>0.571</td>
<td>0.498</td>
<td></td>
</tr>
</tbody>
</table>

are shown in Figure 10.

The above solution required predetermination of $\varepsilon_0$, $S$, $\sigma_u$. The clean bed porosity, $\varepsilon_0$, was calculated as follows:

$$
\varepsilon_0 = 1 - \frac{\text{volume of solids/cm}^3 \text{ of bed volume}}{\text{weight of media present in the 8 in. bed}}
\frac{\text{(total volume of the 8" bed) (sp. gr. of sand media)}}{
\pi (3.0 \times 2.54)^2 (8.0) (2.54) (2.65 \text{ gm/cm}^3)}
$$

$$
= 1 - \frac{1117.33 \text{ gm}}{
\pi (3.0 \times 2.54)^2 (8.0) (2.54) (2.65 \text{ gm/cm}^3)}
$$

$$
= 0.445.
$$
Fig. 10. Comparison between the experimental and theoretical breakthrough curves for full 24 inch filter depth for Saleh's run 10
A sample of media grains was cleaned, dried and each particle was weighed individually. A random choice of about 20 sand grains was analyzed. The volume of each grain, $V_g$, was determined knowing the specific gravity of sand media as follows:

$$V_g \ (\text{in cm}^3) = \frac{\text{weight of grain in gm}}{2.65 \ \text{gm/cm}^3}$$

$$\text{grain radius } \phi_o = \frac{3\sqrt[3]{\frac{V_g}{\pi}}}{\phi_{so}}$$

For each grain the factor $S$ was calculated as follows:

$$S = \frac{\phi_o}{\phi_{so}}$$

where $\phi_{so}$ represents the one half of the average sieve size of the clean media. The values of $S$ for the sample grains were averaged and the mean value was considered for further computations.

The ultimate value of specific deposit, $\sigma_u$, used in the analysis of runs 10 and 11 was the maximum value of specific deposit computed for the top 2.0 inch media layer. The specific deposit values inside each bed layer were found to gradually increase until an ultimate value was reached, beyond which no changes occurred. Hypothetically, all bed layers were assumed to exhibit the same values of $\sigma_u$. To test this hypothesis, the filter runs should have been extended further. The results of the analysis of the experimental data of run 10 are presented in Table 7 and the comparison between the experimental and theoretical curves is shown in Figure 10.
Table 7. Analysis of Saleh's run 10

<table>
<thead>
<tr>
<th>Bed Depth x (inch)</th>
<th>Time to Reach $\frac{\sigma}{\sigma_u} = 0.4$ t, (hr)</th>
<th>X.t (hr/inch)</th>
<th>Slope (hr)</th>
<th>Intercept (inch$^{-1}$)</th>
<th>a (hr$^{-1}$)</th>
<th>b (hr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.2</td>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.8</td>
<td>26.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.5</td>
<td>4.5</td>
<td>51.75</td>
<td>0.396</td>
<td>-1.38</td>
<td>0.1616</td>
<td>0.3787</td>
</tr>
<tr>
<td>16</td>
<td>7.4</td>
<td>118.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.5</td>
<td>10.0</td>
<td>205</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma=57.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma x^2 = 863.75$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The root mean square of velocity gradient, $\bar{a}$, was calculated for various bed depths and filtration times and the resulting graphs are shown in Figure 11 and 12. As shown in Figure 11, the value of $\bar{a}$ through most of the bed ranged between 40-340 sec$^{-1}$. Also, after a few hours of filtration the contact flocculator maintained a stable mode of operation during which no significant changes in headloss of $\bar{a}$ took place. During this steady state the bed continued to produce large size flocs and a clear liquid in between the flocs.

The results of particle counting study involved several serious problems. The Minusil primary particles which were necessary to verify the contact flocculation model were smaller than 1 µm and were thus undetectable by the Hiac particle counter. Therefore, two methods of
Fig. 11. Values of $\bar{G}$, sec$^{-1}$ inside the filter bed at different bed depths during Saleh's run 10
Fig. 12. Values of $\bar{G}$ in sec$^{-1}$ inside the filter bed at different filtration time, $t$, in hours during Saleh's run 10
particle separation were tried in order to remove all undetectable submicron particles from the influent suspension. However, these two attempts at particle separation were not totally successful for several reasons, the details of which are listed in Appendix V.

In addition to the aforementioned shortcoming of the Hiac Particle Counter, the instrument was found to miscount the particles in samples higher than 1.0 mg/l concentration. The cause of such discrepancy was due to the shadowing of small particles by the large particles. The shadowing problem was extensively investigated in order to determine a sample dilution range within which shadowing did not occur. The details of the sample dilution study are presented in Appendix IV.

The problem with submicron particles was also faced in the preparation of distilled water for Minusil suspension dilution in the two large influent tanks. The university distilled water was processed through a cartridge filter without success in the removal of submicron particles. The results of distilled water cartridge filtration data are presented in Appendix III.

As a result of the obstacles to particle counting mentioned above, it was not possible to provide experimental proof for equation 89. However, the results of runs 10 and 11 provided experimental evidence supporting the occurrence of both in-bed flocculation of particles and steady state conditions. Substantial evidence of steady state stage were provided in the analysis of the data of both runs 10 and 11. In run 10 for example, all values of the parameters \( \frac{\sigma}{\sigma_u} \), \( \bar{c} \), \( H \) total, \( \frac{N}{N_0} \), and \( \frac{c}{c_0} \)
Fig. 13. Total bed head loss for Saleh's run 10
Fig. 14. Values of the ratio of influent to effluent particle count vs. filtration time for Saleh's run 10
Fig. 15. Breakthrough curves for the entire length of Saleh's run 10
Fig. 16. Calculated values of $\sigma/\sigma_u$ at different depths for Saleh's run 11
Fig. 17. Comparison of the experimental data of Saleh's run 11 with the theoretical breakthrough curve obtained using method three.
TOTAL BED HEAD LOSS (INCH WATER)

Fig. 18. Total bed head loss for Saleh's run II
approached upper limits towards the end of the run as shown in Figures 9, 11, 13, 14 and 15 respectively. (The data points of Figure 15 are the same as shown in Figure 10, but the line of Figure 15 is a visual best fit to the data.) The same conclusion was true for run 11 as shown in Figures 16, 17, and 18 which represent the relationships $\frac{C}{C_0}$ and $H_{(\text{total})}$ vs time of filtration respectively.

Perhaps what is more important is the relationship between $C$ and the variables $x$, $t$. As pointed out earlier the flow velocity gradient is a function of deposit content $\sigma$, and therefore it exhibits higher values in the filter top layers than deeper inside the bed. This is substantiated by the results of run 10 presented in Figures 11 and 12.

In the literature review section, Bhole and Mhaisalkar (8) mistakenly concluded the opposite, i.e., that the velocity of mixing inside granular media increased with the increase in bed depth.

Cleasby's data shown in his Figure 3 and 6 (18)

In Cleasby's data, four filter depths were used, 3, 5, 9, and 24 inches, and 0.649 mm sand was used for filtration of two types of clay, calcium montmorillonite and kaolinite, at a flow rate of 6.0 gpm/ft$^2$. In the first run, an influent clay concentration of 20 mg/l was filtered without any chemical pretreatment, while in the second run, a dose of 10 mg/l alum was added to an influent clay concentration of 20 mg/l.

The values of the coefficients $a$ and $b$ for each run were determined by trial and error (First Method), based on data from the four filters with the results presented in Tables 8 and 9. During the initial trial,
the experimental values of $\frac{C}{C_0}$ at times 0 and 8 hours were measured
and the coefficients of each filtration run were calculated as follows:

Table 8. Analysis of Cleasby's data presented in his Figure 3

<table>
<thead>
<tr>
<th>$x$ (in)</th>
<th>$\frac{C_i}{C_0}$</th>
<th>$a$ (in$^{-1}$)</th>
<th>$t_f$ (hr)</th>
<th>$\frac{C_f}{C_0}$</th>
<th>$b$ (hr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.60</td>
<td>0.056</td>
<td>8</td>
<td>0.80</td>
<td>0.171</td>
</tr>
<tr>
<td>9</td>
<td>0.44</td>
<td>0.048</td>
<td>8</td>
<td>0.72</td>
<td>0.253</td>
</tr>
<tr>
<td>24</td>
<td>0.14</td>
<td>0.041</td>
<td>8</td>
<td>0.40</td>
<td>0.260</td>
</tr>
</tbody>
</table>

*The ratio $\frac{C_i}{C_0}$ was obtained by extending the experimental breakthrough curve to intersect with the vertical axis at time = 0.

**Coefficients $a, b$ were calculated using equations 40 and 42 respectively.

Notes: Final values of $a, b$ were $a = 0.0525$ inch$^{-1}$, $b = 0.19603$ hr$^{-1}$.

Table 9. Analysis of Cleasby's data presented in his Figure 6

<table>
<thead>
<tr>
<th>$x$ (in)</th>
<th>$\frac{C_i}{C_0}$</th>
<th>$a$ (in$^{-1}$)</th>
<th>$t_f$ (hr)</th>
<th>$\frac{C_f}{C_0}$</th>
<th>$b$ (hr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.21</td>
<td>0.156</td>
<td>8</td>
<td>0.77</td>
<td>0.346</td>
</tr>
<tr>
<td>9</td>
<td>0.08</td>
<td>0.140</td>
<td>8</td>
<td>0.52</td>
<td>0.312</td>
</tr>
<tr>
<td>24</td>
<td>0.01</td>
<td>0.096</td>
<td>8</td>
<td>0.18</td>
<td>0.313</td>
</tr>
</tbody>
</table>

*Final values of $a$ and $b$ were $a = 0.135$ inch$^{-1}$ and $b = 0.3413$ hr$^{-1}$.

The initial values of $a$ and $b$ for each depth shown in Tables 8 and 9 for each run were increased or decreased by small increments until reasonable agreements between theoretical and experimental curves were obtained. These final values of $a$ and $b$ are presented at the bottom of Tables 8 and 9. Once the final values of these coefficients were determined, the
breakthrough curves for all filter depths were calculated and the results are shown in Figure 19 and 20.

**Peterson's runs 13, 14, 18, 20, 21, 22, 23 and 24 (85)**

Peterson's experimental work included several filtration runs through a 52 inch deep single media bed of 2.0 mm sand. Peterson collected only influent and effluent turbidity and head loss data during all of his filtration runs. In the first three runs (No. 13, 14 and 18), high turbidity influent was filtered through the bed and the extent of removal was monitored.

Since the chemical pretreatment applied during these three runs was not the same, it was not possible to predict the theoretical breakthrough curves of any run based on the data of the other runs. Accordingly, the trial and error approach presented in the first method was used to verify each run separately. The results of the trial and error analysis are presented in Table 10, and the theoretical and experimental filtration curves are shown in Figure 21.

**Table 10. Analysis of Peterson's runs 13, 14, 18 and 20**

<table>
<thead>
<tr>
<th>Run #</th>
<th>Initial Value of a</th>
<th>Initial Value of b</th>
<th>Final Value of a</th>
<th>Final Value of b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{C_i}{C_o}$ (inch$^{-1}$)</td>
<td>$t_f$ (hr)</td>
<td>$\frac{C_f}{C_o}$ (hr$^{-1}$)</td>
<td>$\frac{C_f}{C_o}$ (hr$^{-1}$)</td>
</tr>
<tr>
<td>13</td>
<td>0.01</td>
<td>0.04428</td>
<td>14.5</td>
<td>0.48</td>
</tr>
<tr>
<td>14</td>
<td>0.31</td>
<td>0.01126</td>
<td>3.5</td>
<td>0.73</td>
</tr>
<tr>
<td>18</td>
<td>0.12</td>
<td>0.02039</td>
<td>3.5</td>
<td>0.41</td>
</tr>
<tr>
<td>20</td>
<td>0.005</td>
<td>0.051</td>
<td>20</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Fig. 19. Comparison between the experimental and theoretical breakthrough curves for Cleasby's run presented in his Fig. 3
Fig. 20. Comparison between the experimental and the theoretical breakthrough curves for Cleasby's run presented in his Fig. 6
In the next five runs 20-24, the alum and polymer doses in mg/l of influent flow were the same, but the flow rate and influent suspension concentration were different. Therefore, the results of these runs were applicable to the fourth method of data analysis presented earlier. First, the coefficients a and b for run No. 20 were determined using the first method. Second, the coefficients a and b calculated for run 20 were used for the calculation of the coefficients of other runs using equations 38 and 39. The data of these runs are presented in Table 11 and all the theoretical and experimental filtration curves are shown in Figure 22.

Table 11. Analysis of Peterson's runs 20, 21, 22, 23 and 24

<table>
<thead>
<tr>
<th>Run #</th>
<th>(gpm/ft²)</th>
<th>(mg/l)</th>
<th>a (inch⁻¹)</th>
<th>b (hr⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>50</td>
<td>0.0684</td>
<td>0.20262</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>25</td>
<td>0.0684</td>
<td>0.20262(\frac{25}{50}) = 0.10131</td>
</tr>
<tr>
<td>22</td>
<td>7.5</td>
<td>25</td>
<td>0.0684(\frac{5}{7.5}) = 0.0456</td>
<td>0.20262(\frac{25}{50}) = 0.10131</td>
</tr>
<tr>
<td>23</td>
<td>7.5</td>
<td>50</td>
<td>0.0684(\frac{5}{7.5}) = 0.0456</td>
<td>0.20262</td>
</tr>
<tr>
<td>24</td>
<td>10</td>
<td>25</td>
<td>0.0684(\frac{5}{10}) = 0.0342</td>
<td>0.20262(\frac{25}{50}) = 0.10131</td>
</tr>
</tbody>
</table>

Hegg's runs A-13 and 22, B-2, 4 and 15 (41)

Both Series A and Series B experiments were part of Hegg's iron filtration study. The pilot filters contained 1.5 and 9 inches of 0.649 mm
Fig. 21. Comparison between the experimental and theoretical breakthrough curves for Peterson's Runs 13, 14, and 18
Fig. 22. Comparison between the experimental and theoretical breakthrough curves for Peterson's runs 20, 21, 22, 23 and 24
Table 12. Analysis of Hegg's filtration data

<table>
<thead>
<tr>
<th>Run #</th>
<th>Bed Depth (inch)</th>
<th>(\frac{C_i}{C_0})</th>
<th>Initial Value of (a) (inch(^{-1}))</th>
<th>(\frac{C_f}{C_0})</th>
<th>(t_f) (hr)</th>
<th>Initial Value of (b) (hr(^{-1}))</th>
<th>Final Value of (a) (inch(^{-1}))</th>
<th>Final Value of (b) (hr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-13</td>
<td>1</td>
<td>0.72</td>
<td>0.164</td>
<td>0.91</td>
<td>8</td>
<td>0.232</td>
<td>0.1650</td>
<td>0.2800</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.17</td>
<td>0.177</td>
<td>0.59</td>
<td>8</td>
<td>0.267</td>
<td>0.1650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.05</td>
<td>0.166</td>
<td>0.34</td>
<td>8</td>
<td>0.271</td>
<td>0.1650</td>
<td></td>
</tr>
<tr>
<td>A-22</td>
<td>1</td>
<td>0.75</td>
<td>0.144</td>
<td>0.94</td>
<td>12</td>
<td>0.182</td>
<td>0.1196</td>
<td>0.1515</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.26</td>
<td>0.135</td>
<td>0.72</td>
<td>8</td>
<td>0.285</td>
<td>0.1196</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.08</td>
<td>0.140</td>
<td>0.42</td>
<td>6</td>
<td>0.353</td>
<td>0.1196</td>
<td></td>
</tr>
<tr>
<td>B-2</td>
<td>1</td>
<td>0.85</td>
<td>0.163</td>
<td>0.95</td>
<td>6</td>
<td>0.285</td>
<td>0.0750</td>
<td>0.4298</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.40</td>
<td>0.092</td>
<td>0.81</td>
<td>6</td>
<td>0.375</td>
<td>0.0750</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.25</td>
<td>0.077</td>
<td>0.60</td>
<td>4</td>
<td>0.443</td>
<td>0.0750</td>
<td></td>
</tr>
<tr>
<td>B-4</td>
<td>5</td>
<td>0.31</td>
<td>0.117</td>
<td>0.77</td>
<td>7</td>
<td>0.335</td>
<td>0.1133</td>
<td>0.3991</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.14</td>
<td>0.109</td>
<td>0.61</td>
<td>7</td>
<td>0.342</td>
<td>0.1133</td>
<td></td>
</tr>
<tr>
<td>B-15</td>
<td>1</td>
<td>0.69</td>
<td>0.186</td>
<td>0.94</td>
<td>5</td>
<td>0.497</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.21</td>
<td>0.156</td>
<td>0.81</td>
<td>5</td>
<td>0.602</td>
<td>0.1561</td>
<td>0.6073</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.06</td>
<td>0.156</td>
<td>0.60</td>
<td>5</td>
<td>0.601</td>
<td>0.1561</td>
<td></td>
</tr>
</tbody>
</table>
sand. The rest of the information regarding the filtration parameters in Hegg's runs are presented in Table 4.

Since it was not possible to intercept all the $C/C_0$ curves for each depth with a single horizontal line, the linear transformation approach was rejected. The only possible way to verify the model using Hegg's experimental runs was that described in the fifth method of data analysis. A brief summary of the calculations used is presented in Table 12, and both the experimental and theoretical breakthrough curves are shown in Figure 23, 24, 25, 26 and 27.

**Saatci's runs 31, 34a and 38 (89)**

Saatci's filtration experiments were particularly designed to test the similarity between the phenomena of adsorption and filtration through granular media. Therefore, the author used a bank of five filters of the same media size but with depths of 25, 35, 40, 45 and 55 cm. The results of the filtration runs were then used to verify the author's linear transformation model which was very similar to Borhart Adam's (BDST) method.

The chemical pretreatments were the same in all of Saatci's runs, and it was desired to apply the fourth method of data analysis to predict the results of runs 34a and 38 from the data of run 31. First the linear transformation method (second method) was applied to the data of run 31 in Figure 28, to determine constants a and b (Table 13). Then a and b were modified using equations 38 and 39 and the modified values were used to calculate the theoretical breakthrough curves for runs 34a
Fig. 23. Comparison between the theoretical and experimental breakthrough curves for Hegg's run A-13
Fig. 24. Comparison of the theoretical and experimental breakthrough curves for Hegg's Run A-22.
Fig. 25. Comparison between the experimental and theoretical breakthrough curves for Hegg's run B-2
Fig. 26. Comparison between the experimental and theoretical breakthrough curves for Hegg's run B-4.
Fig. 27. Comparison between the experimental and theoretical breakthrough curves for Hegg's run B-15
and 38. However, the results were poor and are not presented.

Table 13. Saatci's run 31

<table>
<thead>
<tr>
<th>( \frac{C}{C_0} ) (cm)</th>
<th>( t ) (min)</th>
<th>Slope (min/cm)</th>
<th>Intercept (min)</th>
<th>( a ) (cm(^{-1}))</th>
<th>( b ) (min(^{-1}))</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.5</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.21</td>
<td>34</td>
<td>2.2737</td>
<td>35.23</td>
<td>0.02997</td>
<td>0.0132</td>
<td>0.927</td>
</tr>
<tr>
<td>49.5</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The lack of reasonable agreement between the two curves is due to the inherent approximation involved in the linear transformation approach as well as the questionable similarity between the five filters. In order to correct for the aforementioned discrepancies, the coefficients \( a \) and \( b \) were redetermined for run 31 by the first method, and the final values are presented in Table 14 along with the calculated values for runs 34a and 38. These constants were used to calculate the breakthrough curves for the three runs which are presented in Figure 29, 30 and 31.

Table 14. Analysis of Saatci's runs 34a and 38

<table>
<thead>
<tr>
<th>Run #</th>
<th>( V ) (m/hr)</th>
<th>( C_0 ) (mg/l)</th>
<th>( a ) (cm(^{-1}))</th>
<th>( b ) (min(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>25</td>
<td>47.9</td>
<td>0.02749</td>
<td>0.021</td>
</tr>
<tr>
<td>34a</td>
<td>25</td>
<td>77.4</td>
<td>0.02749</td>
<td>( \frac{77.4}{47.9} = 0.03232 )</td>
</tr>
<tr>
<td>38</td>
<td>30</td>
<td>81.4</td>
<td>( \frac{0.02749 \times 25}{30} = 0.02291 )</td>
<td>( \frac{81.4}{47.9} = 0.03569 )</td>
</tr>
</tbody>
</table>
Fig. 28. Experimental breakthrough curves for Saatci's run 31
Fig. 29. Comparison between the experimental and theoretical breakthrough curves for Saatci's run 31
Fig. 30. Comparison between the experimental and theoretical breakthrough curves for Saatci's run 34-A
Fig. 31. Comparison between the experimental and theoretical breakthrough curves for Saatci's run 38
Agrawal's filtration of algae through fine sand (0.5 mm) would probably result in substantial surface filtration which cannot be represented by the theoretical model which is intended to predict removal with depth. Agrawal's experiments showed higher than expected removal in the bed surface layer and lower removal at the deep layers. To demonstrate the accuracy of this hypothesis, Agrawal's experimental data were analyzed using the fifth method of data analysis and the results obtained for a and b are given in Table 15. The final values of a and b at the bottom of Table 15 were used to construct Figure 32 which shows the experimental and theoretical breakthrough curves. Figure 32 supports the hypothesis because the theoretical curves show less removal in the upper layers, and higher removal in the bottom layers.

Table 15. Analysis of Agrawal's filtration run

<table>
<thead>
<tr>
<th>Bed Depth, x (inches)</th>
<th>$\frac{C_i}{C_o}$</th>
<th>Coefficient, $a^a$ (inch $^{-1}$)</th>
<th>$t_f$ (hr)</th>
<th>$\frac{C_f}{C_o}$</th>
<th>Coefficient, $b^a$ (hr $^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.68</td>
<td>0.096</td>
<td>4</td>
<td>0.90</td>
<td>0.481</td>
</tr>
<tr>
<td>5</td>
<td>0.48</td>
<td>0.073</td>
<td>4</td>
<td>0.76</td>
<td>0.404</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
<td>0.069</td>
<td>4</td>
<td>0.68</td>
<td>0.532</td>
</tr>
<tr>
<td>20</td>
<td>0.10</td>
<td>0.058</td>
<td>4</td>
<td>0.61</td>
<td>0.667</td>
</tr>
<tr>
<td>30</td>
<td>0.005</td>
<td>0.088</td>
<td>4</td>
<td>0.50</td>
<td>1.034</td>
</tr>
</tbody>
</table>

$^a$The final values of a and b were 0.048 inch $^{-1}$, 0.510 hr $^{-1}$ respectively.
Fig. 32. Comparison between the experimental and theoretical breakthrough curves for Agrawal's filtration run.
Eliassen's run no. 6 (26)

Eliassen's run 6 involved filtration of 0.6 mg/l iron in the form of hydrous ferric oxide floc at 2 gpm/ft² through a bed of 0.6 mm sand media. Filtrate samples were collected at depths of 0.14, 0.39, 0.89, 1.39 and 1.96 ft in the same filter for iron measurements. Since the data included in Eliassen's run are applicable to the second analytical method, the slope and intercept of the linear relationship between time to breakthrough \( \frac{C}{C_0} = 0.34 \) and bed depth was determined as in Table 16. The theoretical and experimental breakthrough curves of this run are shown in Figure 33.

Table 16. Analysis of Eliassen's run 6

<table>
<thead>
<tr>
<th>Bed Depth X (ft)</th>
<th>( \frac{C}{C_0} = 0.34 )</th>
<th>( t_r ) (hr)</th>
<th>( t_r \times )</th>
<th>Slope (hr/ft)</th>
<th>Intercept (hr)</th>
<th>( a ) (ft⁻¹)</th>
<th>( b ) (hr⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>1.0</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.39</td>
<td>13.0</td>
<td>5.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.89</td>
<td>42.0</td>
<td>37.38</td>
<td>48.904</td>
<td>8.092</td>
<td>1.8471</td>
<td>0.03777</td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>62</td>
<td>86.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.96</td>
<td>95</td>
<td>186.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \Sigma x = 4.77 \quad \Sigma t_r = 213 \quad \Sigma t_r \times = 314.97 \)
\( \Sigma x^2 = 6.7375 \)

It is apparent from Figure 33 that a similar surface removal problem existed in Eliassen's work, similar to that discussed previously in Agrawal's work. This might have been expected due to Eliassen's low
Fig. 33. Comparison between the theoretical and experimental breakthrough curves for Eliassen's run 6.
filtration rate.

In order to show the weakness of the turbidity linear transformation approach, Eliassen's run 6 was analyzed using the trial and error method. The results of this analysis are presented in Table I-9 in Appendix I, and a comparison between the experimental and theoretical breakthrough curves developed according to the trial and error method is shown in Figure 34. As shown in both Figures 33 and 34 the linear transformation approach provided worse results.

Hsiung's run 40 (43)

Hsiung's run is another example of iron filtration through sand media. The run included the following filtration conditions: flow rate of 3.0 gpm/ft², influent concentration of 5.75 mg/l iron floc as Fe, media size of 0.649 mm, and bed depths of 1, 5 and 9 inches. The linear transformation approach (Second Method) could not be applied since it was not possible to intercept all of the C/C₀ breakthrough curves with one horizontal line. Consequently, the fifth method of data analysis was adopted in the analysis of Hsiung's run 40. The results are presented in Table 17 and the theoretical and experimental curves are shown in Figure 34.

Table 17. Analysis of Hsiung's run 40

<table>
<thead>
<tr>
<th>Bed Depth (inches)</th>
<th>Cᵢ/C₀ (in)⁻¹</th>
<th>a (hr⁻¹)</th>
<th>tᵣ (hrs)</th>
<th>Cᵢ/C₀ (hr⁻¹)</th>
<th>b (hr⁻¹)</th>
<th>Final Values of Coefficients a (in)⁻¹</th>
<th>b (hr⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.490</td>
<td>0.3567</td>
<td>9.5</td>
<td>0.83</td>
<td>0.2159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.020</td>
<td>0.3912</td>
<td>9.5</td>
<td>0.29</td>
<td>0.2714</td>
<td>0.300</td>
<td>0.220</td>
</tr>
</tbody>
</table>
Fig. 34. Comparison between the experimental and theoretical breakthrough curves for Eliassen's run No. 6 using the trial and error approach.
Fig. 35. Comparison between the experimental and theoretical breakthrough curves for Hsiung run No. 40.
RESULTS AND CONCLUSIONS

The phenomena of particle filtration and contact flocculation occurring simultaneously inside a porous media bed are too complex to be precisely described mathematically. However, as in all theories, analysts consider certain assumptions to simplify the process. The validity of the assumptions and the accuracy of the theory may then be challenged by requiring sound experimental evidence.

The contemporary theories of filtration and contact flocculation are either too complex to be useful for further practical applications or too limited to accommodate all possible operating conditions. Design engineers who are engaged in filtration technology do not always fully understand the theoretical processes and are seldom able to utilize current filtration theories. So far, filter design is based on rules of thumb or requires extensive pilot experiments for each application. Because of the time factor, expense, and human resources involved in pilot plant studies, the experimental approach is not always the best solution. Ultimately, for design purposes, a filtration model should exhibit the following characteristics:

1. Easy to understand
2. Simple to apply
3. Require the least amount of experimental investigation
4. Flexible enough to account for all possible filtration conditions.

In the current study, a great deal of attention was devoted to fulfill the aforementioned characteristics. First, several assumptions
about the flow, bed geometry, particle mode of deposition, and process
mechanisms were required to reach an analytical solution. These
assumptions may be briefly restated as follows:

1. The flow is laminar and is best described in light of a capil-
lary tube model.

2. The filter bed is homogeneous and may be represented by a
group of capillary channels of equal size, and the media shape factor,
and tortosity do not significantly change during filtration.

3. The suspension particles uniformly coat the inside walls of
the capillaries, and the deposits retain the same self porosity through­
out the filter run.

4. The mechanisms of particle detachment, perikinetic flocculation,
and primary particle break-up are not significant and may be ignored.

The theoretical model was challenged using a wide range of experi­
mental conditions. In summary, the experimental data used to evaluate
the model included 24 runs of eight different investigators and involved
seven different types of suspension at influent concentrations ranging
from 0.6 to 300 mg/l. Filter media size range was 0.50 to 2.50 mm and
bed depths were 1 to 52 inches. Flow rates varied from 2 to 10 gpm/ft².
Five different types of chemical treatment were applied to one or more
of the various suspensions prior to filtration. The run length ranged
from 2 to 96 hours of filtration.

The results of filtration experiments were analyzed using the fil­
tration model by five different methods, depending on the type of data
available. The five methods varied in their accuracy, advantages and
disadvantages.
The first and fifth methods involve a trial and error approach based on the information obtained from one curve depicting filtrate quality versus time during a filter run. These methods are the easiest and shortest methods to use.

The second method requires data on the filtrate quality at different filter depths and filtration times. The linear transformation method presented in equation 45, is required to solve for the unknown coefficients. This method not only involves an approximation in the relationship between bed depth and time to reach breakthrough, it also requires the collection of a large amount of filtrate data at different bed depths. If the samples of filtrate are collected from the same filter, there is an inherent risk of disturbing the filter bed during the sampling time. On the other hand, if the samples are collected from several filters of different depths which were supposed to resemble one main bed, the problem of maintaining perfect similarity might be too difficult to resolve.

The advantage of the linear transformation is its useful assistance for design purposes. Based on the relationship given in equation 45, it is possible to determine the bed depth that is required to furnish filtered water of turbidity less than or equal to the breakthrough limit for a specific length of time. Or, it might be used to determine the run length until breakthrough for a certain bed depth. Figures 1 and 2 show the relationship between time to breakthrough and bed depth for both specific deposit, and filtrate quality data respectively.

The relationship between run length on one hand and the variables $C_0$, $V_s$, $K_0$, $\sigma_u$ on the other hand may be better understood based on the
information given in equations 44 and 45. The slope of the linear
equations 44 and 45 increases, and consequently the run length increases
if \( C_0 \) or \( V_s \) decrease or if \( \sigma_u \) increase. Also, if the attachment
coefficient, \( K_o \), increases, the y intercept of the straight line de­
creases toward zero (becomes less negative) and consequently the run
length increases. The effect of \( C_0, V_s, K_o \) and \( \sigma_u \) on the linear trans­
formation relationship is presented in Figures 36, 37, 38, and 39
respectively.

The third method used to evaluate the model is based on measure­
ments of head loss in the filter bed at small intervals of bed depth.
Because head loss measurements can be very accurate and do not cause
any disturbance to the process of filtration, this method provides a
great deal of reliability and convenience. The only disadvantage to this
approach is the lengthy solution and the need for computer assistance.

The fourth method requires the use of equations 38 and 39 and the
data of one filtration run to predict the theoretical breakthrough
curves of any other filtration runs using the same filter and same
chemical pretreatment. This method is very simple and useful in the
design of filtration units without the need for extensive experimental
work. Neither the third nor the fourth method has been cited previously,
and are therefore considered as original developments in the area of
filtration model applications.

The diversity of data processing techniques not only made the
model easy to understand but also simplified the model to the extent
Fig. 36. $C_0$ vs. run depth

Fig. 37. $V_s$ vs. run length
Fig. 38. $k_0$ vs. run length

Fig. 39. $\sigma_u$ vs. run length
that a design engineer without prior experience in the use of mathematical filtration models, can fully utilize the theory. As previously pointed out, it is not necessary to collect massive and hard-to-get experimental data in order to use the filtration model. Influent and effluent turbidity readings of a shallow filter alone are enough to predict the breakthrough curves of deeper filters. However, collection of head loss readings with respect to bed depth and time of filtration becomes necessary if the root mean square of the velocity gradient, $\bar{G}$, and specific deposit content are to be calculated.

Perhaps the best and most useful outcome of the current mathematical model is the ability to predict the process variables such as filtrate quality, head loss, flow velocity gradient, and specific deposit content based on filtration data collected for different bed depths, flow rate, and influent concentration. This conclusion was substantiated with experimental evidence presented in the analysis of both Peterson's (85) and Saatci's (89) filtration runs. Thus, for design purposes according to the filtration model, a minimal number of runs performed with a small pilot filter should be sufficient. The significant reduction in the number of filtration experiments, the number of pilot filters and depths, and the necessary filtration run lengths will optimize to a great extent the use of pilot plants for both research and design purposes.

The use of the head loss data to predict the breakthrough as described in the third method gave good predictions in Saleh's runs 10
and 11. This would indicate that the head loss model, Equation 60a is a valid model. This model could be used in the reverse manner using the breakthrough curve \((C/C_0)\) to calculate the coefficients \(a\) and \(b\), and then \(\sigma\) as function of \(x\) and \(t\), equation 30, and finally using equation 60a to get the head loss for any small depth increment. The total bed head loss at any time would be the summation of the head loss increments for each layer.

Among the problems encountered in the verification of filtration models is the conversion of influent or effluent concentrations in common units such as mg/1 or TU into volume/volume for the purpose of determining specific deposit. The proposed filtration model provides an approximate way to calculate this conversion factor, \(K_f\). Knowing the values of coefficients \(a = \frac{K_0 \sigma_u}{2V_s}\) and \(b = 8 K_0 C_o\) it follows that:

\[
\frac{a}{b} = \frac{\sigma_u}{16V_s C_o}
\]

Therefore,

\[
C_o = \left[ \frac{b}{16aV_s} \right] \sigma_u \quad (85)
\]

In order to render equation 85 dimensionally correct, a conversion factor, \(C_1\), the value of which is unity and its units are mg/1 per vol/vol or TU per vol/vol is necessary, i.e.

\[
C_o = \left[ \frac{C_1 b}{16aV_s} \right] \sigma_u \quad (85a)
\]
Equation 85a was verified with reasonable agreement using the results of runs 10 and 11 and the calculations are presented in Appendix I. The verification of equation 85a required calculation of both the total mass of solids removed inside the entire bed between an interval of time, $\Delta t$, and the increase in total deposit inside the bed for the same interval of time, i.e.

\[
V_s \int_{t_1}^{t_2} (C_o - C) \, dt = K_f \left[ \int_0^L \sigma dx \right]_2 - \left[ \int_0^L \sigma dx \right]_1
\]

\[
V_s (t_2 - t_1) \frac{(C_o - C)_2 + (C_o - C)_1}{2} = K_f \left[ \Delta \sigma_2 - \Delta \sigma_1 \right]
\]

where $\Delta \sigma_1$, $\Delta \sigma_2$ are the total deposit volume in the full bed at the moments $t_1$ and $t_2$ respectively, and $K_f$ is a dimensionless conversion factor and its average value is equal to \( \frac{C_1 b}{16a V_s} \), as given in equation 85a.

The filtration model presented herein provides both analytical and computer solutions to the process of filtration and contact flocculation. The ratio of particle removal, \( \frac{C}{C_0} \), specific deposit content, $\sigma$, and primary particle disappearance, \( \frac{N}{N_0} \), as functions of both filter depth and time of filtration were presented in equations 30, 31, and 80.
respectively.

The analytical solution presented in the current study was developed by solving a new continuity equation. The proposed equation is an alternative solution to the controversial issue of simultaneous particle attachment and detachment. Since the mechanism of detachment is only hypothetical and has not been substantiated with sound experimental proof, the writer is of the opinion that a filtration continuity equation should be free of any detachment terms. It is realistic to assume that during the process of filtration, particles are simultaneously subjected to two groups of forces: one force enhancing removal, equal to $K_s V_s (\sigma_u - \sigma)C$, and the other retarding removal, equal to

$$\left( \frac{K_p}{V_s} \left[ 1 - \frac{\sigma}{\sigma_u} \right] \right)^{-1}.$$ 

The rate of particle removal is thus the product of the two groups of forces as presented in equation 1.

The ratio of clogged bed to clean bed head loss, $\frac{H}{H_o}$, and the mean square root of velocity gradients $\bar{G}_a$, $\bar{G}_w$, as functions of specific deposit $\sigma$ were presented in equations 60a, 61a, and 62b respectively. Since specific deposit $\sigma$ is a function of both $x$ and $t$, it is possible to express $\frac{H}{H_o}$, $\bar{G}_a$, $\bar{G}_w$ as functions of $x$, $t$ by substituting for $\sigma$, as given in equation 30, into equations 60a, 61a, and 62b respectively.

As shown in equations 61a and 61b the value of $\bar{G}$ is a function of the filter media, flow variabilities, and the specific deposit ($\epsilon_o$, $\phi_{so}$
S, V, and C). G is proportional to \( V_s \) and \( C \), and inversely proportional to \( \varepsilon_0, \phi_{so}, S \). The numerical example presented in Table 1 clearly demonstrates the effect of each of the foregoing variables on the value of \( \bar{G} \) for typical filter media. A granular media filter bed of 1.0 mm diameter may show values of the flow weighted \( \bar{G} (\bar{G}_a) \) as high as 322 \( \text{sec}^{-1} \) when a flow rate of 8.0 gpm/ft\(^2\) is used and a degree of clogging at which \( \sigma = 0.3 \) is reached.

The value of \( \bar{G} \) inside a filter bed is a function of the degree of clogging as indicated by the deposit content, \( \sigma \). Generally speaking, at the start of a filter run the bed is clean and \( \sigma = 0 \); therefore, \( \bar{G} \) is at its lowest value. As the run proceeds, more volume of filtrate passes through the bed causing gradual increase in bed clogging, and consequently the value of \( \bar{G} \) increases. Towards the end of the run the filter bed reaches its ultimate capacity for removal and no more particle deposition takes place. At this stage \( \sigma = \sigma_u \) and \( \bar{G} = \bar{G}_{max} \) and no further increase in the value of \( \bar{G} \) occurs as shown in Figure 42.

Figures 40, 41 and 42 show the general behavior of \( \bar{G} \), at a given filter lamina, during a filter run with different filtration rate, grain size, or with specific deposit (\( \sigma \)).

It is interesting to note that equations 63 and 64 show that the rate of change of \( \bar{G} \) or \( \bar{G}_u \), with respect to bed porosity, \( \varepsilon \), is always positive and exhibits no peak values for \( 0 < \varepsilon < \varepsilon_o \). As previously stated, filter beds never reach a point at which the whole bed porosity is zero; therefore, values of \( \bar{G} \) or \( \bar{G}_u \) equal to infinity are never met in
Fig. 40. $G$ vs. time of filtration for two different filtration rates

Fig. 41. $G$ vs. time of filtration for two different grain diameters
Fig. 42. $G$ vs. $\sigma$ during a filter run
practice. However, as $\sigma$ reaches $\sigma_u$, bed porosity approaches the minimum value of bed porosity, $\varepsilon_m$. Operating filters beyond this stage would result in no further removal and the bed would function solely as a contact flocculator at maximum and constant values of $\bar{G}$ and $\bar{G}_c$, as indicated in Figures 43a, b and c.

Because of the numerous obstacles encountered in the particle counting study, it is not possible at the present time to present experimental verification of the primary particle disappearance model. However, the study included several interesting observations in regard to bed contact flocculation. The following section highlights the results of the experimental investigation of bed flocculation.

Evidence of particle flocculation was provided in two different ways. First, towards the end of each run, the contact flocculator produced a continuous flow of large visible flocs interspersed by clear water. Since the influent was generally a cloudy suspension, with particles too small to be visible, it was concluded that particles do flocculate while flowing through the bed capillary channels. During the initial stages of each run, the produced flocs were partially removed inside the bed until the amount of removal gradually deteriorated until all the bed voids were filled up to their ultimate removal capacity.

The second evidence of in-bed flocculation of particles was implicitly found during the analysis of the 1-2 $\mu$m particle counting data. The results of almost all runs indicated higher numbers of 1-2 $\mu$m particles in the effluent than in the influent for each pair of samples. The increase in the 1-2 $\mu$m particle count after filtration may have been
Fig. 43. Relationship between bed porosity ($\varepsilon$) during a filter run and $\bar{G}$, bed residence time, $t_b$, and the product $\bar{G} \cdot t_b$. 
caused by either break-up of larger size particles or flocculation of smaller size particles. The number of larger size particles in the influent was not large enough to account for the significant increase in the number of 1-2 μm particles. Furthermore, the number of larger sized particles did not decrease enough to account for the increase in 1-2 μm particles. Therefore, of the two hypotheses it seems more reasonable to assume that there is flocculation of submicron particles, especially since the prevailing conditions favored flocculation as evidenced by the appearance of large flocs later in the filter run.

As pointed out earlier in the literature review section, contact flocculators go through a steady state period after the exhaustion of bed removal capacity. The steady state is manifested by retaining a constant value of \( \frac{H}{H_0} \), \( \frac{C}{C_0} \), \( \frac{\sigma}{\sigma_u} \), and \( \frac{N}{N_0} \). As shown in runs 10 and 11, towards the end of each run the bed operated under a steady state condition and it was possible to keep the process going.

Reasons for Disagreement Between the Theory and Experiments

There are instances where theory and experiments do not show satisfactory agreement. The reason for such discrepancies is due in part to the approximate assumptions used in developing the theory and the lack of an ideal experimental design which fulfills these assumptions. For instance, the theory assumes that the filter bed is homogeneous and the influent solids concentration, temperature, ion concentration and pH
are constants throughout the run. However, these assumptions are difficult to be fulfilled perfectly in an experiment. Also, in order to simplify the theory, a linear transformation of the breakthrough equation was proposed. The linear transformation is an approximation and results in a poor comparison between the experimental and theoretical curves.

Filtration of a concentrated suspension through coarse media at high flow rates and with inadequate chemical pretreatment often is characterized by a poorer filtrate for a period during the first part of the run. The filtration model developed herein, if applied to the above filtration conditions, would predict higher removal at the beginning of the run and gradual deterioration with filtration time. Thus, the model does not describe the early breakthrough period of a filter run.

Attempts to verify the filtration model using filtration data where surface removal was predominant were usually characterized by noticeable disagreements. In such cases, the theory usually underestimates the rate of removal for the top layers and over estimates the rate of removal for the deeper layers. This was observed for both Eliassen's (26) and Agrawal's (3) runs.
SUMMARY AND CONCLUSIONS

In the current study, two models of filtration and bed contact flocculation were established. The models were investigated theoretically and experimentally. The experimental verification in most of the aspects of this study provided substantial support and proof of the filtration model.

The conclusions of this investigation may be summarized as follows:

1. In order to solve the complexity of the processes of filtration and contact flocculation certain assumptions were considered and an original kinetic equation was proposed. The proposed kinetic equation included the common effect of both attachment and detachment without suffering from any of the errors and deficiencies pointed out in the prior equations cited in the literature review.

2. Both analytical and numerical solutions of the proposed filtration equations were developed and presented in a filtration model which was experimentally verified using the authors own data as well as data of other investigators with a remarkable agreement in most cases.

3. Five different methods were proposed for the analysis of the experimental data using the current filtration model. Some of these methods required collection of very little filtration data in order to predict almost all the information an engineer needs to design a filter. Thus the simplicity and wide utility of the filtration model provides a significant advantage over other models.
4. The method involving linear transformation of turbidity data suffered from two deficiencies, which resulted in poor comparisons between experimental and theoretical breakthrough curves. The reason for such poor results was mainly due to the approximation involved in deriving the linear relationship and the difficulties encountered in conducting the kind of filtration experiments required in this method.

5. For the first time in the history of filtration theories an attempt was made to relate the two processes of particulate removal inside the filter bed and the increase in energy loss. The head loss data were used in predicting the filtrate quality with great success.

6. One of the remarkable applications of the filtration model was the ability to predict the filtration breakthrough curves for the runs at different flow rates, influent suspension concentrations and bed depths based on the information collected from only one run. The advantages of such a technique will be useful in the optimization of pilot plant studies and filter designs.

7. One of the deficiencies of earlier filtration models was inability to convert the influent concentration from turbidity units or mg/l to vol/vol units. The current filtration model provides an easy and reliable way of solving this problem.

8. The root mean square of velocity gradient inside the filter bed during the process of filtration was explicitly given in a mathematical formula as a function of both bed depth and filtration time.
This relationship is the first and only method available so far for calculation purposes. The relationship was derived according to the capillary tube model.

9. A new equation describing the change in head loss as a function of specific deposit at any bed depth and run time was presented earlier in the theory section. The equation was part of the application of the capillary tube model to the process of filtration.

10. A mathematical model describing the process of bed contact flocculation in light of the disappearance of primary particles was deduced after considering certain simplifying assumptions. The model required specific experimental design in order to provide experimental verifications. However, due to numerous problems in particle counting no direct experimental proof was obtained.

11. The experimental work included in the verification of the bed contact flocculation model provided substantial evidence of the production of large flocs coming out of the filter during the final stages of the run. Also, the proposed steady state stage was verified in all the following relationships; $\frac{C}{C_0}$ and $\frac{u}{u_0}$ vs $(x,t)$, $\frac{\Delta H}{\Delta H_0}$ vs $t$ and $\frac{N}{N_0}$ vs $t$.

12. The following limitations of the filtration model are acknowledged. The filtration model was not designed to predict the rate of removal at the beginning of a run. Furthermore, the model is intended to predict removal in depth and can not be successfully applied to cases where substantial surface removal occurs.
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APPENDIX I

Numerical Examples

In the chapter entitled Experimental Investigation, five methods of applying the filtration equations to the experimental data were presented briefly along with the results. The details of each of these five methods are presented below along with an illustrative numerical example of each method.

First method: trial and error

The procedures used in predicting the theoretical filtration curves according to this method are explained using Peterson's filtration run #13 as an example (previously shown in Figure 19). The method of predicting both coefficients $a$, $b$ for this run involves the following steps:

1. Draw the curve that smoothly joins the experimental data points of this run. Ignore both outlier points and the data during initial breakthrough.

2. Determine the removal ratios $\frac{C_i}{C_o}$ and $\frac{C_f}{C_o}$ at 0 and $t_f$ filtration times respectively. $\frac{C_i}{C_o}$ is obtained by extending the lowest portion of the $C/C_o$ curve back to time zero. For run #13 these were,

   \[ \frac{C_i}{C_o} = 0.01, \quad \text{and} \quad \frac{C_f}{C_o} = 0.48 \text{ at time } t_f = 14.5 \text{ hr.} \]

3. Determine the initial value of coefficient $a$ using equation 40 as follows:

   \[ a = \frac{\ln \left( \frac{C_o}{C_i} \right)}{x} \]  

   (40)
where

\[ x = \text{bed depth} = 52 \text{ inch.} \]

\[ \frac{C_o}{C_f} = \frac{1}{0.01} = 100 \]

\[ \therefore a = \frac{\ln(100)}{52} = 0.04428 \text{ inch}^{-1} \]

4. Determine the initial value of coefficient \( b \) using equation 42 as follows:

\[
\begin{align*}
  b &= \ln \left[ \frac{2 \left( \sqrt{\frac{C_o}{C_f}} - 1 + \sqrt{\frac{C_o}{C_f}} \right)}{(\sqrt{\frac{C_o}{C_f}} - 1)} \right] \div t_f \\
  \text{where} & \\
  \frac{C_o}{C_f} &= \frac{1}{0.48} = 2.083 \\
  t_f &= \text{time to breakthrough} = 14.5 \text{ hr.}
\end{align*}
\]
\[ b = \ln \left[ \frac{2 \sqrt{\frac{1}{0.01} - (1 + \frac{1}{0.48})}}{(1 - \frac{1}{0.48})} \right] \div 14.5 \]

\[ = 0.25371 \text{ hr}^{-1}. \]

5. Draw the theoretical breakthrough curve for this run following equation 31 for the above values of \(a\), \(b\). If the predicted filtration curve does not reasonably fit the experimental curve, introduce incremental changes in \(a\) and/or \(b\) until final values are obtained.

The adjustments of \(a\) and \(b\) is facilitated by remembering that \(a\) has major influence in the initial stages of the breakthrough curve, and \(b\) during the latter part of the curve. Increasing \(a\) lowers the initial levels of \(C/C_o\), while increasing \(b\) raises the terminal levels of \(C/C_o\). With some experience, it is not difficult to find the best values of \(a\) and \(b\). For this example the final values for \(a\), \(b\) were 0.04428 inch\(^{-1}\) and 0.25491 hr\(^{-1}\) respectively.

6. The ratio \(C/C_o\) at any depth, \(x\), and filtration time, \(t\), is determined using equation 31 as follows:

\[ \frac{C}{C_o} = \left[ \frac{e^{bt} + 1}{2e^{ax} + e^{bt} - 1} \right]^2 \] (31)

For example, at the point \((x = 52 \text{ in}, t = 10 \text{ hr.})\)
Second method: filtrate quality data

Example  Saatci's (89) run 31. In the analysis of this run using the second method of data analysis the following procedures were followed:

1. Draw the experimental breakthrough curves for each bed depth as explained in the first method as shown in Figure 28.

2. An arbitrary value of \( \frac{C}{C_0} = 0.21 \) was selected at which a horizontal straight line was drawn. The horizontal line intercepted the experimental curves at the points \((x = 24.5 \text{ cm}, t = 26 \text{ min}), (x = 34.5 \text{ cm}, t = 34 \text{ min}), \text{ and } (x = 49.5 \text{ cm}, t = 81 \text{ min})\). The data for filters B and C had been rejected by Saatci because the data were inconsistent with the shallower depth sometimes providing lower \( \frac{C}{C_0} \) than the deeper filter.

3. Determine the slope and intercept of the straight line that joins the above three points. For this purpose the linear regression analysis technique is used as in Table I-1.

Table I-1. Regression analysis of Saatci's data for run 31

<table>
<thead>
<tr>
<th>( x ) cm</th>
<th>( t ) min</th>
<th>( x_t )</th>
<th>( x^2 )</th>
<th>Slope ( \text{min/cm} )</th>
<th>Intercept ( \text{min} )</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.5</td>
<td>26</td>
<td>637</td>
<td>600.25</td>
<td>2.2737</td>
<td>-35.323</td>
<td>0.927</td>
</tr>
<tr>
<td>34.5</td>
<td>34</td>
<td>1173</td>
<td>1190.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.5</td>
<td>81</td>
<td>4009.5</td>
<td>2450.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108.5</td>
<td>141</td>
<td>5819.5</td>
<td>4240.75</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ y = b_0 + b_1 x \]

\[
\text{Slope } b_1 = \frac{\sum x t - \frac{\sum x \cdot \sum t}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{5819.5 - \frac{(141)(108.5)}{3}}{4240.75 - \frac{(108.5)^2}{3}} = 2.2737 \text{ min/cm}
\]

\[
\text{Intercept } b_0 = \frac{\sum t - b_1 \sum x}{n} = \frac{141 - (2.2737)(108.5)}{3} = -35.232 \text{ min.}
\]

4. Determine the values of coefficients \( a, b \) using equation 45 as follows:

\[
b = \frac{-\ln \left( \frac{1}{\frac{C}{C_0} + 1} \right)}{-\ln \left( \frac{1}{\frac{1}{0.21} + 1} \right)} = \frac{\text{Intercept } (b_1)}{-35.23 \text{ min.}} = 0.0132 \text{ min}^{-1}
\]

\[
a = b \cdot \text{slope } (b_1)
= 0.0132 \text{ (min}^{-1}) \cdot 2.2737 \text{ min/cm} = 0.02997 \text{ cm}^{-1}
\]

5. Use equation 31 and the above values of \( a, b \) to draw the theoretical breakthrough curves for this run. The calculations of \( C/C_0 \) vs. \( x, t \) are given in Table I-2.
Table I-2. Calculated breakthrough curves in Saatci's run 31

<table>
<thead>
<tr>
<th>t (Min)</th>
<th>x-24.5 cm</th>
<th>x-34.5</th>
<th>x-39.7</th>
<th>x-44 cm</th>
<th>x-49.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.247</td>
<td>0.138</td>
<td>0.102</td>
<td>0.079</td>
<td>0.057</td>
</tr>
<tr>
<td>30</td>
<td>0.285</td>
<td>0.166</td>
<td>0.124</td>
<td>0.097</td>
<td>0.071</td>
</tr>
<tr>
<td>50</td>
<td>0.331</td>
<td>0.201</td>
<td>0.153</td>
<td>0.122</td>
<td>0.091</td>
</tr>
<tr>
<td>70</td>
<td>0.383</td>
<td>0.243</td>
<td>0.189</td>
<td>0.153</td>
<td>0.116</td>
</tr>
<tr>
<td>90</td>
<td>0.441</td>
<td>0.293</td>
<td>0.234</td>
<td>0.192</td>
<td>0.149</td>
</tr>
<tr>
<td>110</td>
<td>0.502</td>
<td>0.351</td>
<td>0.287</td>
<td>0.241</td>
<td>0.190</td>
</tr>
</tbody>
</table>

Third method: head loss data

Example Saleh run 10. The procedures involved in verifying the filtration model, using the head loss data of the writer's run 10 previously shown in Figure 9 were as follows:

1. Insert the piezometer readings for different bed depths and filtration times into computer program no. 1, (see Appendix II). The computer will then execute a trial and error solution to equation 69.

\[
\frac{\Delta H}{\Delta H_o} = \left( \frac{1 - \varepsilon_o + \sigma}{1 - \varepsilon_o} \right)^{1/3} \frac{\varepsilon_o}{(\varepsilon_o - \sigma)}^{2}
\]

and prints out the values of \( \sigma \) vs. x, t. The computer input includes also, the value of the clean bed porosity, filter media sieve size \( \phi_{so} \), value of the shape factor \( S \), and the approach velocity \( V_s \). The output
includes values of $\Delta H(x, t)$, $\frac{\Delta H}{\Delta h_0}$, $\sigma(x, t)$, $G(x, t)$ and $\sigma(x)$ at the selected filtration times, $t$.

2. Determine the value of $\sigma_u$ which is the highest value of $\sigma$ in the top few inches of the filter bed. Select several depths and for each depth obtain the ratios of $\frac{\sigma}{\sigma_u}$ at all the filtration times used in the step 1 above. The values of $\frac{\sigma}{\sigma_u}$ vs. $t$ for depths 2.5, 7, 11.5, 16 and 20.5 in., were given previously in Table 6.

3. Draw the curves $\frac{\sigma}{\sigma_u}$ vs. $t$ for the selected depths. These curves were shown previously in Figure 9.

4. Intersect the above curves with a horizontal line at an arbitrary value of $\frac{\sigma}{\sigma_u}$. In this example $\frac{\sigma}{\sigma_u}$ was equal to 0.4. The values of $x$, $t$ for the points of intersection were (2.5, 2.45), (7, 3.8), (11.5, 4.5), (16, 7.4) and (20.5, 9.7). The slope and intercept of the straight line which joins the above points were determined using the linear regression analysis and the results are shown in Table 1-3.

Table 1-3. Regression analysis of Saleh's run 10

<table>
<thead>
<tr>
<th>Bed Depth x Inch</th>
<th>Time to Reach $\frac{\sigma}{\sigma_u}$ - 0.4 t hr</th>
<th>$x.t$</th>
<th>$x^2$</th>
<th>Slope $b_0$ hr/inch</th>
<th>Intercept $b_1$ hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.45</td>
<td>6.125</td>
<td>6.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>3.85</td>
<td>26.15</td>
<td>49.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.5</td>
<td>4.5</td>
<td>51.75</td>
<td>132.25</td>
<td>0.396</td>
<td>0.3787</td>
</tr>
<tr>
<td>16.0</td>
<td>7.4</td>
<td>118.4</td>
<td>256.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.5</td>
<td>9.70</td>
<td>198.85</td>
<td>420.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Σ57.5</td>
<td>27.9</td>
<td>402.075</td>
<td>863.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using equation 44

\[ b = \ln \left( \frac{2}{\frac{1}{b_0} + 1} \right) \]

\[ b = \ln \left( \frac{2}{0.4 + 1} \right) = 0.3738 \text{ hr}^{-1} \]

\[ a = b \cdot b_1 = 0.1616 \text{ inch}^{-1} \]

4. The values of coefficients \( a \) and \( b \) are subsequently used to calculate the theoretical breakthrough curve for a depth of 24 inches using equation 31 with the results given in Table I-4, and plotted previously in Figure 9.
Table I-4. Calculated breakthrough curve for Saleh's run 10

<table>
<thead>
<tr>
<th>t/hr</th>
<th>C/C₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>0.010</td>
</tr>
<tr>
<td>10</td>
<td>0.098</td>
</tr>
<tr>
<td>14</td>
<td>0.443</td>
</tr>
<tr>
<td>18</td>
<td>0.807</td>
</tr>
<tr>
<td>20</td>
<td>0.901</td>
</tr>
<tr>
<td>22</td>
<td>0.951</td>
</tr>
<tr>
<td>26</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Fourth method: extrapolating the data of other runs

Example Peterson runs 20, 21, 22, 23, and 24.

1. The coefficients a, b for were determined using the data in run 20 only (previously shown in Figure 22) by the trial and error approach discussed in the first method. In this case the initial values of a, b =

\[ a = \frac{\ln \left( \frac{1}{C_i} \right)}{x} = \frac{\ln \left( \frac{1}{0.005} \right)}{52.0 \text{ inch}} \approx 0.051 \text{ inch}^{-1} \]

\[ b = \ln \left[ \frac{\sqrt{\frac{1}{C_i}}}{\sqrt{\frac{1}{C_f}}} \right] \div t_f \]
Table I-5. Calculated breakthrough curves for Peterson's runs 20-24

<table>
<thead>
<tr>
<th>Run #</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>5.0 gpm/ft²</td>
<td>5.0</td>
<td>7.5</td>
<td>7.5</td>
<td>10</td>
</tr>
<tr>
<td>C₀</td>
<td>50 mg/l</td>
<td>25</td>
<td>25</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>a</td>
<td>0.0684</td>
<td>0.0684</td>
<td>0.0456</td>
<td>0.0456</td>
<td>0.0342</td>
</tr>
<tr>
<td>b</td>
<td>0.20262</td>
<td>0.10131</td>
<td>0.10131</td>
<td>0.20262</td>
<td>0.10131</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time, hr</th>
<th>C/C₀</th>
<th>C/C₀</th>
<th>C/C₀</th>
<th>C/C₀</th>
<th>C/C₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.001</td>
<td>0.010</td>
<td>0.010</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.001</td>
<td>0.001</td>
<td>0.012</td>
<td>0.015</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>0.002</td>
<td>0.001</td>
<td>0.013</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>0.004</td>
<td>0.002</td>
<td>0.016</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
<td>0.002</td>
<td>0.020</td>
<td>0.06</td>
<td>0.061</td>
</tr>
<tr>
<td>10</td>
<td>0.013</td>
<td>0.003</td>
<td>0.026</td>
<td>0.09</td>
<td>0.076</td>
</tr>
<tr>
<td>12</td>
<td>0.025</td>
<td>0.004</td>
<td>0.034</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.044</td>
<td>0.005</td>
<td>0.044</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.08</td>
<td>0.007</td>
<td>0.057</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.13</td>
<td>0.009</td>
<td>0.073</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.22</td>
<td>0.013</td>
<td>0.094</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td></td>
<td></td>
<td></td>
<td>0.92</td>
<td></td>
</tr>
</tbody>
</table>

266
Here $t_f = 20$ hr,

\[
\frac{c_4}{c_o} = 0.005,
\]

\[
\frac{c_f}{c_o} = 0.33
\]

\[
.. b = \ln \left( \frac{2 \sqrt{\frac{1}{0.005} - (\sqrt{\frac{1}{0.33} + 1})}}{\left( \sqrt{\frac{1}{0.33} - 1} \right)} \right) \div 20
\]

\[= 0.177 \text{ hr}^{-1}\]

However, the final values of $a$, $b$, for the best fit of run 20 were found by trial and error to be $a = 0.0684$ inch$^{-1}$, $b = 0.20262$ hr$^{-1}$.

2. Since the only differences among the variables included in runs 20-24 were the approach velocity and influent concentration, it was possible to estimate the coefficients $a$, $b$ for runs 21-24 based on their values in run 20. The calculations are as follows:

a) Run 21.

\[V_s = 5 \text{ gpm/ft}^2, \quad c_o = 25 \text{ mg/l}\]

the approach velocity is the same as in run 20, therefore the value of coefficient, $a$, for this run is the same as in 20. Also, coefficient $b$ for this run is reduced by the ratio $\frac{c_o \text{ of run 21}}{c_o \text{ of run 20}}$ i.e. the new
coefficient is $b_{21} = b_{20} \left( \frac{25}{50} \right) = 0.20262$, $\frac{25}{50} = 0.10131 \text{ hr}^{-1}$.

Similarly, the coefficients $a$, $b$ for runs 22, 23 and 24 were calculated and their values were presented previously in Table 12. Also, the theoretical breakthrough curves for each run were predicted for a depth of 52 inches and values of $C/C_0$ vs. $t$ are presented in Table I-5.

Fifth method: Extrapolating the data collected for one filter depth

Example Hegg's Run B-15. The coefficients $a$ and $b$ for this run were calculated based on the experimental data of the 5-inch filter as shown previously in Figure 25. Since these coefficients should be the same for all depths, it is possible to predict the filtration curves at other depths using equation 31 by changing the values of bed depth, $x$.

Values of $a$ and $b$ have been calculated for each of the 3 bed depths used by Hegg and were presented earlier in Table 13. However, only values for the 5-inch depth have been used in predicting the breakthrough curves for all depths in the following table.

Table I-6. Calculated breakthrough curves for Hegg's run B-15

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x = 1 \text{ in.}$</th>
<th>$x = 5 \text{ in.}$</th>
<th>$x = 9 \text{ in.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.73</td>
<td>0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>1</td>
<td>0.80</td>
<td>0.30</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>0.86</td>
<td>0.42</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>0.91</td>
<td>0.56</td>
<td>0.28</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>0.70</td>
<td>0.43</td>
</tr>
<tr>
<td>5</td>
<td>0.97</td>
<td>0.81</td>
<td>0.60</td>
</tr>
</tbody>
</table>
In determining coefficients a, b for the 5-inch depth, the trial and error method was used as in the first method. Therefore, the initial values were determined as follows:

\[ a(5 \text{ in. bed}) = \frac{\ln \frac{1}{C_1/C_0}}{x} = 0.156 \text{ inch}^{-1} \]

\[ b(5 \text{ in. bed}) = \ln \left[ \frac{2 \sqrt{\frac{1}{C_1/C_0}} - (\sqrt{\frac{1}{C_f/C_0}} + 1)}{\left(\frac{1}{C_f/C_0} - 1\right)} \right] \div t_f \]

\[ b = \ln \left[ \frac{2 \sqrt{\frac{1}{0.21}} - (\sqrt{\frac{1}{0.81}} + 1)}{\left(\sqrt{\frac{1}{0.81}} - 1\right)} \right] \div 5 \]

\[ = 0.602 \text{ hr}^{-1} \]

However, values of \( a = 0.1561 \text{ inch}^{-1} \), \( b = 0.6073 \text{ hr}^{-1} \), determined by trial and error, provide better agreement with the experimental data.
Verification of Equation 85a

In order to verify equation 85a, using the data collected in runs 10 and 11, a mass balance as given in equation 86 was followed. The values of $\sigma_u$ and $A\sigma_1$, $A\sigma_2$, ..., $A\sigma_i$ for each run were given in the output of the first computer program for the filtration times $t_1$, $t_2$, ..., $t_i$ respectively. The amount of solids removed in the entire depth of a unit area bed was calculated by subtracting the areas under the influent and effluent turbidity curves for the interval of time $t_i + 1$, and $t_i$ using the trapezoidal rule. The area entrapped between the two curves was then multiplied by the approach velocity $V_s$. The conversion factor $k_f$ is thus

$$k_f = \frac{(A\sigma_2 - A\sigma_1)}{V_s (t_2 - t_1) \left\{ \frac{(C_o - C) + (C_o - C_1)}{2} \right\}}$$

$$= \frac{\Delta A\sigma}{V_s \Delta t \Delta C}$$

where

$\Delta A\sigma = (A\sigma_2 - A\sigma_1)$ inch (Vol/Vol)

$\Delta t = (t_2 - t_1)$ hr

and

$$\Delta G = \left\{ \frac{(C_o - C) + (C_o - C_1)}{2} \right\} \text{ mg/l}$$
\[ V_s = \text{approach velocity inch/hr.} \]

The dimensions of the conversion constant \( k_f \) is therefore

\[
k_f = \frac{\text{inch}\cdot\text{vol}/\text{vol}}{(\text{inch/hr})(\text{hr})(\text{mg/l})}
\]

\[
= \frac{\text{vol}/\text{vol}}{\text{mg/l}}
\]

In both run 10 and 11, the approach velocity was 10 gpm/ft\(^2\) or 965.24 inch/hr. The values of \( \Delta C, \Delta t \) are given in Tables I-7 and I-8. Also, values \( \Delta C, \Delta t \) and \( \sum \Delta C \Delta t \) (in hours turbidity units) were calculated for both runs and the results are presented in columns 7 and 8 of each table respectively.
Table I-7. Amount of solids removed between sampling intervals for Saleh's run no. 10

<table>
<thead>
<tr>
<th>time t (hr)</th>
<th>Δt (hr)</th>
<th>C₀ (FTU)</th>
<th>C (FTU)</th>
<th>C₀ - C</th>
<th>( \frac{[C_{o_{i+1}} - C_{i+1}] + [C_{o_i} - C_i]}{2} )</th>
<th>(6) (2)</th>
<th>Σ(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>78</td>
<td>61</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>27.750</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.750</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The increase in the total deposit content inside the whole bed during each interval of time was divided by the total amount of solids removed inside the entire bed for the same intervals of time to determine the values of \( k_F \). The results of the mass balance for both runs 10 and 11 are presented in Tables I-9 and I-10 respectively. In both runs each turbidity unit (FTU) was equivalent to ~ 4.0 mg/l of solids concentration based on average influent turbidity compared with known influent clay concentration.

The conversion factor, \( k_F \), was also obtained using equation 85a as follows:

For run 10:

\[
k_F = \frac{bq_u}{16aC_0V_s}
\]
Table I-9. Saleh's run 10 mass balance calculations

<table>
<thead>
<tr>
<th>Δσ (inch vol/vol)</th>
<th>ΔΔσ (inch vol/vol)</th>
<th>ΔCΔt (hr. mg/l)</th>
<th>Weighting</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2546</td>
<td>0.2546</td>
<td>242.8</td>
<td>0.075</td>
<td>0.8184</td>
</tr>
<tr>
<td>0.3731</td>
<td>0.1185</td>
<td>575.5</td>
<td>0.179</td>
<td>0.3809</td>
</tr>
<tr>
<td>0.4835</td>
<td>0.1104</td>
<td>736.5</td>
<td>0.229</td>
<td>0.3549</td>
</tr>
<tr>
<td>0.5229</td>
<td>0.0394</td>
<td>460.0</td>
<td>0.143</td>
<td>0.1267</td>
</tr>
<tr>
<td>0.5509</td>
<td>0.0280</td>
<td>440.0</td>
<td>0.137</td>
<td>0.0900</td>
</tr>
<tr>
<td>0.5690</td>
<td>0.0180</td>
<td>295.0</td>
<td>0.092</td>
<td>0.0579</td>
</tr>
<tr>
<td>0.5722</td>
<td>0.0032</td>
<td>207</td>
<td>0.064</td>
<td>0.0103</td>
</tr>
<tr>
<td>0.5865</td>
<td>0.0143</td>
<td>266</td>
<td>0.083</td>
<td>0.0450</td>
</tr>
</tbody>
</table>

Σ = 3222.8

k_F = 1.885 x 10^{-7} b

\[ k_F = 3.2146 \times 10^{-7} \Delta \sigma \]

\[ k_F = \Sigma \left( \frac{\Delta C \Delta t}{\Sigma \Delta C \Delta t} \right) \cdot \frac{\Delta \sigma}{V \Delta C \Delta t} \]

Table I-10. Saleh's run 11 mass balance calculations

<table>
<thead>
<tr>
<th>Δσ (inch vol/vol)</th>
<th>ΔΔσ (inch vol/vol)</th>
<th>ΔCΔt (hr. mg/l)</th>
<th>Weighting</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4686</td>
<td>0.4686</td>
<td>431.80</td>
<td>0.246</td>
<td>2.7638</td>
</tr>
<tr>
<td>0.5299</td>
<td>0.0610</td>
<td>285.10</td>
<td>0.162</td>
<td>0.3598</td>
</tr>
<tr>
<td>0.5778</td>
<td>0.0479</td>
<td>268.90</td>
<td>0.153</td>
<td>0.2825</td>
</tr>
<tr>
<td>0.6046</td>
<td>0.0268</td>
<td>450.24</td>
<td>0.256</td>
<td>0.1581</td>
</tr>
<tr>
<td>0.6218</td>
<td>0.0172</td>
<td>125.50</td>
<td>0.071</td>
<td>0.1014</td>
</tr>
<tr>
<td>0.6394</td>
<td>0.0176</td>
<td>111.00</td>
<td>0.063</td>
<td>0.1038</td>
</tr>
<tr>
<td>0.6482</td>
<td>0.0088</td>
<td>47.00</td>
<td>0.027</td>
<td>0.0519</td>
</tr>
<tr>
<td>0.6509</td>
<td>0.0027</td>
<td>37.00</td>
<td>0.021</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

Σ1756.54

k_F = 3.839 x 10^{-7}
\[
\begin{align*}
  k_F &= \frac{0.2554 \text{ hr}^{-1} \cdot 0.3106 \text{ vol/vol}}{0.13076 \text{ inch}^{-1} \cdot (16 \times 280 \text{ mg/1}) \cdot (965.24 \text{ inch/hr})} \\
  &= 1.403 \times 10^{-7} \text{ vol/vol mg/1} \\

\text{also, for run 11} \\
  k_F &= \frac{0.167 \text{ hr}^{-1} \cdot 0.3306 \text{ vol/vol}}{0.0534 \text{ inch}^{-1} \cdot (16 \times 160 \times 965.24)} \\
  &= 4.184 \times 10^{-7} \text{ vol/vol mg/1}
\end{align*}
\]

In both runs the calculated values of the conversion factor, \( k_F \), based on equation 85a were very close to the corresponding values of \( k_F \) given in Tables I-9 and I-10 respectively. This demonstrates the validity of equation 85a which could be used independently as a simple and quick method of calculating the conversion factors from turbidity units or mg/1 concentration to volume of deposit per unit bed volume.

Evaluation of the Turbidity Linear Transformation Approach

In order to show the weakness of the turbidity linear transformation approach, Elissen's run 6 was analyzed using the trial and error method, (No. 1). The results of this analysis are presented in Table I-11 and the comparison between the experimental and theoretical breakthrough curves developed according to the trial and error method is shown in Fig. 34. As shown in both Fig. 33 and 34, the linear transformation approach was not in as good agreement with the
experimental data as the trial and error method.

As previously mentioned, the reason the linear transformation of turbidity data approach was not the best method is due to the experimental difficulties as well as the approximate assumptions involved.

Table I-ll. Analysis of Eliassen's run 6 using the trial and error approach

<table>
<thead>
<tr>
<th>ft.</th>
<th>( \frac{C_i}{C_o} )</th>
<th>( \frac{a}{ft.} )</th>
<th>( \frac{t_f}{hr} )</th>
<th>( \frac{b}{hr^{-1}} )</th>
<th>( \frac{a}{ft.} )</th>
<th>( b_{hr^{-1}} )</th>
<th>Final</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.33</td>
<td>3.960</td>
<td>96</td>
<td>0.999</td>
<td>0.155</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.39</td>
<td>0.16</td>
<td>2.35</td>
<td>96</td>
<td>0.96</td>
<td>0.052</td>
<td>0.9</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>0.89</td>
<td>0.12</td>
<td>1.19</td>
<td>&quot;</td>
<td>0.56</td>
<td>0.024</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>0.09</td>
<td>0.866</td>
<td>&quot;</td>
<td>0.41</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.96</td>
<td>0.04</td>
<td>0.821</td>
<td>&quot;</td>
<td>0.33</td>
<td>0.024</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Computer Program No. 1

Program No. 1 was used to calculate the values of specific deposit, $\sigma(x,t)$, the total volume of deposits inside the full depth, $AO(t)$, and the mean square root of velocity gradient $\overline{v_x}(x,t)$. The input data include such information as piezometer number, $J$, and readings, $H$, filtration time at which piezometer readings were recorded, $I$, clean bed porosity, $\varepsilon_o$, filtration approach velocity, $V_s$, media sieve radius $\phi_{so}$ and the shape factor, $S$. The symbols used in this program were defined as follows:

- $EO = \varepsilon_o = 0.445$
- $VS = V_s = 0.681 \text{ cm/sec}$
- $B = \frac{3.784 V_s (1 - \varepsilon_o)^{1/3}}{S \phi_{so}}$

$$B = \frac{3.784 V_s (1 - \varepsilon_o)^{1/3}}{(1.23) \left( \frac{0.2596}{2} \right)}$$

$$= 23.65 V_s (1 - \varepsilon_o)^{1/3}$$

$H(I,J) =$ piezometer readings $(t,x)$

$DH = \Delta H$

$HO = \Delta H_o$
\[ DHR(I,J) = \frac{\Delta H}{\Delta H_0} (t,x) \]

\[ SIG(I,J) = YL (I,J) = \sigma (t,x) \]

\[ G(I,J) = \tilde{G}_a (t,x) \]

\[ AREA (I) = A\sigma (t) \]

Computer program No. 1 follows:

```
$JOB 'SALEH'TIME = 5, PAGES = 10
DIMENSION H(9,16), DH(9,15), DHR(9,15), DHP(9,15), SIG(9,15),
       SUM2(9), SUM1(9), AREA(9), G(9,15), H1(9,16)
J DEPTH CODE
I TIME CODE
EO = 0.445
VS = 0.6804
B = 23.65 * VS * (1 - EO)**(1/3.0)
DO 10 I = 1, 9
  READ(5,*) (H1(I,J), J = 1, 16)
10 WRITE(6,11) (H1(I,J), J = 1, 16)
11 FORMAT ('0', 16F6.3)
DO 300 I = 1, 9
  DO 300 J = 1, 16
    H(I,J) = H1(I,17-J)
300 WRITE(6,11)(H(I,J), J = 1, 16)
C DHR=DH/H0
DO 30 I = 1, 9
  DO 30 J = 1, 15
    DH(I,J) = H(I,J) - H(I,J+1)
30 DHR(I,J) = DH(I,J)/DH(I,J)
DO 40 I = 1, 9
40 WRITE(6,41)(DHR(I,J), J = 1, 15)
41 FORMAT ('0', 15(F6.3,2X))
DO 110 I = 2, 9
  DO 110 J = 1, 15
    DHR(I,J) = H(I,J) - H(I,J-1)
110 WRITE(6,101) I, J, SIG(I,J), G(I,J)
```

\[ G(I,J) = B*(1-EO+SIG(I,J))**(1/6.0)*(EO-SIG(I,J))**(-1.5) \]

\[ C DHR=DH/H0 \]
281

101 FORMAT('0',2(12,2X),F10.5,5X,F10.3)
110 CONTINUE
0 SUMMATION OF SIG(I,J) AT CONSTANT I
1 X1=2.5
2 X=1.5
D) 250 I=2,9
3 SUM1(I)=SIG(I,1)*X1
4 SUM2(I)=0
5 DO 200 J=2,15
6 SUM2(I)=SUM2(I)+SIG(I,J)
7 AREA(I)=SUM1(I)+SUM2(I)*X
8 WRITE(6,*),I,AREA(I)
9 STOP
END

SUBROUTINE ROOTS(SIG1,EO,DHR1)
1 FUNC(YL,EO,DHR1)=(1+YL/(1-EO))**(1/3.0)*(EO/(EO-YL))**2-DHR1
2 ITER=ALOG(0.05/1.0E-4)/ALOG(2.0)+1
3 YL=-0.001
4 IF(FUNC(YL,EO,DHR1)*FUNC(YL+0.05,EO,DHR1).LT.0.0)GO TO 3
5 YL=YL+0.05
6 GO TO 1
7 YR=YL+0.05
8 FYL=FUNC(YL,EO,DHR1)
9 DO 6 K=1,ITER
10 YHALF=(YL+YR)/2.0
11 FYHALF=FUNC(YHALF,EO,DHR1)
12 IF(FYHALF*FYL.LE.0.0)GO TO 5
13 YL=YHALF
14 FYL=FYHALF
15 GO TO 6
16 YR=YHALF
17 CONTINUE
18 SIGL=(YL+YR)/2.0
19 RETURN
END

$ENTRY

The second computer program was used in calculating values of $\frac{C}{C_0}$ (x,t), $\frac{\sigma}{\sigma_u}$ (x,t), $\frac{\Delta H}{\Delta H_0}$ (x,t) and $\bar{G}_w (x,t)$. Also, the computer plots the graphs $\frac{C}{C_0}$ (x,t), $\frac{\sigma}{\sigma_u}$ (x,t), $\frac{\Delta H}{\Delta H_0}$ (x,t) and $\bar{G}_w (x,t)$ for bed depths, X = 2, 5, 10, 15, 20, 25 inch, and filtration time 0, 1, 2, .... 20 hr.
In order to run this program, such information as the values of $v_s$ (cm/sec), $\varepsilon_0$, $S\phi_{so}$ (cm), the slope and intercept of the turbidity data linear transformation relationship in hr/inch and inch respectively, value of breakthrough level $\frac{C}{C_o}$ for which the linear transformation was developed, and the ultimate deposit value, $\sigma_u$.

The program includes the following operations:

\[ \text{PHIS} = S\phi_{so} \]
\[ \text{XINT} = \text{intercept (hr)} \]
\[ \text{SLOPE} = \text{slope (hr/inch)} \]
\[ \text{SIGMAU} = \sigma_u \]
\[ C = \frac{C}{C_o} \]
\[ B = b(\text{hr}^{-1}) = \frac{\ln \left( \frac{C}{C_o} + 1 \right)}{\text{intercept}} \]
\[ A = a(\text{inch}^{-1}) = B \times \text{slope} \]
\[ B_1 = \frac{4.73 v_s (1 - \varepsilon_0)^{1/3}}{S\phi_{so}} \]
\[ S(I) = e^{ax} \]
\[ P(J) = e^{bt} \]
\[ \text{COC}(I,J) = \frac{C}{C_o} (x,t) \]
\[ = \left[ \frac{e^{bt} + 1}{2e^{ax} + e^{bt} - 1} \right]^2 \]
SOEO(I,J) = \frac{\sigma}{\sigma_u} (x,t)

= \left[ \frac{e^{bt} - 1}{2e^{ax} + e^{bt} - 1} \right]^2

SIG(I,J) = \sigma (x,t)

= \sigma_u \text{ SOEO }

= \sigma_u \left( \frac{\sigma}{\sigma_u} \right)

DHR(I,J) = \frac{\Delta H}{\Delta H_o} (x,t)

= \left[ 1 + \frac{\sigma(x,t)}{1 - \varepsilon_o} \right]^{1/3} \left( \frac{\varepsilon_o - \sigma(x,t)}{\varepsilon_o - \sigma(x,t)} \right)^{2/3}

G(I,J) = \bar{G}_w (x,t)

= B_1 \frac{1 - \varepsilon_o + \sigma(x,t)}{[\varepsilon_o - \sigma(x,t)]^{3/2}}

Computer program no. 2 follows:

$\text{JOB } 'SALEH', \text{TIME}=5, \text{PAGES}=10$

DIMENSION COC(10,20), SOEO(10,20), SIG(10,20), DHR(10,20), G(10,20), T1 + (20), T(20), XI(10), X(10), COC1(19), COC2(19), COC3(19), COC4(19), COC5(1 + 9), COC6(19), COC7(19), SOE1(19), SOE2(19), SOE3(19), SOE4(19), SOE5(19), +SOE6(19), SOE7(19), DHR1(19), DHR2(19), DHR3(19), DHR4(19), DHR5(19), DHR +6(19), DHR7(19), G1(19), G2(19), G3(19), G4(19), G5(19), G6(19), G7(19), P( +20), S(10)

E0=0.445

VS=0.6764
PHIS=0.2
XINT=33.6
SIGMAU=0.3301
SLOPE=10.5
C=2.5
B=ALOG((SORT(C)+1)/2.0)/XINT
A=B*SLOPE
B1=4.73*VS*(1-E0)**(1/3.0)/PHIS
READ(5,*) (X(I),I=1,7)
READ(5,*) (T(J),J=1,19)
WRITE(6,11) (X(I),I=1,7)
11 FORMAT('0',7(F5.2,5X))
WRITE(6,16) (T(J),J=1,19)
16 FORMAT('0',19(F4.1,2X))
DO 30 I=1,7
30 X(I)=2.54*X1(I)
DO 40 J=1,19
40 T(J)=60*T1(J)
DO 50 I=1,7
DO 50 J=1,19
S(I)=EXP(A*X(I))
P(J)=EXP(B*T(J))
COC(I,J)=((P(J)+1)/(2*S(I)+P(J)-1))**2
SOEO(I,J)=((P(J)-1)/(2*S(I)+P(J)-1))**2
SIG(I,J)=SIGMAU*SOEO(I,J)
DHR(I,J)=(1+SIG(I,J)/(1-E0))**(1/3.0)*(EO/(EO-SIG(I,J)))**2
G(I,J)=B1*(1-E0+SIG(I,J))**(-1.5)
50 WRITE (6,51) COC(I,J),SOEO(I,J),DHR(I,J),G(I,J),X1(I),T1(J)
51 FORMAT(2(F8,4,3X),2(F15.3,3X),2(F5.2,3X))
DO 60 J=1,19
60 COC1(J)=COC(1,J)
SOEO1(J)=SOEO(1,J)
DHR1(J)=DHR(1,J)
DO 70 J=1,19
70 G1(J)=G(1,J)
DO 80 J=1,19
80 COC2(J)=COC(2,J)
SOEO2(J)=SOEO(2,J)
DHR2(J)=DHR(2,J)
DO 90 J=1,19
90 G2(J)=G(2,J)
DO 100 J=1,19
100 COC3(J)=COC(3,J)
SOEO3(J)=SOEO(3,J)
DHR3(J)=DHR(3,J)
DO 110 J=1,19
110 G3(J)=G(3,J)
DO 120 J=1,19
120 COC4(J)=COC(4,J)
SOEO4(J)=SOEO(4,J)
DHR4(J)=DHR(4,J)
285

90  G4(J)=G(4,J)
   DO 100 J=1,19
   COC5(J)=COC(5,J)
   SOE5(J)=SOEO(5,J)
   DHR5(J)=DHR(5,J)
100  G5(J)=G(5,J)
   DO 110 J=1,19
   COC6(J)=COC(6,J)
   SOE6(J)=SOEO(6,J)
   DHR6(J)=DHR(6,J)
110  G6(J)=G(6,J)
   DO 120 J=1,19
   COC7(J)=COC(7,J)
   SOE7(J)=SOEO(7,J)
   DHR7(J)=DHR(7,J)
120  G7(J)=G(7,J)
   CALL GRAPH(19,T1,0001,11,7,12.0,9.0,0.0,0.0,0.0,0.0,0.0,0.0,'TIME;','C/C +;','CONC VS TIME;','SUBROUTINE GRAPH;')
   CALL GRAPHS(19,T1,0002,11,7,'DEPTH=2 ;')
   CALL GRAPHS(19,T1,0003,11,7,'DEPTH=5 ;')
   CALL GRAPHS(19,T1,0004,11,7,'DEPTH=10;')
   CALL GRAPHS(19,T1,0005,11,7,'DEPTH=15;')
   CALL GRAPHS(19,T1,0006,11,7,'DEPTH=20;')
   CALL GRAPHS(19,T1,0007,11,7,'DEPTH=25;')
   CALL GRAPH(19,T1,SOE1,11,7,12.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,'TIME;','SIG/E);','CONC VS TIME;','SUBROUTINE GRAPH;')
   CALL GRAPHS(19,T1,SOE2,11,7,'DEPTH=2 ;')
   CALL GRAPHS(19,T1,SOE3,11,7,'DEPTH=5 ;')
   CALL GRAPHS(19,T1,SOE4,11,7,'DEPTH=10;')
   CALL GRAPHS(19,T1,SOE5,11,7,'DEPTH=15;')
   CALL GRAPHS(19,T1,SOE6,11,7,'DEPTH=20;')
   CALL GRAPHS(19,T1,SOE7,11,7,'DEPTH=25;')
   CALL GRAPH(19,T1,DHR1,11,7,12.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,'TIME;','DH/DH +;','HEADLOSS VS TIME;','SUBROUTINE GRAPH;')
   CALL GRAPHS(19,T1,DHR2,11,7,'DEPTH=2 ;')
   CALL GRAPHS(19,T1,DHR3,11,7,'DEPTH=5 ;')
   CALL GRAPHS(19,T1,DHR4,11,7,'DEPTH=10;')
   CALL GRAPHS(19,T1,DHR5,11,7,'DEPTH=15;')
   CALL GRAPHS(19,T1,DHR6,11,7,'DEPTH=20;')
   CALL GRAPHS(19,T1,DHR7,11,7,'DEPTH=25;')
   CALL GRAPH(19,T1,G1,11,7,12.0,9.0,0.0,0.0,0.0,0.0,0.0,0.0,'TIME;','G +;','G VS TIME;','SUBROUTINE GRAPH;')
   CALL GRAPHS(19,T1,G2,11,7,'DEPTH=2 ;')
   CALL GRAPHS(19,T1,G3,11,7,'DEPTH=5 ;')
   CALL GRAPHS(19,T1,G4,11,7,'DEPTH=10;')
   CALL GRAPHS(19,T1,G5,11,7,'DEPTH=15;')
   CALL GRAPHS(19,T1,G6,11,7,'DEPTH=20;')
   CALL GRAPHS(19,T1,G7,11,7,'DEPTH=25;')
STOP
END
$ENTRY
Computer Program No. 3

The third computer program was designed to study the effect the attachment coefficient, \( k_o \), on the breakthrough curve for a given flow rate, \( V_s \), bed depths, \( x \) and ultimate specific deposit \( \sigma_u \). The depths used were 0, 10, 20, 30, 40 and 50 cm, flow rate \( V_s = 10 \text{ gpm/ft}^2 = 2449.3 \text{ cm/hr} \) and \( \sigma_u = 0.3301 \), as given in run #10. The program was executed for values of \( k_o = 1200, 1300, ..., 2000 \) and the corresponding graphs of \( \frac{C}{C_0} (x,t) \) were drawn by the computer.

In this program the following abbreviations were used:

\[
A = e^{8k_oC_0t}, \quad B = e^{\frac{k_o\sigma_u}{2V_s}x},
\]

\[
C(I,J) = \frac{C}{C_0} (x,t) = \left[ \frac{A + 1}{2B + A - 1} \right]^2
\]

Computer program no. 3 follows:

```
$JOB SALEH,TIME=5,PAGES=60
DIMENSION CO(50),OL(50),C2(50),03(50),03(50),C5(50),C6(50),C0(50) +,CO1(50),IT1(50),XK0(10),0(50,10),C7(50),C8(50),C9(50),C10(50),IT2 ++(50),IT3(50),IT4(50),IT5(50),IT6(50),IT7(50),IT8(50),IT9(50),IT10( ++50)
C IT=TIME
C IX=LENGTH
C SIG=SIGMAU
C CO,C1,C2,C3,C4,C5,A RE CONCENTRATION RATIOS AT 0,10,20,30,40,50CM +DEPTH
```
SIG=0.3301
VS=2449.3
READ(5,*) (CO(J), J=1,33)
WRITE(6,300)(C)1(J),J=1,33)
300 FORMAT('0',F6.3)
DO 6 J=1,33
6 CO(J)=CO(J)*1.0E-4
DO 200 K=1,6
IX=10*(K-1)
WRITE (6,10)IX
10 FORMAT('0',15X,'X=',12)
DO 50 I=1,33
IT1(I)=I-1
DO 50 J=1,10
XK0(I)=(10+J)*100
A=EXP(B*XK0(J)*C0(I)*IT1(I))
B=EXP(XK0(J)*SIG*IX/(2*VS))
50 C(I,J)=((A+1)/(2*B+A-1))**2
DO 52 I=1,33
52 WRITE(6,53)(C(I,J),J=1,10)
53 FORMAT('0',10(F5.3,5X))
DO 60 I=1,33
IT1(I)=I-1
60 C1(I)=C(I,1)
DO 70 I=1,33
IT2(I)=I-1
70 C2(I)=O(I,2)
DO 80 I=1,33
IT3(I)=I-1
80 C3(I)=C(I,3)
DO 90 I=1,33
IT4(I)=I-1
90 C4(I)=C(I,4)
DO 100 I=1,33
IT5(I)=I-1
100 C5(I)=C(I,5)
DO 110 I=1,33
IT6(I)=I-1
110 C6(I)=C(I,6)
DO 120 I=1,33
IT7(I)=I-1
120 C7(I)=C(I,7)
DO 130 I=1,33
IT8(I)=I-1
130 C8(I)=C(I,8)
DO 140 I=1,33
IT9(I)=I-1
140 C9(I)=C(I,9)
DO 150 I=1,33
IT10(I)=I-1
150 C10(I)=C(I,10)
WRITE(6,160)(IT1(I),I=1,33)
160 FORMAT(2X,33(12, IX))
C PLOTTING GRAPHS
CALL GRAPH (33,IT1,01,11,7,12.0,9.0,0.0,0.0,0.0,0.0,0.0,0.0,'TIME;','C/CO;+
', 'CONC VS TIME;','SUBROUTINE GRAPH;')
CALL GRAPHS(33,IT2,C ,11,7,'K=1200;')
CALL GRAPHS(33,IT3,C3,11,7,'K=1300;')
CALL GRAPHS(33,IT4,C4,11,7,'K=1400;')
CALL GRAPHS(33,IT5,C5,11,7,'K=1500;')
CALL GRAPHS(33,IT6,C6,11,7,'K=1600;')
CALL GRAPHS(33,IT7,C7,11,7,'K=1700;')
CALL GRAPHS(33,IT8,C8,11,7,'K=1800;')
CALL GRAPHS(33,IT9,C9,11,7,'K=1900;')
CALL GRAPHS(33,IT10,010,11,7,'K=2000;')
200 CONTINUE
STOP
END
$ENTRY
APPENDIX III: THE PROBLEM OF PARTICLES IN THE DISTILLED WATER

In order to test the validity of the flocculation models developed herein, it was necessary to conduct filtration experiments by preparing known suspensions in a distilled water with no particles in size range below 2 μm. Unfortunately, the distilled water supply of the Town Engineering Building at Iowa State University did not meet this criterion and was quite variable in its particle concentration. The following data are presented to illustrate that problem and the attempts to resolve it using cartridge filtration of the distilled water.

The distilled water showed a rather high particle count in the smallest size range, 1 to 2 μm. The amount of submicron particles for this water could not be counted using the HIAC particle analyzer due to the instrument's lower size limitation of 1 μm. However, these numbers were speculated to be even higher than in the one μm range. Therefore, in an attempt to eliminate their presence, which might be confused with the minusil particles which were added to the water during the filtration studies the distilled water was filtered through a single shell cartridge filter model no. BRX10 3/4SD manufactured by Fulflo Filter, Division of Carborundum. The results of particle counting of the filtered distilled water are shown in Table III-1. A sample of unfiltered distilled water
collected from Room 177 in the same building at the same time was counted for comparison purposes.

As shown in Table III-1, the cartridge filter was partially effective in removal of 1-2 μm particles after a period of 20 to 30 minutes of continuous filtration. The deterioration in filtrate quality during the first 30 minutes was probably due to a release of some of the filter fibers which were counted and added to the count of the distilled water particles. However, it didn't take long before the filtrate particle count became nearly the same as that of the distilled water. The failure of the cartridge filter to remove all the 1-2 μm particles was one of the obstacles to experimentally verifying the mathematical model of bed contact flocculation.
Table III-1. Result of distilled water filtration through the cartridge filter

<table>
<thead>
<tr>
<th>Filtration Time (Minutes)</th>
<th>Average number of particles in each channel/30 ml sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1622</td>
</tr>
<tr>
<td>20</td>
<td>421</td>
</tr>
<tr>
<td>35</td>
<td>172</td>
</tr>
<tr>
<td>50</td>
<td>533</td>
</tr>
<tr>
<td>65</td>
<td>636</td>
</tr>
<tr>
<td>80</td>
<td>575</td>
</tr>
<tr>
<td>Dist. water counting</td>
<td>832</td>
</tr>
<tr>
<td>(from room 177)</td>
<td></td>
</tr>
</tbody>
</table>

Channel Size Range μm

1-2  2-3  3-4  4-5  5-6  6-7  7-9  9-13  13-17  17-20  20-25  25-60
APPENDIX IV: INVESTIGATION OF THE SHADOWING PROBLEM

Shadowing was one of the problems encountered with the HIAC particle analyzer. Since the instrument operates according to a light blockage technique, a cluster of small particles flowing through the HIAC channel might be counted as one large particle. Also, a larger particle may shield one or more smaller particles as they pass through the instrument. Therefore, dilution of the sample was necessary to prevent or reduce both shadowing and clustering problems.

The instrument manufacturer recommended a total particle count of not more than 20000 particles per ml as the total of all 12 channels to minimize these problems. However, even within this limit, the problem was present and detected in parallel observation using the transmission electron microscope. Therefore, it was decided to try to evaluate the maximum concentration of minusil which would minimize these counting problems.

To determine the concentration at which shadowing ceases to exist, several samples of suspension at different levels of dilution were prepared. In the first series of dilution, an original sample of 50 mg/l was diluted with distilled water in the following ratios of ml distilled water/ml suspension: 0.250, 50/200, 100/150, 150/100, 200/50 and 250/0.

The procedures used in preparation of the original suspension and the rest of the diluted samples were as follows:
1. Soak all the glassware and sample bottles in a chromic acid bath and wash with distilled water.

2. Prepare a 4.0 liter stock of dilution distilled water in a 4.0 liter beaker.

3. Pipette 0, 50, 100, 150, 200 and 250 milliliters of dilution distilled water and transfer each to 300 ml glass sampling bottles. Cover the mouths of all sampling bottles with Saran Wrap and then screw on the lids.

4. Add 75.0 mg of dry minusil to 1500 milliliters of dilution water in a two liter beaker.

5. Mix the suspension for two minutes using the particle dispersion unit at 6000 rpm speed.

6. Pipette 250, 200, 150, 100, 50 and 0 milliliters of suspension and transfer each to the sampling bottles prepared in Step 3 using the same order.

7. Run all the samples through the HIAC analyzer for particle count. For each sample, four replicates of 30 milliliters each were counted.

The results of particle counting are presented in Table IV-1, and the calculated number of particles in each channel of the HIAC are given in Table IV-2. The method of calculating particle numbers in each channel was as follows:

\[
N_s = \frac{[(D + S)N_i - D N_c]}{S}
\]
where

\[ D = \text{ml of distilled water} \]
\[ S = \text{ml of suspension (the 50 mg/l sample)} \]
\[ N_D = \text{number of particles in any channel present in distilled water} \]
\[ N_i = \text{number of particles in any channel present in diluted sample} \]
\[ N_S = \text{number of particles in any channel present in suspension before dilution.} \]

In the above series of dilution samples, solid concentrations were 50, 40, 30, 20, 10 and 0 mg/l. To fully cover the range of suspension concentration between 0 and 10 mg/l, another set of samples, 0, 0.1, 0.2, 0.5, 1, 2, 3, 4 and 5 mg/l, was prepared. The original suspension was 5.0 mg/l and was diluted according to the following ratios of ml distilled water/ml suspension: 250/0, 245/5, 240/10, 225/25, 50/200, 100/150, 150/100, 200/50 and 250/0.

The procedures used in the preparation and calculation of particle count in each channel were the same as before except in the preparation of the original suspension. The 5.0 mg/l suspension was prepared by mixing 100 ml of the 50 mg/l suspension prepared previously into 900 ml of the dilution water. The mixing time was 1.0 minute at a speed of 4000 rpm. The results of the particle count for these series of dilutions are presented in Tables IV-3 and IV-4.

The results of particle count of diluted samples indicate the general trend of particle number increase with regard to the extent of dilution. The most diluted samples exhibited the least degree of shadowing. The
<table>
<thead>
<tr>
<th>Dilution Ratio</th>
<th>Average Number of Particles Per Channel No./30 ml</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0/250</td>
<td>1,400</td>
</tr>
<tr>
<td>50/200</td>
<td>1,200</td>
</tr>
<tr>
<td>100/150</td>
<td>1,800</td>
</tr>
<tr>
<td>150,100</td>
<td>2,600</td>
</tr>
<tr>
<td>200/50</td>
<td>4,500</td>
</tr>
<tr>
<td>250/0</td>
<td>3,600</td>
</tr>
</tbody>
</table>

*aStarting suspension 50 mg/l minusil.*
### Table IV-2. Results of sample dilution particle count (1.0 mg/l)

<table>
<thead>
<tr>
<th>Dilution Ratio (ml dist. w./ml suspn.)</th>
<th>Calculated Values of Particle Count for Each Channel No./30 ml&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0/250</td>
<td>1,400</td>
</tr>
<tr>
<td>50/200</td>
<td>600</td>
</tr>
<tr>
<td>100/150</td>
<td>700</td>
</tr>
<tr>
<td>150/100</td>
<td>1,200</td>
</tr>
<tr>
<td>200/50</td>
<td>7,800</td>
</tr>
</tbody>
</table>

<sup>a</sup>Bases of calculations are: If D = ml of dist. water, S = ml of suspension; and N<sub>D</sub> = # of particles in any channel present in the dist. water; N<sub>i</sub> = # of particles in the same channel present in the diluted sample; N<sub>s</sub> = # of particles in the same channel present in the suspension before dilution; N<sub>i</sub> = (D.N<sub>D</sub> + S.N<sub>s</sub>)/(D + S), or

\[
\frac{[(D + S)N_i - DN_D]}{S}
\]

\[
N_s = \frac{[(D + S)N_i - DN_D]}{S}
\]
results of particle count of the 0.1, 0.2 mg/1 concentration samples were reasonably similar to each other and quite different from the results of the other samples. Therefore, it was concluded that the optimum recommended concentration at which the degree of shadowing is acceptable is 0.1 to 0.2 mg/1.

To substantiate these conclusions, the data of particle count for all the second series of samples were converted to mass of solids per 250 ml. To do this calculation, all the particles were considered perfect spheres with diameters equivalent to the average of the upper and lower particle size range in each channel of the HIAC. Also, the specific gravity of minusil particles was considered 2.65, as given by the manufacturer. The total mass of solids calculated for each dilution was compared with the actual value which was 0.150 mg. The results of this mass balance are presented in Table IV-3.

Because particles were assumed to be perfect spheres which is not really true, the calculated total solids mass per each dilution should be higher than the actual value of 0.150 mg. However, the total solids calculated for the 3, 4 and 5 mg/1 samples was lower than 0.150 mg because of the shadowing problem. The high values of mass of solids in the 0.1 and 0.2 mg/1 samples is basically due to the aforementioned assumption of particle sphericity. Based on both the counting data and the mass data, it is concluded that the 0.1 and 0.2 mg/1 samples gave the best counting results.

The results of this study show that the manufacturers limit of
20000 particles per ml does not necessarily ensure that shadowing problems do not exist. In fact, with both minusil and kaolinite clay, substantial undercounting in the small channels was observed even though the total count was within the 20000 per ml limit.
Table IV-3. Results of suspension dilution particle count (0.1 mg/l)

<table>
<thead>
<tr>
<th>Dilution Ratio</th>
<th>Average Number of Particles Per Channel No./30 ml</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>D/ml DW</td>
<td></td>
</tr>
<tr>
<td>S/ml SS</td>
<td></td>
</tr>
<tr>
<td>250/0</td>
<td>900</td>
</tr>
<tr>
<td>245/5</td>
<td>15,800</td>
</tr>
<tr>
<td>240/10</td>
<td>28,600</td>
</tr>
<tr>
<td>225/25</td>
<td>40,800</td>
</tr>
<tr>
<td>200/50</td>
<td>63,800</td>
</tr>
<tr>
<td>150/100</td>
<td>84,200</td>
</tr>
<tr>
<td>100/150</td>
<td>92,500</td>
</tr>
<tr>
<td>50/200</td>
<td>94,000</td>
</tr>
<tr>
<td>0/250</td>
<td>89,200</td>
</tr>
</tbody>
</table>

SS = Stock suspension containing 5 mg/l minusil.
DW = Distilled water.
S = ml of suspension (5 mg/l)
D = ml of distilled water.
Table IV-4. Calculated number of particles for each dilution (0.1 mg/1)

<table>
<thead>
<tr>
<th>Dilution Ratio</th>
<th>Average Number of Particles Per Channel No./30 ml (N_s)^a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>250/0 Dist. Wat.</td>
<td>900</td>
</tr>
<tr>
<td>245/5 0.1 mg/1</td>
<td>774,000</td>
</tr>
<tr>
<td>240/10 0.2 mg/1</td>
<td>691,800</td>
</tr>
<tr>
<td>225/25 0.5 mg/1</td>
<td>399,700</td>
</tr>
<tr>
<td>200/50 1 mg/1</td>
<td>315,100</td>
</tr>
<tr>
<td>150/100 2 mg/1</td>
<td>209,200</td>
</tr>
<tr>
<td>100/150 3 mg/1</td>
<td>153,200</td>
</tr>
<tr>
<td>50/200 4 mg/1</td>
<td>117,300</td>
</tr>
<tr>
<td>0/250</td>
<td>89,200</td>
</tr>
</tbody>
</table>

\[
N_s = \frac{(D + S)N_L - DN_D}{S}
\]

N_s = calculated particle count per channel in 30 ml before dilution.
Table IV-5. Mass balance calculations for the diluted samples

<table>
<thead>
<tr>
<th>Channel</th>
<th>Size Range (µm)</th>
<th>Average Particle Size in Channel (m)</th>
<th>Volume of Average Particle (in (µm)^3)</th>
<th>Total Weight of Particles Per Channel for the Following Dilutions in mg/1 x 10^-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>1-2</td>
<td>1-5</td>
<td>1.8</td>
<td>3.48</td>
</tr>
<tr>
<td>2</td>
<td>2-3</td>
<td>2-5</td>
<td>8.2</td>
<td>15.51</td>
</tr>
<tr>
<td>3</td>
<td>3-4</td>
<td>3.5</td>
<td>22.4</td>
<td>15.60</td>
</tr>
<tr>
<td>4</td>
<td>4-5</td>
<td>4.5</td>
<td>48.0</td>
<td>21.52</td>
</tr>
<tr>
<td>5</td>
<td>5-6</td>
<td>5.5</td>
<td>87.0</td>
<td>20.31</td>
</tr>
<tr>
<td>6</td>
<td>6-7</td>
<td>6.5</td>
<td>144.0</td>
<td>19.28</td>
</tr>
<tr>
<td>7</td>
<td>7-9</td>
<td>8</td>
<td>268.0</td>
<td>38.93</td>
</tr>
<tr>
<td>8</td>
<td>9-13</td>
<td>11</td>
<td>697.0</td>
<td>63.16</td>
</tr>
<tr>
<td>9</td>
<td>13-17</td>
<td>15</td>
<td>1,767.0</td>
<td>49.17</td>
</tr>
<tr>
<td>10</td>
<td>17-20</td>
<td>18.5</td>
<td>3,315.0</td>
<td>24.62</td>
</tr>
<tr>
<td>11</td>
<td>20.25</td>
<td>22.5</td>
<td>5,964.0</td>
<td>21.75</td>
</tr>
<tr>
<td>12</td>
<td>25-60</td>
<td>33.75</td>
<td>20,129.0</td>
<td>14.78</td>
</tr>
</tbody>
</table>

Total Weight in mg

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>0.162</td>
</tr>
<tr>
<td></td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>0.136</td>
</tr>
</tbody>
</table>

Actual weight in all channels is 0.150 mg.

Weight of particles/channel = 2.65 gm/cm^3 x no. of particles in the channel x vol. of average particle x 10^{-12}.
APPENDIX V

Particle Separation Study

Among the problems encountered using the minusil, was the presence of particles which were either too small or too large. The small particles, less than one micron, were not detectable by the HIAC particle analyser, and the large particles were often responsible for clogging the HIAC's aperture. In order to successfully deal with the minusil, removal of those undesirable particles was necessary.

Two different upflow separation columns were proposed to solve this problem. In both columns, particles were separated according to the difference between their downward settling velocity and the upward flow velocity in the column. The suspension concentration inside each column was kept below the level past which hindered settling occurred; otherwise, large particles surrounded by small ones might get carried away. Also, it was necessary to maintain medium stability by continuously feeding a solution of a sequestering agent (or a dispersant) in order to avoid particle agglomerations during the process of separation.

The following section includes a description of each column as well as the problems encountered of each. Also, a schematic drawing of the first and second particle separation systems are shown in Figures V-1 and V-2 respectively.

Description of the first elutriation column

The elutriator consisted of three glass tubes of different diameters arranged vertically. The lower tube had a diameter of 1.5 inches, the
middle tube, 3 inches, and the upper tube, 6 inches. Distilled water flowed from a constant head tank into the bottom of the lower tube and up through the three tubes in series to where it was discharged from the top of the upper tube. The flow rate was controlled by a valve located between the constant head tank and the bottom tube.

The elutriator operated on the basis of Stoke's law of sedimentation of spherical particles:

\[ V_p = \frac{2(G_s - G_l)g r^2}{9 \nu} \]

where

- \( V_p \) = settling velocity of particle (LT\(^{-1}\))
- \( G_s \) = specific gravity of particle (dimensionless)
- \( G_l \) = specific gravity of liquid (dimensionless)
- \( g \) = gravitational constant (LT\(^{-2}\))
- \( r \) = radius of particle (L)
- \( \nu \) = kinematic viscosity (LT\(^{-1}\)).

The settling velocity of a 1.0 micron minusil particle was calculated as follows:

- \( r = 0.5 \mu m \)
- \( G_s = 2.65 \)
- \( G_l = 1.00 \)
- \( \nu = 1.059 \times 10^{-5} \text{ ft}^2/\text{sec at 70° F} \)
- \( g = 980.5 \text{ cm/sec}^2 \)
\[
\frac{v}{p} = \frac{2(2.65 - 1.00) \times 980 \text{ cm/sec}^2 (0.5 \times 10^{-4})^2 \text{ cm}^2}{1.05 \times 10^{-5} \text{ ft}^2/\text{sec} \times (30.54)^2 \frac{\text{cm}^2}{\text{ft}^2} \cdot 9} = 0.92 \times 10^{-4} \text{ cm/sec}.
\]

In order to separate the 1.0 micron particles inside the middle tube, a flow rate equal to or slightly higher than (0.063 ml/min) had to be maintained.

Before each elutriation run, the particle dispersion unit was used to mix 100 to 125 grams of dry minusil with distilled water for two minutes at a speed of 6,000 rpm. The mixed minusil was immediately transferred to the lowest tube where the liquid velocity is greatest, since it has the smallest diameter. All particles which have a settling velocity less than the liquid velocity will flow upward to the next larger tube. The particles which have a settling velocity greater than the fluid velocity will remain in the tube. Because the succeeding tubes have increasingly larger diameters, the fluid velocity becomes proportionately smaller. Thus, the smaller particles are theoretically washed higher and higher in the column until their settling velocity is greater than the fluid velocity. Only the particles which have a settling velocity less than the fluid velocity in the outlet will escape from all the tubes.

As soon as the minusil suspension was placed inside the lowest tube, the flow of distilled water was started. The flow of suspension began to move from the lower tube to the upper tubes until the whole
Figure V-1. Water elutriator used for elutriating powdered quartz
column was completely filled and was running under a constant head. The flow was then adjusted by turning the rate control valve until the desired flow was reached. The feed rate of sodium hexa-meta phosphate was adjusted by changing the elevation of the dispersant container. The feed rate used during all the runs was around 0.1 mg/l.

Once the flow had stabilized, the elutriator was left to run continuously. The apparatus was watched very closely to prevent any flow fluctuations or total blockages due to minusil accumulation inside the lower portion of each tube.

The results of particle separation were totally unsuccessful. The cause of failure was partially due to the nature of minusil and partially due to the design of the elutriator itself. In the following paragraphs, some of the problems are briefly discussed.

First, the elutriated minusil settled rather quickly to the bottom of the container, regardless of the energy exerted by the stirring device. The settled minusil formed a stiff layer which was hard to break or re-disperse later on. For this reason, storage of elutriated minusil was rather difficult. Second, neither separation of submicron particles nor reproduction of the particle size results were achieved within a reasonable elutriation time. Because of the high demand for fresh elutriated minusil (500 gm of minusil required for one full load of suspension tank every 6.0 hours during a filter run.) The production of this small separation column was not sufficient. Third, the minusil sticks badly to the surfaces of glassware and the HIAC channels causing serious contamination problems.
Some difficulty was experienced in maintaining constant flow rate. These problems could have been corrected but the use of the device was discontinued due to its inadequate production capability. In order to solve the production problem, a second elutriator was designed as discussed below.

**Second elutriation column**

The elutriation column was a 5.50' I.D. plexiglass column six feet tall. Tap water was used due to high demand for distilled water for other purposes. The tap water pressure and flow were kept constant using a pressure regulator unit and a needle valve rate controller respectively as shown in Figure V-2. To take flow rate readings, a rotometer was connected between the vertical column outlet and the inlet to the rate controller. A dispersant feeder was also used to maintain a constant flow of sodium hexa-meta-phosphate at approximately 0.1 mg/l.

To operate the elutriator, the water was allowed to rise inside the separation column up to a level below the suspension inlet connection. The suspension was then prepared using the same procedures as in the first method; however, a larger amount of minusil was used per each run (1000 gm/run). The dispersed minusil was transferred to the separation column through the inlet cone and an upward flow of tap water was started at a very low rate. The pressure regulator's outlet was adjusted to 5.0 psi, and feeding of the dispersant solution was started. At the same time, the rate controller chamber was filled with tap water and the outlet needle valve was kept shut. As soon as the water overflowed the separation column and started flowing into the rate controller chamber,
the needle valve was gradually turned on until the selected flow rate was reached.

After 3 to 4 hours of elutriation, the flow rate was stopped and the minusil suspension was withdrawn through a bypass line attached to the bottom of the separation column. The contents of the bottom one-third of the separation column, believed to include the larger sized particles, were therefore wasted. The contents of the middle third were then collected mixed and with distilled water in one of the two large suspension tanks using the propeller mixer which was left running continuously. In order to obtain enough elutriated minusil for one full tank, the above elutriation process was repeated three or four times. Each time the elutriated suspension was transferred to the large influent suspension tank until a desired influent turbidity was reached.

In the second elutriation system, it was possible to resolve all the design problems encountered in the first one. The system pressure, flow rate, and dispersant feed rate were maintained constant during each elutriation run. Also, fewer elutriation runs were needed to produce the required amount of suspension for filtration experiments. However, in spite of this limited success, it was not possible to achieve a complete separation of the desired particle size range, nor to achieve reproducible influent particle size results for each elutriation run.
Fig. V -2. Schematic diagram of the second elutriation column
APPENDIX VI:  TYPICAL DATA FROM ONE EXPERIMENTAL FILTRATION
RUN COLLECTED WHILE FILTERING MINUSIL.
Table VI-1. Saleh’s filtration run 10

<table>
<thead>
<tr>
<th>Time hr.</th>
<th>Influent C₀</th>
<th>Effluent C</th>
<th>Piezometer Readings (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
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<td>52</td>
<td>4.206 4.212 4.218 4.224 4.230 4.236 4.242</td>
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</tr>
<tr>
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<td>68</td>
<td>47.5</td>
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<tr>
<td>12</td>
<td>65</td>
<td>30</td>
<td>2.940 2.955 2.970 2.987 3.018 3.028 3.047</td>
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</tr>
<tr>
<td>17</td>
<td>88</td>
<td>51</td>
<td>3.729 3.746 3.765 3.780 3.798 3.826 3.856</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>18.5</td>
<td>91</td>
<td>59</td>
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</table>

Q = 10 gpm/ft².

Polymer dose 1.0 mg/l.

Percol LT-22.
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<th></th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>ΔH₀⁺</th>
<th>C/C₀</th>
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Table VI-2. Influent and effluent particle count for Saleh's run 10

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<td>S^ Inf</td>
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<tr>
<td>S^ 2 Inf</td>
<td>3801</td>
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<tr>
<td>S^ 3 Inf</td>
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<td>S^ 4 Eff</td>
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</tr>
<tr>
<td>S^ 4 Inf</td>
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<tr>
<td>S^ Eff</td>
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<tr>
<td>S^ Inf</td>
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</tr>
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<td>S^ 6 Eff</td>
<td>55295</td>
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<tr>
<td>S^ 6 Inf</td>
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<tr>
<td>S^ 7 Eff</td>
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<td>S^ 7 Inf</td>
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<td>S^ 8 Inf</td>
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Table VI-2. continued

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<th>Sample Iden.</th>
<th>Avg. No. of Particles in Channel No. (No/30 ml Sample)</th>
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</tr>
<tr>
<td>( S_{17} ) Inf</td>
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</tr>
<tr>
<td>( S_{18} ) Eff</td>
<td>14,777</td>
</tr>
<tr>
<td>( S_{18} ) Inf</td>
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</tr>
<tr>
<td>( S_{19} ) Eff</td>
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<tr>
<td>( S_{19} ) Inf</td>
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</tr>
<tr>
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<tr>
<td>Sample #</td>
<td>Ratio of Eff./Inf. Particle Count in Channels</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>S1</td>
<td>1.348 1.304 1.195 1.275 1.300 1.321 1.329 1.088 0.498 0.145 0.054 0.014</td>
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<tr>
<td>S2</td>
<td>1.224 1.210 1.154 1.174 1.189 1.214 1.209 1.150 0.850 0.461 0.206 0.150</td>
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<td>S3</td>
<td>2.057 3.091 3.325 3.631 3.052 2.094 0.864 0.102 0.009 0.005 0.009 0.025</td>
</tr>
<tr>
<td>S4</td>
<td>9.281 5.090 4.709 4.008 2.697 1.474 0.450 0.055 0.025 0.077 0.380 4.690</td>
</tr>
<tr>
<td>S5</td>
<td>14.002 6.734 6.102 5.067 3.192 1.629 0.443 0.039 0.005 0.003 0.005 0.010</td>
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<td>S6</td>
<td>11.558 5.997 5.395 4.723 3.293 1.906 0.671 0.113 0.042 0.058 0.139 0.597</td>
</tr>
<tr>
<td>S7</td>
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</tr>
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<td>2.678 1.683 1.173 0.563 0.202 0.083 0.051 0.210 0.289 1.000 0.567 0.222</td>
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<td>S10</td>
<td>2.185 1.533 1.179 0.665 0.296 0.143 0.091 0.318 0.137 0.867 1.111 0.867</td>
</tr>
<tr>
<td>S11</td>
<td>2.238 1.670 1.470 0.996 0.499 0.219 0.092 0.081 0.383 0.045 0.459 0.130</td>
</tr>
<tr>
<td>S12</td>
<td>1.996 1.531 1.417 1.056 0.608 0.300 0.133 0.086 1.515 3.333 2.333 0.394</td>
</tr>
<tr>
<td>S13</td>
<td>1.755 1.425 1.354 1.099 0.730 0.427 0.222 0.125 1.400 1.500 3.308 1.538</td>
</tr>
<tr>
<td>S14</td>
<td>1.756 1.460 1.377 1.089 0.675 0.352 0.160 0.072 0.741 0.000 0.000 0.321</td>
</tr>
<tr>
<td>S15</td>
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</tr>
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<td>1.620 1.330 1.284 1.086 0.784 0.519 0.306 0.191 1.270 8.571 0.000 4.714</td>
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<td>0.866 0.798 0.816 0.827 0.846 0.857 0.809 0.650 0.660 3.400 4.000 1.500</td>
</tr>
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<td>1.155 1.081 1.089 1.093 1.087 1.071 0.020 0.825 0.480 0.339 0.400 0.438</td>
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<td>1.389 1.333 1.337 1.358 1.353 1.359 1.276 0.874 0.402 0.241 0.324 0.385</td>
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<td>S21</td>
<td>1.389 1.304 1.227 1.283 1.284 1.318 1.304 1.146 0.729 0.399 0.251 0.109</td>
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<td>1.389 1.304 1.227 1.283 1.284 1.318 1.304 1.146 0.729 0.399 0.251 0.109</td>
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