ESTIMATES OF EDDY CURRENT RESPONSE TO SUBSURFACE CRACKS FROM 2-D FINITE ELEMENT CODE PREDICTIONS

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ABSTRACT

Using a two dimensional finite element code, the response of a U-core eddy current probe was computed for a subsurface flaw in a stainless steel medium. Next, using a three dimensional scattering model, the change in coil impedance was calculated for the same situation. From a comparison of these two results, it was concluded that the two dimensional finite element code overestimates the eddy current sensor response for the practical problem at hand by a factor of 10. This agreed well with the result obtained using an approximate technique described in this paper to estimate the true response from two dimensional calculations. Application of such desensitization factor should allow the two dimensional calculations to be effectively used in design studies.

INTRODUCTION

The selection or development of a reliable eddy current inspection system for a specified flaw requires accurate determination of the sensor characteristic. Traditionally, this is achieved theoretically by predicting the coil response (i.e., the change in impedance, ΔZ) to cracks in the given test specimen. In the last few years, one of the popular techniques employed in such analysis is the two-dimensional finite element model. This numerical method calculates ΔZ_{2D} in ohms for unit length of the coil since the current source, conducting medium and the flaw are assumed to be infinitely long in the third direction. The physical significance of the 'infinitely long' flaw must be interpreted with some caution.

In two dimensional analysis the eddy currents flow perpendicular to the plane of the figure, so that they are obstructed by the crack area. However, in this analysis, the currents cannot flow together behind the crack as they would in a three dimensional calculation. Hence, it appears likely that these calculations overestimate somewhat the probe sensitivity values ($\Delta Z/Z$). It is useful to know the error associated with such predictions. This paper describes an approximate technique to estimate the true response of an eddy current sensor (ΔZ_{3D}) to a finite subsurface crack from the results of two dimensional calculations (ΔZ_{2D}). Both ΔZ_{2D} and ΔZ_{3D} were theoretically computed for a specific case using finite element and scattering models, respectively. The results are found to be in good agreement with the estimate.

2-D FINITE ELEMENT ANALYSIS

In the present study, a horseshoe ferrite core eddy current probe is used to detect subsurface flaws in a semi-infinite stainless steel medium. This situation is similar to the sleeved bolt hole inspection problem reported in references 1 and 2. In the present analysis, we deal with a single conducting medium as opposed to the layered medium in the sleeved bolt hole geometry. The U-core eddy current probe and the cross-section of the test geometry are depicted in Fig. 1.

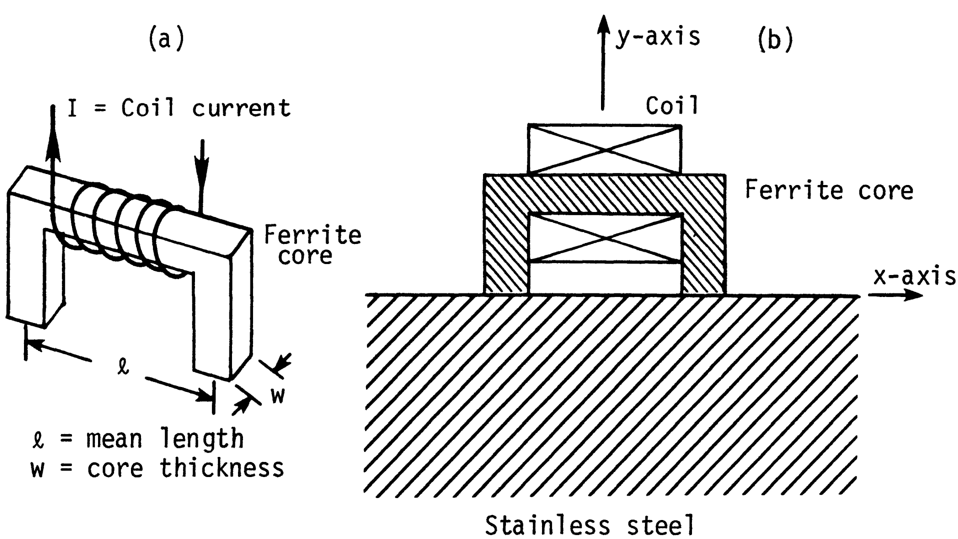

Fig. 1. a) U-core eddy current probe.
b) Cross-section of U-core probe over a semi-infinite conducting (stainless steel) medium.

The material of the semi-infinite medium is assumed to be stainless steel (nonmagnetic, and the electrical conductivity = 1.494×10^6 (ohm-m)$^{-1}$)). The relative permeability and electrical conductivity of ferrite core are 5000 and 1×10^{-6} (ohm-m)$^{-1}$, respectively. The flaw (0.03" wide and 0.015" deep) is assumed to be located at a depth of 0.05" from the surface. The excitation frequency is 40 kHz. Finite element solution of quasi-static magnetic fields in materials, and the prediction of eddy current probe impedance using this technique are presented in references 3 and 4, respectively. As the U-core probe slides over the slab, maximum impedance change (ΔZ) occurs when the probe is directly above the flaw which is located at a depth of 0.05" below the surface.[2] Taking advantage of the symmetry about y-axis (Fig. 1b), finite element analysis was performed only for one half of the region. Contours of constant magnetic vector potential values (absolute) are plotted in Fig. 2. The results of this two dimensional analysis are discussed later in this article.

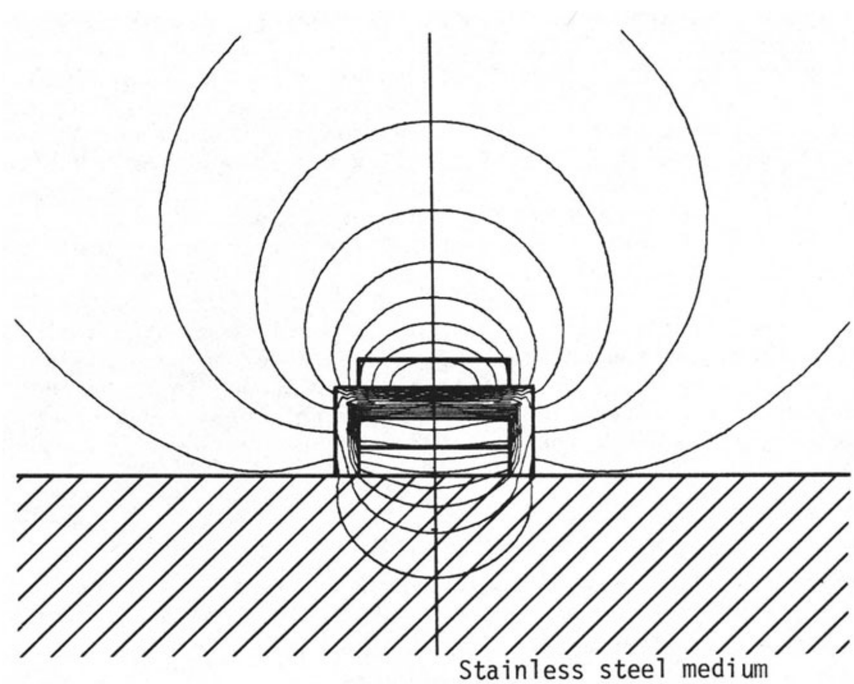

Fig. 2. Contours of constant magnetic vector potential (absolute) at 40 kHz.

3-D FIELD SCATTERING THEORY

The change in eddy current sensor impedance is found from the incident and scattered fields of the flaw by using the reciprocity theorem.[5] For a void flaw in a linear, homogeneous, isotropic, conducting medium with free space permitivity and permeability, the change ΔZ in sensor impedance is given by

$$\Delta Z = \frac{1}{I^2} \int_{v_f} \sigma \; (\bar{E} \cdot \bar{E}')dv \qquad (1)$$

where, σ = electrical conductivity of the medium
I = current at the sensor terminals
E = electric field without the flaw
E' = electric field with the flaw
v_f = volume of the flaw
dv = differential volume element.

It is apparent that, to compute ΔZ, it is necessary to know $\bar{E}$ and $\bar{E}'$ within the boundaries of the flaw.[6,7] The strategy for computing these electric fields is to approximate the incident field in the vicinity of the flaw by its constant plus linearly varying components. The respective scattered fields are then approximated for an ellipsoidal flaw by the dipole and quadrupole field solutions to the static form of Maxwell's equations. This quasi-static approximation is good when the spatial variations of the fields in the vicinity of the flaw take place over a distance small compared to a skin depth (i.e., the flaw size does not exceed a skin depth). Making use of these approximations, Kincaid[6,7] has derived expressions to calculate ΔZ for surface and subsurface flaws.

As a special case, let us consider a single component incident field which has constant direction and amplitude over a semi-infinite conductor surface. In Fig. 3, this field is chosen to be in the z direction with constant amplitude E_o. Within the conductor, the field equation along the positive x direction is

$$E_z = E_o e^{-(1+j)\frac{x}{\delta}} \qquad (2)$$

For this situation, Kincaid[7] has obtained an expression for ΔZ due to an ellipsoidal void flaw at a distance d from the surface of the conductor (Fig. 3).

$$\Delta Z = \frac{8}{3}\sigma ab^2 \left(\frac{E_o}{I}\right)^2 e^{-(1+j)\frac{2d}{\delta}} \{1+j \frac{12}{45} \left(\frac{b}{\delta}\right)^2\} \qquad (3)$$

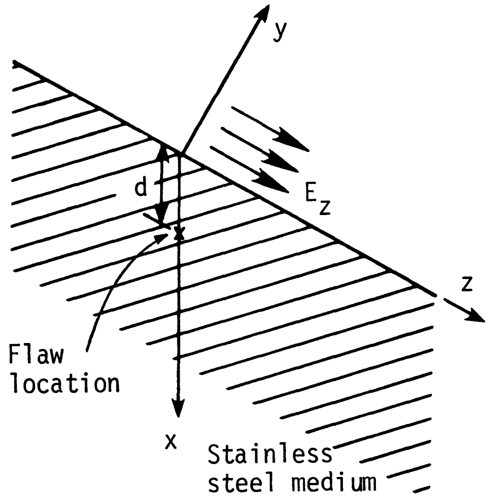

Fig. 3. Single component (E_Z) uniform incident field over a semi-infinite medium with a subsurface flaw.

where, δ = skin depth in conductor
a, b = major and minor axes of ellipsoidal void flaw (a and b are parallel to x and y axes, respectively, and b/a > 1/2).

ESTIMATE OF Z_{3D} FROM Z_{2D}

Figure 4 illustrates in greater detail the assumptions made in the interpretation of the $\Delta Z/Z$ calculations obtained using the 2-D finite element code. Since the currents flow parallel to the infinite dimension of the crack, no ΔZ will be predicted if the width, w, is set equal to zero. The best simulation of the 3-D crack case appears to be obtained when the width w and depth d of the 2-D crack are set equal to the length 2a and depth c of the 3-D crack. As shown in Fig. 4., the disturbance of the current flow in the 2-D claculation is comparable to that in the plane of the crack in the 3-D case. However, since the crack induced current disruption is less in other planes, one would expect the 2-D calculation to provide a systematic overestimate of $\Delta Z/Z$. A rough estimate of the error which this produces follows.

The inductance (L_{3D}) of a coil is given by

$$L_{3D} = \frac{1}{I^2} \iiint \mu H^2 dxdydz \qquad (4)$$

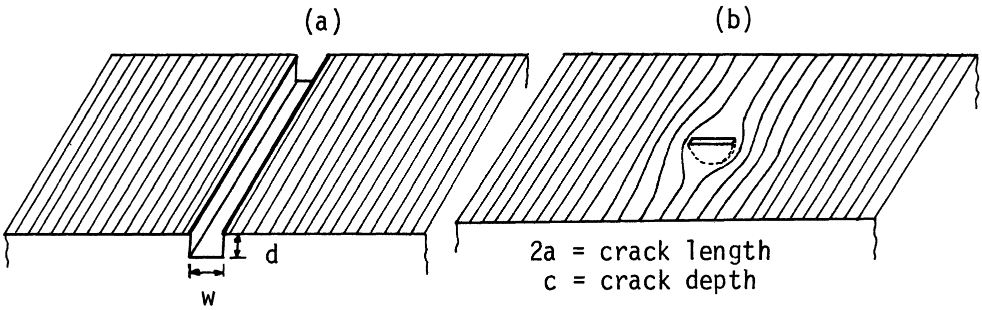

Fig. 4. a) Lines of current flow in two dimensional calculation.
b) Lines of current flow in three dimensional calculation.

where I, H and μ are current in the coil, magnetic field and magnetic permeability, respectively. In 2-D numerical computation the impedance is obtained as 'ohms/unit length'. That is

$$L_{2D} = \frac{1}{I^2} \iint \mu \bar{H}^2 dxdy \tag{5}$$

where $\bar{H}$ is the magnetic field in the 2-D analysis.

The flaw induced fractional change in inductance can be expressed as

$$\left(\frac{\Delta L}{L}\right)_{3D} = \frac{\iiint (H_o - H_1)^2 dxdydz}{\iiint H_o^2 dxdydz} \tag{6}$$

where, $H_o = H(x,y,z)$, flaw free H field
$H_1 = H_1(x,y,z)$, H field with flaw

or,

$$\left(\frac{\Delta L}{L}\right)_{3D} \simeq \frac{2\iiint H_o \Delta H dxdydz}{\iiint H_o^2 dxdydz} \tag{7}$$

where $H = H_o - H_1$ and $H/H_o \ll 1$.

Let ℓ and w (Fig. 1) be the mean distance between the pole faces, and the core thickness, respectively. Suppose that $\bar{H}_0(x,y)$ and $\Delta\bar{H}(x,y)$ in the vicinity of the flaw are known from the results of the finite element calculation. Then, when the probe is centered over the flaw, one expects that, near the flaw position,

$$H_o(x,y,z) \simeq \bar{H}_o(x,y)f(w/\ell) \tag{8}$$

where $f(w/\ell)$ is a factor, accounting for field decrease due to finite length effects, which varies from zero when $w/\ell = 0$ to unity when $w/\ell\to\infty$. Similarly, one expects that near the flaw position,

$$\Delta H(x,y,z) \simeq \Delta\bar{H}(x,y)f(w/\ell)g(z/a) \tag{9}$$

where, the factor $g(z/a)$ accounts for the field variation in the z direction. That is, at the flaw where $z = 0$, $g(0) = 1$, and when z increases, $g(z/a)\to 0$ as the fields return to their flaw free values.

Finally, it is assumed that the L_{3D} is related to the L_{2D} by the expression

$$\frac{L_{3D}}{w} = \iint \mu\bar{H}_o^2(x,y)dxdy\ h(w/\ell) \tag{10}$$

where, $h(w/\ell)$ is a factor accounting for finite length effects which varies from zero when $w/\ell = 0$ to unity when $w/\ell\to\infty$. Substitution of Eqs. (8)-(10) into Eq. (7) yields the desired result

$$\left(\frac{\Delta L}{L}\right)_{3D} \simeq \left[\frac{f^2(w/\ell)}{(\frac{w}{\ell})h(\frac{w}{\ell})}\right] \frac{\int_{-\infty}^{\infty} g(z/a)dz}{\ell} \left(\frac{\Delta L}{L}\right)_{2D} \tag{11}$$

The term inside the square brackets describes the finite length effects of the coil and is expected to be slowly varying when $w\sim\ell$. Note, for example, that both the numerator and denominator increase with w/ℓ and these variations would tend to cancel one another. Thus, for the purpose of this rough estimate, this factor is set equal to unity. If one further assumes that when $a\simeq c$, the spatial extent of the field perturbation by a crack is on the order of the crack radius, then

$$\int_{-\infty}^{\infty} g(z/a)dz \simeq 2a \tag{12}$$

and one obtains the final result

$$(\frac{\Delta L}{L})_{3D} \simeq \frac{2a}{\ell}(\frac{\Delta L}{L})_{2D} \tag{13}$$

RESULTS AND CONCLUSION

Taking a = 0.015" and = 0.16" as predicted by the finite element calculations,[2] one obtains $(\Delta L/L)_{3D} \simeq 0.2(\Delta L/L)_{2D}$. From the 2-D finite element prediction, $(\Delta L/L)_{2D} = 1.2 \times 10^{-3}$. Therefore, $(\Delta L/L)_{3D} = 2.4 \times 10^{-4}$. Using the scattering model, ΔZ_{3D} was calculated for an oval shaped crack (major and minor axes were assumed as 0.03" and 0.015", respectively) at a depth of 0.05" from the surface. That is, $\Delta Z_{3D} = (.17 \times 10^{-5})/I^2$ ohms, where I is the drive current required to produce a current density of 10^6 amps/m^2 in the coil whereas the 2D finite element code predicted $\Delta Z_{2D} = (0.425 \times 10^{-4})/I^2$ ohms/m. Assuming that w=0.16" (= 0.004 meter), these results imply

$$\frac{\Delta L_{3D}}{(\Delta L_{2D})w} = \frac{0.17 \times 10^{-5}/I^2}{(0.425 \times 10^{-4}/I^2) \times 0.004} = 0.1 \tag{14}$$

By way of comparison, the order of magnitude calculations leading to the estimate in Eq. (13) can be recast in the form

$$\frac{\Delta L_{3D}}{(\Delta L_{2D})w} \simeq f^2 (\frac{2a}{w}) \tag{15}$$

which has the value $0.2f^2$ for the case under consideration. The two are considered to be in excellent agreement, particularly when one would expect f to be somewhat less than unity for a coil in which $\ell = w$. Hence, it is concluded that the 2-D finite element calculations overestimate the sensitivity by a factor of 10 for the practical problem at hand. Application of this desensitization factor should allow those calculations to be effectively used in design studies in which the properties of the materials are varied.

ACKNOWLEDGEMENT

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