

EDDY CURRENT RESPONSE TO THREE-DIMENSIONAL FLAWS BY THE BOUNDARY ELEMENT
METHOD

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INTRODUCTION

In planning an inspection procedure, or in designing parts with flaw detectability as a design goal, it is essential that the engineer have available some form of model for estimating the probability of flaw detection. In the past this need has been met, with varying degrees of success, by relying on experience in the inspection of similar parts, sometimes supplemented by experimental testing. With the rapid advances in computer technology in recent years, it is now feasible to consider replacing, or at least enhancing, such practices with predictions based on numerical simulation of the flaw detection process [1].

For eddy current NDE, numerical simulation requires the solution of Maxwell's equations as a step toward predicting the response of a probe to the presence of a flaw [2]. In general this poses a very difficult problem because flaw, part and probe geometries are not amenable to analytic treatment. This means that computer simulation of eddy current flaw detection will almost always require the numerical solution of Maxwell's equations in a complex geometry. To further complicate matters, one needs not just a single calculation of probe response, but a multitude of such calculations, one for each probe position as it is moved over the surface of the part [1,3]. Finally, it is important to note that the inspection simulation problem is inherently three-dimensional because, at a general point in the course of a scan, the induced eddy current field is not symmetrical with respect to the flaw position.

In this paper we describe a hybrid approach to the three-dimensional simulation of an eddy current inspection. Our approach makes use of the boundary element method (BEM) [4] for solving the boundary integral form of Maxwell's equations for the current density and tangential magnetic field on the surface of a flaw in a known incident field. Incident field data, i.e., the current density and magnetic field in the material in the absence of a flaw, are provided by analytic solutions [5,6] for simple part geometries, or by an additional boundary element calculation if the part geometry is complex. Probe response is then calculated by means of the reciprocity theorem [7], with receiver field data provided again by analytic or boundary element calculations for the unflawed part.

By formulating the problem in this way it is possible to separate the calculation of flaw surface fields from calculations of incident fields and receiver response. The formal solution for the flaw surface field is the product of a solution matrix, which depends only on the flaw geometry and skin depth, and a column matrix representation of the incident field. Thus, to simulate a scan of the eddy current probe, only one calculation of the solution matrix is required. Changes in probe response caused by changes in probe position are then completely determined by calculations of the transmitter and receiver fields for the unflawed part. This makes for an efficient simulation of flaw signals as a function of position in a scan pattern for the purpose of determining the probability of flaw detection.

The theoretical elements that comprise this hybrid approach are developed elsewhere [8] and are therefore reviewed only briefly in the next section. The principal purposes of this paper are to provide illustrations of inspection simulations for a simple geometry, and to discuss extensions of the method to more complex, three-dimensional applications.

THEORY

The reciprocity theorem [7], which is given by (1), shows that the flaw signal can be expressed as an integral over the flaw surface of a vector product of certain fields which are labeled here with subscripts T and R.

$$\Delta Z = \frac{1}{I^2} \int_{S_F} [\vec{E}_R \times \vec{H}_T - \vec{E}_T \times \vec{H}_R] \cdot d\vec{S} \quad (1)$$

The T fields are those produced on the flaw surface when coil T, the transmitter or induction coil, is activated. The R fields are those that would be produced in an unflawed part if the receiver coil were activated. If we expand the vector product in the integrand, we see that only the tangential components of the T and R fields are involved. The reciprocity theorem therefore tells us that we need only the tangential components of the T and R fields on the flaw surface to determine the response of an eddy current probe. The boundary element method (BEM) is a numerical procedure for calculating these tangential fields.

Before describing the BEM as used here, it is first necessary to introduce a class of functions called dyadic Green's functions [9]. Physically, these functions relate the electric and magnetic fields at an arbitrary point \vec{x} within a conductor to the current in the conductor at another point \vec{x}' . In general, the Green's functions must be dyads, or, equivalently, tensors, in order to satisfy the boundary conditions at the surface of the conductor. Also, except in very simple geometries, dyadic Green's functions are complicated functions of position that cannot be expressed in terms of simple analytic functions. However, if the Green's dyads are known for a particular geometry in the unflawed conductor it is possible to simplify the calculation of flaw surface fields. So, for the present, let us assume that the dyadic Green's functions for the unflawed conductor are known.

In this case, starting with Maxwell's equations, it is possible to develop a set of coupled integral equations that involve only the tangential components of the fields on the flaw surface. The result is [8]

$$\frac{\Omega c}{4\pi} \mathbf{q}_i(\vec{x}) = \mathbf{q}_i^o(\vec{x}) + \sum_{j=1}^3 \int_{S_F} [T_{ij}^E(\vec{x}, \vec{x}') \mathbf{q}_j(\vec{x}') + U_{ij}^E(\vec{x}, \vec{x}') \mathbf{h}_j(\vec{x}')] dS \quad (2)$$

$$\frac{\Omega c}{4\pi} \mathbf{h}_i(\vec{x}) = \mathbf{h}_i^o(\vec{x}) + \sum_{j=1}^3 \int_{S_F} [T_{ij}^H(\vec{x}, \vec{x}') \mathbf{h}_j(\vec{x}') + U_{ij}^H(\vec{x}, \vec{x}') \mathbf{q}_j(\vec{x}')] dS$$

with $\vec{q}(\vec{x}) = \vec{n}(\vec{x}) \times \vec{j}(\vec{x})$, $\vec{h}(\vec{x}) = \mathbf{n}(\vec{x}) \times \vec{H}(\vec{x})$, where $\vec{n}(\vec{x})$ is the normal to the flaw surface at the point \vec{x} , and \vec{j} and \vec{H} are the current density and magnetic field intensity, respectively. In these equations the kernels U and T are determined by the dyadic Green's functions, Ω_c is the solid angle subtended by the conductor at the point \vec{x} , and \mathbf{q}_i^o and \mathbf{h}_i^o are the tangential components of the unperturbed fields, i.e., the fields that would exist at the point \vec{x} if no flaw were present. We assume, for the present, that the unperturbed fields can be calculated by another method. The simplification that results from use of the dyadic Green's functions is that the integrals are over points on the flaw surface only. If we had used simpler Green's functions that do not satisfy the boundary conditions on the surface of the conductor, then additional integrals over the surface of the conductor would appear on the right sides of these equations.

The boundary element method is a numerical technique for solving integral equations of this form. To develop an approximate set of algebraic equations, the surface of the flaw is divided into surface elements as shown in Figure 1, and each element is defined by a number of nodal points around the periphery of the element. Integration over the element is accomplished by expressing the fields inside the element in terms of their values at the nodal points, and evaluating the resulting integral by double Gaussian quadrature. The end result is a set of simultaneous algebraic equations for the fields at the nodal points, which can be written in matrix form. The solution for the tangential fields on the flaw surface is therefore of the form

$$\begin{bmatrix} \mathbf{q} \\ \mathbf{h} \end{bmatrix} = \mathbf{Z}^{-1} \begin{bmatrix} \mathbf{q}_o \\ \mathbf{h}_o \end{bmatrix} \quad (3)$$

where \mathbf{q} and \mathbf{h} are column matrices containing the components of the vectors \vec{q} and \vec{h} at each node. An important property of this solution, which is demonstrated elsewhere [8], is that the inverse matrix, which we will call the solution matrix, is a function only of the flaw geometry and skin depth. It is independent of the unperturbed field and is therefore independent of probe geometry and position. The vector containing \mathbf{q}_o and \mathbf{h}_o is, on the other hand, independent of the flaw geometry and depends only on the probe configuration and position. The final solution therefore involves a factor that depends only on the flaw and another factor that depends only on the probe. For a given flaw geometry and skin depth, this means that we need compute the solution matrix only once to determine the probe response as a function of probe position and/or configuration, which simplifies the simulation of an inspection as a function of scan pattern and probe geometry. Examples of such applications are given in the next section.

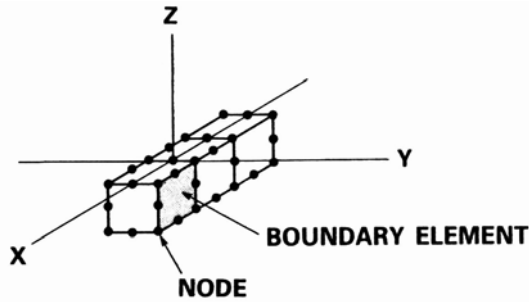


Fig. 1. Discretization procedure for the solution of integral equations by the boundary element method

FLAW SIGNAL PREDICTIONS

We now consider a simple example that illustrates the simulation of an eddy current inspection. In this case the flaw is a cube, about 8 mils on an edge, located about one skin depth below a plane surface as shown in Figure 2. We choose the skin depth to be large compared to flaw dimensions because, as we have shown elsewhere [8], this allows us to uncouple the electric and magnetic field solutions. Also, because the flaw is a skin depth below the surface, we use the infinite medium Green's dyad to calculate current densities on the flaw surface, and thus ignore effects of the plane surface in the BEM solution. To further simplify the calculation we also assume that the magnetic field is unperturbed by the presence of the flaw. It should be noted that none of these approximations are essential to the application of the boundary element method; they are introduced only as a way of saving computer time in these illustrative examples.

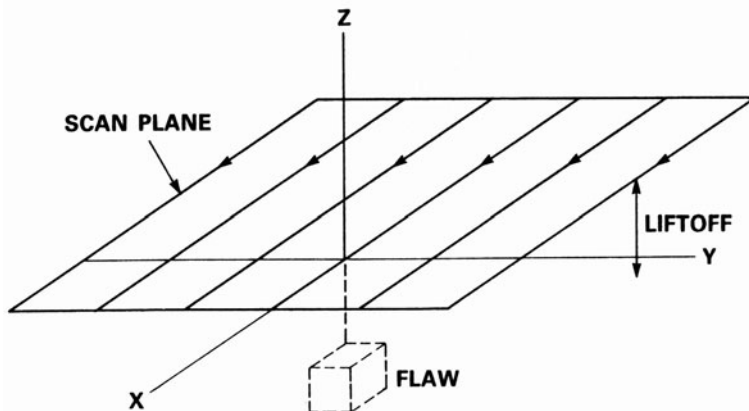


Fig. 2. Geometry for the calculation of eddy current flaw signals

Our illustrations are plots of the magnitude of the probe impedance as different probes are scanned over a raster pattern like that shown in Figure 2. To provide data on the unperturbed fields and the E_R and H_R fields of (1) we use a one-dimensional Fourier transform model equivalent to that of Dodd and Deeds [5], and, when necessary, a two-dimensional generalization of that theory [6].

Figure 3 shows the absolute value of the complex impedance of an absolute probe as a function of probe position; the flaw is located at the center of the pattern and the response shows the expected symmetry for a circular coil over a cubic flaw. Another calculation for an absolute probe of smaller diameter produces a similar pattern (not shown), the only difference being that the signal is better localized, as one would expect. The results of a third calculation show that an asymmetric signal is obtained when we use a separate receiver located adjacent to the transmitter and displaced in the positive X direction of Figure 2.

When the receiver coil is rotated so that its axis is parallel to the surface of the conductor and along the X axis, we see a different type of asymmetric signal as shown in Figure 4. Finally, in the fifth calculation of this series, still another asymmetric pattern (not shown) is obtained when the receiver coil axis is parallel to the Y axis of Figure 2.

The flaw model used for these examples is, of course, a very simple one and several approximations have been introduced to reduce computation time. Still, the results should serve as a first illustration of what can be done with a single boundary element calculation in the simulation of eddy current flaw detection. To apply the method to more practical problems the next steps we must take are to remove the various approximations we have introduced and extend the program to the treatment of more realistic part and flaw geometries.

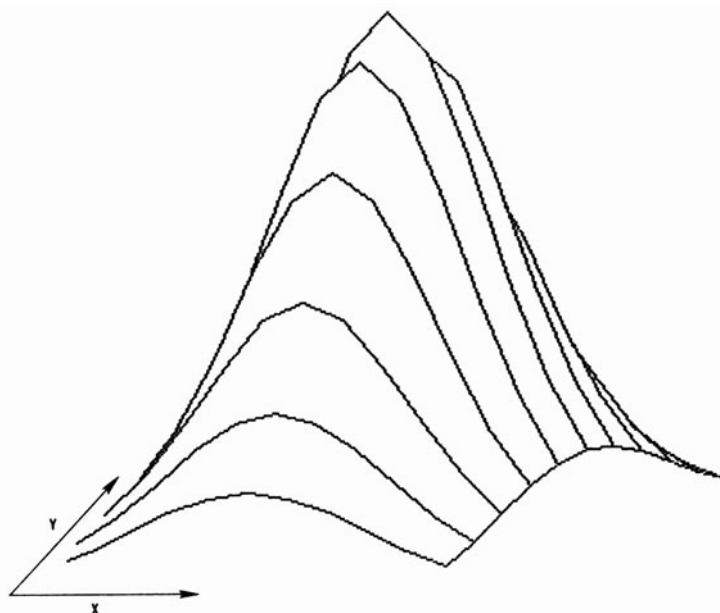


Fig. 3. Flaw signal magnitude for an absolute probe in the geometry of Figure 2

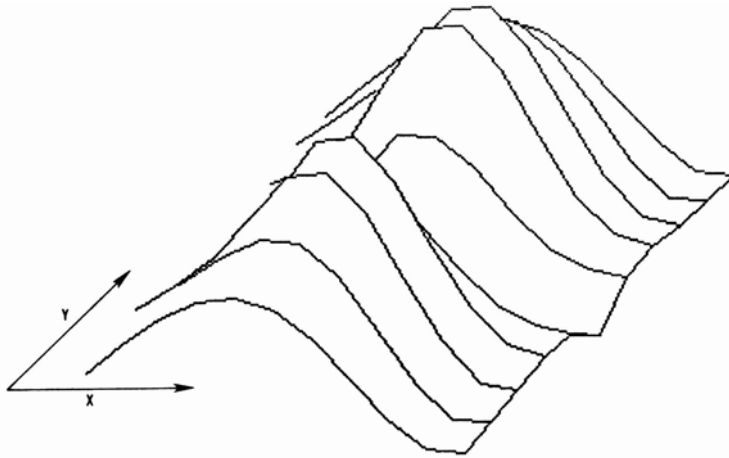


Fig. 4. Flaw signal magnitude for a probe with a separate receiver coil with axis in the X direction

EXTENSIONS OF THE MODEL

An important special case is the surface crack in a half-space or a flaw near a plane surface. Because the Green's dyads are known for the half-space [10], this class of problems can be handled in much the same way as the infinite medium calculations just illustrated. All we need do is replace the infinite medium kernels with half-space kernels in the existing code; there is no need to introduce boundary elements on the plane surface because boundary conditions on that surface are automatically satisfied through the use of the half-space Green's dyads. We expect that this version of the program will be useful mostly for studies of the effects of probe and flaw geometry on flaw detectability under ideal, flat-surface conditions.

If the part to be inspected has a complicated shape, then the calculation of appropriate Green's dyads becomes so complex as to be prohibitive. In such cases it becomes necessary to introduce boundary elements on the surface of the part, as well as the flaw, and solve for both the unperturbed and flaw surface fields by the boundary element method. It is still possible to present the solution for flaw surface fields as the product of a solution matrix times an incident field vector, but in this case the incident field is the field in air, rather than the unperturbed field in the conductor. Modification of the program to treat this, the most general case, will require considerable effort. However, development of such a code is considered feasible, and its implementation should be practical with existing computer capabilities.

Our plans for the immediate future therefore call for completion of the half-space version of the boundary element model, and, over a somewhat longer period of time, development of a general, complex geometry code. We also believe it would be advantageous to explore various approximations, such as those used in the calculations presented here, which could greatly reduce computational requirements while providing accuracy adequate for most purposes.

CONCLUSION

Our early experience with the boundary element method indicates that it should prove to be an efficient approach to modeling complex, three-dimensional eddy current problems. Because the desired solution can be expressed as the matrix product of a flaw-dependent factor and a probe-dependent factor, the formulation presented here appears to be well suited to the simulation of eddy current scanning operations and the evaluation of candidate probe designs.

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