Order Number 9321189

The excess smoothness puzzle in consumption

Lee, Taeyol, Ph.D.

Iowa State University, 1993

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

The excess smoothness puzzle in consumption

by

Taeyol Lee

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Department: Economics
Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1993

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ACKNOWLEDGMENTS

Thank you God for blessing me!

I would like to express my deepest appreciation to my major professor, Dr. Barry Falk. His kind guidance and encouragement have given me confidence about the successful completion of my study. I would also like to deeply appreciate my acting major professor, Dr. Walter Enders, for his nice caring. In addition, I also wish to thank Dr. Wayne Fuller, Dr. Jay Breidt, Dr. Donald Liu and Dr. Peter Orazem for valuable advises and helpful discussions.

Special thanks go to my wife, Insook Nam, for her love and sacrifice. I do believe this is our achievement rather than mine. Finally, I would like to dedicate this dissertation to my parent who love me most.

1 INTRODUCTION

1.1 Preview

Friedman's famous permanent income hypothesis was reconsidered by Hall (1978) under the framework of rational expectations. Hall showed that if consumption is the outcome of a certain type of optimization problem, current consumption must be approximately unpredictable based on past information other than one period lagged consumption. When Flavin (1981) analyzed Hall's idea formulating permanent income as the annuity value of the weighted sum of human and non-human wealth, she found significant explanatory power of lagged labor income with respect to current consumption and so rejected Hall's optimization hypothesis. This rejection is the so called "excess sensitivity" puzzle. Another violation of the optimization hypothesis was proposed by Deaton (1987). He argued that permanent income is much too volatile to predict actual consumption based on Flavin's formulation of permanent income. This is well known as the "excess smoothness" puzzle.

The random walk property of consumption and the smoothness of permanent income have been considered as testable hypothesis of optimal behavior, the rejection of these hypothesis seeming to imply a violation of rational expectations. However, a number of researchers have successfully explained excess sensitivity under the assumption of rational expectations. "Liquidity constraints" and "durability of consumption goods" are good examples. Even though excess sensitivity is well explained by rational behavior, excess smoothness has not been explained as well. There is even no consensus about the existence of excess smoothness and the measure of permanent income volatility. The purpose of this paper is providing information about these conflicts under the framework of rational expectations.

There have been two major explanations of the "excess smoothness" puzzle. First, Campbell and Deaton (1989) and Gali (1991) said the variance of consumption can be smaller than the variance of permanent income when there is excess sensitivity. It is clear that the existence of excess sensitivity spreads the effect of the permanent income innovation over multiple periods. Gali (1991) formulated permanent income in terms of consumption and set up a general consumption model embodying the existence of excess sensitivity. He concluded that permanent income is more volatile than consumption and that the existence of excess smoothness implies excess sensitivity. Second, Quah (1990) argued that the variance of permanent income itself can be small enough to resolve the "excess smoothness" puzzle if labor income contains a permanent and transitory component. He showed there are an infinitely large number of such decompositions of labor income which match any one univariate process and concluded that there are lots of possible decompositions of labor income which give sufficiently small volatility of permanent income. His result seems to resolve the "excess smoothness" without requiring any other possible explanation of the puzzle.

There are two interesting points this paper is going to highlight. First, these two explanations of excess smoothness differ about whether this puzzle can be explained without mentioning excess sensitivity. The paper criticizes Quah by arguing that his idea implies too small a covariance between the first differences of observed consumption and labor income. If his bivariate labor income process is restricted by the sample covariance between the first differences of consumption and labor income, it gives even more volatile permanent income than he showed This means that excess smoothness might be impossible to be resolve by employing a bivariate labor income process alone.

Second, there are two kinds of permanent income formulations by Flavin and by Gali. This paper finds that these imply different volatility of permanent income, and tries to reconcile this difference by constructing a model which embodies the two formulations. The model also embodies the existence of excess sensitivity and bivariate labor income process to analyze a general feature of consumption behavior.

Generalized Method of Moments (GMM) is suggested as an estimation method. Since excess smoothness is a problem from the second moment properties of consumption and permanent income, and the bivariate labor income process needs to be embodied without violating the second moment relationship between labor income and consumption, GMM which can fully utilize the second moment properties of variables might be a good way to estimate and test the model. To perform GMM, the paper suggests an algorithm which calculates weighted criteria values efficiently.

1.2 Organization

Chapter 2 provides a review of the rational expectations version of the consumption function focusing on the "excess smoothness" puzzle. This chapter provides mathematical explanations of some important papers: Hall (1978), Flavin (1981), Quah (1990) and Gali (1991).

The theoretical framework is considered in Chapter 3. This chapter criticizes Quah (1990) and concludes that his decomposition is not enough to resolve the puzzle. Then, it will reproduce major works which suggest the existence of excess smoothness based on our data set and show that two permanent income formulations imply different volatility of consumption. To reconcile this difference and analyze the excess smoothness phenomena, a general consumption model will be constructed. This model combines Gali's model for excess sensitivity and Quah's bivariate labor income process to derive a structural basis for time series equations.

Chapter 4 discusses the estimation method. First, statistical properties of the equations from chapter 3 are discussed. Second, Generalized Method of Moments (GMM) is suggested as an estimation method. This chapter also provides the background information for the application of GMM. Finally it will develop an algorithm which performs GMM efficiently.

Chapter 5 discusses empirical results. It performs econometric analysis to test whether the model is meaningful and estimated parameters are significant. Based on the estimated model, the chapter analyzes the difference in volatility of permanent income suggested in Chapter 3 and explains excess smoothness. It also analyzes a general feature of consumption behavior and discusses its implication. Chapter 6 provides a summary of general results.

2 LITERATURE REVIEW

2.1 Rational Expectations Interpretation of Consumption

The study of consumption has been profoundly influenced by the development of the rational expectations hypothesis in macroeconomics. After Lucas (1976) criticized traditional structural macroeconomic models, the consumption function was reconsidered in terms of economic an agent's optimization behavior. Hall (1978) first reexamined the life-cycle permanent income hypothesis under the assumption of rational expectations. His model led to a testable implication of the optimization behavior of consumers based on an Euler equation. The idea is that if consumption is the outcome of an optimization problem, current consumption should be nearly unpredictable based on past information other than one-period lagged consumption.

The consumer's optimization problem is to maximize

$$E_t \sum_s \left(\frac{1}{1+\delta}\right)^s U\left(C_{t+s}\right)$$

subject to $W_{t+1} = (1+r)W_t + Y_t - C_t$

where E_t = mathematical expectation conditional on all information in t

δ = rate of subjective time preference

r = real rate of interest, a constant over time

$U(\bullet)$ = one - period utility function, strictly concave

C_t = consumption

Y_t = labor income

W_t = assets apart form human capital

From the problem above, he derived the Euler equation.

$$E_t U'(C_{t+1}) = \frac{1+\delta}{1+r} U'(C_t) \qquad (2.1)^1$$

He approximated the Euler equation as a linear function between C_{t+1} and C_t and concluded that consumption must follow a simple random walk process.[2]

---

[1]We can consider W_t as the choice variable, and since $C_t = (1+r)W_t + Y_t - W_{t-1}$ and $C_{t+1} = (1+r)W_{t+1} + Y_{t+1} - W_t$, then,

$\partial E_t \sum_s \left(\frac{1}{1+\delta}\right)^s U\left(C_{t+s}\right) / \partial W_t = 0$ is the first order necessary condition of life time utility maximization of infinite-horizon agent. Since $\partial C_t / \partial W_t = 1 + r$ and $\partial C_{t+1} / \partial W_t = -1$, the F.O.C. is

$\left(\frac{1}{1+\delta}\right)^t (1+r) U'\left(C_t\right) - E_t \left(\frac{1}{1+\delta}\right)^{t+1} U'\left(C_{t+1}\right) = 0$. This is the same as (2.1).

[2]He argued that the Euler equation could be approximated closely by assuming a quadratic utility function. If the utility function is

Hall found that lagged real disposable income and lagged consumption except one-period lagged consumption have little predictive power for current consumption. Even though he showed that the Euler equation is rejected for lagged stock prices, he concludes that the optimization hypothesis is generally holds.

2.2 The Standard PIH

Flavin (1981) reconsidered Hall's work based on a formulation of permanent income. She formulated permanent income as the annuity value of the weighted sum of human and non-human wealth to derive the random walk property of optimal consumption. The advantage of her approach is that it provides a relationship among consumption, labor income and wealth.

When consumption exactly equals permanent income, Flavin's PIH gives a simple form of the consumption process. We call this the standard PIH. Mathematically, the standard PIH is

$$C_t = Y_t^p = r\left[W_t + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t Y_{t-1+i}\right] \tag{2.2}$$

---

quadratic, $U(C_t) = -\frac{1}{2}(\overline{C} - C_t)^2$ where $\overline{C}$ is the bliss level of consumption, then consumption has the exact regression, $C_{t+1} = \beta_0 + \gamma C_t - \varepsilon_{t+1}$, with $\beta_0 = \overline{C}(r - \delta)/(1 + r)$.

subject to $W_{t+1} = (1+r)W_t + Y_t - C_t$

where ΔY_t^p is the innovation in permanent income

By solving (2.2) with the intertemporal budget constraint, we can derive a simple random walk process of consumption:

$$\begin{aligned}\Delta C_t = \Delta Y_t^p &= r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i (E_t - E_{t-1})Y_{t-1+i} \\ &= \xi_t\end{aligned} \tag{2.3}$$

However, her empirical research rejected the optimal hypothesis by finding significant explanatory power of lagged labor income in explaining consumption changes.[3] This rejection is called the "excess sensitivity" puzzle.

2.3 Finite Horizon Life-cycle Model

Under the framework of rational expectations, the permanent income hypothesis and life-cycle income hypothesis have been treated as the same thing. However, Gali (1990) developed a finite-horizon model whose essential features are closer to that of aggregated life-cycle

[3]Sargent (1978) also rejected the optimazation hypothesis. However, an error in Sargent's paper was corrected by Flavin (1981). Other findings and resolutions of excess sensitivity are discussed in Section 2.4.

models developed by Franco Modigliani and his coauthors. Appendix A provides a detailed mathematical description of his model.

His idea is that even if the consumption of each agent satisfies the optimization hypothesis, the change of generation can lead to a violation of random walk property of consumption. More specifically, the consumption of an agent who died at period t and that of agent who is born at period t + 1 will be a random walk process.

In his model, agents have finite life times but are not concerned about the next generation. Mathematically, he assumes p portion of the total population dies each period and is replaced by the same number of new agents. Another important feature of his model is the assumption of an annuity market. Since an annuity firm inherits its customers' wealth or debt at their death, the annuity rate is higher than the market interest rate. The annuity rate is assumed as z ($z = (1+r)(1-p)^{-1} - 1 > r$).

Assuming Flavin's formulation of permanent income, he derives an individual's consumption to be

$$C_{s,t} = z\left[W_t + \sum_{j=1}^{\infty}(1+z)^{-j} E_{s,t} Y_{s,t-1+j}\right] \tag{2.4}$$

subject to $W_{s,t+1} = (1+z)W_{s,t} + Y_{s,t} - C_{s,t}$.

Therefore,

$$\Delta C_{s,t} = z\sum_{j=1}^{\infty}(1+z)^{-j}(E_{s,t} - E_{s,t-1})Y_{s,t-1+j}$$

This equation implies that the random walk property holds for individual consumption and it is consistent to with (2.3).

However, this random walk process can hold only for a consumer who survives both in period t and period t - 1. Since the probability of death is p each period, (1 - p) portion of aggregate consumption is following random walk. However, consumption of agents who died at period t and that of agents who are born at period t + 1 will not have the random walk property. Therefore, the expectation of aggregate consumption is

$$E_{t-1}C_t = (1-p)C_{t-1} + pE_{t-1}C_{t,t} \tag{2.5}$$

Therefore, (2.4) and (2.5) give

$$\Delta C_t = -pC_{t-1} - pz\sum_{j=1}^{\infty}(1+z)^{-j}E_{t-1}Y_{t,t-1+j} + \eta_t \tag{2.6}$$

where $\eta_t = C_t - E_{t-1}C_t$ is the innovation in aggregate consumption. There is no non human wealth in (2.6), because a new born agent has no wealth. If p = 0, (2.6) turns out to be the same as (2.3).

He said the difference between (2.3) and (2.6) can be tested by cointegration. More specifically, Gali formulates (2.6) as

$$\Delta C_t = -pC_t + \Gamma\beta Y_{t-1} + (\Gamma(1+z)/(1-\alpha))[\Omega + u_{t-1}] + \eta_t$$

where $\beta \equiv z/(z+\alpha), \Omega \equiv \beta\mu(1-\alpha)/(z+\alpha)$ and $\Gamma \equiv [1-(1-\alpha)(1-p)]$,

assuming $E(\Delta Y)$ exists and is equal to μ. Then, he showed that C_t and Y_t are cointegrated with cointegrating vector $[1, \Gamma\beta/p]$, if his model holds.

The infinite horizon model does not imply that consumption and labor income are cointegrated, but the finite horizon life-cycle model does. However, Gali concluded that there is no cointegration between labor income and consumption. This result weakens the evidence for the life-cycle model. His empirical research also could not find any evidence that the life-cycle consideration explains excess sensitivity or excess smoothness significantly.

2.4 Explanations of Excess Sensitivity

Following Campbell and Deaton (1989) and Gali (1991), excess sensitivity and excess smoothness might be closely related. This is the reason why this section reviews research regarding excess sensitivity. However, since the purpose of this paper is to explain the "excess smoothness" puzzle rather than the "excess sensitivity" puzzle, just a brief introduction about each idea is provided.

When income is abnormally low, some consumers may not be able to maintain optimal consumption by withdrawing their financial asset or borrowing. Since these liquidity-constrained consumer's consumption is predictable based on abnormally low lagged income, they can be the source of excess sensitivity. This idea was developed by Muellbauer (1983), Runkle (1991) and Zeldes (1989).

Current durables consumption is a flow of services from durable goods in the current period. Thus some consumption is provided by durables which were purchased in past periods. Since the purchase of these durables was based on the information available at that period, current durables consumption is partially determined by past information. Mankiw (1982), Bernanke (1985) and other papers developed this idea to explain excess sensitivity.

Flavin (1981) and Hall and Mishkin (1982) have noted that consumption contains a stochastic component independent of permanent income. This component may be a transitory component of consumption due to shifts in preferences, and can lead to the rejection of the simple random walk property of consumption.

Time-variable interest rates have also been suggested as the source of excess sensitivity based on the framework of Hansen and Singleton (1983). From (2.1), we can easily see that time-variable interest rate can violate a simple random walk property of consumption.

Time aggregation (Chistiano, Eichenbaum, and Marshall (1991)) has also been suggested as a possible explanation of excess sensitivity. Working (1960) showed that the a random walk process can be observed as an autocorrelated process when observation is based on aggregated periods. Christiano, et al, argued that violation of the random walk property based on quarterly data can be explained by time aggregation bias.

2.5 The "Excess Smoothness" Puzzle

Deaton (1987) showed that if labor income is difference stationary, the variance of consumption predicted by the permanent income hypotheses is much bigger than the sample variance of consumption. West (1988) analyzed the variance of permanent income for nine possible univariate ARMA representations of ΔY, and confirmed the existence of excess smoothness.[4]

Campbell and Deaton (1989) provided a very simple example of this puzzle using an ARMA(1,0) representation of ΔY. Their data period runs from 1953(2) to 1984(4), so that with differencing and lags there are 125 observations. By OLS, they estimated the following labor income process:

$$\Delta Y_t = 8.2 + 0.442\Delta Y_{t-1} + \varepsilon_t \qquad \sigma_\varepsilon = 25.2$$

[4]Those are ARMA (0,0), (1,0), (0,1), (1,1), (2,0), (0,2), (2,1), (1,2) and (2,2).

Quah (1990) showed that if $\Delta Y_t = \alpha + A(L)\varepsilon_t$,then (2.3) implies that $\Delta C_t = A\left(\frac{1}{(1+r)}\right)\varepsilon_t$. If $A(L) = 1 + 0.442L + 0.442^2L^2 + 0.442^3L^3 + \cdots\cdots$as in Campbell and Deaton's estimated model, the formula gives

$$\Delta C_t = \frac{(1+r)}{0.558+r}\varepsilon_t$$

The multiplier is 1.79 when r is zero, and is 1.76 when r is 10% per annum. Even though the theoretical standard deviation of consumption is at least 1.76 times larger than the standard deviation of the innovation of labor income, the sample standard deviation of consumption is 15.8 which is only about 60% of that of labor income This volatility of permanent income is called the "excess smoothness" puzzle.

2.6 Explanations of the Excess Smoothness Puzzle

There have been two major explanations of the "excess smoothness" puzzle. First, Campbell and Deaton (1989) criticized the widely accepted proposition that permanent income is smoother than measured income. They empirically showed that permanent income is less smooth than measured income and argued that the reason for observing smooth consumption is excess sensitivity. Later, Gali (1991) provided a more general model explaining the relationship between excess smoothness and excess sensitivity. Second, Quah (1990) argued that permanent income appears to be too volatile because we model labor income as a univariate process. He considered economic agents who use a more sophisticated labor income process than the univariate process.

This section covers the theoretical frameworks of Quah's bivariate labor income process and Gali's model for excess smoothness, respectively.

Bivariate labor income process

Quah assumed that consumers identify a permanent and transitory innovation in the labor income process. We call this idea "non-fundamentalness". The labor income process is assumed to be

$$Y_t = Y_{1t} + Y_{0t}$$

Y : labor income

Y_1 : permanent component

Y_0 : transitory component

Under the assumption that the process ΔY has finite time-invariant second moments, it necessarily has a unique Wold representation:

$$\Delta Y_t = \sum_{k=0}^{\infty} a_k \varepsilon_{t-k} = A(L)\varepsilon_t \qquad a_0 = 1 \qquad (2.7)$$

where ε is a serially uncorrelated innovation.

The permanent component Y_1 and the transitory component Y_0 are assumed to be a difference stationary process and a covariance stationary process, respectively. They are also assumed to be mutually uncorrelated processes. Quah writes their Wold decompositions as

$$\Delta Y_{1t} = \sum_{k=0}^{\infty} a_{1,k}\varepsilon_{1,t-k} = A_1(L)\varepsilon_{1,t} \qquad a_{1,0} = 1$$

$$Y_{0t} = \sum_{k=0}^{\infty} a_{0,k}\varepsilon_{0,t-k} = A_0(L)\varepsilon_{0,t} \qquad a_{0,0} = 1$$

Therefore, ΔY can be expressed as follows:

$$\Delta Y_t = A_1(L)\varepsilon_{1,t} + (1-L)A_0(L)\varepsilon_{0,t} \tag{2.8}$$

Since (2.7) and (2.8) are time series representations of an identical variable ΔY, they must satisfy the following condition:

$$\mathrm{var}(\varepsilon)\left|A\left(e^{-iw}\right)\right|^2 = \mathrm{var}(\varepsilon_1)\left|A_1\left(e^{-iw}\right)\right|^2 + \mathrm{var}(\varepsilon_0)\left|1-e^{-iw}\right|^2\left|A_0\left(e^{-iw}\right)\right|^2 \tag{2.9}$$

This condition means that the spectral densities of ΔY_1 and ΔY_0 sum pointwise to equal the spectral density of ΔY. Quah noted that there are an infinite number of combinations between $A_1(L)$ and $A_0(L)$ which can be identified as the same univariate process A(L).

By inserting (2.8) into (2.3), He gets ΔC under the assumption of the standard PIH:

$$\Delta C_t = \Delta Y_t^p = A_1(\beta)\varepsilon_{1,t} + (1-\beta)A_0(\beta)\varepsilon_{0,t} \qquad \beta = \frac{1}{1+r} \tag{2.10}$$
$$= \xi_t$$

(2.10) implies that the variance of the permanent income innovation is

$$\mathrm{var}(\zeta) = A_1(\beta)^2\,\mathrm{var}(\varepsilon_1) + (1-\beta)^2 A_0(\beta)^2\,\mathrm{var}(\varepsilon_0) \tag{2.11}$$

Quah calculates (2.11) under the condition (2.9) for different specifications of $A(L)$, $A_1(L)$ and $A_0(L)$.

Let us discuss briefly his strategy of calculation. The MA process of the first difference of the permanent component of labor income is assumed by Quah to be:

$$A_1(L) = \frac{(1+L)^q}{A_{1d}(L)} \tag{2.12}$$

The polynomial $A_{1d}(L)$ is $(1-0.8L)(1-0.85L)$.[5] The order of the MA process in ΔY_1 is q. When frequency ω is zero, (2.6) yields a simple condition between var(ε) and var(ε_1) as follows:

---

[5]Quah explains that this polynomial fixes the dominant root in the autoregressive part of A_0 at 0.85. If this is not done, it might seem that

$$\mathrm{var}(\varepsilon_1) = 4^{-q} A_{1d}(1)^2 A(1)^2 \times \mathrm{var}(\varepsilon) \tag{2.13}$$

Since $1 - e^{-i\omega}$ is zero at frequency zero, $\mathrm{var}(\varepsilon_0)$ varnishes. (2.9) also allows derivation of the dynamics in Y_0 which is

$$\begin{aligned} &\mathrm{var}(\varepsilon_0)\cdot(1-z)(1-z^{-1})A_0(z)A_0(z^{-1}) \\ &= \mathrm{var}(\varepsilon)\cdot A(z)A(z^{-1}) - \mathrm{var}(\varepsilon_1)\cdot A_1(z)A_1(z^{-1}) \end{aligned} \tag{2.14}$$

From (2.13) and (2.14), he obtains

$$\begin{aligned} &\mathrm{var}(\varepsilon_0)\cdot(1-z)(1-z^{-1})A_0(z)A_0(z^{-1}) \\ &= \mathrm{var}(\varepsilon)\left[A(z)A(z^{-1}) - 4^{-q}A_{1d}(1)^2A(1)^2\cdot\frac{(1-z)^q(1-z^{-1})^q}{A_{1d}(z)A_{1d}(z^{-1})}\right] \end{aligned}$$

Then he wrote

$$\begin{aligned} &\mathrm{var}(\varepsilon_0)\cdot A_0(z)A_0(z^{-1}) \\ &= \mathrm{var}(\varepsilon)\frac{\left[A(z)A(z^{-1}) - 4^{-q}A_{1d}(1)^2A(1)^2\cdot\frac{(1-z)^q(1-z^{-1})^q}{A_{1d}(z)A_{1d}(z^{-1})}\right]}{(1-z)(1-z^{-1})} \end{aligned}$$

---

the procedure simply trades off a declining importance in the permanent component for the transitory component approaching nonstationarity.

For sufficiently large q, the right-hand side is the covariogram of a real covariance stationary process. He can therefore factor it to obtain var(ε_1) and A_0, because A, A_{1_d} and var(ε) can be identified.[6]

He found that a bivariate labor income process with a long MA permanent component gives smooth permanent income. This finding holds for the nine possible ARIMA representations of U.S. aggregate labor income processes considered by Quah. However, Chapter 3 will show that a long MA permanent component implies too small a covariance between consumption and labor income.

<u>Excess smoothness and excess sensitivity</u>

Gali (1991) suggests an alternative formulation of permanent income to Flavin's permanent income. He said that an infinite-lived representative consumer faces a sequence of dynamic budget constraints of the form

$$W_{t+1} = (1+r)W_t + Y_t - C_t \qquad (2.15)$$

The consumer's transversality condition takes the form

[6]Since $A_0(0) = 1$ and A, A_1 and var(ε) are known, var(ε_0) can be easily calculated. Then Quah explained that the lag distribution A_0 is, in fact, simply the series expansion of a rational function. The denominator and numerator parts can therefore be obtained separately by a standard algorithm, such as that in Wilson (1969).

$$\lim_{T\to\infty} W_T^{'}(1+r)^{-T} \geq 0 \tag{2.16}$$

Assuming nonsatiation, (2.15) and (2.16) make it possible to obtain the intertemporal budget constraint:

$$\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t C_{t-1+i} = W_t + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t Y_{t-1+i}$$

Therefore, this constraint yields alternative formulation of permanent income. That is

$$Y_t^p = r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t C_{t-1+i} \tag{2.17}$$

Applying the law of iterated expectation, he obtains

$$\begin{aligned} Y_t^p - Y_{t-1}^p &= r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i (E_t - E_{t-1}) C_{t-1+i} \\ &= \xi_t \end{aligned} \tag{2.18}$$

His formulation has the advantage that permanent income can be analyzed without any assumption about labor income. Since estimating a labor income process may involve complicated problems like Quah's "non-fundamentalness", it may be more convenient to analyze permanent income.

Gali assumes a general consumption process as follows:

$$\Delta C_t = \Delta C_t^* + n_t \tag{2.19}$$

where ΔC_t^* = first difference of fundamental component of consumption or permanent consumption.

$$= \alpha + \sum_{i=0}^{\infty} b_i \xi_{t-i}$$

n_t = transitory consumption.

$$= \sum_{i=0}^{\infty} d_i u_{t-i} \qquad d_0 = 1$$

where u is transitory innovation in consumption. Excess sensitivity means that at least one of b_i or d_i (i ≥ 0) is not zero. If only b_0 and d_0 are not zero, consumption is random walk.

From (2.19), the consumption innovation is derived as follows:

$$\text{var}(\Delta C) = \sum_{i=0}^{\infty} b_i^2 \,\text{var}(\xi) + \sum_{i=0}^{\infty} d_i^2 \,\text{var}(u) \tag{2.20}$$

Then, by inserting (2.19) into (2.18), he obtains the following condition:

$$\sum_{i=0}^{\infty}(1+r)^{-i}b_i\xi_t + \sum_{i=0}^{\infty}(1+r)^{-i}d_iu_t = \xi_t \tag{2.21}$$

For (2.21) to hold, two restrictions can be derived:

$$\sum_{i=0}^{\infty}(1+r)^{-i}b_i = 1 \tag{2.22}$$

$$\sum_{i=0}^{\infty}(1+r)^{-i}d_i = 0 \tag{2.23}$$

From (2.20), (2.21) (2.22) and (2.23), we can derive a necessary condition for excess smoothness which is

$$\sum_{i=0}^{\infty}b^2{}_i < \left(\sum_{i=0}^{\infty}(1+r)^{-i}b_i\right)^2 \tag{2.24}$$

where $\left(\sum_{i=0}^{\infty}(1+r)^{-i}b_i\right)^2 = 1.$

This condition implies an important proposition. Excess smoothness implies the existence of excess sensitivity, but the converse is not true. To satisfy the condition (2.24), b_0 must be less than one. If b_0 is less than one, at least one of b_i (i ≥ 1) is not zero. This means excess smoothness implies excess sensitivity. However, we can find some set of b's which contain non zero b_i (i ≥ 1) but violate condition (2.24). If b_0 is bigger than one, (2.24) is clearly violated and at least one lagged b_i

is not zero. This means there is excess sensitivity but no excess smoothness. Therefore, excess sensitivity does not always imply excess smoothness.

Empirically, Gali reported that permanent income cannot be smoother than consumption no matter what representation of labor income is employed. His empirical results show that the standard deviation of permanent income innovations is from 1.25 to 1.67 times greater than the standard deviation of consumption innovations. He concluded that there are both excess smoothness and excess sensitivity.

Issue

Here, we need to highlight some interesting points. First, there is a conflict between Quah and Gali about whether permanent income can be smoother than consumption. Quah said it is possible by employing a bivariate labor income process which contains a transitory component. Gali showed that there is no evidence to say permanent income is as smooth as consumption. Chapter 3 will analyze this by reexamining Quah's "non-fundamentalness". Second, there is a big difference between permanent income volatility predicted by Flavin's permanent income and Gali's alternative formulation. Campbell and Deaton (1989) said permanent income is about 2.9 times more volatile than consumption based on Flavin's permanent income, but Gali (1991) reported that it is from 1.25 to 1.67 times more volatile than consumption. To discuss this issue, a general model which embodies Flavin's permanent income (2.2), Gali's alternative formulation (2.17) and consumption process (2.19) and Quah's bivariate labor income process (2.8) will be set up.

3 THEORETICAL FRAMEWORK

3.1 Critique of Quah (1990)

In this section, we will discuss why Quah's "non-fundamentalness" is not enough to resolve the "excess smoothness" puzzle. Gali (1991) already showed empirically that permanent income cannot be smooth based on a permanent income formulation in terms of consumption. This paper will reconcile their difference in opinion about the smoothness of permanent income by indicating the shortcomings of Quah's decomposition.

As mentioned in Chapter 2, Quah derived a lower-bound on the variance of the permanent income innovation. Technically, under condition (2.6), the lower-bound on the variance of permanent income innovation is

$$\inf_{A_1, A_0, \mathrm{var}(\varepsilon_1), \mathrm{var}(\varepsilon_0)} A_1(\beta)^2 \mathrm{var}(\varepsilon_1) + (1-\beta)^2 A_0(\beta)^2 \mathrm{var}(\varepsilon_0)$$

His analysis is done for nine possible ARMA representations of ΔY which West (1988) used. The result is that a long MA process of the permanent component gives a variance of the permanent income that is small enough to resolve the excess smoothness puzzle no matter which ARMA form of the labor income process is assumed. He reports that the excess smoothness can be resolved when the length of the MA process in ΔY_1 is at least 100 or 250 for the nine possible representations of the univariate process of labor income.

However, this chapter argues that the long MA process of the permanent component implies too small a covariance between consumption and labor income. Let us analyze the case when ΔY is an ARMA (1,0) process. The estimated volatility of ΔC and that of ΔY^p are equal to one another when q = 201. Quah reported,

$$\begin{aligned}
&\text{var}(\varepsilon) = 636.1\\
&\text{var}(\Delta C)\ \textit{due to}\ \varepsilon_1\text{: } A_1(\beta)^2\,\text{var}(\varepsilon_1) = 228.9\\
&\text{var}(\Delta C)\ \textit{due to}\ \varepsilon_0\text{: } (1-\beta)^2 A_0(\beta)^2\,\text{var}(\varepsilon_0) = 17.8\\
&\text{var}(\varepsilon_0) = 591.2
\end{aligned}$$

From (2.9), we can obtain the variance of the permanent component innovation which is

$$\text{var}(\varepsilon_1) = 4^{-q} A_{1d}(1)^2 A(1)^2 \times \text{var}(\varepsilon) = 4^{-201} \times 1.79$$[1]

---

[1] $A_{1d}(1)^2 = [(1-0.8)(1-0.85)]^2 = 0.0009$, and $A(1)^2 = 3.1888$. Therefore, $\text{var}(\varepsilon_1) = 4^{-201} A_{1d}(1)^2 A(1)^2 \times \text{var}(\varepsilon) = 4^{-201} \times .0009 \times 3.19 \times 636.1 = 4^{-201} \times 1.79$.

Then, we can derive $A_1(\beta)^2$ by dividing the variance of ΔC due to ε_1 by the variance of ε_1.

$$\begin{aligned} A_1(\beta)^2 &= (\text{var}(\Delta C)\ \textit{due to}\ \varepsilon_1) \div \text{var}(\varepsilon_1) \\ &= 228.9 \div 4^{-201} \div 1.79 = 4^{201} \times 127.6 \end{aligned}$$

$(1-\beta)^2 A_0(\beta)^2$ is derived by dividing variance of ΔC due to ε_0 by variance of ε_0.

$$\begin{aligned} (1-\beta)^2 A_0(\beta)^2 &= (\text{var}(\Delta C)\ \textit{due to}\ \varepsilon_0) \div \text{var}(\varepsilon_0) \\ &= 17.8 \div 591.2 = 0.03 \end{aligned}$$

It is clear that a large q implies extremely small var(ε_1). This property becomes a problem, when we consider the second moment between consumption and labor income. The covariance between ΔC_t and ΔY_t is

$$\begin{aligned} \text{cov}(\Delta C_t, \Delta Y_t) &= \text{cov}[(A_1(\beta)\varepsilon_{1,t} + (1-\beta)A_0(\beta)\varepsilon_{0,t}), (A_1(L)\varepsilon_{1,t} + (1-L)A_0(L)\varepsilon_{0,t})] \\ &= A_1(\beta)\text{var}(\varepsilon_1) + (1-\beta)A_0(\beta)\text{var}(\varepsilon_0) \\ &= 4^{-100.5} \times 20.4469 + 102.569 \\ &\cong 102.569. \end{aligned}$$

Since $A_1(\beta)\text{var}(\varepsilon_1)$ is negligibly small, $\text{cov}(\Delta C_t, \Delta Y_t)$ comes almost entirely from the transitory component. Is this covariance realistic?

The following simple regression may provide some intuitive information.

$$\Delta Y_t = \alpha_0 + \alpha_1 \Delta C_t + u_t$$

Since $\hat{\alpha}_1$ is cov(ΔC, ΔY)/var(ΔC) and var(ΔC) = 246.7, it should be 0.4157, if cov(ΔC, ΔY) = 102.569. However, it turns out to be 1.03 with consumption of nondurables and services.[2] That is, $\hat{\alpha}_1$ is twice as large as Quah implies. Even if we use total consumption expenditure, it is 0.65. When we think consumption expenditure implies bigger var(ΔC) which is a denominator of $\hat{\alpha}_1$ than consumption does, those results are significant evidence that Quah implies too small covariance between the first difference of consumption and labor income. Table 3-1 shows properties of parameters and second moments implied by Quah's decomposition for nine different univariate labor income processes. All of them support the arguments above.

From the discussion above, we can conclude that Quah's "non-fundamentalness" underestimates the relationship between consumption and labor income as long as his resolution is based on extremely small permanent innovation and extremely large parameters of that innovation. The covariance between ΔC and ΔY is even larger than his

[2]Consumption here is consumption expenditure on nondurables and services. Labor income is compensation of employees. The data period runs from 1954(1) to 1989(4). The next section discusses more about these data.

Table 3-1 Parameters and Second moments implied by Quah[a]

ARMA	q	$\mathrm{var}(\varepsilon_1)$	$A_1(\beta)^2$	$(1-\beta)^2 A_0(\beta)^2$	$\mathrm{var}(\varepsilon_0)$	$\frac{\mathrm{cov}(\Delta C, \Delta Y)}{\mathrm{var}(\Delta C)}$
(0,0)	99	$4^{-99} \times 0.71$	$4^{99} \times 345.0$	0.008	726.5	0.25
(1,0)	201	$4^{-201} \times 1.79$	$4^{201} \times 127.6$	0.030	591.2	0.41
(0,1)	151	$4^{-151} \times 1.16$	$4^{151} \times 206.0$	0.016	610.0	0.32
(1,1)	212	$4^{-212} \times 1.99$	$4^{212} \times 114.2$	0.034	595.7	0.44
(2,0)	206	$4^{-206} \times 1.81$	$4^{206} \times 121.4$	0.030	598.4	0.44
(0,2)	169	$4^{-169} \times 1.34$	$4^{169} \times 174.5$	0.022	586.9	0.34
(2,1)	209	$4^{-209} \times 1.86$	$4^{209} \times 118.0$	0.031	600.9	0.44
(1,2)	213	$4^{-213} \times 1.99$	$4^{213} \times 114.4$	0.034	595.5	0.44
(2,2)	216	$4^{-216} \times 1.98$	$4^{216} \times 110.5$	0.033	603.8	0.46

[a] This is produced based on Quah (1990)'s empirical results.

theory implies. Quah's decomposition of labor income cannot resolve the excess smoothness puzzle while satisfying a sample second moment property between consumption and labor income. This result supports Gali (1991) who said that excess smoothness cannot be resolved no matter what labor income process is employed. He reported that the standard error of the permanent income innovation may be from 1.25 time to 1.67 times larger than that of consumption based on permanent income formulated in terms of consumption and showed that this volatility can be explained by excess sensitivity. However, there remains a gap between the permanent income volatility predicted by Flavin' permanent income with univariate labor income process and by Gali's alternative formulation. In section 3.3, we will discuss this problem.

3.2 Data

The data consist of quarterly seasonally adjusted U.S. aggregate labor income and consumption from 1954(1) to 89(4). This section therefore discusses the source of these data, its treatment and some econometric properties

Source and treatment of data

First, this paper chooses quarterly seasonally adjusted compensation of employees from the U.S. National Income and Product Accounts to measure labor income. Since national income consists of compensation of employees, proprietor's income, rental income of persons, corporate profits before tax, and net interest, it is a good choice for a measure of labor income.[3] However, the concept of labor income in the model is disposable labor income rather than total labor income. To get this measure, this paper used annual total income tax data from Statistics of Income Bulletin. This source provides total income tax as a percentage of the personal income measure of NIPA. Assuming the tax rate is the same for labor income and capital income, the paper applies this tax rate to get disposable labor income. When we recognize that half of personal income is non-taxable income, this treatment may be better than applying the marginal tax rate.[4] All of measures above are nominal base. This paper uses the consumer price index for all urban consumers from NIPA as a deflator. The base of this measure is from 1982 to 1984 (1982-84 = 100). The data are divided by total population to be per capita base. The population measure was from Current Population

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[3]West (1988) and Deaton and Campbell (1989) used Blinder and Deaton's (1986) data whose sample period from 1953(2) to 1984(4). Their data set is basically the same as the data set of this paper, but they tried to distribute some portion of proprietor's income into labor income. They assumed that proprietor's income consists of labor income and capital income with the same ratio as the sum of wages and salaries plus other labor income to the sum of interest, dividends, and rental income. Since the magnitute of proprietor's income is about one tenth of compensation of employees and there may be a bigger portion of capital income in proprietor's income than their assumption, this paper excludes proprietor's income from the measure of labor income.

[4]Following Statistics of Income Bulletin, only 49.6 percentage of personal income was taxable income in 1989.

Report. Therefore, the measure of labor income can be formulated as follows:

$$\text{Labor income} = \frac{(\text{compensation employee})(1 - \dfrac{\text{total income tax}}{\text{personal income}})}{(\text{consumer price index})(\text{total population})}$$

Second, this paper uses consumption of nondurables and services as the measure of consumption.[5] The data were collected from the U.S. National Income and Product Account and were quarterly and seasonally adjusted. Since labor income uses the consumer price index as a deflator, the nominal consumption data are collected and divided by the same deflator. This treatment can protect this paper from using different base year data sets. The base year for consumer price index is from 1982 to 1984, but that of real consumption data is 1982. Total population is applied to get per capita base data. The consumption data here can be formulated as follows:

$$\text{Consumption} = \frac{\begin{gathered}(\text{consumption of expenditure nondurables}\\ + \text{consumption expenditure of services})\end{gathered}}{(\text{consumer price index})(\text{total population})}$$

[5]Gali (1991) used consumption of nondurables and services, but the sample period which is 1947(1) to 1988(3). West (1988) used the Blinder and Deaton (1986) data. The consumption data is for nondurables and services, excluding shoes and clothing. This sample period is from 1953(2) to 1984(4).

Time series properties of the data

Figure 3-1 shows time series for consumption and labor income. Both series show upward trend, but labor income fluctuates more than consumption. Neither series shows any sign of structural change. To analyze properties of the data more closely, this paper performs unit root tests and a cointegration test. Gali (1990) performed these tests for an updated version of the Blinder and Deaton (1985) data. He failed to reject either null hypothesis, the existence of a unit root and the no cointegration between labor income and consumption.

A number of studies of rational expectations versions of the consumption function assumed the existence of unit roots in labor income and consumption. This means that there are difference stationary process. Such a process is said to have a permanent component, because every shock to the process has a permanent effect on the future movement of that variable.

Dickey and Fuller (1979) derived the distribution of a statistic for the unit root null hypothesis under the assumption that first differences are uncorrelated and homoscedastic. As Figure 3-2 shows there is significant autocorrelation in the first differences of both variables. Therefore, this paper performs augmented Dickey-Fuller (ADF) tests to account for this autocorrelation. The unit root test is performed as follows:

——: consumption - - -: labor income annual rate $\times 10^3$

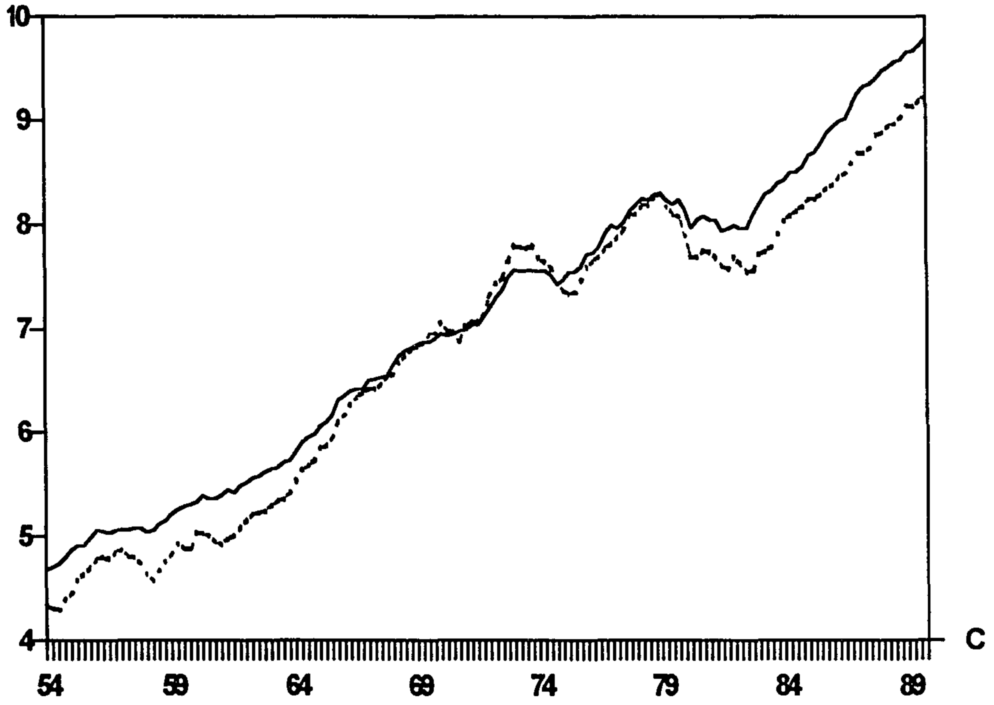

Figure 3-1 Data plot for consumption and labor income.

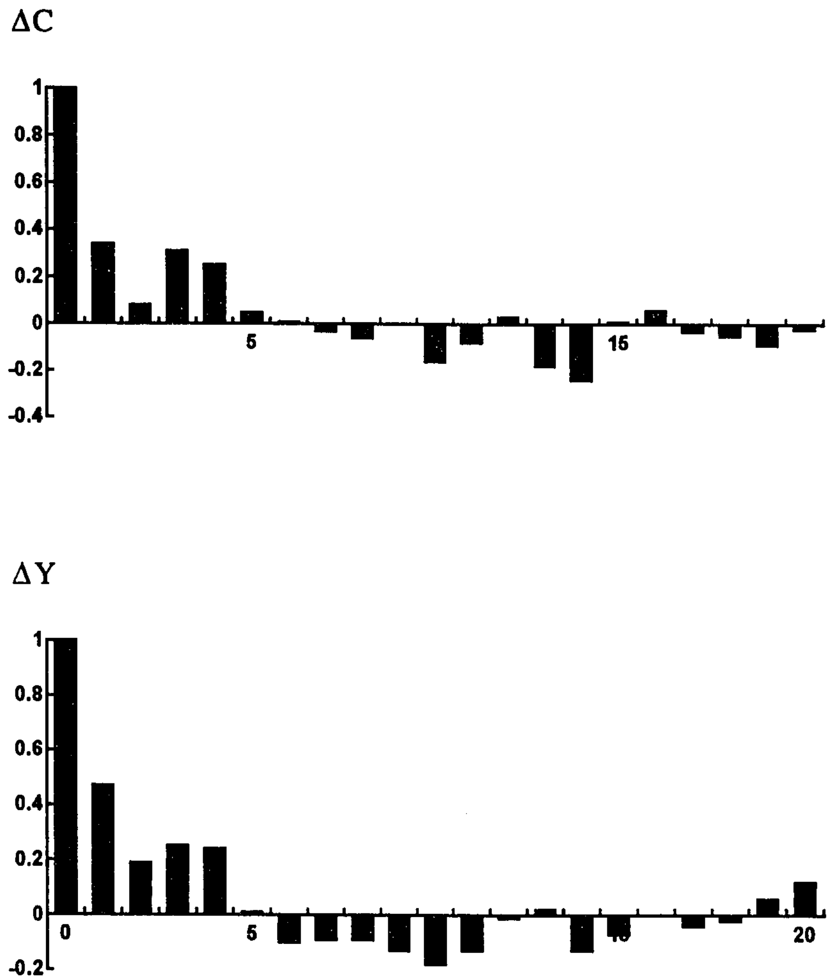

Figure 3-2 autocorrelation of ΔC and ΔY.

$$\Delta Y_t = \alpha + \beta t + (\rho - 1)Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-i} + \varepsilon_t \qquad ADF(\tau_\tau) \ = \ -1.68$$

$$\Delta C_t = \alpha + \beta t + (\rho - 1)C_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta C_{t-i} + \varepsilon_t \qquad ADF(\tau_\tau) \ = \ -1.90$$

The autocrrelation adjustment p here is 2. If p = 0, this test turns out to be the simple Dickey-Fuller (DF) test. Fuller (1976) provides critical values of $ADF(\tau_\tau)$ under the null hypothesis that $\rho = 1$, i.e., that there is a unit root.

	significance level			
n	0.01	0.025	0.05	0.10
250	-3.99	-3.69	-3.43	-3.13

The null hypothesis is not rejected even at the 10% significance level. This result implies that there are permanent components in the consumption and labor income processes.

Granger (1981) and Engle and Granger (1987) developed a cointegration test. If two stochastic processes are nonstationary in their levels but a linear combination of their levels is stationary, we say these processes are cointegrated. This means that these processes share a common persistent component. If consumption is cointegrated with labor income, the first difference of consumption must be predictable by some linear combination of consumption and labor income (error correction term). In other words, a rational economic agent uses this linear combination to decide optimal consumption. Theoretically, (2.3) and (2.19) imply that a rational econnomic agent does not need to use an error correction term to decide his optimal consumption.[6]

This paper performs a cointegration test by doing a unit root test for the residuals from two kinds of cointegration regressions. If we reject the null hypothesis of a unit root, it means there is cointegration. The ADF test is adopted for the unit root test.

$$Y_t = \alpha + \beta C_t + e_t$$

$$\Delta e_t = (\rho - 1)e_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta e_{t-i} + \varepsilon_t \qquad ADF(\tau) \; = \; -1.46$$

$$C_t = \alpha + \beta Y_t + e_t$$

$$\Delta e_t = (\rho - 1)e_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta e_{t-i} + \varepsilon_t \qquad ADF(\tau) \; = \; -1.22$$

The autocorrelation adjustment p here is 4. Engle and Yoo (1987) provides a critical value for ADF test with p = 4.

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[6]Section 2-3 discussed the implication of cointegration in a finite horizon life-cycle model. Even though Gali (1990) theoretically showed that the change of generations can give prediction power to the error correction term, he could not find supporting empirical evidence.

	significance level		
n	0.01	0.05	0.1
200	-3.78	-3.25	-2.98

We fail to reject the null hypothesis even at the 10% significance level. This is a consistent result with Gali (1990). Therefore, this paper does not consider the finite horizon model and the implied error correction term.

3.3 Excess smoothness revisited

Based on Flavin's permanent income and univariate labor income process, West (1988) reported that permanent income is from 2.87 times to 3.11 times volatile than consumption.[7] This means that Flavin's permanent income formulation assuming a univariate labor income process is even more volatile than that of Gali. However, these figures are produced based on different data sets. This section is going to reproduce their results based on the same data set.

Table 3-2 shows the volatility of permanent income under the assumptions of Flavin's permanent income and a univariate labor income process. Results are compared with those of West based on ARMA (1,0), $(1-\phi_1 L)\Delta Y_t = \varepsilon_t$, and ARMA (2,0),.$(1-\phi_1 L-\phi_2 L^2)\Delta Y_t = \varepsilon_t$. κ is

[7]Since very short MA processes are not good representations of the labor income process, the results based on MA (0,0), (0,1), (0,2) are disregarded.

square root of the ratio of the variance of permanent income to variance of consumption. If $\kappa > 1$, there is excess smoothness. The estimation method is described in Section 2.5. Every estimate of κ confirms there is strong excess smoothness. West predicts a bigger κ than this paper does. This difference may be caused by his exclusion of shoes and clothing from consumption data.

Next, we analyze volatility of permanent income based on Gali's measure of permanent income. From (2.19), (2.22) and (2.23), Gali obtains simple relationship between the variance of the permanent income innovation and the spectral density of consumption as follows (See Appendix B):

$$\lim_{r\to 0}[2\pi h_{\Delta C^*}(0) - \mathrm{var}(\xi)] = 0 \tag{3.1}$$

$$\lim_{r\to 0} h_n(0) = 0 \tag{3.2}$$

Orthogonality at all leads and lags between ΔC and n implies $h_{\Delta C}(\omega) = h_{\Delta C^*}(\omega) + h_n(\omega)$ for all $0 < |\omega| < \pi$, and the following result follows directly form (3.1) and (3.2).

$$\lim_{r\to 0}[2\pi h_{\Delta C}(0) - \mathrm{var}(\xi)] = 0 \tag{3.3}$$

Table 3-2 Volatility of permanent income with univariate labor income process

	This paper	West
ARMA (1,0)		
ϕ_1	0.46	0.44
ψ[a]	1.84	1.79
κ[b]	2.36	2.87
ARMA(2,0)		
ϕ_1	0.47	0.43
ϕ_2	-0.03	0.01
ψ	1.78	1.78
κ	2.30	2.93

[a] Under the assumption of the standard PIH $\Delta C_t = \psi\varepsilon_t$.

[b] The square root of the ratio of variance of permanent income to variance of consumption.

Let $f_{\Delta C}(\omega) \equiv h_{\Delta C}(\omega)/\operatorname{var}(\Delta C)$ be the normalized spectral density of ΔC and γ be var(ξ)/var(ΔC). From (3.3), we can obtain

$$\lim_{r \to 0}[2\pi f_{\Delta C}(0) - \gamma] = 0 \tag{3.4}$$

$2\pi f_{\Delta C}(0)$ in (3.4) is a consistent estimate for the variance ratio γ, and an estimate for $f_{\Delta C}(\omega)$ is

$$\hat{f}_{\Delta C}(\omega) = (2\pi)^{-1} \sum_{s=-(N-1)}^{N-1} \lambda(s)\hat{\rho}(s)\cos(s\omega)$$

where

$$\hat{\rho}_{\Delta C}(s) = \frac{\sum_{t=s+1}^{N}(\Delta C_t - \overline{\Delta C})(\Delta C_{t-s} - \overline{\Delta C})}{\sum_{t=1}^{N}(\Delta C_t - \overline{\Delta C})^2}$$

$$\overline{\Delta C} = \left(\frac{1}{N}\right)\sum_{t=1}^{N} \Delta C_t$$

We use a Bartlett window to estimate γ. The "Bartlett window" $\{\lambda_B(s)\}$ is

$$\lambda_B(s) = \begin{cases} 1 - |s|/M & |s| \le M \\ 0 & |s| > M \end{cases}$$

Table 3-3 shows the volatility of permanent income based on Gali's permanent income formulation. κ is simply the square root of λ. The estimates are generally consistent with those of Gali. Table 3-2 and Table 3-3 shows that the two formulations predict different levels of volatility of permanent income. Table 3-2 predicts permanent income is about 2.3 times more volatile than consumption based on Flavin's permanent income and a univariate labor income process. However, Table 3-3 predicts permanent income is from 1.31 times to 1.58 times more volatile than consumption. Both results show evidence of excess smoothness, but estimates of volatility are different from each other. The second purpose of this paper is to reconcile this difference. The next section constructs a general model which embodies two different permanent income formulations, excess sensitivity and the existence of a bivariate labor income process.

3.4 General PIH Model

This section will construct a model of the Permanent Income Hypothesis which embodies the excess sensitivity phenomena, a bivariate labor income process and the permanent income formulations of Flavin and of Gali. The model will consist of two equations: a labor

Table 3-3 Estimates of κ based on Gali's permanent income.

Window size	This paper	Gali
2	1.156	1.114
3	1.225	1.190
4	1.319	1.272
5	1.409	1.336
6	1.471	1.370
7	1.515	1.401
8	1.545	1.433
9	1.564	1.451
10	1.580	1.467
20	1.494	1.539
30	1.480	1.535
40	1.363	1.582
50	1.336	1.624

income process and a consumption process.

Background of the Model

Following Flavin (1981), Quah (1990) and Gali (1991), this paper analyzes aggregate consumption based on an infinite-horizon representative agent model. An agent has full information about current and past economic variables and can fully utilize the information available. Labor income is assumed to be an independent stochastic process. This means that a consumer does not need to utilize any information other than the current and past history of labor income to predict future labor income. Interest rate is assumed to be constant. At the beginning of each period, an agent is assumed to know his current labor income which will be paid at the end of the period. A consumer calculates his permanent income and decides the amount of consumption at the beginning of each period, but he earns labor income and consumes it at the end of period. This is the reason why current labor income is discounted by the interest rate in the formulation of permanent income.

The characteristic of this model can be defined as an infinite horizon representative agent model with exogeneous labor income process and a constant interest rate. The reason of adopting a very restricted model is that the model can not gain much explanatory power comparing to the complexity caused by relaxing restrictions. Section 2-3 and Section 3-2 already showed that the consideration of a finite horizon life time does not have any evidence. Therefore, this section discusses the restrictions of a constant interest rate, exogenous labor income process and a representative agent.

Constant interest rate

Capital return has been considered as an important factor in the consumption decision. Summers (1982) argued that the expected real interest rate has a significant role in explaining the current change of consumption. However, Hall (1988) reversed this finding using an autoregressive transformation proposed by Hayashi and Sims (1983). He concludes that there is no significant influence from the interest rate to consumption change. If the model embodies a stochastic movement of interest rate, the additively separable linear function (2.2) is going to become highly nonlinear. Thus, there may not be much gain from employing time variable interest rate.

The assumption of a constant interest rate implies that the only source of unexpected fluctuation of income comes from labor income. This is the reason that permanent income is formulated as a function of innovations in only labor income process.

Exogenous labor income

Endogenous labor supply is another possible factor which may affect the consumption decision. If a current increase (decrease) of the wage rate causes a decrease (increase) of future labor supply, the estimated volatility of permanent income is smoother than this model predicts. Lucas and Rapping (1969) discussed various aspects of the U.S.

aggregate labor market based on annual data. Especially, this paper is interested in whether wage rate changes cause changes of labor supply. They concluded that labor supply is elastic to the wage rate. However, this paper recreates their causality test based on U.S. quarterly data. Employee hours in nonagricultural establishments is adopted as labor supply. This data set is divided by total population to exclude the increase of labor supply due to population change. The wage rate is measured as the index of real average hourly earnings of production on nonsupervisory workers on private nonagricultural payrolls.[8] The base year of this measure is 1977. These data sets are collected from NIPA and are seasonally adjusted. The sample period is from 1964 (1) to 1988 (4). Since Lucas and Rapping analyze the effect to an labor supply from current and one period lagged annual wage rate and one period lagged annual labor supply, this paper includes up to 8 lags (p = 8) for the causality test. The null hypothesis is that all γ_i are zero in the regression equation:

$$\Delta L_t = \alpha + \sum_{i=1}^{p} \beta_i \Delta L_{t-i} + \sum_{i=1}^{p} \gamma_i \Delta W_{t-i} + u_t$$

$F^* = 1.420$ $\quad n = 91 \quad F(8,90,0.05) = 2.04$

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[8]In Lucas and Rapping, labor supply is man hours per year divided by population over fourteen years of age with constant age-sex distribution, and the wage rate is real compensation per man-hour. All data are annual and logged. The differences between their work and this paper is that this paper uses quarterly non-logged data and excludes the implicit GNP deflator from the explanatory variables.

where L = Employee Hours per Capita, and W = Average Hourly Earnings.

This result fails to reject the hypothesis that the wage does not cause labor supply. Therefore, this paper assumes labor supply and, therefore, labor income are exogenous.

A representative agent

Cross aggregation is a possible problem for a representative agent model. Runkle (1991) argued that the results from aggregate data are possibly due to aggregation bias. He concluded that the Euler equation does hold for panel data. However, it is very difficult to collect enough panel data without encountering a measurement problem which can misguide the empirical research.

This section discusses the aggregation problem in terms of the structure of the model. Since the economic decision rules and processes of variables are additively separable linear functions, aggregation seems not to be a serious problem for this model. This section discusses this by showing that average aggregate permanent income can be expressed in terms of an individual agent's decision rule with average aggregate human and nonhuman wealth. The average aggregate permanent income is

$$\sum_{j=1}^{N} y^p_{j,t} / N = r\sum_{j=1}^{N}[w_{j,t} + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_{j,t}y_{j,t-1+i}] / N$$

$$= r[\sum_{j=1}^{N} w_{j,t} + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i \sum_{j=1}^{N} E_{j,t}y_{j,t-1+i}] / N$$

where N is the number of population.

Assuming that the labor income of every agent can be different in its levels but has the same additively separable linear processes, $\sum_j E_{j,t}y_{j,t-1+i}$ can be expressed as $E_t\sum_j y_{j,t-1+i}$. Therefore,

$$\sum_{j=1}^{N} y^p_{j,t} / N = r[\sum_{j=1}^{N} w_{j,t} / N + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t(\sum_{j=1}^{N} y_{j,t-1+i} / N)]$$

According to this result, the sum of individual agents' permanent income divided by the population is the same as the permanent income calculated based on individual decision rules with average aggregate human and nonhuman wealth. Theoretically, this model does not cause any aggregate bias problem so that the assumption of a representative agent is pretty reasonable.

Recalling of main results

To set up the general model, this section recalls the main results of these three authors. Some of them will be modified slightly for the econometric model. First, Flavin's formulation of permanent income and Gali's alternative formulation are recalled. Flavin's permanent income is

$$Y_t^p = r\left[W_t + \sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t Y_{t-1+i}\right] \tag{2.2}$$

and

$$\begin{aligned}\Delta Y_t^p &= r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i (E_t - E_{t-1})Y_{t-1+i} \\ &= \xi_t\end{aligned} \tag{2.3}$$

where r is the constant real interest rate. Gali's expression for permanent income is

$$Y_t^p = r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i E_t C_{t-1+i} \tag{2.17}$$

$$\begin{aligned}Y_t^p - E_{t-1}Y_t^p &= r\sum_{i=1}^{\infty}\left(\tfrac{1}{1+r}\right)^i (E_t - E_{t-1})C_{t-1+i} \\ &= \xi_t\end{aligned} \tag{2.18}$$

Since Y^p is a random walk process, $Y_t^p - E_{t-1}Y_t^p$ is the same as ΔY_t^p.

Second, Quah's labor income process is recalled with some modification. First, we change expression of parameters for convenience of estimation. Quah expresses the parameters of $\Delta Y_{0,t}$ as $(1-L)A_0(L)$, but this paper will express this as $A_0(L)$ with the restriction, $A_0(1)=0$. Both expressions are identical. Second, labor income is assumed to be an infinite-order bivariate MA process in Quah's original version, but this section assumes that it is a finite order bivariate moving average process. That is, we rewrite (2.8) as

$$\begin{aligned} \Delta Y_t &= \sum_{i=0}^{h_1} a_{1,i}\varepsilon_{1,t-i} + \sum_{i=0}^{h_0} a_{0,i}\varepsilon_{0,t-i} \\ &= \alpha_Y + A_1(L)\varepsilon_{1,t} + A_0(L)\varepsilon_{0,t} \qquad a_{0,0},\, a_{1,0} = 1,\ A_0(1) = 0 \end{aligned} \tag{2.8}$$

By inserting (2.8) into (2.3), we can obtain

$$\xi_t = A_1(\beta)\varepsilon_{1,t} + A_0(\beta)\varepsilon_{0,t} \qquad A_0(1) = 0 \tag{2.10}$$

Finally, Gali's consumption process is presented. Gali embodies the existence of excess sensitivity by saying that consumption is a function of current and past permanent income innovations and independent transitory innovations. Assuming (2.19) consists of two finite order MA processes, we recall his consumption process as follows;

$$\begin{aligned}\Delta C_t &= \Delta C_t^* + n_t \\ &= \alpha_c + \sum_{i=0}^{k} b_i \xi_{t-i} + \sum_{i=0}^{m} d_i u_{t-i} \\ &= \alpha_c + B(L)\xi_t + D(L)u_t\end{aligned} \tag{2.19}$$

where d_0 is one. b_0 does not need to be restricted to be one, because ξ is the innovation of permanent income rather than ΔC's own innovation. This section also recalls the implied condition and restrictions on the parameters of (2.19):

$$\begin{aligned}\xi_t &= \sum_{i=0}^{k}\left(\tfrac{1}{1+r}\right)^i b_i \xi_{t-i} + \sum_{i=0}^{m}\left(\tfrac{1}{1+r}\right)^i d_i u_{t-i} \\ &= B(\beta)\xi_t + D(\beta)u_t\end{aligned} \tag{2.21}$$

$$B(\beta) = 1 \tag{2.22}$$

$$D(\beta) = 0 \tag{2.23}$$

<u>**General PIH Model**</u>

We can now construct a general model which consists of a labor income process and a consumption process. The consumption process is obtained by inserting (2.7) into (2.16). Since (2.7) is derived from Flavin's permanent income and Quah's labor income process, and (2.16) is from Gali's consumption model, it can be regarded as a general consumption process. The labor income process is the same as (2.5).

Therefore, our model is

Therefore, our model is

$$\Delta C_t = \alpha_C + B(L)\left[A_1(\beta)\varepsilon_{1,t} + A_0(\beta)\varepsilon_{0,t}\right] + D(L)u_t \tag{3.5}$$

$$\Delta Y_t = \alpha_Y + A_1(L)\varepsilon_{1,t} + A_0(L)\varepsilon_{0,t} \tag{2.8}$$

restrictions: $B(\beta) = 1, D(\beta) = 0 \textit{ and } A_0(1) = 0$

Since there are three independent innovations ε_1, ε_0 and u, this model has a two equation structure with three disturbance terms.

4 ESTIMATION METHOD

4.1 Estimation method and lag length.

There have been two major estimation methods calculating the volatility of permanent income. One is a parametic approach based on the ARIMA representation of labor income. Deaton (1987), West (1988) and Gali (1990) used this method, and Section 2.5 provides the idea of this approach. The advantage of this method is that we can easily see the relationship between labor income and permanent income. However, there are some disadvantages. First, this method is based on a univariate labor income process. If there is a significant transitory component, it may overestimate the variance of permanent income. This is the reason why Quah (1990) criticized the univariate approach. Even though this paper argues that his decomposition can not resolve excess smoothness, it does not mean the ARIMA representation approach is adequate. Second, the method disregards long lagged second moment properties. Deaton and Campbell (1989) show both logged labor income and disposable income have negative autocorrelations after four or five lags. They analyzed the possibility that these negative second moments can imply smaller volatility of permanent income than the simple ARIMA representation does. Figure 2-2 shows similar negative second moments properties.

The other way is a non-parametic method. Deaton and Campbell (1989), West (1988) and Gali (1991) have used this method. An advantage of this method is the full utilization of the autocorrelation structure of the process. However, this advantage is very questionable when we think Quah's "non-fundamentalness". We can calculate any volatility of permanent income without violating the second moment properties of labor income.

To overcome these problems, this paper suggests three ideal requirements the estimation method should meet. First, the method needs to estimate a bivariate labor income process. Since the role of transitory labor income is regarded as an important factor in calculating the variance of permanent income, it may be necessary to analyze this component. Second, the utilization of long lagged second moments of labor income is necessary. The series of negative autocorrelations in labor income process may have some interesting implications. Third, the method needs to utilize not only second moments of labor income but also correlations between labor income and consumption. Section 3-1 showed the importance of this covariance when we use a bivariate labor income process. This is the reason why this paper chooses the Generalized Method of Moment estimator with a long labor income process.

The choice of the lag lengths is a difficult issue, because the choice of long-lag lengths causes a big computation burden. However, this paper needs to choose long lags as far as possible to overcome the disadvantage of the simple ARIMA representation approach. This paper therefore balances these two factors. Figure 2-2 shows the existence of negative autocorrelation series from lag 6 to lag 11. Since it may be very interesting to analyze the implication of these negative second moments, lag length of labor income is chosen as 11. However, consumption process does not show such evidence. Even though there are some negative autocorrelations at lag 10, 11, 13 and 14, this paper decides that it is not worth enough to pay big computation burden. Therefore, the lag length of consumption process is decided as 5.

4.2 Identification

To estimate the free parameters in the econometric model, we must have at least the same number of information as that of unknown parameters. This section checks this identification condition.

The econometric model specified at the end of Chapter 3 has unknown parameters as follows;

σ_u^2, $\sigma_{\varepsilon_0}^2$, $\sigma_{\varepsilon_1}^2$	3
$a_0(1), \cdots\cdots, a_0(h_0)$	h_0
$a_1(1), \cdots\cdots, a_1(h_1)$	h_1
$b(0), \cdots\cdots, b(k)$	k + 1
$d(1), \cdots\cdots, d(m)$	m

restrictions: $B(\beta) = 1, D(\beta) = 0 \; and \; A_0(1) = 0$

Since the econometric model contains $h_0 + h_1 + k + m + 4$ parameters and three restrictions, the number of unknown parameters is $h_0 + h_1 + k + m + 1$. This paper will assume that the permanent component and transitory component of labor income process each has lag length 11, and those of the consumption process each have lag length 5. Therefore, the number of unknown parameters is 33.[1]

Since the econometric model is made up of two finite MA processes, we can easily show the information to be used. There are three sources of information: autocovariances of ΔC and ΔY and the covariance's between ΔC and ΔY. The information is as follows;

$$
\begin{array}{ll}
\mathrm{cov}(\Delta Y_t, \Delta Y_{t-i}) & i = 0, \cdots, \max\{h_1, h_0\} \\
\mathrm{cov}(\Delta C_t, \Delta C_{t-i}) & i = 0, \cdots, \max\{k, m\} \\
\mathrm{cov}(\Delta Y_t, \Delta C_{t-i}) & i = 0, \cdots, \max\{h_1, h_0\} \\
\mathrm{cov}(\Delta C_t, \Delta Y_{t-i}) & i = 1, \cdots, \max\{k, m\}
\end{array}
$$

The number of information is $2 \cdot \max\{h_1, h_0\} + 2 \cdot \max\{k, m\} + 3$. For any positive integers, h_1, h_0, k and m, the number of information is always bigger than the number of unknown parameters. This means that the model is over-identificated. The number of information of the model here is 35.

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[1] $h_0 = 11$, $h_1 = 11$, k = m = 5. However, k and m will be changed in modifed model later.

4.3 Summary of GMM

Basic Idea of GMM

Since there is an over-identification problem, this paper will estimate parameters by minimizing the difference between the sample second moments and the theoretical ones. However, we need to impose weights to these differences to get consistent estimates. This is the reason why this paper adopts the generalized method of moments estimator. We can call this method a weighted numerical optimization. Hartley and Walsh (1992) provide a nice explanation of the properties of GMM which was originally proposed by Hansen (1982).

Let $f(\underline{x}_n, \underline{b})$ be the vector of differences between the sample second moments in period n and the corresponding theoretical second moments[2], and

$$g_N(\underline{b}) = \frac{1}{N}\sum_{n=1}^{N} f(\underline{x}_n, \underline{b}).$$

The GMM estimates can be obtained by minimizing a weighted sum of squares which is

[2] $\underline{x}_n$ here is for sample second moments and b is for theoretical second moment.

$$g_N(b)'Wg_N(b) \tag{4.1}$$

for a symmetric weighing matrix, W.

Property of GMM

This section discusses statistical properties of the GMM estimator. We assume that data are generated by stationary processes, i.e., processes with a constant mean, finite variance, and an autocorrelation function that is only a function of the time difference. The data set of this paper is assumed to meet this condition by eliminating trends through first differences.

If the vector $\hat{\underline{b}}$ minimizes (4.1), $\sqrt{N}(\tilde{\underline{b}} - \underline{b})$ will converge in distribution to a random vector with mean zero and covariance matrix

$$(d'S_N{}^{-1}d)^{-1} \tag{4.2}$$

S_N here is the covariance matrix of theoretical second moment, and $\hat{S}_N{}^{-1}$ is a consistent estimate of W and (4.2) can be estimated by $(\hat{d}'\hat{S}_N{}^{-1}\hat{d})^{-1}$, where

$$d = E\left[\frac{\partial f}{\partial \underline{b}}(\underline{b})\right] \qquad and \qquad \hat{d} = \left[\frac{\partial g}{\partial \underline{b}}(\hat{\underline{b}})\right]$$

and

$$\hat{S}_N = \frac{1}{N}\sum_{j=-z+1}^{z-1}\sum_{n=1+j}^{N} f(\underline{x}_n, \hat{\underline{b}}) f(\underline{x}_{n-j}, \hat{\underline{b}})'. \tag{4.3}$$

$E\left[f(\underline{x}_n, \hat{\underline{b}}) f(\underline{x}_{n-j}, \hat{\underline{b}})'\right] = 0$ for $|j| \geq z$ for some integer z.

In this model, z is max{k, m, h_1, h_0} + 1 which is 12 here.

The test statistic of the over-identifying restrictions is:

$$N g_N(\hat{\underline{b}})'(\hat{S}_N)^{-1} g_N(\hat{\underline{b}}) \rightarrow \chi^2_{p-q}$$

where p is the number of moment conditions and q is the number of estimated parameters. If the restriction is accepted, we can conclude that the estimated model can explain the observed sample second moments.

4.3 ALGORITHM OF NONLINEAR OPTIMIZATION

The model may be very difficult to be estimate, because it contains 33 unknown parameters, and a weight matrix W that is unknown. This section therefore develops an algorithm which can calculate the weighted criteria value efficiently.

As section 4.2 said the GMM estimates minimizes (4.1) Setting $g_N(b) = F(s) - F(t)$, (4.1) is expressed as

$$(F(s) - F(t))'W(F(s) - F(t)). \qquad (4.4)^3$$

Since W is a symmetric matrix, (4.4) can be transformed as

$$(F(s) - F(t))'A'A(F(s) - F(t))$$
$$(AF(s) - AF(t))'(AF(s) - AF(t)) \qquad (4.5)$$

The first order condition minimizing (4.5) is

$$-Af(t)'(AF(s) - AF(t)) = 0 \qquad (4.6)$$

where $f(t)$ is the derivative of F(t).

Applying the Gauss-Newton method, this section derives an algorithm for weighted nonlinear optimization. The idea of this method is to approximate satisfying the first order condition by minimizing $AF(s) - AF(t)$. We can express this idea as follows;

$$AF(s) = AF(t) + \varepsilon.$$

Minimizing the sum of squares of ε can approximately satisfy the first order condition (4.6).

As the first step, $AF(t)$ is replaced by a first order Taylor series approximation.

---

[3]F(s) is a vector of sample moments, and F(t) is a vector of theoretical moments.

$$AF(t) \cong AF(t_1) + Af(t_1)(t - t_1)$$

Then,

$$AF(s) \cong AF(t_1) + Af(t_1)(t - t_1) + \varepsilon$$
$$AF(s) - AF(t_1) + Af(t_1)t_1 \cong Af(t_1)t + \varepsilon \tag{4.7}$$

The method of least squares is applied to (4.7) to satisfy the F.O.C.. The estimate of t minimizing the sum of squares of ε is

$$\begin{aligned}\hat{t} &= [f(t_1)'A'Af(t_1)]^{-1} f(t_1)'A'[AF(s) - AF(t_1) + Af(t_1)t_1] \\ &= t_1 + [f(t_1)'A'Af(t_1)]^{-1} f(t_1)'A'[AF(s) - AF(t_1)] \\ &= t_1 + [f(t_1)'Wf(t_1)]^{-1} f(t_1)'W[F(s) - F(t_1)]\end{aligned}$$

Therefore, the weighted criteria value is

$$\textit{Criteria value} = [f(t_1)'Wf(t_1)]^{-1} f(t_1)'W[F(s) - F(t_1)] \tag{4.8}$$

This algorithm offers some good properties. First, this algorithm does not require second derivatives to calculate the criteria value. Calculation of the second derivative matrix may be extremely burdensome. Second, deriving the first derivatives is independent of calculating the weight matrix, because we do not need to calculate $Af(t_1)$

to derive the weighted criteria value. In the case of a model which contains a lot of unknown parameters, this property will provide a big efficiency to get the criteria value.[4]

4.4 Strategy of convergence

Getting a good initial value is critical for success of the estimation. However, there is no formal rule to calculate a good initial value. In general, a non-weighted convergence point is regarded as a good initial point. A lot of points around a non-weighted convergence points are tried to get convergence.

If an initial point is chosen, (4.3) gives the covariance matrix of the second moments. Then, we can minimize (4.4) by applying our numerical algorithm. The criteria value is calculated by (4.8). The steplength is s in

$$\underline{b}_1 = \underline{b}_0 + s \times (\textit{critetia value})$$

where $\underline{b}_0$ is the vector of the initial point for the iteration and $\underline{b}_1$ is the new initial point for the next iteration. Steplength is chosen by line search. First, weighted sums of squares are compared when s = 1 and when s = 0.5. If the former yields the smaller value of weighted sum of

[4]Numerically, calculation of $f(t_1)'Wf(t_1)$ may be faster than calculation of $f(t_1)A'Af(t_1)$.

square, 1 is chosen as the steplength for that iteration. If the latter yields the smaller one, the weighted sums of square are compared again between when s = 0.5 and when s = 0.25. This procedure is continued until the larger steplength yields the smaller weighted sum of square than the smaller steplength. Once the steplength for the iteration is decided, the initial point for the next iteration is determined. Then, a new weight matrix and criteria value are calculated based on these new points.

The tolerance level is 10^{-4} or 10^{-3} in this paper. If the steplength muliplied by the criteria value is smaller than the tolerance level for every estimate, iteration is stopped. The estimates and weight matrix from the last iteration become the converged estimates and weight matrix. This estimates is regarded as the vector of points which minimize (4.1). From these estimates and weight matrix, we can get the covariance matrix of the estimated parameters by (4.2).

Approach to modified consistent weighting matrix

This section discusses a modified method which is more convenient than the standard method described in this chapter. First, the covariance matrix (4.3) shows bad numerical properties in our research. The converged covariance matrix shows some negative numbers on the diagonal elements. This is econometrically meaningless. Newey and West (1987) imposed weights to informations in S_N to improve numerical properties and keep consistency of this matrix. Their $\hat{S}_N$ is

$$\hat{S}_N = \hat{\Omega}_0 + \sum_{j=1}^{J} \omega(j, J)[\hat{\Omega}_j + \hat{\Omega}'_j] \tag{4.9}$$

where $\omega(j, J) = 1 - [j/(J+1)]$

and $\hat{\Omega}_j = \frac{1}{N} \sum_{n=1+j}^{N} f(\underline{x}_n, \hat{\underline{b}}) f(\underline{x}_{n-j}, \hat{\underline{b}})'$.

This paper adopts this weighting matrix.

Second, if $\hat{\underline{b}}$ is consistent estimate, theoretical second moments also consistent estimate of sample second moments. That is, in (4.3), $f(\underline{x}_n, \underline{b})$ is the vector of differences between the sample second moments in period n and the corresponding theoretical second moments. Even if we express $f(\underline{x}_n, \underline{b})$ as the vector of differences between the sample second moments in period n and the mean of corresponding second moments, the measured weighting matrix is a consistent matrix for the true weighting matrix. The reason is that both the theoretical second moments and the mean of second momets are consistent estimates of sample second moments. Since this matrix does not require convergence, it provides better convergent properties. Major estimations of this paper are done by this consistent measure of weighting matrix.[5] Chaper5 will empirically show the consistency of this weighting matrix.

[5]If we used this fixed weighting matrix, our algorithm is just Gauss-Newton method. However, this paper performs our algorithm for key estimation.

4.5 Numerical aspect of estimation

The GMM method provides the strong advantage that we can directly observe bivariate processes based on sample second moments. However, the GMM method in this model is numerically very difficult. The rest of this chapter is going to discuss the numerical difficulty of this model and the treatment to cure the problem. The sources of numerical problems can be summarized as three aspects.

First, long moving average processes are assumed in the labor income process. The long moving average process is hard to be identify, even if we use a different econometric method. The parameters in this process are generally highly correlated with each other.

Second, bivariate processes are assumed in the labor income and consumption processes. As Quah indicated, the relative importance of both sub-processes can be changed without violating the second moments of the univariate process. Even though this model is also restricted by the covariance between the two main processes, it is very difficult to be fully independent from this problem, which can complicate identification.

Third, there are parameter restrictions for the two transitory components. These restrictions have bad statistical properties. The restriction on the transitory component of labor income is $A_0(1) = 0$ and that on the transitory component of consumption is $D(1/1.01) = 0$. The transitory consumption is assumed to be totally independent from labor income. The transitory labor income has little contribution to permanent income which is $A_1(1/1.01)\varepsilon_{1,t} + A_0(1/1.01)\varepsilon_{0,t}$. However, $A_0(1/1.01)$ is very small, because $A_0(1) = 0$. This means these two components are restricted only by the autocovariance of their own processes. This characteristic causes a "trade off" relationship between the variance of transitory innovation and its parameters: Decrease (increase) of variance of innovation can be offset by increase (decrease) of parameters without significantly violating autocovariance of the process. Numerically, there are many sets of transitory estimates which have numerically similar properties.

The identification problem of transitory component was the most serious problem in estimation so that treatment to cure numerical problems is focused on these components. Decreasing the number of parameter is a key strategy to resolve the numerical problem. First, this paper tried to decrease lags of the processes. This treatment was successful for the transitory component of consumption. The transitory consumption is expressed as $u_t - 1.01u_{t-1}$. However, transitory labor income is very difficult to be simplify. Decreasing lags does not work for this component.

This paper also tried to approximate moving average parameters in these transitory component as simple forms keeping long lags. However, these components seem to be too complicated moving average process to be simplified. Therefore, this paper restricts the variance of transitory innovation. This treatment significantly improves the properties of the estimates without losing consistency. Chapter 5 will discuss this treatment with empirical results.

5 EMPIRICAL RESULTS

5.1 Introduction

Convergence is evaluated based on three aspects. First, the test statistic is a important factor to evaluate convergence. Optimal points provide very insignificant values of the test statistic. Second, the difference between sample second moments and predicted theoretical second moments are evaluated. Even though there is no clear standard about how close a good prediction should be, we can easily see that optimal convergence points yield good predictions of sample second moments. Third, the properties of individual parameters are evaluated. Since there are numerical problems in transitory components, it is hard to get good t ratios for every coefficient. However, restrictions on these components improve the properties of individual parameters. Even though this property itself is not critical for determination of convergence, too bad properties of individual parameter can cause the overestimation of the test statistic.

The analysis is started from the original model which consists of (3.5) and (2.8). However, this model would not converge. None of the treatments on transitory components described in Section 4.5 help provide good convergence. Therefore, this paper will focus on restrictions on economic behavior. There are two restrictions of economic behavior, (2.10) and (2.22).[1]

$$\Delta C_t = \Delta Y_t^p = A_1(\beta)\varepsilon_{1,t} + (1-\beta)A_0(\beta)\varepsilon_{0,t} \qquad \beta = \frac{1}{1+r} \tag{2.10}$$
$$= \xi_t$$

$$\sum_{i=0}^{\infty}(1+r)^{-i}b_i = 1 \tag{2.22}$$

(2.10) is from Flavin's permanent income and (2.22) is from Gali's permanent income. However, (2.22) is derived under the assumption of Flavin's permanent income. If this restriction holds, excess smoothness is fully explained by the parameters and innovations of the permanent component of consumption. Since these parameters represent the existence of excess sensitivity, the restriction means that excess smoothness is fully explained by excess sensitivity. Therefore, this paper relaxes the restriction (2.22) to check whether the restriction is empirically acceptable. If this modification yields good convergence but estimated parameters are not consist with the restriction, we need to reconsider whether excess sensitivity can fully explain excess smoothness.

[1](2.23) is also an economic behavioral restriction on transitory consumption. However, it is basically the restriction of transitory component so that we do not discuss this restriction.

5.2 Converged results

The modified model is

$$\Delta C_t = \alpha_C + B(L)\left[A_1(\beta)\varepsilon_{1,t} + A_0(\beta)\varepsilon_{0,t}\right] + u_t - 1.01u_{t-1}$$

$$\Delta Y_t = \alpha_Y + A_1(L)\varepsilon_{1,t} + A_0(L)\varepsilon_{0,t}$$

restriction: $A_0(1) = 0$

There were three major changes to the original model. First, transitory consumption is simplified as $u_t - 1.01u_{t-1}$ satisfying the restriction (2.23), D(1/1.01) = 0. Second the economic behavioral restriction (2.22) is relaxed so that model does not have the restriction that excess smoothness is fully explained by excess sensitivity. Third, $\mathrm{var}(\varepsilon_{0,t})$ is fixed at some constant in estimation. This paper chose the magnitude of $\mathrm{var}(\varepsilon_{0,t})$ by estimating the model without any restriction of this variance. Then, some numbers around the estimated value are chosen as restrictions. The first treatment decreases the number of unknown parameters by 4, and the third treatment does that by 1. However, the second treatment adds one unknown parameter by releasing one restriction. Therefore, the total number of unknown parameters is 29 which is 4 less than the original model. Therefore, the test statistic of over-identifying restrictions has 6 degrees of freedom.

<u>**Consistency**</u>

Table 5-1 shows the comparison between when var(ε_0) is unrestricted and when var(ε_0) is restricted as 1 which is the closest perfect number to the estimated var(ε_0). The restrictions significantly improve the properties of individual parameters and test statistic keeping consistency of estimates. Since the unrestricted model has too high a correlation among parameters, the test statistic seems to be overestimated. The restricted model yields a very insignificant test statistic, 1.134. Since χ^2_6 is 1.64 at p-value 0.05, the modified model can not be rejected even at the 95-percent significant level . However, the parameters in the transitory component of labor income still do not have good properties. When we estimated the model without a transitory component, it did not converge at all. These phenomena mean that transitory labor income is jointly significant but its individual parameters are hard to identify.

Table 5-2 shows that the restriction on var(ε_0) does not significantly affect the consistency of the estimates and the test statistic. This is a good example of the "trade off" relationship discussed in Section 4.5. The change of restrictions on transitory innovation does not significantly affect estimates of other components, implied volatility of permanent income and the test statistic, but it is almost absorbed by the change of parameters in its own component.

All estimates in Table 5-1 and Table 5-2 are based on the fixed weighting matrix method discussed Section 4.5. Table 5-3 shows that this modified method well-approximate results from the converged weighting matrix. The difference between the two weighting matrices is that the converged matrix uses theoretical second moments and the fixed matrix uses the mean of sample second moments. Table 5-3 can be interpreted as indirect evidence that the theoretical second moments are very close to the mean of the sample second moments. Figure 5-1 provides the comparison between the mean of sample second moments and the corresponding theoretical second moments predicted by the converged weighting matrix method with the restriction, $\mathrm{var}(\varepsilon_0) = 1$. We can easily see that estimated parameters well predict sample second moments.

This section also performs simulations to see how well this method represents the true process. Simulated data are generated based on the parameter estimates in Table 5-3 which are derived by the converged weighting matrix method. The number of data is 230. Table 5-4 reports estimates of parameters from the sample and simulated data sets and the implied impulse response parameters which are the standard deviation of parameter multiplied by its parameters. Even though estimated parameters of the permanent component of labor income shows some proportional differences, the implied impulse response parameters show the consistency of our estimates. Figure 5-2 shows how this estimation method well simulates the assumed true process.[2]

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[2]Since trasitory consumption process is assumed as very simple process, we exclude impulse response analysis about this component. Figure 5-6 also exclude this component for the same reason.

Table 5-1 Estimates of parameters (restriction vs. no restriction)[a]

parameter	no restriction estimates	no restriction t ratio	restricted $var(\varepsilon_0)$ estimates	restricted $var(\varepsilon_0)$ t ratio
$var(\varepsilon_1)$	11.984	0.024	12.912	2.607
$var(\varepsilon_0)$	0.966	1.3×10^{-4}	(1)	
$var(u)$	1.021	1.403	1.467	2.456
a_1: 1	0.630	0.027	0.657	3.223
2	0.190	0.004	0.116	1.158
3	0.284	0.003	0.305	1.736
4	0.708	0.005	0.699	4.483
5	0.293	0.003	0.330	2.849
6	0.330	0.003	0.295	2.743
7	-0.002	1.7×10^{-5}	0.029	0.220
8	0.157	0.002	0.176	1.189
9	-0.185	-0.003	-0.074	-0.413
10	-0.219	-0.006	-0.236	-1.478
11	-0.309	-0.012	-0.257	-1.282
a_0: 1	1.454	1.9×10^{-4}	1.261	1.258
2	-0.885	-2.1×10^{-4}	-0.827	-0.393
3	2.094	1.8×10^{-4}	1.802	0.354
4	0.656	2.3×10^{-5}	0.593	0.120
5	-1.461	-6.3×10^{-5}	-1.243	-0.263
6	-1.647	-1.4×10^{-4}	-1.708	-0.350
7	-0.849	-3.4×10^{-5}	-1.084	-0.230
8	0.764	4.3×10^{-5}	0.617	0.149
9	-0.371	-3.1×10^{-5}	-0.115	-0.052
10	0.162	1.8×10^{-5}	0.300	0.158
b : 0	0.346	5.803	0.307	5.996
1	0.197	4.027	0.184	3.794
2	-0.033	-1.604	-0.031	-1.636
3	0.151	5.082	0.129	4.093
4	0.129	3.296	0.136	3.702
5	0.071	1.980	0.061	1.657
Test Statistic		19.819		1.134

[a] Parenthesis means the restricted parameter.

Table 5-2 Estimates of parameters (for various restrictions)

parameter $var(\varepsilon_0)$ = 0.5		1	1.5	2	3
$var(\varepsilon_1)$	12.905	12.912	12.854	12.803	12.697
$var(u)$	1.435	1.467	1.477	1.493	1.445
a_1: 1	0.649	0.657	0.660	0.661	0.663
2	0.121	0.116	0.113	0.110	0.116
3	0.289	0.305	0.321	0.336	0.286
4	0.689	0.699	0.709	0.715	0.689
5	0.320	0.330	0.338	0.343	0.317
6	0.295	0.295	0.299	0.300	0.293
7	0.024	0.029	0.033	0.039	0.019
8	0.168	0.176	0.183	0.188	0.163
9	-0.080	-0.074	-0.068	-0.061	-0.085
10	-0.234	-0.236	-0.236	-0.238	-0.238
11	-0.252	-0.257	-0.260	-0.258	-0.247
a_0: 1	1.962	1.261	0.952	0.733	-0.434
2	-1.279	-0.827	-0.621	-0.482	0.472
3	2.239	1.802	1.493	1.302	1.096
4	1.527	0.593	0.202	-0.018	-0.076
5	-1.545	-1.243	-1.015	-0.829	-0.742
6	-2.367	-1.708	-1.408	-1.249	-1.136
7	-1.947	-1.084	-0.696	-0.453	-0.023
8	0.661	0.617	0.550	0.460	-0.175
9	-0.285	-0.115	-0.034	0.019	0.163
10	0.487	0.300	0.166	0.063	-0.105
11	(-0.453)	(-0.596)	(-0.589)	(-0.546)	(-0.040)
b : 0	0.310	0.307	0.305	0.302	0.311
1	0.187	0.184	0.181	0.179	0.189
2	-0.031	-0.031	-0.031	-0.031	-0.030
3	0.130	0.129	0.129	0.129	0.130
4	0.135	0.136	0.136	0.136	0.135
5	0.062	0.061	0.062	0.062	0.061
Test Statistic	1.124	1.134	1.256	1.448	1.101
Implied $var(\xi)$:	112.586	116.375	119.668	122.388	109.881
Implied κ^a:	2.211	2.248	2.280	2.306	2.185

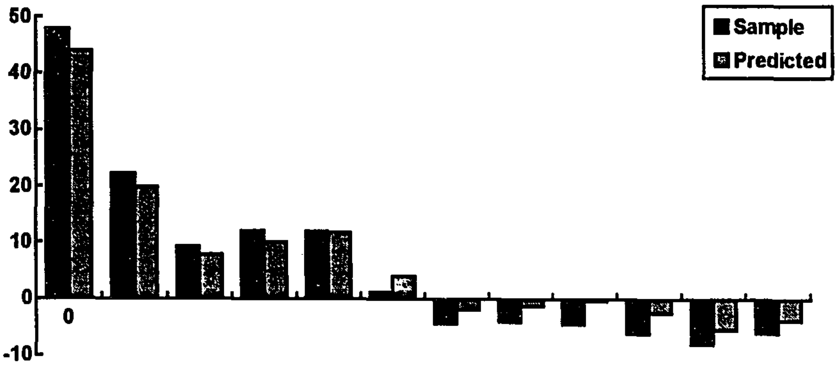
Table 5-3 Estimates of parameters (converged W vs. fixed W)

parameter	Converged W estimates	t ratio	Fixed W estimates	t ratio
var(ε_1)	12.980	2.641	12.912	2.607
var(u)	1.418	2.347	1.467	2.456
a_1: 1	0.652	3.338	0.657	3.223
2	0.116	1.163	0.116	1.158
3	0.297	1.693	0.305	1.736
4	0.691	4.553	0.699	4.483
5	0.331	2.844	0.330	2.849
6	0.297	2.833	0.295	2.743
7	0.032	0.245	0.029	0.220
8	0.176	1.187	0.176	1.189
9	-0.068	-0.384	-0.074	-0.413
10	-0.230	-1.454	-0.236	-1.478
11	-0.247	-1.265	-0.257	-1.282
a_0: 1	1.299	1.384	1.261	1.258
2	-0.834	-0.370	-0.827	-0.393
3	1.765	0.337	1.802	0.354
4	0.603	0.118	0.593	0.120
5	-1.254	-0.258	-1.243	-0.263
6	-1.728	-0.343	-1.708	-0.350
7	-1.099	-0.228	-1.084	-0.230
8	0.628	0.150	0.617	0.149
9	-0.113	-0.049	-0.115	-0.052
10	0.313	0.163	0.300	0.158
b : 0	0.306	6.107	0.307	5.996
1	0.182	3.839	0.184	3.794
2	-0.031	-1.643	-0.031	-1.636
3	0.130	4.233	0.129	4.093
4	0.132	3.640	0.136	3.702
5	0.063	1.736	0.061	1.657
Test Statistic		1.009		1.134
Implied var(ξ):	117.316		116.375	
Implied κ:	2.257		2.248	

Table 5-4 Estimates of parameters (sample vs. simulation)

parameter	Estimates sample	simulation	Impulse response sample	simulation
var(ε_1)	12.980	17.190		
var(u)	1.418	1.157		
a_1: 0	(1)	(1)	3.603	4.146
1	0.652	0.566	2.349	2.347
2	0.116	0.015	0.418	0.062
3	0.297	0.119	1.070	0.493
4	0.691	0.704	2.490	2.919
5	0.331	0.239	1.193	0.991
6	0.297	0.284	1.070	1.177
7	0.032	0.056	0.115	0.232
8	0.176	0.144	0.634	0.597
9	-0.068	0.008	-0.245	0.033
10	-0.230	-0.323	-0.829	-1.339
11	-0.247	-0.181	-0.890	-0.750
a_0: 0	(1)	(1)	1	1
1	1.299	1.311	1.299	1.311
2	-0.834	-0.827	-0.834	-0.827
3	1.765	1.224	1.765	1.224
4	0.603	0.889	0.603	0.889
5	-1.254	-0.955	-1.254	-0.955
6	-1.728	-1.637	-1.728	-1.637
7	-1.099	-1.603	-1.099	-1.603
8	0.628	0.106	0.628	0.106
9	-0.113	0.239	-0.113	0.239
10	0.313	0.497	0.313	0.497
11	(-0.580)	(-0.244)	-0.580	-0.244
b : 0	0.306	0.325	3.314	3.503
1	0.182	0.209	1.971	2.253
2	-0.031	-0.052	-0.336	-0.560
3	0.130	0.129	1.408	1.390
4	0.132	0.157	1.430	1.692
5	0.063	0.015	0.682	0.162
Test Statistic	1.009	1.066		
Implied var(ξ):	117.316	116.178		
Implied κ:	2.257	2.121		

$\mathrm{cov}(\Delta Y_t, \Delta Y_{t-j})$



$\mathrm{cov}(\Delta Y_t, \Delta C_{t-j})$

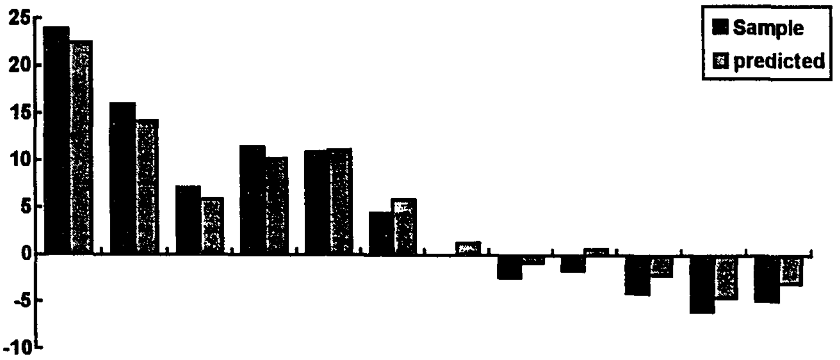
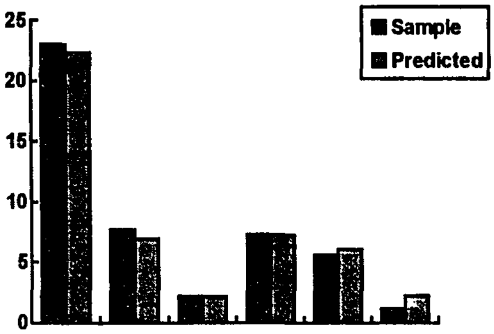


Figure 5-1 Sample and estimated theoretical second moment

$\text{cov}(\Delta C_t, \Delta C_{t-j})$



$\text{cov}(\Delta C_t, \Delta Y_{t-j})$

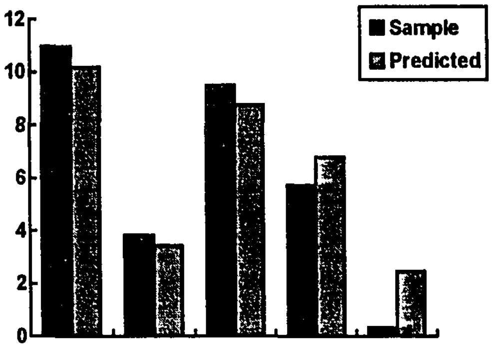
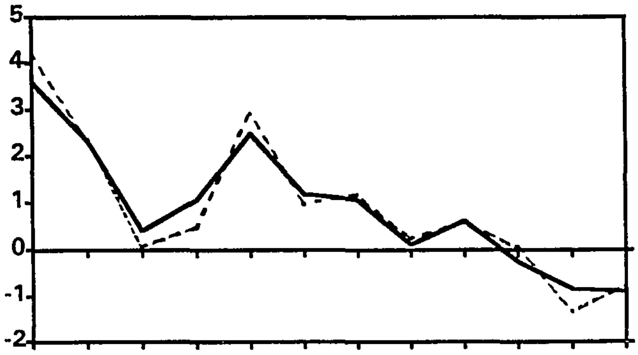


Figure 5-1. (Continued)

——— sample data

-------- simulated data

Permanent labor income



Transitory labor income

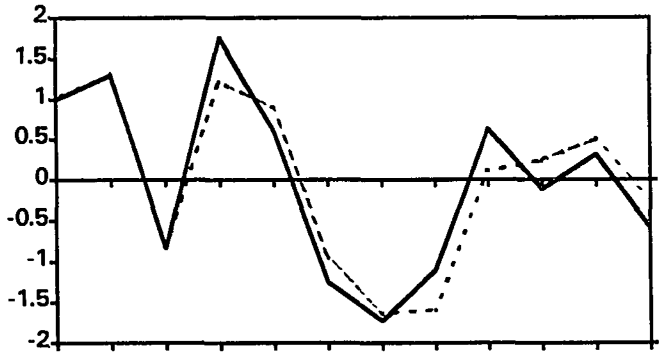


Figure 5-2. Comparison of the impulse response (sample vs. simulation)

Permanent consumption

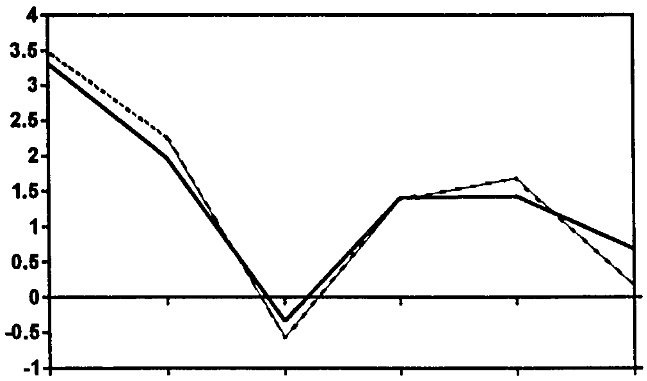


Figure 5-2 (Continued)

The results above provide the evidence that our estimation has good property of consistency. All estimates from Table 5-1 through Table 5-4 show consistency not only about estimates and permanent income volatility but also about the measured volatility of permanent income. Predicted second moments also show good fits to sample second moments. Section 5.3 will discuss implications of these estimates.

Prediction

The process we estimate is basically an unobservable bivariate process. This means that we can not predict sample data based on an estimated process. However, Brocknell and Davis (1991) provides a prediction algorithm which is based on the second moments of the process and prediction errors. Their algorithm is as follows;

Proposition (The multivariate Innovation Algorithm): Let $\{X_t\}$ be an m-dimensional time series with mean $EX_t = 0$ for all t and with covariance function $K(i,j) = E(X_i, X_j')$. If the covariance matrix of the nm components of $X_1, \ldots, X_n$ is nonsingular for every $n \geq 1$, then the one-step predictors $\hat{X}_{n+1}$, $n \geq 0$, and their prediction error covariance matrices V_n, $n \geq 1$, are given by

$$\hat{X}_{n+1} = \begin{cases} 0, & \text{if } n = 0, \\ \sum_{j=1}^{n} \Theta_{n,j}(X_{n+1-j} - \hat{X}_{n+1-j}) & \text{if } n \geq 1, \end{cases}$$

and

$$V_0 = K(1,1)$$

$$\Theta_{n,n-k} = (K(n+1,k+1) - \sum_{j=0}^{k-1}\Theta_{n,n-j}V_j\Theta'_{k,k-j})V_k^{-1}$$

$$V_n = K(n+1,k+1) - \sum_{j=0}^{n-1}\Theta_{n,n-j}V_j\Theta'_{n,n-j}$$

(The recursion are solved in the order V_0; $\Theta_{1,1}$,V_1; $\Theta_{2,2}$ $\Theta_{2,1}$ V_2; $\Theta_{3,3}$ $\Theta_{3,2}$ $\Theta_{3,1}$ V_3;.....)

This paper applies this algorithm based on the predicted second moments. Therefore, the K(i, j)'s in the algorithm are substituted by predicted second moments. V's and Θ's are recursively estimated and are converge to some 2×2 matrices.[3] The paper calculates parameters up to 34 periods, and 34th estimated parameters, $\Theta_{34,j}$'s, are used as parameters for prediction. Estimated parameters are

$$\Theta_{34,1} = \begin{bmatrix} 0.14 & 0.42 \\ 0.20 & 0.12 \end{bmatrix}, \Theta_{34,2} = \begin{bmatrix} -0.14 & 0.24 \\ -0.05 & 0.02 \end{bmatrix}$$
$$\Theta_{34,3} = \begin{bmatrix} -0.19 & 0.54 \\ 0.05 & 0.30 \end{bmatrix}, \Theta_{34,4} = \begin{bmatrix} 0.03 & 0.52 \\ 0.07 & 0.27 \end{bmatrix}$$
$$\Theta_{34,5} = \begin{bmatrix} -0.06 & 0.40 \\ 0.04 & 0.11 \end{bmatrix}, \Theta_{34,6} = \begin{bmatrix} -0.13 & 0.30 \\ 0 & 0 \end{bmatrix}$$

---

[3] X here is $\begin{bmatrix} \Delta Y - \mu_Y \\ \Delta C - \mu_C \end{bmatrix}$

$$\Theta_{34,7} = \begin{bmatrix} -0.01 & 0.07 \\ 0 & 0 \end{bmatrix}, \Theta_{34,8} = \begin{bmatrix} -0.02 & 0.13 \\ 0 & 0 \end{bmatrix}$$
$$\Theta_{34,9} = \begin{bmatrix} 0.05 & -0.11 \\ 0 & 0 \end{bmatrix}, \Theta_{34,10} = \begin{bmatrix} -0.06 & -0.16 \\ 0 & 0 \end{bmatrix}$$
$$\Theta_{34,11} = \begin{bmatrix} -0.09 & -0.11 \\ 0 & 0 \end{bmatrix}$$

Figure 5-3 and Figure 5-4 are the plots of predicted ΔY and ΔC against sample ΔY and ΔC. Figure 5-5 is the plot of residuals which seems to be a random walk. All sample data here are deviations from the means, because the algorithm is for data which have mean zero. R-square is 0.24 for labor income and 0.21 for consumption. Since we predict the differenced data not just Y and C, we can not expect high R-square. However, Figure 5-3 shows that predicted values well keep track of the movement of sample data. Figure 5-4 implies sample data and predicted value are positively correlated. This section concludes that the predicted second moments well explain movements of sample data.

5.3 Implications

This section discusses some economic implications of the empirical results in the previous section. Since every estimate of Table 5-1 through Table 5-4 provides consistent results, we will mainly cite the

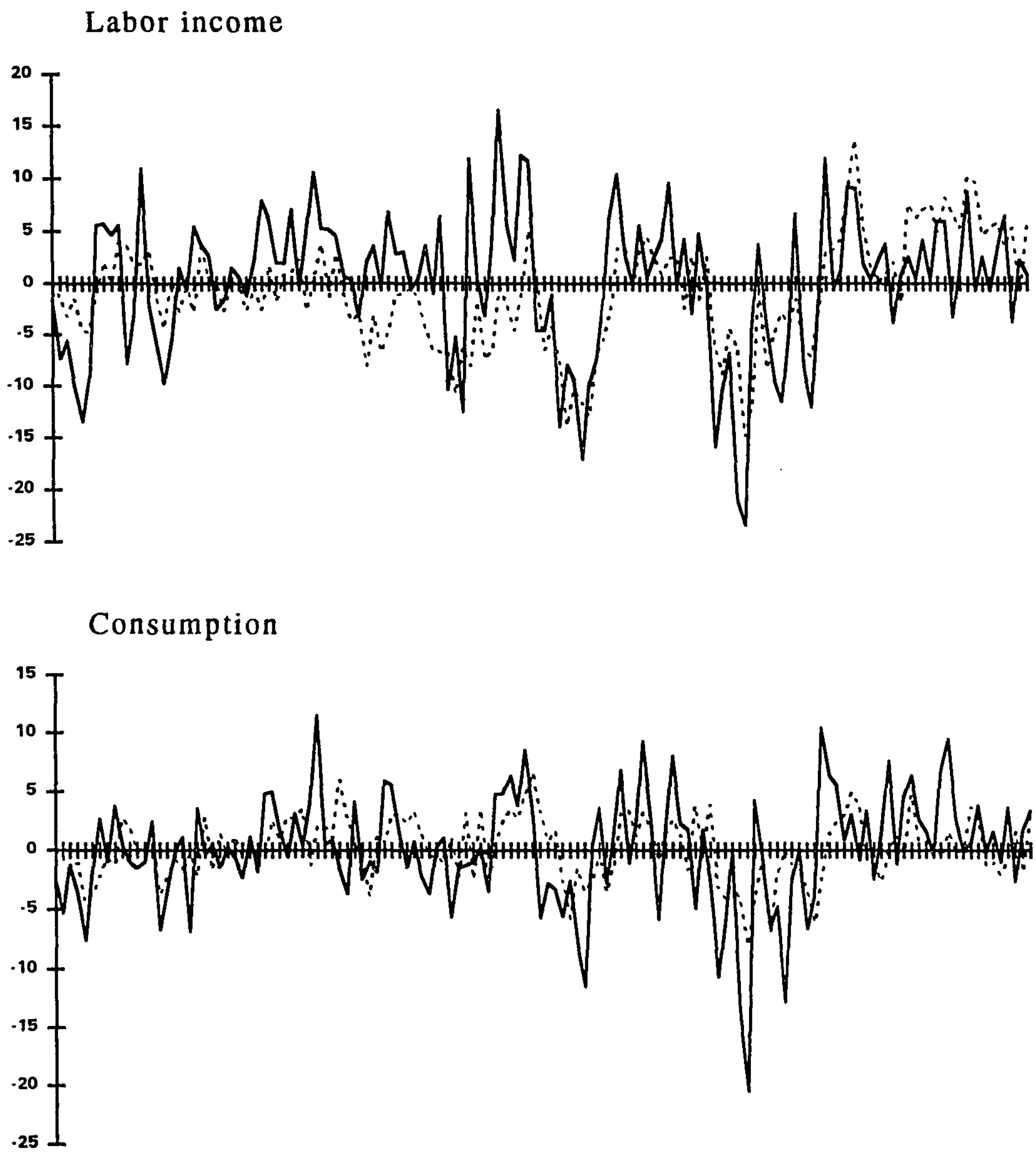
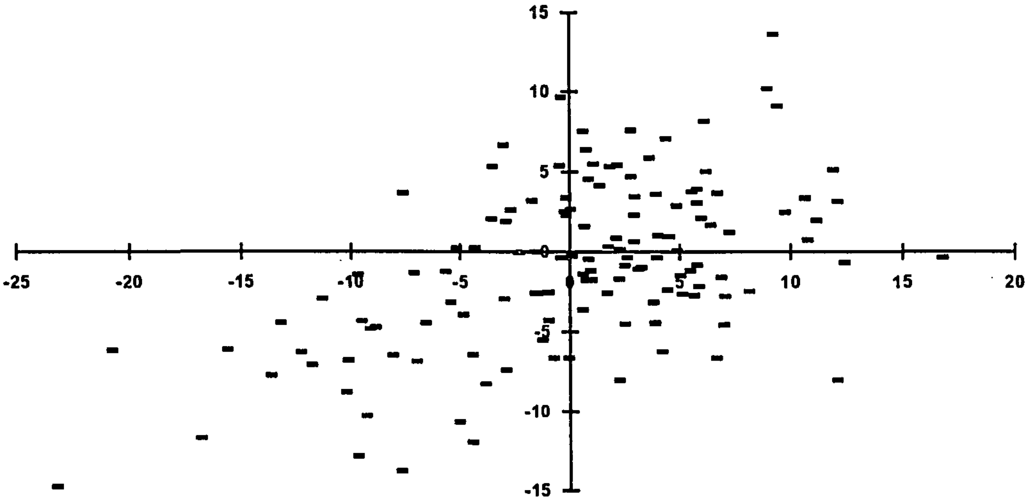


Figure 5-3 Time plot (sample v.s. prediction)

Labor income



Consumption

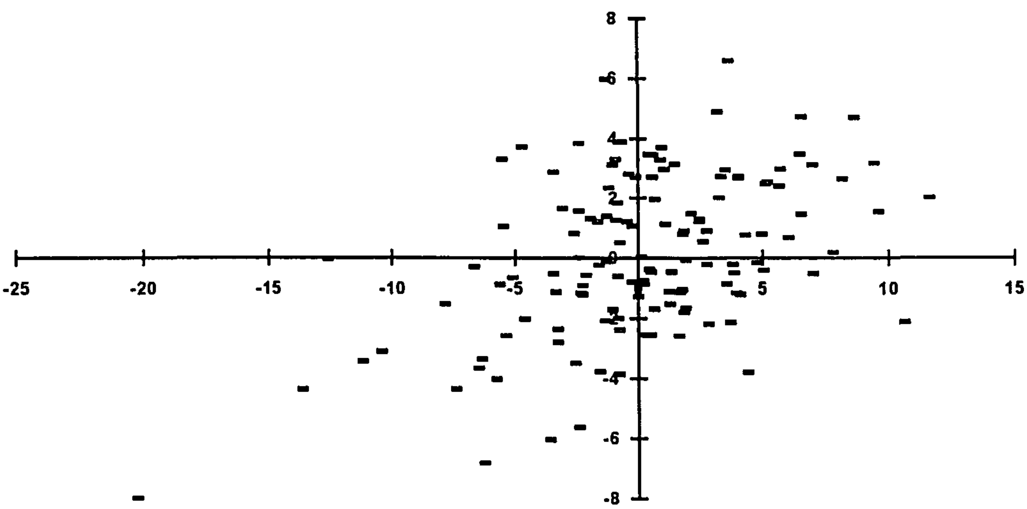
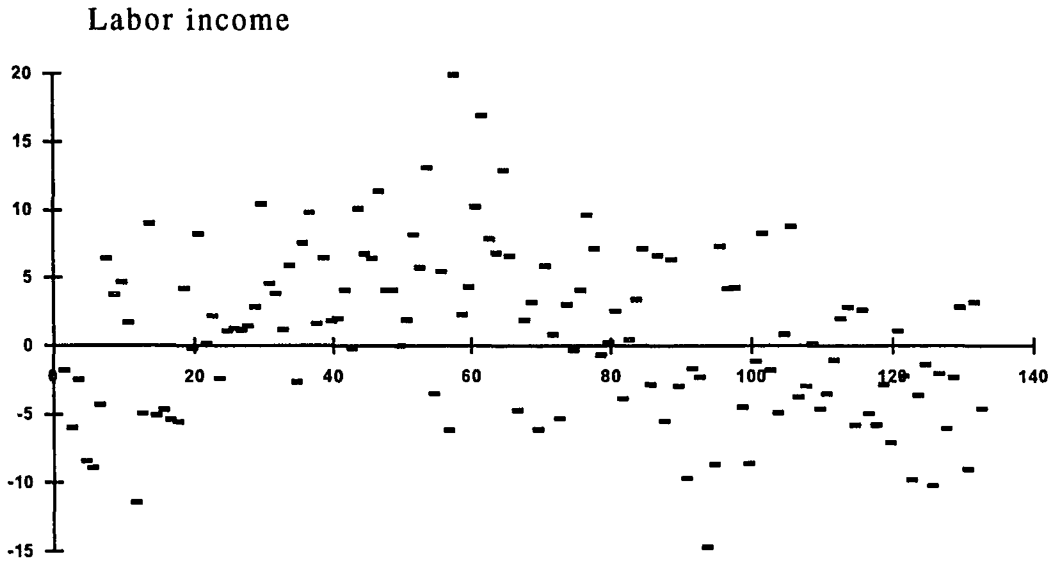


Figure 5-4. Plot (prediction vs. sample)

(horizontal: sample, vertical: prediction)



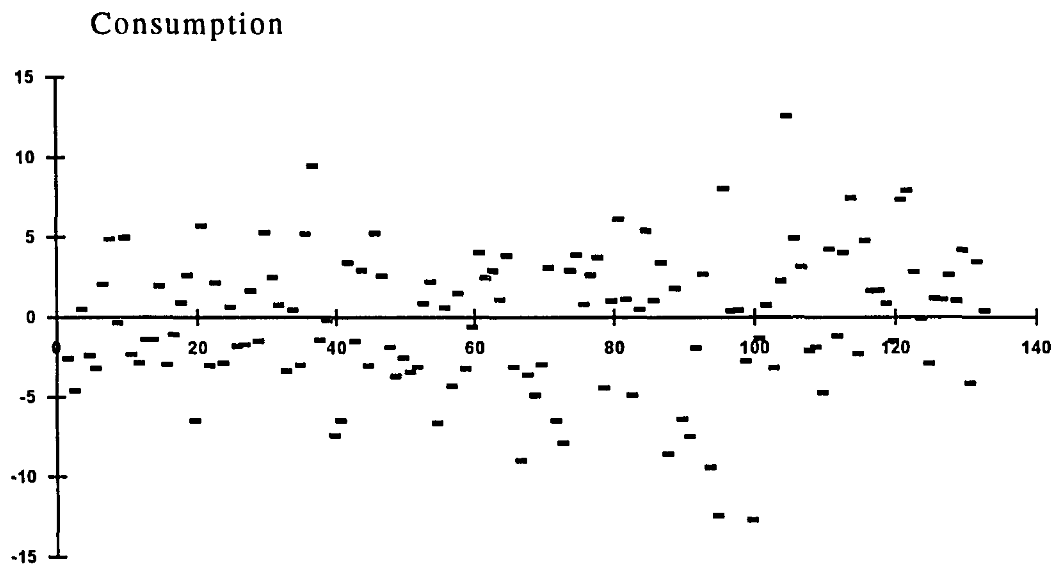


Figure 5-5 Plot of residuals.

results from the converged weighting matrix method.

First, there is a significant transitory component in labor income, even though its individual parameters are hard to identify. Without a transitory component, the labor income process could not converge to something that satisfies the autocovariances of its own process and covariances. Every estimate in Section 5.2 confirms the significant existence of transitory movements in the labor income process. However, its contribution to consumption and permanent income is negligible. Table 5-5 shows that about one third of ΔY movement is explained by innovation in this component. However, its contributions to ΔC and $\Delta\xi$ are negligible. This table also shows that the covariance between consumption and labor income is mainly determined by the permanent innovation in labor income. This is clearly a different result from Quah who implies that it is mainly determined by the transitory innovation in labor income. Table 5-6 shows that about one third of forcast error is determined by the transitory component of labor income. Figure 5-6 also confirms that there are the significant existences of transitory movements in Y and ΔY.

Second, this paper confirms the existence of excess smoothness. The implied κ is consistently higher than 2.1 based on Flavin's permanent income. This means that the measured volatility of permanent income is even larger than the prediction of Gali's nonparametic method and is very close to that of a simple ARIMA representation of labor income approach. Even though we embody the existence of transitory labor income and negative autocovariance of labor income, these do not imply significantly smoother permanent income than a simple ARIMA representation of labor income does.

Third, this paper estimates a permanent consumption process which is assumed to be a moving average process of the permanent income innovation. The paper confirms the existence of excess sensitivity by finding significant explanatory power of lagged permanent consumption innovations (permanent income innovations). However, our empirical analysis raise questions about Gali's formulation of permanent income. His parameter restriction on permanent consumption does not provide meaningful convergence, and the modified model implies the violation of his restriction. The model with a converged weighting matrix shows that $\sum_{j=0}(1+r)^{-j}b_j$ is 0.769 which is less than 1. If we put the measured consumption process into his formulation of permanent income, the variance of permanent income is 69.376 (implied κ = 1.734) which is significantly less than 117.316 from labor income and Flavin's permanent income. This means that the measured permanent income is too volatile to satisfy the restriction (2.22). Our estimation just recreates the gap between the two different formulations of permanent income, even if we consider the possible sources of excess smoothness which has been suggested by two major explanations of this puzzle. The gap between the two measured volatilities is the volatility of permanent income which can not be explained by excess sensitivity and a transitory component of labor income. There may be another source to explain excess smoothness which has not yet been considered by economists.

Table 5-5 Contribution of innovations to second moments

	ε_1	ε_0	u	Total
var(ξ)	117.30	0.02	0	117.32
var(ΔC)	19.85	0.00	2.86	22.71
var(ΔY)	30.53	13.47	0	44.00
cov(ΔC, ΔY)	22.38	0.05	0	22.43

Figure 5-6. Forecast Error Decomposition of ΔY[a]

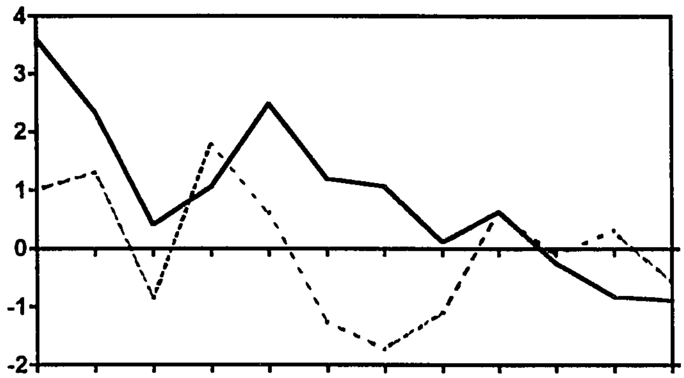
	ε_1	ε_0	Total
1 period ahead	12.98 (92.8)	1 (7.2)	13.98
2	18.50 (87.3)	2.69 (12.7)	21.19
3	18.67 (84.7)	3.38 (15.3)	22.05
4	19.81 (75.3)	6.50 (24.7)	26.31
5	26.00 (79.9)	6.86 (20.1)	32.86
6	27.43 (76.5)	8.44 (23.5)	35.87
7	28.57 (71.4)	11.42 (28.6)	39.99
8	28.58 (69.4)	12.62 (30.6)	41.20
9	28.98 (69.0)	13.02 (31.0)	42.00
10	29.05 (69.0)	13.03 (31.0)	42.08
11	29.74 (69.4)	13.13 (30.6)	42.87
12	30.53 (69.4)	13.47 (30.6)	44.00

[a] The numbers in parenthesis are percentage out of total forcase error.

——— innovation of permanent component of labor income

-------- innovation of transitory component of labor income

ΔY



ΔC

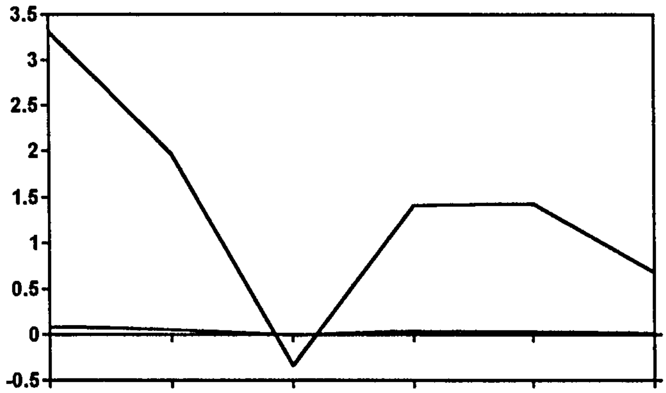
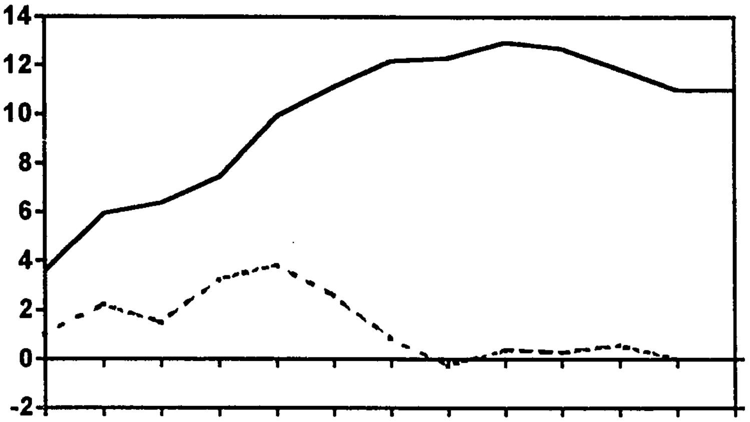


Figure 5-6. Impulse response.

Y



C

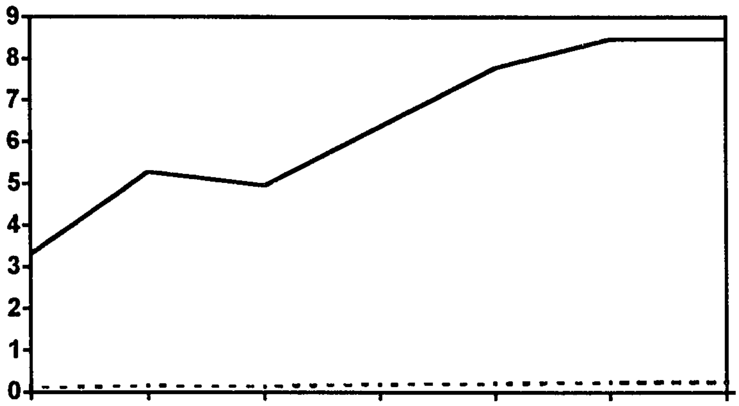


Figure 5-6 (Continued)

6. CONCLUSION

This paper considered the "excess smoothness" puzzle which means that permanent income is more volatile than consumption. Deaton argued that permanent income is much too volatile to predict actual consumption based on a difference stationary labor income process and Flavin's formulation of permanent income. There have been two major explanations about this puzzle. One is the existence of excess sensitivity. The existence of excess sensitivity will spreads the effect of the permanent income innovation over multiple periods. The other is the existence of a transitory component in labor income. If there is significant transitory component which is independent with permanent income, implied permanent income may be much smoother than the univariate labor income process predicts.

Two questions are suggested. The first question is whether excess smoothness is an independent phenomena with excess sensitivity. Quah (1990) indicated that the existence of excess smoothness can be resolved by the existence of transitory component of labor income. The second question is why two formulations of permanent income which are mathematically the same imply different volatility of permanent income. There is a significant difference in the measured volatility of permanent income when a simple ARIMA representation of labor income process is applied to Flavin's permanent income and when a nonparametic approach is applied to Gali's alternative formulation. The former yields even more volatile permanent income.

This paper analyzed Quah's decomposition based on the covariance between the first difference of observed consumption and labor income. He argued that a negligibly small permanent innovation with an extremely large parameter can imply a small enough variance of permanent income. However, this paper argued that this process can not satisfy the sample covariance between consumption and labor income.

To answer the second question, this paper constructed a general model which embodies not only two formulations permanent income but also the existence of excess sensitivity and a bivariate labor income process which have been suggested as possible explanations for excess smoothness. However, there were serious numerical problems in estimation. One is mainly from the identification of the transitory component. The other is from the empirical inadequacy of Gali's parametic restriction on the permanent component of consumption. Therefore, this paper gets converged estimates only by imposing restrictions on transitory component and releasing restrictions on permanent consumption.

From the empirical results, we found significant transitory labor income and the existence of excess sensitivity. However, these factors can not explain the difference between the two formulations of permanent income. If we put estimated labor income into Flavin's permanent income and the consumption process into Gali's alternative formulation, the difference we questioned is just recreated. This means that even if there is significant transitory component in labor income, it does not significantly smooth Flavin's permanent income. We can also say that excess sensitivity probably can not fully explain excess smoothness. There might other sources which should be embodied to explain excess smoothness.

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APPENDIX A FINITE HORIZON LIFE-CYCLE MODEL

This appendix provides a description of key aspects of Gali's finite life time model.

Demographics:

The size of each cohort at birth is normalized to p, and is assumed to decline deterministically over time, at a rate also given by p. The size of the cohort born in period s is

$$N_{s,t} = p(1-p)^{(t-s)}$$

Thus, total population at time t is

$$N_t \equiv \sum_{s=-\infty}^{t} N_{s,t} \equiv 1.$$

Annuity market:

Annuity firms make (receive) every period an annuity payment to (from) each consumer holding positive (negative) financial wealth, and inherit the wealth of that consumer at his death. A zero-profit condition and the population structure imply the effective gross return of nonhuman wealth is 1 + z, where $(1+z)=(1+r)(1-p)^{-1}$.

Individual behavior:

Let us analyze the behavior of an agent who was born at period s and lives at period t. Following Flavin's permanent income,

$$C_{s,t} = z\left[W_{s,t} + H_{s,t}\right]$$

where

$$H_{s,t} \equiv (1+z)^{-1}\sum_{j=0}^{\infty}(1+z)^{-j}E_t Y_{s,t+j}.$$

Labor supply:

The amount of labor services supplied by an individual consumer, denoted by $L_{s,t}$, is assumed to decline geometrically over his lifetime at a rate α, reflecting underlying changes in his productivity and/or hours supplied.

$$L_{s,t} = (\Gamma / p)(1-\alpha)^{(t-s)},$$

where $0 \leq \alpha < 1$, and $\Gamma \equiv [1-(1-\alpha)(1-p)]$ satisfies the obvious aggregation/normalization restriction $1 \equiv L_t \equiv \sum_{s=-\infty}^{t} L_{s,t}N_{s,t}$. They drop the subscript to denote an aggregate variable.

Therefore,

$$Y_{s,t} = L_{s,t}Y_t = (\Gamma / p)(1-\alpha)^{(t-s)}Y_t$$

Aggregate behavior:

Expressions for the main aggregates can now be easily derived.

Aggregate nonhuman wealth, $W_t \equiv \sum_{s=-\infty}^{t} W_{s,t}N_{s,t}$ will satisfy

$$W_t = (1+r)W_{t-1} + Y_{t-1} - C_{t-1}$$

which satisfies the fact that, in the aggregate, the return on financial wealth is r instead of z, since annuity payments represent pure transfers among consumers. Aggregated human wealth, $H_t \equiv \sum_{s=-\infty}^{t} H_{s,t}N_{s,t}$ will be given by

$$H_t \equiv (1+z)^{-1}\sum_{j=0}^{\infty}(1+z)^{-j}E_tY_{t+j}$$

Aggregate consumption, $C_t \equiv \sum_{s=-\infty}^{t} H_{s,t}C_{s,t}$, is given by

$$C_t = z[W_t + H_t]$$

Assuming that E(ΔY) exists and is equal to μ, we can rewrite C_t in the following convenient way:

$$C_t = \Omega + zW_t + \beta Y_t + u_t$$

where $\beta \equiv z/(z+\alpha), \Omega \equiv \beta\mu(1-\alpha)/(z+\alpha)$

and $u_t \equiv \beta\sum_{j=0}^{\infty}(1+z)^{-j}(1-\alpha)^j(E_t\Delta Y_{t+j} - \mu)$.

Long-run behavior of consumption:

Aggregate consumption in period t can be decomposed as follows:

$$C_t \equiv \sum_{s=-\infty}^{t-1} C_{s,t}N_{s,t} + N_{t,t}C_{t,t}$$

Applying the E_{t-1} operator to both sides of the previous expression using the martingale result, we get:

$$E_{t-1}C_t = (1-p)C_{t-1} + pE_{t-1}C_{t,t}.$$

Thus, E_{t-1} has two components. The first one corresponds to expected one-period-ahead consumption by those alive at t - 1 who will remain alive at t. The second component reflects the level of consumption by the new cohort born at t, as expected at t - 1. The assumption that individuals are born with zero nonhuman wealth implies that $C_{t,t} = zH_{t,t}$. Using the expression for individual human wealth above,

$$C_t = (1 - p)C_t + \Gamma\beta Y_{t-1} + (\Gamma(1 + z)/(1 - \alpha))[\Omega + u_{t-1}] + \eta_t$$

where $\eta_t \equiv C_t - E_{t-1}C_t$ is the innovation in aggregate consumption. Therefore, we can derive

$$\Delta C_t = -pC_t + \Gamma\beta Y_{t-1} + (\Gamma(1 + z)/(1 - \alpha))[\Omega + u_{t-1}] + \eta_t$$

APPENDIX B SPECTRAL DENSITIES IN CONSUMPTION PROCESS

From the properties of spectral densities and the specification of the ΔC, ΔC^*, and n processes in (2.16), it is known that

$$h_{\Delta C^*}(\omega) = (2\pi)^{-1}\left|\sum_{s=0}^{\infty} b_s e^{-i\omega s}\right|^2 \mathrm{var}(\xi)$$

$$h_n(\omega) = (2\pi)^{-1}\left|\sum_{s=0}^{\infty} d_s e^{-i\omega s}\right|^2 \mathrm{var}(\xi)$$

where $|\bullet|$ is the usual product by complex conjugate operator.

Since spectral density of ΔC^* and n at frequency zero is thus given by

$$h_{\Delta C^*}(0) = (2\pi)^{-1}\left(\sum_{s=0}^{\infty} b_s\right)^2 \mathrm{var}(\xi)$$

$$h_n(0) = (2\pi)^{-1}\left(\sum_{s=0}^{\infty} d_s\right)^2 \mathrm{var}(\xi)$$

we can derive useful properties of spectral densities as followings:

$$\lim_{r \to 0} \left[(2\pi) h_{\Delta C^*}(0) - \left(\sum_{s=0}^{\infty} (1+r)^{-s} b_s \right)^2 \mathrm{var}(\xi) \right]$$
$$= 0$$

and

$$\lim_{r \to 0} \left[(2\pi) h_n(0) - \left(\sum_{s=0}^{\infty} (1+r)^{-s} d_s \right)^2 \mathrm{var}(u) \right]$$
$$= 0$$

Using these properties and restriction (2.19) and (2.20), we can obtain (3.1) and (3.2).