Time–Varying Risk and Return in the Bond Market: A Test of a New Equilibrium Pricing Model

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Abstract

This paper uses bond market data to empirically test the asset pricing model of Kazemi (1992). According to this model, the rate of return on a long-term pure-discount default-free bond will be perfectly correlated with changes in the marginal utility of the representative investor. The covariability between financial asset returns and returns on such a bond can, therefore, serve as a measure of the riskiness of assets. The aim of this study is to determine whether the model can explain cross-sectional differences in the monthly returns of bonds with different maturity dates. We estimate and test the restrictions imposed by the model on returns of default-free bonds, while allowing the conditional distribution of bond returns to be time-varying. The model is rejected during the full sample period (1973-1995) and the sub-period (1973-1980) when the Federal Reserve's focus is on interest rates, while the model is not rejected during the sub-period (1981-1995) when the Federal Reserve's focus is on money supply.

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The basic outcome of *equilibrium* asset pricing models is summarized by the result that asset prices adjusted by the marginal utility of the representative investor follow a martingale process. From this it follows that the covariability between asset returns and changes in this marginal utility would measure the riskiness of asset returns.[1] By making suitable assumptions, the return on aggregate wealth, the growth rate of aggregate consumption, or returns on a number of hedge portfolios become perfectly correlated with changes in the investor's marginal utility. Empirical implications of these models have been extensively tested using stock market data and, in a few cases, using bond market data.[2]

Kazemi (1992) shows that if the variables describing the state of the economy have long-run stationary distributions, the inverse of the per period return on a long-term zero-coupon default-free bond will be proportional to the representative investor's marginal rate of substitution. Therefore, asset prices adjusted by the inverse of the per period return on such a bond will follow a martingale process, and further, the covariability between asset returns and the return on such a bond measures the riskiness of the assets. This paper offers empirical tests of this Bond-based Capital Asset Pricing Model (BCAPM) using bond market data. Our objective is to determine whether the model can explain cross-sectional differences in the monthly returns of bonds with different maturity dates.

Under the above assumptions, the rate of return on a long-term bond will be perfectly correlated with the pricing kernel. This paper presents a discrete–time version of this model and shows that the inverse of one plus the per period rate of return on a long-term bond will be the same as the pricing kernel. Since the rate of return on the long-term bond is not observable, we use the rate of return on a 20-year bond to approximate its behavior.[3] In this regard the results reported in this paper continue the recent research efforts of Bansal and Viswanathan (1993), Bansal, Hsieh, and Viswanathan (1993) and Chapman (1997) in approximating the pricing kernel. The first two papers use neural nets to approximate the pricing kernel, and asset returns are chosen to represent the relevant state variables. Chapman (1997) uses orthonormal polynomials to approximate the pricing kernel and aggregate consumption to represent state variables.

Similar to these papers we use the GMM approach of Hansen (1982) and Hansen and Singleton (1982) and the estimation procedure suggested by Newey and West (1987) to test the model's implications for bond returns. This procedure allows for moments of conditional distributions of bond returns to be time-varying. This is consistent with the evidence presented by Keim and Stambaugh (1986), Stambaugh (1988), Campbell (1987), Fama and French (1989), and Harvey (1989).[4]

Using the full sample period, 1973-1995, the empirical results show that the average pricing error is very small (about 0.0001) but significantly different from zero when the rate of return on the long-term bond is used as a proxy for the pricing kernel. In addition, the mispricing for each bond class is considered during this time period and the results indicate that the proxy for the pricing kernel could not completely measure the riskiness of bonds. In particular, the covariability between a bond's rate of return and the pricing kernel tends to underestimate the riskiness of bonds. During the sub-period when the Federal Reserve's focus is on interest rates, 1973-1980, the model is completely rejected as well. Not only is the average pricing error significantly different from zero, but also the pricing kernel grossly underestimates the riskiness of bonds. However, during the sub-period when the Federal Reserve's focus is on the money supply, 1981-1995, the model *cannot* be rejected with the average pricing error being -0.00003. Although the pricing errors are not significantly different from zero, during this sub-period the pricing kernel has a tendency to overestimate the riskiness of bonds.

The paper is organized as follows. In section 1 we summarize Kazemi's (1992) continuous-time model and obtain new results in a discrete-time framework. Section 2 presents the paper's methodology and section 3 describes the data. In section 4 we report our empirical results and section 5 offers some concluding comments.

1. The Model

Consider an exchange economy in which a representative investor exists. It is well known that the time-t price of a *nominal* bond that will pay one unit of account at maturity, T, and the representative investor's marginal utility of *nominal* wealth satisfy the relationship (e.g., see Lucas (1978), Cox, Ingersoll, and Ross (1985a) and Breeden (1986))

$$B_{t,T} = \frac{E[J_w(T)|\xi_t]}{J_w(t)}, \tag{1}$$

where $B_{t,T}$ is the time-t price of the bond, $J_w(t)$ is the representative investor's marginal utility of wealth, $E[\cdot|\cdot]$ is the conditional expectation operator, and ξ_t is a vector of variables that define the state of the economy at time t. Consider the behavior of this bond's price as t increases to t+1. The random *nominal* one plus the rate of return on the bond will be

$$\begin{aligned} R_{t+1,T} &= \frac{B_{t+1,T}}{B_{t,T}}, \\ &= \frac{J_w(t)}{J_w(t+1)} \times \frac{E[J_w(T)|\xi_{t+1}]}{E[J_w(T)|\xi_t]}. \end{aligned} \tag{2}$$

The return has two parts. The first part is equal to the *inverse* of one plus the growth rate of the investor's marginal utility of wealth, and it represents the *current* effect of changes in the state variables. The second part is equal to one plus the current growth rate of the expected value of time T marginal utility of wealth, and it represents the *information* effect of current changes in the state variables.

Suppose the variables that describe the state of the economy have a long-run stationary joint distribution function. Then if T is sufficiently large, $E[J_w(T)|\xi_t]$ remains unchanged as ξ_t changes to ξ_{t+1}, and therefore equation (2) can be rewritten as

$$R_{t+1} = \lim_{T\to\infty} R_{t+1,T} = \frac{J_w(t)}{J_w(t+1)}. \tag{3}$$

In a continuous-time framework equation (3) indicates that the long-term bond's rate of return is perfectly *negatively* correlated with the rate of change in the investor's current marginal utility of wealth.[5] Cox, Ingersoll, and Ross (1985a) report the general result that the negative of the covariance between an asset's rate of return and the rate of change in the representative investor's marginal utility of wealth measures the riskiness of the asset. Given equation (3), Kazemi (1992) shows that an asset's risk premium is equal to the covariance between its rate of return and the rate of return on a long-term default-free pure-discount bond. Since we use discrete-time data in this paper, equation (3) is used to obtain a different equilibrium relationship in a discrete-time framework.

Lucas (1978) has shown that the equilibrium random *excess* rate of return on asset *j* can be expressed as

$$E\left[\frac{J_w(t+1)}{J_w(t)} \times r_{jt+1} \middle| \xi_t\right] = 0 \tag{4}$$

where r_{jt+1} is the *excess* rate of return on asset *j* over the time interval $(t,t+1)$. Given the result of equation (3), equation (4) can be written as

$$E\left[k_{t+1} \times r_{jt+1} | \xi_t\right] = 0 \tag{5}$$

where k_{t+1} is the inverse of one plus the rate of return on the long-term bond, R_{t+1}^{-1}. Using the definition of covariance and noting the fact that $E[k_{t+1} | \xi_t] = B_{t,t+1}^{-1}$, equation (5) can be written in a CAPM type form

$$E[r_{jt+1} | \xi_t] = -B_{t,t+1}^{-1} \times Cov[r_{jt+1}, k_{t+1}], \tag{6}$$

where the negative of covariability of securities' rates of return with k_{t+1} is a measure of risk and $B_{t,t+1}^{-1}$ is the market price of risk.

Since the empirical tests are conducted using a subset of the information set available to investors, we use the law of iterative expectations to obtain the following expression, which will be used as the basis of our tests:

$$E\left[k_{t+1} \times r_{jt+1} | Z_t\right] = 0 \tag{7}$$

Here Z_t is a subset of ξ_t and it represents the information available to us at time *t*.

2. Methodology

To test the model, equation (7) is applied to returns on zero coupon bonds of different maturity dates. Our time-*t* information set, Z_t, is represented by a set of instrumental variables. In particular, let Z_t denote a $1 \times q$ vector of instrumental variables, and let r_t denote the $1 \times n$ vector of bond excess returns (i.e., it includes r_{jt} for $j = 1, \ldots, n$). The $1 \times n$ vector of error terms from the model is expressed as

$$\varepsilon_{t+1} = k_{t+1} \times r_{t+1} - \lambda \,. \tag{8}$$

The pricing model of this paper along with rational expectations implies that $E(\varepsilon_{t+1} \otimes Z_t) = 0$ and $\lambda = 0$. In system (8) we have n model errors ε_t. With q instrumental variables, the number of orthogonality conditions is $n \times q$ with one parameter to estimate.

The parameter of system (8) is estimated using Hansen's (1982) GMM estimators. We form the vector of orthogonality conditions

$$g_T(\lambda) = vec(\varepsilon' Z)/T \tag{9}$$

where Z is a $T \times q$ matrix of observations on the instrumental variables and ε is a $T \times n$ matrix of model errors for T time periods. The GMM estimator is obtained by minimizing the criterion function, $J_T(\lambda) = g_T' S_T^{-1} g_T$, where S_T is a consistent estimator of the optimal weighting matrix derived by Hansen (1982). We use the two-step procedure suggested by Hansen and Singleton (1982) to arrive at this weighting matrix. Since we have $n \times q$ orthogonality conditions and one parameter, one linear combination of these orthogonality conditions will be set equal to zero in estimating the parameter. There are $n \times q - 1$ orthogonality conditions (over-identifying restrictions) that are not set equal to zero in the estimation, but should be close to zero if the model is well specified. Hansen (1982) shows that the statistic $T\left[\min J_T(\hat{\lambda})\right]$ is asymptotically chi-square with $n \times q - 1$ degrees of freedom under the null hypothesis when the model is well specified.

3. Data

This paper uses Bliss's (see Bliss and Ronn (1988, 1989)) updated monthly holding period returns data over the period January 1973 through December 1995 for pure default-free discount bonds with maturity dates of four, six, and nine months; and one, two, three, four, five, eight, ten, 15 and 20 years.[6] The excess returns are calculated with respect to the rate of interest on the one-month Treasury Bills. We use the bond with 20 years to maturity as a proxy for the long-term default-free zero-coupon bond.

Forward rates have been shown to contain information of the expected returns of discounted default-free securities (see, e.g., Fama and Bliss (1987) and Stambaugh (1988)). Thus, three of the instrumental variables are the one-month forward rates for two, 12 and 24 months in the future calculated using Bliss’s data. Since we do not use bonds with maturity dates of three, 13 and 25 months, the forward rates do not match the maturity dates of the bonds in the sample. The forward rates are lagged one period to avoid any errors in the returns of the pure discount bonds being correlated with any errors in the forward rates (see Stambaugh (1988)). Two additional instrumental variables are the first and second order lags of the inverse of one plus the monthly holding period return of the 20-year bond. The remaining four instrumental variables are chosen on the basis of their usefulness in predicting asset returns of broader classes of assets.[7] These variables are the junk bond premium measured as the yield on Moody’s BAA-rated bonds less the yield on Moody’s AAA-rated bond, the capital gain yield of the S&P 500 index, the dividend yield on the S&P 500 index, and the inflation rate. This data is obtained from Ibbotson & Associates and each of these instrumental variables are lagged one period.

4. Empirical results

The empirical results of our tests of the model appear in Table 1. For the first set of results appearing in Panel A, data from the entire time period is used. The pricing error (i.e., the estimate of the conditional expected value of ε_t) is 0.0001, which is rather small. However the *t*-statistic is 2.797, indicating that it is significantly different from zero at the 1% level using a two-tailed test. To determine if the mispricing differs across bond maturities, the GMM method is applied to each bond maturity separately. The results indicate that the mispricing is not uniform. In fact, the average pricing error is significantly less than zero using a two-tailed test at the 5% level for the four-month bond while it is significantly greater than zero at the 10% level for the one-year bond. The average pricing errors for other bond maturities are not significantly different from zero.

[Insert Table 1]

The next issue addressed is whether the model captures the riskiness of the bonds. In other words, does higher covariability with the long-term bond mean a higher excess rate of return on a given bond. If covariability with the long-term bond is a measure of risk, the estimate of the conditional expected value of ε_t should be unrelated to the average rate of return on each asset class. To see this, consider equation (7), with the right-hand-side set equal to λ_j rather than zero. Using the definition of covariance, we have

$$E[k_{t+1}|Z_t]E[r_{jt+1}|Z_t] = -Cov[k_{t+1}, r_{jt+1}|Z_t] + \lambda_j$$

or,

$$\begin{aligned} E[r_{jt+1}|Z_t] &= -E[k_{t+1}|Z_t]^{-1}Cov[k_{t+1}, r_{jt+1}|Z_t] + E[k_{t+1}|Z_t]^{-1}\lambda_j \\ &= -B_{t,t+1}{}^{-1}Cov[k_{t+1}, r_{jt+1}|Z_t] + B_{t,t+1}\lambda_j, \end{aligned} \tag{10}$$

where $B_{t,t+1}$ is the current price of a one-period bond.[8] As can be seen, if there is a strong *positive* cross-sectional relationship between λ_j and $E[r_{jt+1}|Z_t]$, then one can conclude that the covariance term does not explain a large portion of cross-sectional differences in expected returns. On the other hand, if there is a cross-sectional relationship between $E[r_{jt+1}|Z_t]$ and $Cov[k_{t+1}, r_{jt+1}|Z_t]$, then changes in the pricing error λ_j should not be related to cross-sectional differences in expected returns. Table 2 reports the average realized rates of return for each bond maturity. For the full sample (1973-1995) the realized rates of return tend to increase with time to maturity. As can be seen from Table 1, the average pricing error appears to be related to expected excess returns and there is a positive correlation coefficient of 0.311 between the average pricing errors and realized rates of return. Thus, the covariance underestimates the riskiness of the bonds. Therefore, not only is the average pricing error during this period different from zero, the model's pricing kernel cannot explain all of the cross-sectional differences in expected rates of return during this time period.

[Insert Table 2]

To consider the possibility that different monetary regimes may alter the results, we sub-divide the sample into two periods based on the Federal Reserve's monetary policy: 1) March 1973 through December 1980 when the Federal Reserve's focus is on interest rates; and 2) January 1981 through November 1995 when the Federal Reserve's focus is on money supply. These test results appear in Panels B and C, respectively, in Table 1.

During the time period when the Federal Reserve's focus is on interest rates rather than money supply, the model is completely rejected. The average pricing error is –0.0005 with a *t*-statistic of –4.354 and with a *p*-value of 0.0124, the orthogonality test is soundly rejected. For tests using individual bond maturities, the coefficient for the four-month and six-month bonds are significantly negative at the 1% and 5% significance levels, respectively, using a two-tailed test. Six of the eleven coefficients are negative while five are positive. The only significantly positive coefficient (at the 10% level) is for the five-year bond. Consistent with the rejection of the model in this time period using all bond maturities there is a strong positive correlation coefficient of 0.612 between the average pricing errors and the average realized rates of return on bonds, indicating a stronger relation between the two than when the entire sample period is used. Thus the model's pricing kernel underestimates the riskiness of the assets and explains an even smaller portion of cross-sectional differences in expected returns than when the entire time period is used.

Using more recent data when the Federal Reserve's focus is on money supply the model is *not* rejected. The average pricing error is only –0.00003 with a *t*-statistic of –0.89 and the over-identifying restrictions test has a *p*-value of 1.0. When GMM is applied to each bond maturity separately, individual pricing errors are *not* significantly different from zero. Moreover, the average pricing error is uniformly negative and it is *negatively* correlated with realized returns with a correlation coefficient of –0.761. A negative correlation coefficient indicates the pricing kernel is overestimating the riskiness of the assets. However, unlike the tests on the entire time period and the earlier sub-period, the average pricing error when all bond maturities are used, although negative, is not significantly different from zero. Neither can the model be rejected using the over-identifying restrictions test. Therefore, the evidence supports the argument that in more recent years, higher covariability with the rate of return on the long-term bond indicates a higher expected rate of return.

The fact that the estimated pricing error in BCAPM is significantly different from zero in some of the tests suggests two possibilities. First, the basic assumption of the model that the state variables have a limiting stationary joint distribution is not consistent with the data and thus the BCAPM is of little use. However, given the differing results under the different monetary regimes, it may be that the Federal Reserve's policy may impact the joint distribution of the state variables. Second, the return on the 20-year bond may not be an appropriate proxy for the return on the long-term bond. However, we repeated the same tests using the 30-year bond. The overall results are not different, indicating that for this data set using a longer maturity bond as the proxy for the long-term bond does not affect the results. This casts doubt on the long-term default-free discount bond being an inappropriate proxy for the return on the long-term bond.

5. Conclusions

In this paper we empirically test the asset pricing model of Kazemi (1992) using bond market data. According to this model, if the state variables have long-run stationary joint distributions, the rate of return on a long-term pure-discount default-free bond will be perfectly correlated with changes in the marginal utility of the representative investor. We examine whether this model can explain cross-sectional differences in the monthly returns of bonds with different maturity dates. We use the GMM procedure to estimate and test the model, while allowing the moments of conditional distributions of asset returns to be time-varying. Our results are mixed. When the entire sample is used, and particularly for the sub-period 1973-1980 when the Federal Reserve's focus is on interest rates the model performs poorly. However, the model cannot be rejected when more recent data is used when the Federal Reserve's focus is on the money supply. During this time period the average pricing error is close to its predicted value of zero with a statistically insignificant estimated coefficient of –0.00003. Moreover, the *p*-value for the over-identifying restrictions test is 1.0 indicating the model cannot be rejected.

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Table 1
Results of testing the Bond-based Capital Asset Pricing Model: $E[k_{t+1} \times r_{jt+1} | Z_t] = 0$.

	Bond maturities used in the test of the Bond-based CAPM											
	All maturities	Four months	Six months	Nine months	One Year	Two Years	Three years	Four years	Five years	Eight years	Ten years	Fifteen years
Panel A: Sample time period is March 1973 – November 1995												
Average Pricing Error (λ)	0.0001	–0.0003	–0.0000	0.0001	0.0005	0.0003	0.0007	0.0011	0.0009	0.0004	–0.0002	0.0004
(*t*-statistic)	(2.797)[a]	(–2.119)[b]	(–0.317)	(0.328)	(1.738)[c]	(0.612)	(0.874)	(1.130)	(0.810)	(0.249)	(–0.124)	(0.151)
χ^2 of model	65.249	15.635	13.218	13.777	15.678	13.359	15.464	13.520	12.912	14.865	16.843	13.966
(*p*-value)	(0.9997)	(0.075)	(0.153)	(0.131)	(0.074)	(0.147)	(0.079)	(0.141)	(0.167)	(0.095)	(0.051)	(0.124)
Panel B: Sample time period is March 1973 – December 1980												
Average Pricing Error (λ)	–0.0005	–0.0006	–0.0005	–0.0004	–0.0003	0.0000	0.0012	0.0020	0.0025	0.0009	–0.0012	–0.0058
(*t*-statistic)	(–4.354)[a]	(–4.377)[a]	(–2.437)[b]	(–1.303)	(–0.766)	(0.038)	(1.112)	(1.474)	(1.656)[c]	(0.476)	(–0.559)	(–0.192)
χ^2 of model	144.769	10.551	8.780	7.114	4.844	6.9394	9.325	10.474	8.725	8.744	10.706	8.699
(*p*-value)	(0.0124)	(0.308)	(0.458)	(0.625)	(0.848)	(0.644)	(0.408)	(0.314)	(0.463)	(0.461)	(0.296)	(0.466)
Panel C: Sample time period is January 1981 – November 1995												
Average Pricing Error (λ)	–0.00003	–0.0003	–0.0001	–0.0001	–0.0004	–0.0004	–0.0005	–0.0002	–0.0002	–0.0014	–0.0033	–0.0019
(*t*-statistic)	(–0.890)	(–1.636)	(–0.492)	(–0.455)	(–0.874)	(–0.491)	(–0.483)	(–0.169)	(–0.133)	(–0.760)	(–1.483)	(–0.621)
χ^2 of model	44.302	13.825	11.787	11.451	11.506	10.593	11.393	10.034	9.40545	11.747	12.457	10.926
(*p*-value)	(1.0000)	(0.129)	(0.226)	(0.246)	(0.243)	(0.305)	(0.250)	(0.348)	(0.401)	(0.228)	(0.189)	(0.281)

k_{t+1} is the inverse of one plus the rate of return on the long-term bond. r_{jt+1} is the *excess* rate of return on asset *j* over the time interval (*t*,*t*+1). The time-*t* information set, Z_t, denotes a 1×*q* vector of instrumental variables with the instrumental variables being the one period lagged forward rates of months two to three, months 12 to 13 and months 24 to 25, the first and second order lags of the inverse of one plus the monthly holding period return of the 20-year bond, the one period lagged junk bond premium measured as the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bond, the one period lagged capital gain yield of the S&P 500 index, the one period lagged dividend yield on the S&P 500 index, and the one period lagged inflation rate. The 1×*n* vector of error terms from the model is expressed as $\varepsilon_{t+1} = k_{t+1} \times r_{t+1} - \lambda$. The pricing model along with rational expectations implies that $E(\varepsilon_{t+1} \otimes Z_t) = 0$ and $\lambda = 0$. The t-statistic tests that the average pricing error is not significantly different from zero. Superscripts *a*, *b*, and *c* represent statistical significance under a two-tailed test at the 1%, 5% and 10% levels, respectively. The reported *p*-value is for the over-identifying restrictions test. The time period is March 1973 through November 1995 (272 observations).

Table 2
Average percentage monthly rates of return on bonds with different maturity dates for the three sample periods used in the tests of the Bond-based Capital Asset Pricing Model.

	Bond maturity										
Sample period	Four months	Six months	Nine months	One year	Two years	Three years	Four years	Five years	Eight years	Ten Years	Fifteen years
March 1973 – November 1995	0.6323	0.6556	0.6728	0.7016	0.6905	0.7631	0.7956	0.8088	0.8843	0.8655	0.9766
March 1973 – December 1980	0.6450	0.6522	0.6342	0.5450	0.5397	0.5219	0.4528	0.4663	0.2717	0.1165	-0.1085
January 1981 – November 1995	0.6256	0.6573	0.6930	0.7838	0.7697	0.8897	0.9757	0.9887	1.2060	1.2588	1.5464

Notes

1. This is the theoretical underpinning of the single-period Sharpe-Lintner model, the intertemporal models of Merton (1973), Lucas (1978), Breeden (1979), Cox, Ingersoll, and Ross (1985a,1985b), and the equilibrium arbitrage pricing model of Connor and Korajczyk (1989).

2. Many of these empirical tests are reviewed in Huang and Litzenberger (1988), Schwert and Smith (1992), and Campbell, Lo and MacKinlay (1997). Empirical tests of these models using bond market data appear in Friend, Westerfield, and Granito (1978), Campbell (1987), Stambaugh (1988), and Chang and Huang (1990).

3. The rate of return on the long-term bond that is used to model the pricing kernel in Kazemi (1992) and in this paper is defined as $\lim_{T \to \infty}\left[B_{t+1,T} / B_{t,T} \right]$.

4. The assumption that bond returns follow time-varying conditional distributions does not conflict with the BCAPM's assumption that *state variables* follow processes with long-run stationary distributions. Bond prices may be non-linear functions of state variables and time, and thus their distributions could be non-stationary.

5. The rate of return on the long-term bond has a number of other properties. For instance, one can show that it is equal to the rate of return on the *growth optimum portfolio* discussed in Cox and Huang (1989) and Merton (1990).

6. Bliss uses treasury quotations obtained from the CRSP Government Bond Files, excluding any bonds with special features such as being callable, flower bonds, or with special tax provisions to extract the implied term structure from the prices of coupon notes and bonds. The extracted term structure is then used to construct pseudo-discount bonds of the desired maturity. Each month a term structure of forward rates is calculated and each successive maturity is used to calculate an additional forward rate. If there are multiple bonds for a given maturity, forward rates for each of them are calculated and the average forward rate is used. Forward rates from shorter-maturity bonds

are used to price the coupons on the bond for the next available maturity. Coupon dates are unlikely to correspond exactly to forward rate dates. Therefore, for coupons falling within the period covered by a forward rate, the pricing assumption is that the daily continuously compounded forward rate for the period between successive maturities is the relevant discount rate for each day in the period, so that it can be used for any sub-period. Similarly, there may be coupons or a principal payment during the period from the maturity of the last included bond to the maturity of the next longer bond. Again, the pricing assumption is that the (solved for) incremental forward rate for the incremental period to the maturity of the next longer bond is the relevant discount rate for each day in that incremental period. Using the assumption that a forward rate applies to each day of the period it covers, forward rates can be summed to get the implied end-of-month time t discount bond yield for any maturity. The yields are used to calculate discount bond prices and returns are then calculated from the prices. The parameters of the discount function is an extension of the function employed by Nelson and Siegel (1987), and is estimated using a constraint non-linear method. The constraints are that the short-term rate, the long-term rate, and all forward rates are positive. This methodology explicitly takes into account the bid and ask spread in bond prices, thus avoiding the need to use the average of the bid and the ask prices as the "true" price. The estimated discount function is then applied to out of sample bonds to measure its accuracy. For additional details see Bliss and Ronn's (1988) working paper prior to the 1989 published version.

7. See e.g., Ferson and Harvey (1991) for a general discussion of these variables in relation to predicting asset returns.

8. For the sample period the average value of $B_{t,t+1}$ is 0.9945.