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Citation: AIP Conf. Proc. 1511, 1578 (2013); doi: 10.1063/1.4789230
View online: http://dx.doi.org/10.1063/1.4789230
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1511&Issue=1
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AN INTERDIGITAL CAPACITIVE SENSOR FOR QUANTITATIVE CHARACTERIZATION OF WIRE INSULATION

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ABSTRACT. An interdigital capacitive sensor has been developed to characterize the permittivity of wire insulation. A theoretical model has been developed to calculate the capacitance of such a sensor that is in intimate contact with the surface of a double-layer cylindrical dielectric surrounding a conductive core. The cylindrical form of the electrostatic Green’s function due to an exterior point source is utilized, with the final capacitance value being calculated using the Method of Moments. Example calculations are performed and a field optimization method is developed to improve the sensing efficiency of the electrodes.

Keywords: Interdigital Electrodes, Dielectric Materials, Cylinders, Conductors, Capacitance Measurement, Permittivity Measurement, Green’s Function, Numerical Modeling

PACS: 41.20.Cv, 02.70.-c, 77.22.Ch, 84.37.+q

INTRODUCTION

All insulated wires are subject to a variety of environmental exposures, such as moisture, extreme temperatures, and mechanical stress. Failure of these wires in critical systems, such as aircraft and nuclear reactors, as a result of insulation degradation can be potentially hazardous and even fatal as these wires can transmit power, navigation and control signals. Although several wire testing instruments are commercially available, most are designed to seek “hard” faults in the wire conductor and others can test the insulation only with access to the central conductor.

Capacitive sensing is ideally suited for characterizing dielectric materials due to the linear relationship with the relative permittivity (dielectric constant) of the material, there being no need to access the central conductor of the wire, and independence from the applied voltage. For test-pieces that are of cylindrical geometry, a numerical calculation method was developed in [1] that utilizes cylindrical Green’s functions and the Method of Moments to solve for the electrode charge distribution and, hence, the total capacitance. The electrodes themselves consist of two curved patch electrodes that conform to the surface of an infinitely-long dielectric rod. A central conductor is added to the dielectric rod in [2] to create a geometry that most closely resembles that of a real wire. This simple electrode pair can be expanded to an array in which many parallel electrode fingers interdigitate with one another. This structure has the advantage being less susceptible to external noise while also greatly increasing the capacitance. In this paper, the numerical model in [2] is extended to treat two dielectric layers and interdigital sensors. A method
for optimizing the sensor penetration depth is presented and benchmark experimental results, to be published in a future paper, have shown mean agreement to within 5.1% of calculations made with the numerical model.

MODELING

Figure 1 shows a perspective view of the configuration of the capacitive sensor. As this sensor is designed to be affixed to two separate faces of a clamp that conforms to opposite sides of a wire, the individual electrode digits are periodically spaced only around a half circumference of the wire and exist on the other half circumference in mirrored positions yet opposing polarity. Each half of the sensor is composed of $N_E$ positive or negative digits enclosed by $N_E + 1$ negative or positive digits, guaranteeing an odd number of digits per half of the sensor and an even number of total digits, defined, therefore, as $N_{ET} = 4N_E + 2$. For one half of the sensor, each electrode has an arc-width $w$ and arc-spacing $s$ in the $\phi$-direction, and length $l$ in the $z$-direction. There are, therefore, two gaps $g$ that occur between the end digits of each half of the sensor, which represent the discontinuity in full circumferential periodicity of the digits that is likely to occur in the actual sensing clamp, and is given by $g = \pi c - w - 2N_E(s + w)$. To charge each of the electrodes in laboratory experiments, a thin 0.1 mm power bus connects all positive electrodes together and all negative electrodes together. The power bus is thinner than any of the digits and separated from them by spacing somewhat greater than the interdigital spacing $s$, in order to reduce any stray capacitance contribution to the overall measurement.

In the theoretical model, the wire is modeled as a cylinder that is infinitely long and comprised of a central perfect conductor of radius $a$ and two surrounding dielectric layers of radii $b$ and $c$, and permittivities $\varepsilon_1$ and $\varepsilon_2$, as shown in Fig. 2. This cylinder is embedded in an infinite region of permittivity $\varepsilon_3$. In laboratory experiments, the two dielectric layers are used to model a single insulation layer of a wire enclosed by the substrate layer that supports the electrodes, but in practice wires have multiple insulation layers that could be modeled using this method if the substrate layer contribution can be ignored.
Knowledge of the electrostatic potential $\Psi$ at a point $(\rho, \phi, z)$ due to a point charge placed at $(\rho', \phi', z')$ is required to calculate the capacitance of the sensor. The potential is then related to the volume charge density $\rho_v$ by Poisson’s equation [3]:

$$\nabla^2 \Psi = -\rho_v / \varepsilon,$$

where, for electrodes on a cylindrical surface, $\rho_v$ will reduce to a surface charge density $\sigma$. In order to find a suitable solution to this inhomogeneous partial differential equation, the electrostatic Green’s function $G$ at a point $(\rho, \phi, z)$ due to a point charge placed at $(\rho', \phi', z')$ must be derived and employed, and is found by solving the following Poisson equation:

$$\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] G(\rho, \phi, z) = -\frac{1}{\rho} \delta(\rho - \rho') \delta(\phi - \phi') \delta(z - z').$$

The general solution to this equation for both source and observation points exterior to the cylinder is derived in [2] and is of the form:

$$G(r|r') = \frac{1}{2\pi^2} \left\{ \int_0^\infty \tilde{G}_0(\rho, \rho', \kappa) \cos[\kappa(z - z')] d\kappa ight. + \left. 2 \sum_{i=1}^\infty \cos[i(\phi - \phi')] \int_0^\infty \tilde{G}_i(\rho, \rho', \kappa) \cos[\kappa(z - z')] d\kappa \right\}.$$

For a central conductor that is perfectly conducting and surrounded by two concentric dielectric layers, the Green’s function kernel $\tilde{G}_t$ in each region has the general form:

$$\tilde{G}_t^{(1)}(\rho, \rho', \kappa) = A_t(\kappa)[I_t(\kappa \rho)K_t(\kappa \rho') - \alpha_t(\kappa)K_t(\kappa \rho)K_t(\kappa \rho')],$$
$$\tilde{G}_t^{(2)}(\rho, \rho', \kappa) = B_t(\kappa)I_t(\kappa \rho)K_t(\kappa \rho') - \alpha_t(\kappa)C_t(\kappa)K_t(\kappa \rho)K_t(\kappa \rho'),$$
$$\tilde{G}_t^{(3)}(\rho, \rho', \kappa) = I_t(\kappa \rho)K_t(\kappa \rho') - \alpha_t(\kappa)D_t(\kappa)K_t(\kappa \rho)K_t(\kappa \rho').$$
where the superscripts (1), (2), and (3) denote regions $a < \rho < b$, $b < \rho < c$ and $\rho > c$, respectively,

$$
\alpha_t(\kappa) = I_t(\kappa a)/K_t(\kappa a),
$$

and $I_t(\kappa \rho)$ and $K_t(\kappa \rho)$ are the modified Bessel functions of the first and second kind, respectively, of order $t$. Each of these kernels are composed of two terms that represent incident and reflected fields due to a point source external to the cylinder, with $C_t(\kappa)$ and $D_t(\kappa)$ acting as reflection coefficients and $A_t(\kappa)$ and $B_t(\kappa)$ acting as transmission coefficients. These four coefficients can be obtained by applying four interface conditions:

$$
\begin{align*}
\tilde{G}^{(3)}_{t}(\rho = c) &= G^{(2)}_{t}(\rho = c), \\
\epsilon_3 \frac{\partial}{\partial \rho} \tilde{G}^{(3)}_{t}(\rho = c) &= \epsilon_2 \frac{\partial}{\partial \rho} G^{(2)}_{t}(\rho = c), \\
\tilde{G}^{(2)}_{t}(\rho = b) &= G^{(1)}_{t}(\rho = b), \\
\epsilon_2 \frac{\partial}{\partial \rho} \tilde{G}^{(2)}_{t}(\rho = b) &= \epsilon_1 \frac{\partial}{\partial \rho} G^{(1)}_{t}(\rho = b).
\end{align*}
$$

For an arrangement of electrodes on the surface $\rho = c$, the source and observation points exist solely on this surface and are considered to be in region 3. As such, the only coefficient from region 3 that needs to be utilized in Equation (3) is $D_t(\kappa)$, which, by solving for the boundary conditions, is given as:

$$
D_t(\kappa) = \frac{1}{\alpha_t(\kappa)} \frac{\epsilon_1 I'_t(\kappa c) D_{t1}(\kappa) + \epsilon_2 I'_t(\kappa c) D_{t2}(\kappa)}{\epsilon_1 K'_t(\kappa c) D_{t1}(\kappa) + \epsilon_2 K'_t(\kappa c) D_{t2}(\kappa)},
$$

where

$$
\begin{align*}
D_{t1}(\kappa) &= \epsilon_1 [I_t(\kappa c) K'_t(\kappa b) - I'_t(\kappa c) K_t(\kappa c)] [I'_t(\kappa b) - \alpha_t(\kappa) K'_t(\kappa b)] \\
&\quad - \epsilon_2 [I_t(\kappa c) K'_t(\kappa b) - I'_t(\kappa b) K_t(\kappa c)] [I_t(\kappa b) - \alpha_t(\kappa) K_t(\kappa b)], \\
D_{t2}(\kappa) &= \epsilon_2 [I'_t(\kappa c) K'_t(\kappa b) - I'_t(\kappa b) K'_t(\kappa c)] [I_t(\kappa b) - \alpha_t(\kappa) K'_t(\kappa b)] \\
&\quad - \epsilon_1 [I'_t(\kappa c) K_t(\kappa b) - I'_t(\kappa b) K_t(\kappa c)] [I'_t(\kappa b) - \alpha_t(\kappa) K'_t(\kappa b)],
\end{align*}
$$

and $I'_t(\kappa \rho)$ and $K'_t(\kappa \rho)$ are the first derivatives of the modified Bessel functions with respect to $\rho$.

**NUMERICAL IMPLEMENTATION**

The total capacitance $C$ between the electrodes is calculated in the following sequence. The Green’s function for any pair of source and observation points on the outer dielectric surface $\rho = c$ derived in the previous section is used to set up the integral equations relating the Green’s function and surface charge density with the potential in the Method of Moments calculations. Because of the axisymmetry of the electrode arrangement, only the Green’s functions for positive-positive and positive-negative source-observation point pairs, or vice versa, need be calculated. Therefore, the surface charge density requires calculation on either the positive or negative electrodes only. The total capacitance $C$ is then calculated from

$$
C = Q/V,
$$
FIGURE 3. Calculated sensor capacitance as a function of insulation permittivity. The default sensor configuration is $w = s = 0.1$ mm, $l = 25.4$ mm, and $N_{ET} = 6$ unless otherwise indicated in the legend.

where the total charge $Q$ on either the positive or negative electrodes is obtained by integrating the surface charge density over each electrode surface, and $V$ is the potential difference, or voltage, between the electrodes [2].

EXAMPLE CALCULATIONS

The dependence of the sensor capacitance on the various electrode configuration parameters is investigated as follows. In Fig. 3, the output capacitance is plotted as a function of the insulation relative permittivity $\varepsilon_{r1}$ where different electrode configurations are considered. The wire is modeled as a cylinder with $a = 0.4953$ mm, $b = 1.0795$ mm and $c = 1.1049$ mm with a substrate relative permittivity value of $\varepsilon_{r2} = 2.84$. The default electrode configuration is $w = s = 0.1$ mm, $l = 25.4$ mm and $N_{ET} = 6$. For any of the electrode configurations given in Fig. 3, a linear relationship between the capacitance and the permittivity is clearly observed. This slope, $\Delta C/\Delta \varepsilon$, can be viewed as the sensitivity of the capacitive sensor to changes in the insulation permittivity and is listed, for each configuration, in Table 1. It should be noted that this definition of sensitivity has units of length, which is related to the ratio of the area of the electrodes to the distance between them. Increasing the length $l$ of the electrodes by a factor of two doubles the output capacitance and the sensitivity due to the doubling of the total charge. Increasing the electrode spacing $s$ by a factor of two decreases the output capacitance as would be expected, but the sensitivity decreases only slightly, due to the effect of the penetration depth that will be discussed in the next section. Increasing the electrode width $w$ by a factor of two also doubles the total area of the electrodes allowing more charge to be stored, but the dimensions of the interelectrode region remain the same and so the capacitance increases only slightly but with higher sensitivity. Adding four more electrodes increases the total electrode area and sensing area between the electrodes, which increases the total capacitance and sensitivity.
TABLE 1. Sensitivity values of the electrode configurations given in Fig. 3. The default sensor configuration is \( w = s = 0.1 \text{ mm}, \ l = 25.4 \text{ mm}, \) and \( N_{ET} = 6 \) unless otherwise indicated in the “Parameter” column.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \Delta C / \Delta \varepsilon_1 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>28.49</td>
</tr>
<tr>
<td>( l = 50.8 \text{ mm} )</td>
<td>57.32</td>
</tr>
<tr>
<td>( s = 0.2 \text{ mm} )</td>
<td>28.40</td>
</tr>
<tr>
<td>( w = 0.2 \text{ mm} )</td>
<td>40.82</td>
</tr>
<tr>
<td>( N_{ET} = 10 )</td>
<td>47.79</td>
</tr>
</tbody>
</table>

FIELD PENETRATION OPTIMIZATION

Penetration depth is an important sensor characteristic when localized flaws could occur at various depths or layers within the insulation of a wire. Also, because the electric field generated by the interdigital sensor fringes between oppositely-charged electrodes and couples perpendicularly to the central conductor, the goal is to design the most efficient sensor that balances both of these effects. In the interaction of a dynamic electric field with a lossy material, such as a real conductor, penetration depth is defined as the distance into the conductor at which the magnitude of a transmitted electromagnetic wave falls to \( 1/e \) (about 37%) of its value at the material boundary. In an electrostatic or electroquasistatic interaction with a low-loss dielectric, however, penetration depth must be defined differently. In prior work [4], the penetration depth for a coplanar capacitive sensor (designed for characterizing flat, laminar dielectrics) is defined as the thickness of the dielectric material for which the measured capacitance has experienced a 10% change from the constant value \( C_0 \) measured on a half-space of the same material. In this case of a test-piece with cylindrical geometry and central conductor, a similar definition of penetration depth \( \delta \) can be made as

\[
\delta = b - a_{10}
\]

where, referring to Fig. 2, \( a_{10} \) is the radius of the central conductor for which the capacitance of the sensor has increased by 10% compared with that for \( a = 0 \), i.e. for the case in which there is no central conductor. Essentially, \( \delta \) represents the theoretical thickness of the wire insulation by which the capacitance has increased by 10% from that for \( a = 0 \).

Here, results are presented of numerical simulations for the theoretical wire described in the previous section, but in which the radius of the central conductor was varied from 0 to \( c \) while holding all other parameters constant. Because the penetration depth of the interdigital configuration depends on the separation between the electrodes, calculations were also conducted for five different values of \( s \) and \( N_{ET} \). The results of this simulation are shown in Fig. 4.

Considering the data plotted in Fig. 4 it can be seen that, as \( a \) vanishes, the total capacitance approaches a constant value \( C_0 \), which is to be expected since the electric field no longer couples with the central conductor, leaving only the fringing field between neighboring digits. When \( a \rightarrow c \), the capacitance tends toward infinity, which is characteristic of the inverse distance relationship in a parallel plate capacitor. In essence, \( a_{10} \) represents one point of balance between the fringing and parallel fields, and an efficient sensor may be designed to have \( \delta = t = b - a \), where \( t \) is the thickness of the wire insulation.
FIGURE 4. Calculated $C$ for sensors with several different total digits and electrode spacing as a function of conductor radius $a$. In this case, $b = 1.0795$ mm, $c = 1.1049$ mm, $\varepsilon_{r1} = 4.015$, $\varepsilon_{r2} = 2.84$, and $w = 0.1$ mm. The point at which the capacitance has increased by 10% from $C_0$ is indicated by *.

TABLE 2. Results of the penetration depth simulation in Figure 4.

<table>
<thead>
<tr>
<th>$N_{ET}$</th>
<th>$C_0$ (pF)</th>
<th>$C_{10}$ (pF)</th>
<th>$a_{10}/c$</th>
<th>$a_{10}$ (mm)</th>
<th>$\delta$ (mm)</th>
<th>$\delta/t$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.56</td>
<td>1.72</td>
<td>0.73</td>
<td>0.81</td>
<td>0.27</td>
<td>47</td>
</tr>
<tr>
<td>10</td>
<td>2.99</td>
<td>3.29</td>
<td>0.81</td>
<td>0.89</td>
<td>0.18</td>
<td>32</td>
</tr>
<tr>
<td>14</td>
<td>4.63</td>
<td>5.10</td>
<td>0.85</td>
<td>0.94</td>
<td>0.14</td>
<td>24</td>
</tr>
<tr>
<td>18</td>
<td>6.76</td>
<td>7.44</td>
<td>0.88</td>
<td>0.97</td>
<td>0.11</td>
<td>18</td>
</tr>
<tr>
<td>30</td>
<td>14.0</td>
<td>15.4</td>
<td>0.94</td>
<td>1.0</td>
<td>0.06</td>
<td>11</td>
</tr>
</tbody>
</table>

The data from Fig. 4 are provided in detail in Table 2. It can be observed from this data that $a_{10} \rightarrow c$ as the spacing between the digits decreases, meaning that the penetration depth also decreases. This is to be expected since decreasing electrode spacing will result in a greater concentration of the electric field nearer to the surface coplanar with the electrodes. From Table 2 it can also be seen from the penetration efficiency $\delta/t$ that the most efficient sensor at penetrating the wire insulation is the one with the greatest electrode spacing, but this occurs with a significantly reduced output capacitance. Conversely, the sensor with the greatest output capacitance displays the lowest penetration efficiency. Ultimately, the chosen sensor design should balance maximum capacitance and penetration efficiency, guided by the particular inspection need.

CONCLUSION AND FUTURE WORK

A quasielectrostatic numerical model, based on the Green’s function due to a point source exterior to an infinitely long cylindrical conductive cylinder coated with two different dielectric materials, has been developed to quantitatively evaluate the permittivity of cylindrical test-pieces via the measured capacitance of interdigital electrodes that conform to the wire insulation surface. This model has been validated by large-scale benchmark capacitive measurements which show agreement with the model to within 5%, the results of which are to be published in a future paper. Clamps with integrated interdigital sensors are currently being developed that will be used to nondestructively evaluate wire insulation in critical structures, primarily aircraft. The electrodes for these clamp fixtures have been designed to optimize the electric field penetration depth, in
accordance with the method described in this paper, to improve sensor accuracy and efficiency for measuring degradation in wire insulation as indicated by changes in the complex permittivity of the insulation material.

ACKNOWLEDGMENTS

This work is supported by The Boeing Company under contract No. 476184 at Iowa State University’s Center for Nondestructive Evaluation.

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