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# Natural Selection of Asphalt Mix Stiffness Predictive Models with Genetic Programming

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**ABSTRACT**
Genetic Programming (GP) is a systematic, domain-independent evolutionary computation technique that stochastically evolves populations of computer programs to perform a user-defined task. Similar to Genetic Algorithms (GA) which evolves a population of individuals to better ones, GP iteratively transforms a population of computer programs into a new generation of programs by applying biologically inspired operations such as crossover, mutation, etc. In this paper, a population of Hot-Mix Asphalt (HMA) dynamic modulus stiffness prediction models is genetically evolved to better ones by applying the principles of genetic programming. The HMA dynamic modulus ($|E^*|$), one of the stiffness measures, is the primary HMA material property input in the new Mechanistic Empirical Pavement Design Guide (MEPDG) developed under National Cooperative Highway Research Program (NCHRP) 1-37A (2004) for the American State Highway and Transportation Officials (AASHTO). It is shown that the evolved HMA model through GP is reasonably compact and contains both linear terms and low-order non-linear transformations of input variables for simplification.

## Introduction

Artificial intelligence (AI) techniques are promising alternatives to conventional statistical (e.g. multivariate regression analysis) or mathematical (e.g. differential equation) approaches to system modeling with complex and uncertain conditions. In recent years, they have been successfully applied in different areas of science, engineering, medicine, etc.

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One of the broadest subfields in AI is the machine learning (ML) method which focuses on the development of data modeling techniques and algorithms that learn from data. Genetic programming (GP), introduced by Koza (1992), is one of the evolutionary computation (EC) techniques that employ Darwin's natural selection theory of evolution to solve complex engineering problems through computers. The early major types of EC include Genetic Algorithms (GA) (Holland, 1975), Evolutionary Programming (EP) (Fogel et al., 1966) and Evolutionary Strategy (ES) (Schwefel, 1981). GP is the extended variant of GA. However, GP optimizes functional relation (functional set) of models with best model coefficients (terminal set) while GA searches the best value for a given set of model parameters (Khu et al., 2001; Rezania and Javadi, 2007). The other major difference is that GP has flexibility in length of solution resulting in increase of search space (Khu et al., 2001).

The asphalt concrete mixture, or hot mix asphalt (HMA) is a composite material consisting of aggregate, sand, and filler, bound by asphalt binder. The HMA mechanical behavior is affected by individual component properties but shows very different response with respect to individual component responses. As a result, the prediction of HMA mechanical properties involves a high degree of complexity and uncertainty. The stiffness of HMA is an important mechanical property used in determining pavement response and performance under loading. The HMA dynamic modulus ($|E^*|$), one of the stiffness measures, is the primary HMA material property input in the new Mechanistic Empirical Pavement Design Guide (MEPDG) developed under National Cooperative Highway Research Program (NCHRP) 1-37A (2004) for the American State Highway and Transportation Officials (AASHTO).

The MEPDG currently employs a purely statistical regression model, namely Witczak $|E^*|$ predictive model developed in 1999 (Andrei et. al., 1999). This Witczak $|E^*|$ model was based on conventional multivariate regression analysis of laboratory test data. The researchers at Iowa State University (ISU) (Ceylan et al., 2007; Ceylan et al., 2008; Ceylan et al., 2009) are the first to introduce AI techniques in developing $|E^*|$ predictive models. The next-generation predictive $|E^*|$ models developed at ISU are based on backpropagation neural networks (BPNN) approach and were found to be more accurate compared to existing multivariate regression based model (Ceylan et al., 2007; Ceylan et al., 2008; Ceylan et al., 2009).

In Genetic Symbolic Regression (GSR), a special application of GP in the area of symbolic regression, the goal is to find a mathematical expression in symbolic form to provide an optimal fit between values of the independent variable and their counterparts of the dependent variable (Koza, 1992). A big advantage of developing GP-based $|E^*|$ prediction models over the BPNN approach is that the end product is a mathematical equation which can be physically understood and more easily applied by practitioners. The primary objective of this study is to explore the feasibility of employing GP to develop HMA stiffness predictive models. The development and performance of GP based $|E^*|$ predictive models are discussed in the following sections.

## Brief Overview of GP Algorithm

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Figure 1 illustrates a simplified procedure of GP for explanation. The GP procedure randomly generates the initial population of functions and terminals from a given data set. The GP can represent an algebraic expression of individual combination of functions and terminals as a parse tree composed of nodes. The nodes are elements from either functional sets or terminals sets. The performance of each individual combination of functions and terminals in population is evaluated under given criteria. The combinations with better performance among evaluated ones can have the greater probabilities of opportunity in matching and producing new individual combinations called as offspring. This selection procedure is inspired by Darwinian principle of the "survival of the fittest." Two genetic operations involved in producing offspring are crossover and mutation. Crossover interchanges substructures of each selected combinations to produce offspring while mutation is the random alteration of the individual combination at the node or branch level. New individual combinations generated by crossover and mutation in Figure 1 can be introduced into new population pool. The GP operation process described here is repeated until the given criteria are met.

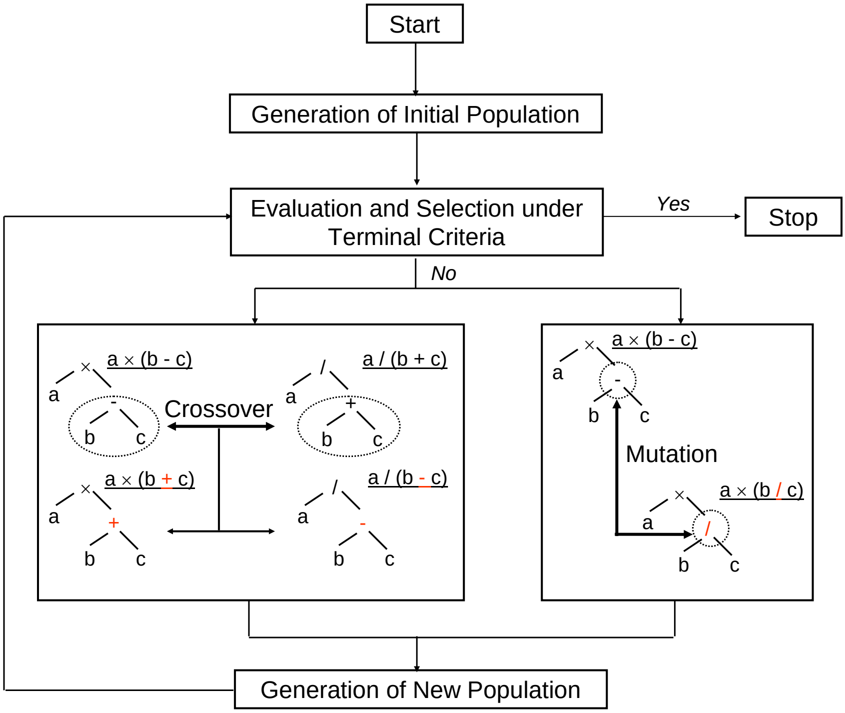


**Figure 1. Simplified genetic programming schematic**

## Genetic Evolution of |E*| Prediction Models

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Data used in this study were retrieved from the National Cooperative Highway Research Program (NCHRP) Report 567 DVD (*CRP-CD - 46*) "Simple Performance Tests: Summary of Recommended Methods and Database."(Witczak, 2005). The *CRP-CD-46* included as an appendix in the NCHRP report 567 contains a total of 7,400 data records from 346 HMA mixtures (Bari and Witczak, 2006). The new pavement design guide software in US, namely the MEPDG software, employs Witczak $|E^*|$ predictive model developed in 1999 as one of the user options depending on the availability of input parameter data.

The eight input parameters for the 1999 version of Witczak $|E^*|$ model include:

(1) *x1* - percent of aggregates retained 19-mm sieve ($\rho_{19mm}$), %
(2) *x2* - percent of aggregates retained 9.5-mm sieve ($\rho_{9.5mm}$), %
(3) *x3* - percent of aggregates retained #4 sieve ($\rho_{\#4}$), %
(4) *x4* - percent of aggregates passing #200 sieve ($\rho_{\#200}$), %
(5) *x5* - air void ($V_a$), %
(6) *x6* - effective binder content ($V_{beff}$), %
(7) *x7* - *log*(viscosity of the asphalt binder ($\eta$)), poise
(8) *x8* - loading frequency (f), Hertz

The eight input parameters of the Witczak $|E^*|$ predictive model were used in the development of GP-based models with one output variable, *log* $|E^*|$ in psi. GPTIPS (Searson, 2009), a MATLAB toolbox for performing multigene symbolic regression, was adopted in this study to develop GP-based $|E^*|$ prediction models. The data were divided randomly into three different subsets: the training data subset containing 6,800 data vectors, the validation data subset containing 100 data vectors, and the testing data subset which consisted of 500 data vectors.

The GPTIPS parameter settings include population size (*p*), number of generations (*g*), optimization type (minimization or maximization), natural selection method options (tournament, elitism, etc.), tree depth (*t*), maximum number of genes per individual (*mg*), and active function nodes ('plus', 'minus', 'tanh', 'exp', etc.). The plain lexicographic tournament selection proposed by Luke and Panait (2002) was always used and the tournament size was set to 10 as recommended by Searson (2009). There is also an option to use a 'holdout' validation set during training to minimize the effects of overfitting which was used in this study.

Since this is an exploratory study, few different GP parametric configurations were initially evaluated which resulted in prediction models with different accuracies as listed in Table 1. To maximize prediction accuracy, both the input and output data were scaled to zero mean and unit variance.

The *goodness-of-fit* statistics for the GP model predictions in arithmetic scale were performed using statistical parameters such as the correlation coefficient ($R^2$ ), the standard error of predicted values divided by the standard deviation of measured values ($S_e/S_y$). The $R^2$ is a measure of correlation between the predicted and the measured values and therefore, determines accuracy of the fitting model (higher $R^2$ equates to higher accuracy). The $S_e/S_y$ indicates the relative improvement in accuracy and thus a smaller value is indicative of better accuracy.

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**Table 1. Summary of GP $|E^*|$ predictive models**

| Case | GP Parameter Configuration | GP $\|E^*\|$ Prediction Model | Accuracy |
|---|---|---|---|
| 1 | $p$ = 450, $g$ = 150, $t$ = 4, $mg$ = 5 | (a) $y = 1.009\,x_7 - 0.0742\,x_5 - 0.0399\,\mathrm{plog}(x_5\,x_7\,(x_7 + x_8)) + 0.1774\,\mathrm{plog}(\mathrm{plog}(x_8 + 1.827)) - 0.07923\,\mathrm{plog}(\mathrm{plog}(1.896 - x_3)) - 0.0742\,\mathrm{plog}(x_2) - 0.0742\,x_7\,x_8 - 0.2261\,{x_7}^2 + 0.3309$ | $R^2$ = 0.79; $S_e/S_y$ = 0.46 |
| | | (b) $y = 0.4364\,\mathrm{plog}(x_8 + \tanh(x_7) + 1.941) - 0.07545\,\tanh(x_6 - x_7) - 0.0009606\,e^{e^{x_3}} - 0.099\,x_5 - \frac{0.9407}{e^{\tanh(x_7)}} + 0.099\,e^{x_3} - 0.07545\,\mathrm{plog}(x_2) - 0.099\,\mathrm{plog}(x_8) - 0.07545\,\tanh(x_3) + 0.8419$ | $R^2$ = 0.79; $S_e/S_y$ = 0.45 |
| 2 | $p$ = 500, $g$ = 200, $t$ = 6, $mg$ = 4 | (a) $y = 0.0309\,x_4 - 0.05522\,x_1 - 0.09354\,x_5 - 0.8224\,x_7 - 0.05522\,x_8 + 0.0309\,e^{\mathrm{plog}(x_8)} - 0.4112\,e^{\mathrm{plog}(x_7)} - 0.4112\,e^{\mathrm{plog}(x_3) - 2.702} - 0.0309\,\mathrm{plog}(\mathrm{plog}(x_4)) - 0.09354\,e^{\mathrm{plog}(x_7 - 2.672) - x_8} - 0.05522\,\mathrm{plog}(x_2 + x_5) + 0.0309\,e^{x_2} - 0.09354\,e^{x_7} + 0.0309\,\mathrm{plog}(x_1) - 0.05522\,\mathrm{plog}(x_3) - 0.0309\,\mathrm{plog}(x_4) - 0.05522\,e^{e^{\mathrm{plog}(x_8)} - x_8} - 0.09354\,\mathrm{plog}(x_7 - 2.672) - 0.0618\,x_7\,\mathrm{plog}(-2.737)\,(\mathrm{plog}(x_7) - 28.87) + 0.9616$ | $R^2$ = 0.81; $S_e/S_y$ = 0.44 |
| | | (b) $y = 5.422\,\tanh(\tanh(\tanh(e^{x_7}))) + 3.858\,\tanh(x_8 + \tanh(x_2 + 3.142) + \tanh(x_2 + 3.249)) - 0.006585\,(\tanh(e^{x_7}) + \tanh(x_2 + 2.868))\,(x_3 + 2\,x_5 + x_8 + 6.848)\,(x_1 + x_5 - 2\,x_7 - x_8 + 0.1292) - 0.2369\,\tanh(x_8 + e^{x_7})\,e^{\tanh(x_7)}\,\tanh(x_8) - 6.179$ | $R^2$ = 0.81; $S_e/S_y$ = 0.44 |

Compared to the predictive accuracy of existing multivariate 1999 Witczak $|E^*|$ predictive model ($R^2$ = 0.73; $S_e/S_y$ = 0.52), all GP-based multigene symbolic regression $|E^*|$ predictive models show better performance, with Case 2 showing the highest accuracy. The output, $y$, corresponds to log $|E^*|$ in these equations whereas the reported predictive accuracies are for $|E^*|$ values. Generally, with higher population and higher user-defined tree depth, more predictive accuracy is achieved, but at the cost of model complexity. Since this is an illustrative study, results for few specific cases are presented for demonstrating the successful implementation of the concept. In the GP-based final regression equations presented in Table 1, *plog* refers to protected natural log (plog(x) = ln(|x|)) and *tanh* refers to hyperbolic tangent.

**Results and Discussions**

Results are first graphically presented for Case 2b (last row in Table 1). The best fitness and mean fitness values over the course of the run is shown in Figure 2. Figure 3 displays the population of evolved models in terms of their complexity (number of nodes) as well as their fitness. Figure 3 can be used to identify symbolic models that perform reasonably well and at the same time are much less complex than the "best" model in the population highlighted in red. The green circles represent the pareto-optimal models in the population which refer to models that are not strongly dominated by other models in the whole population both in terms of fitness and complexity. In Figure 4, GP model predictions are shown for the training, testing, and validation set using Case 2b model. Note that the $y$ values correspond to log $|E^*|$ values in these plots.

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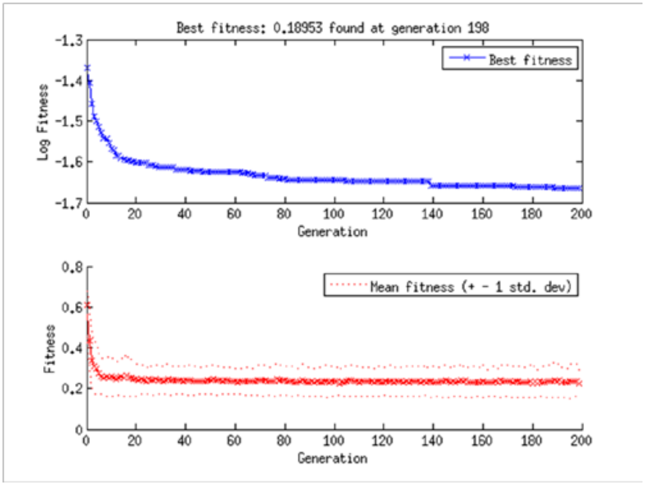


**Figure 2. Fitness values during genetic evolution of Case 2b model**

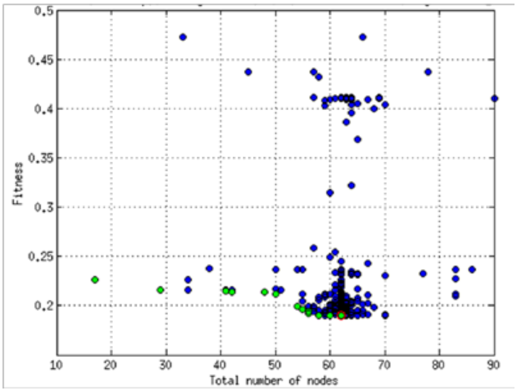


**Figure 3. Population of evolved models for Case 2b**

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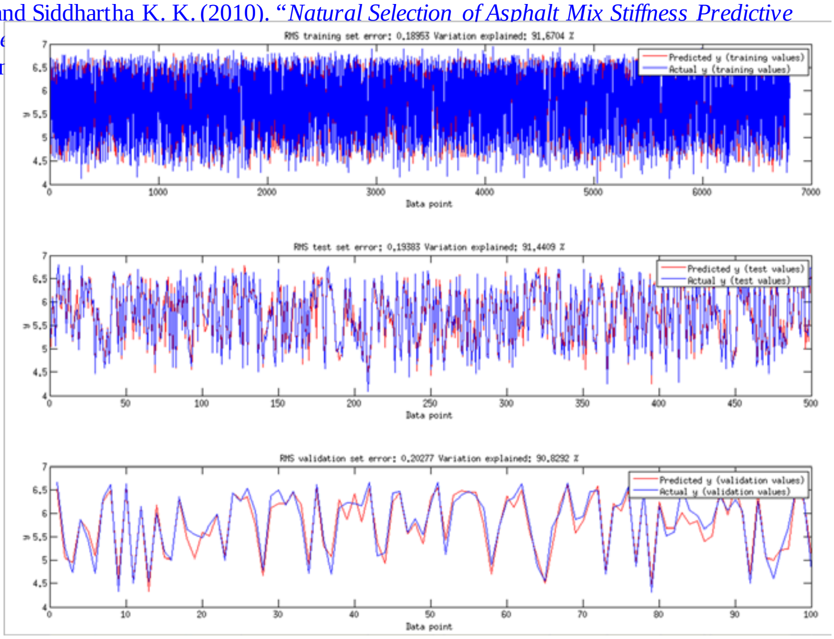


**Figure 4. Best performance GP (Case 2b) model predictions**

The $|E^*|$ prediction accuracies are compared between the existing Witczak and developed GP (Case 2b) models in Figure 5 for the 500 testing data points. As mentioned previously, the 500 test vectors form an independent dataset which was not used in training the GP and it was used to test the accuracy of the trained GP. The final GP $|E^*|$ regression model for which the results are discussed here is as follows:

$$y = 5.422\ \tanh(\tanh(\tanh(e^{x_7}))) + 3.858\ \tanh(x_8 + \tanh(x_2 + 3.142) + \tanh(x_2 + 3.249)) - 0.006585\ (\tanh(e^{x_7}) + \tanh(x_2 + 2.868))\ (x_3 + 2\,x_5 + x_8 + 6.848)\ (x_1 + x_5 - 2\,x_7 - x_8 + 0.1292) - 0.2369\ \tanh(x_8 + e^{x_7})\ e^{\tanh(x_7)}\ \tanh(x_8) - 6.179$$

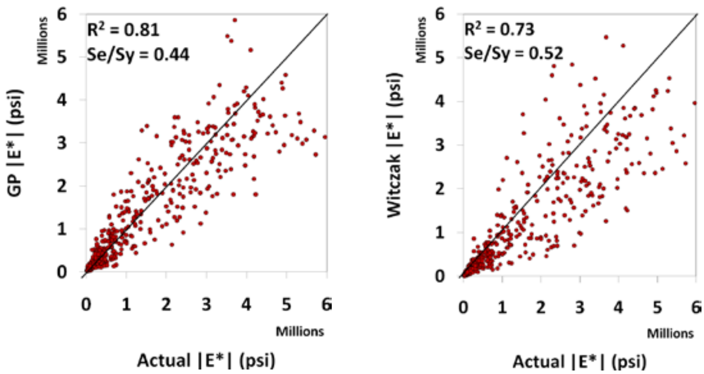


**Figure 5. Predicted versus observed $|E^*|$ for Witczak and GP models**

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## Conclusions

Genetic Programming (GP) is a systematic, domain-independent evolutionary computation technique that stochastically evolves populations of computer programs to perform a user-defined task. This paper explored the feasibility of applying Genetic Programming (GP) technique for developing hot mix asphalt (HMA) dynamic modulus ($|E^*|$) predictive models. GP-based prediction models were developed using the latest comprehensive $|E^*|$ database that is available to the researchers containing 7,400 data points from 346 HMA mixtures. GP models showed significantly better performance compared to existing multivariate regression-based Witczak model for $|E^*|$ prediction although they are not as accurate as BPNN based models developed by the authors. A big advantage of developing GP-based $|E^*|$ prediction models over the BPNN approach is that the end product is a mathematical equation which can be physically understood and more easily applied by practitioners. Future research efforts will focus on determination of optimal GP model configuration to improve the performance of GP based $|E^*|$ prediction models.

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