ULTRASONIC DETERMINATION OF THE ELASTIC PROPERTIES OF UNIDIRECTIONAL COMPOSITES

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INTRODUCTION

In recent years, extensive efforts have been made in developing ultrasonic nondestructive evaluation (NDE) methods for the characterization of the elastic properties of composite materials [1, 2]. A systematic procedure proposed by Karim, Mal and Bar-Cohen [3] by inverting the leaky Lamb wave (LLW) data has been found to be accurate and effective in characterizing the elastic constants of composites. However, this method can accurately determine the matrix dominated constants c_{22}, c_{23}, and c_{55} only. The fiber dominated constants, c_{11} and c_{12} can not be determined accurately, due to the fact that the Lamb wave velocity is insensitive to c_{11} and c_{12} in the range in which the dispersion data are reliable. In this paper we introduce a new technique which can determine all the material constants by analyzing the times of flight of the reflected acoustic waves in a pulsed LLW experiment. The procedure requires clear separation and identification of the reflected pulses. A generalized ray theory described in [4] is used to identify the modality and ray path of each arrival. The time of flight of each ray is then related to the elastic constants of the composite which are determined from these relations. The method requires access from one side of the specimen and is accurate as well as efficient.

THE ULTRASONIC EXPERIMENT

The experiment is based on an oblique insonification of the specimen immersed in water. The acoustic wave is transmitted from a broadband transducer and the reflected signal is recorded by a second transducer in a pitch-catch arrangement as shown in Fig. 1. A detailed description of the experiment can be found in Bar-Cohen [1] and will be omitted.

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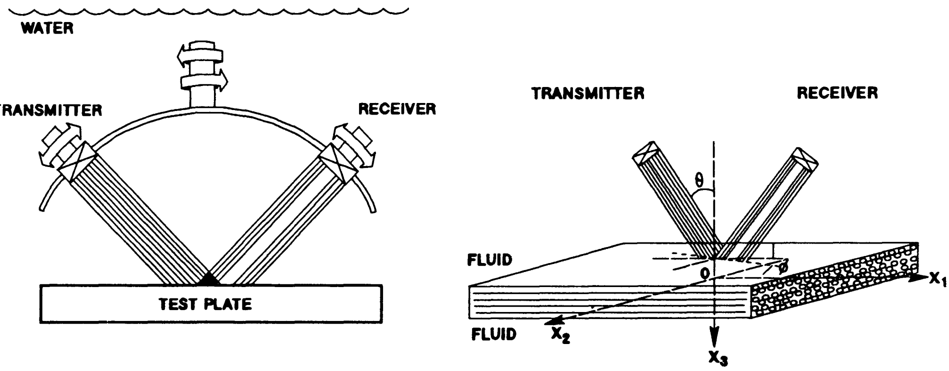

Fig. 1. The experimental setup.

TRAVEL TIMES OF THE REFLECTED RAYS

Consider a unidirectional composite plate, with thickness H and density ρ, immersed in a fluid as shown in Fig. 1. Assume that the material is homogeneous and transversely isotropic with five independent stiffness constants, c_{11}, c_{12}, c_{22}, c_{23}, and c_{55}. The Cauchy's equation of motion for the composite material is

$$\sigma_{ij,j} - \rho \ddot{u}_i = 0 \tag{1}$$

where u_i is the displacement vector and σ_{ij} is the stress tensor. Assume plane wave solutions of (1) in the form

$$u_i = U_i e^{i(k_1x_1 + k_2x_2 + k_3x_3) - i\omega t} \tag{2}$$

where k_1, k_2 and k_3 represent the wave numbers along the x_1, x_2 and x_3 directions, respectively, and ω is the circular frequency. From (1), (2) and the constitutive relation for the material, we obtain the eigenvalue problem

$$\begin{bmatrix} a_2\xi_1^2 + a_5(\xi_2^2 + \zeta^2)-1 & a_3\xi_1\xi_2 & a_3\xi_1\zeta \\ a_3\xi_1\xi_2 & a_5\xi_1^2 + a_1\xi_2^2 + a_4^2\zeta^2-1 & (a_1 - a_4)\xi_2\zeta \\ a_3\xi_1\zeta & (a_1 - a_4)\xi_2\zeta & a_5\xi_1^2 + a_4\xi_2^2 + a_1\zeta^2-1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

where

$$\begin{aligned} &a_1 = c_{22}/\rho, \; a_2 = c_{11}/\rho, \; a_3 = (c_{12} + c_{55})/\rho \\ &a_4 = (c_{22} - c_{23})/2\rho, \; a_5 = c_{55}/\rho \\ &\xi_1 = k_1/\omega, \; \xi_2 = k_2/\omega, \; \zeta = k_3/\omega \end{aligned} \tag{4}$$

In the ultrasonic experiment, ξ_1 and ξ_2 are related to the wave incident angle θ and fiber orientation ϕ (Fig. 1) in the form

$$\xi_1 = \frac{\sin\theta\ \cos\phi}{\alpha_0}, \quad \xi_2 = \frac{\sin\theta\ \sin\phi}{\alpha_0} \tag{5}$$

where α_0 is the acoustic wave speed in water ($\approx$1.485 mm/μs). Then ζ is given by the condition of nontrivial solutions of (3). It can be shown that there are three values of ζ given by

$$\zeta_k = \sqrt{b_k - \xi_2^2} \quad (k = 1, 2, 3) \tag{6}$$

where

$$b_1 = -(\beta/2\alpha) - \sqrt{(\beta/2\alpha)^2 - \gamma/2\alpha}, \quad b_2 = -(\beta/2\alpha) + \sqrt{(\beta/2\alpha)^2 - \gamma/2\alpha}$$

$$b_3 = \frac{1 - a_5\xi_1^2}{a_4} \tag{7}$$

$$\alpha = a_1 a_5, \quad \beta = (a_1 a_2 + a_5^2 - a_3^2)\xi_1^2 - (a_1 + a_5), \quad \gamma = (a_2\xi_1^2 - 1)(a_5\xi_1^2 - 1)$$

The ray diagram for a plane wave transmitted into a unidirectional composite plate is shown in Fig. 2a. Here R^0 indicates the first reflected wave from the top surface of the plate, the rays labeled 1, 2, 3 indicate the three transmitted waves inside the plate in a decreasing order of their speeds, the rays labeled 11, 12, ..., and 33 indicate the waves reflected from the bottom of the plate, and T_1, T_2, T_3 indicate the waves transmitted into the fluid through the bottom of the plate. From Snell's law, the velocities V_k, V_l and the angles θ_k, θ_l in the diagram are related through

$$\frac{\sin\theta}{\alpha_0} = \frac{\sin\theta_k}{V_k} = \frac{\sin\theta_l}{V_l} \tag{8}$$

Two possible ray paths leading to the same point (center) on the receiver are sketched in Fig. 2b. If we denote the difference in the arrival times between rays along paths DO and O′BO by t_{kl}, then t_{kl} can be expressed as

$$t_{kl} = t_k + t_l - t^D \tag{9}$$

where

$$t_k = \frac{H}{V_k \cos\theta_k}, \quad t_l = \frac{H}{V_l \cos\theta_l}, \quad t^D = \frac{H(\tan\theta_k + \tan\theta_l)\sin\theta}{\alpha_0} \tag{10}$$

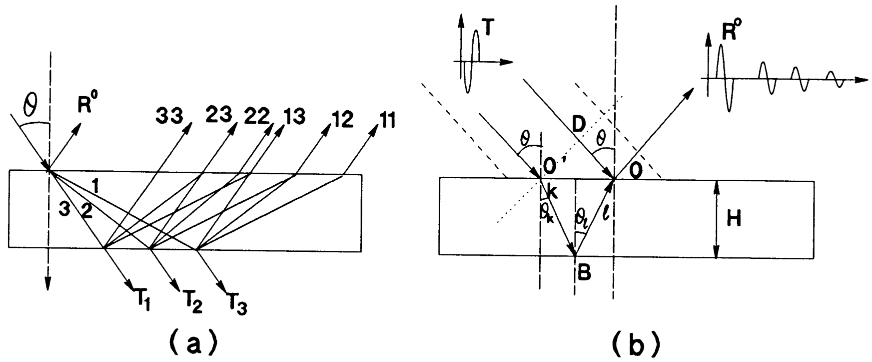

Fig. 2. Ray diagrams of the reflected waves in a unidirectional composite plate.

From (9)-(11), t_{kl} can be expressed as

$$\begin{aligned} t_{kl} &= \frac{H}{V_k \cos\theta_k} + \frac{H}{V_l \cos\theta_l} - \frac{H(\tan\theta_k + \tan\theta_l)\sin\theta}{\alpha_0} \\ &= \frac{H\cos\theta_k}{V_k} + \frac{H\cos\theta_l}{V_l} \\ &= H(\zeta_k + \zeta_l) \end{aligned} \tag{11}$$

It should be noted that equation (11) is valid for homogenous waves only, i.e., when ζ_k, ζ_l are real. In general, there are three possible bulk wave speeds in a composite material, and the recorded time history should contain a reflected pulse from the top surface followed by nine reflected rays from the bottom of the plate and their multiple reflections. However, for a fixed orientation ϕ to the fibers, a certain homogeneous wave will disappear if the incident angle θ is larger than the critical incident angle θ_c. Similarly, for a fixed θ, a certain homogenous wave will disappear for orientation angles ϕ smaller than the critical angle ϕ_c. These phenomena are used in the determination of the elastic constants.

MODEL VERIFICATION

Fig. 3 shows a typical reflected signal from a unidirectional graphite/epoxy specimen for incident angle $\theta = 20°$ and orientation $\phi = 45°$ measured in the ultrasonic experiment. Also shown in this figure is the calculated reflected signal using the theory described in [4]. The stiffness constants used in the calculations are $c_{11} = 160.73$, $c_{12} = 6.44$, $c_{22} = 13.92$, $c_{23} = 6.92$, $c_{55} = 7.07$ (GPa). Material damping was modeled through the use of complex and frequency-dependent moduli as described in [5]. It can be seen that there is excellent agreement between the measured and calculated signals and that at least two pulses reflected from the interior of the plate can be easily identified. Examples of this type of agreement for other unidirectional graphite epoxy specimens can be found in [5].

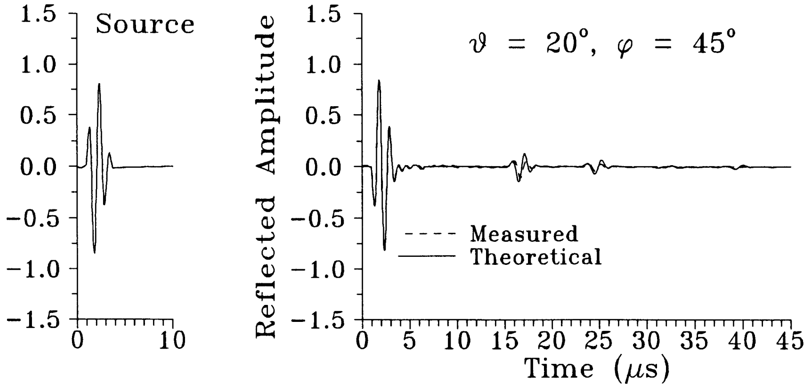

Fig. 3. Measured and calculated reflected signals from a composite laminate.

DETERMINATION OF THE ELASTIC CONSTANTS

We now describe the experiments and the associated formulas that are needed to determine the five stiffness constants with access from one side of the specimen.

a) Pulse-echo experiment

In this case, $\xi_1 = \xi_2 = 0$, so that from (4)-(7)

$$\zeta_1 = \sqrt{\rho/c_{22}}, \quad \zeta_2 = \sqrt{\rho/c_{55}}, \quad \zeta_3 = \sqrt{2\rho/(c_{22} - c_{23})} \tag{12}$$

and the corresponding eigenvectors for ζ_1, ζ_2, and ζ_3 are (0, 0, 1), (0, 1, 0), (1, 0, 0). Since only the longitudinal wave can be transmitted into the fluid from the composite, only the rays associated with ζ_1 exist. Hence the first pulse must be 11, and its arrival time is t_{11}. From (11) and (12),

$$c_{22} = \rho/\zeta_1^2 = 4\rho H^2/t_{11}^2 \tag{13}$$

Thus the pulse-echo experiment provides the constant c_{22}. A simulated result is shown in Fig. 4, where t_{11} = 16.83 μs. Then c_{22} is found to be 13.92 μs, in agreement with the value used in the theoretical calculation.

b) Oblique insonification with ϕ = 90° and incident angle θ > 0°.

In this case,

$$\xi_1 = 0, \; \xi_2 = \sin\theta/\alpha_0, \; b_1 = 1/a_1, \; b_2 = 1/a_5, \; b_3 = 1/a_4 \tag{14}$$

and

$$\zeta_1^2 = -\xi_2^2 + 1/a_1, \; \zeta_2^2 = -\xi_2^2 + 1/a_5, \; \zeta_3^2 = -\xi_2^2 + 1/a_4 \tag{15}$$

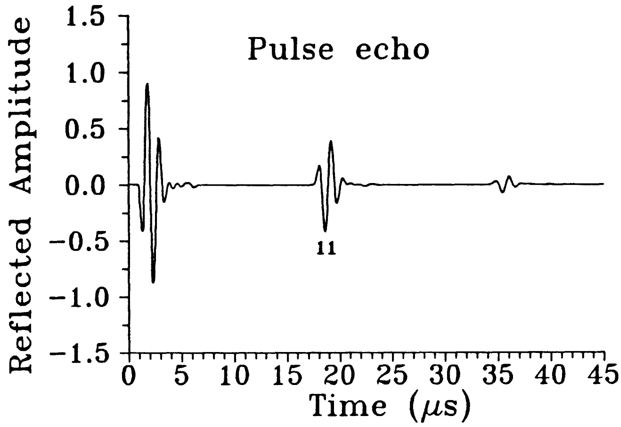

Fig. 4. Reflected signal from a composite laminate in the pulse-echo mode.

It should be noted that the eigenvector associated with ζ_2 is (1, 0, 0), indicating that the particle motion is parallel to the fiber, and this transverse wave can not be transmitted into the fluid. Hence, there is no pulse associated with the corresponding ray path, and the arrived pulses should be in the sequence of "11", "13", and "33". In this experiment, the direction ϕ is kept fixed and the incident angle is increased from 0° until the pulses "11" and "13" can be identified clearly and t_{11} and t_{13} can be measured. The constants c_{22} and c_{23} can be determined from the formulas,

$$c_{22} = \frac{\rho}{\left(\frac{t_{11}}{2H}\right)^2 + \frac{\sin^2\theta}{\alpha_0^2}}, \quad c_{23} = c_{22} - \frac{2\rho}{\left(\frac{t_{13}}{H} - \frac{t_{11}}{2H}\right)^2 + \frac{\sin^2\theta}{\alpha_0^2}} \tag{16}$$

A simulated result for $\theta = 20°$ is shown in Fig. 5, where $t_{11} = 12.28$ μs and $t_{13} = 21.91$ μs. Then c_{22} and c_{23} can be calculated from (16) as 13.92 GPa and 6.92 GPa, respectively.

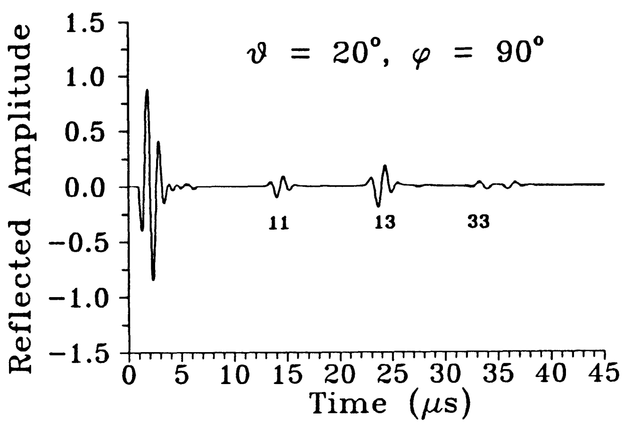

Fig. 5. Reflected signal from a composite laminate for $\theta = 20°$ and $\phi = 90°$.

c) Oblique insonification with ϕ less than the critical angle ϕ_c.

After c_{22} and c_{23} have been determined, the constant c_{55} can be found as follows. With fixed incident angle θ, adjust ϕ such that the pulses "22" and "23" can be identified clearly. Then from measured t_{22}, c_{55} can be determined from the formula,

$$c_{55} = \rho\left\{1 - \frac{(c_{22} - c_{23})}{2\rho\xi_1^2}\left[\left(\frac{t_{23}}{H} - \frac{t_{22}}{2H}\right)^2 + \frac{\sin^2\theta\,\sin^2\phi}{\alpha_0^2}\right]\right\} \tag{17}$$

A simulated result for $\theta = 20°$ and $\phi = 30°$ is shown in Fig. 6, from which $t_{22} = 22.42\ \mu s$, $t_{23} = 14.94\ \mu s$. Hence c_{55} can be determined as 7.07 Gpa.

d) Oblique insonification with $\phi = 0°$

The remaining two unknowns c_{11} and c_{12}, can be determined from this procedure. The time of fight t_{11}, t_{12}, can be identified by changing θ for 0° to an angle less than the first critical angle θ_c. Then ζ_1 and ζ_2 can be calculated from (6) and c_{11} and c_{12} can be determined from the equations,

$$c_{11} = \left\{\frac{\alpha_0^2 c_{22} c_{55}}{\rho(\rho c_{55}\sin^2\theta - \rho\alpha_0^2)}\left(\frac{t_{11}}{2H}\right)^2\left(\frac{t_{12}}{H} - \frac{t_{11}}{2H}\right)^2 + 1\right\}\frac{\rho\alpha_0^2}{\sin^2\theta}$$

$$c_{12} = \left\{(c_{11}c_{22} + c_{55}^2) - \frac{\alpha_0^2}{\sin^2\theta}\left[\rho(c_{22} + c_{55}) - c_{11}c_{55}\left[\left(\frac{t_{11}}{2H}\right)^2 + \left(\frac{t_{12}}{H} - \frac{t_{11}}{2H}\right)^2\right]\right]\right\}^{1/2} - c_{55}$$

The simulated results obtained for $\theta = 8°$ is shown in Fig. 7, with $t_{22} = 17.95\ \mu s$, $t_{23} = 25.43\ \mu s$. Then c_{11} and c_{12} are calculated to be 161.8 GPa and 6.46 GPa, respectively.

CONCLUDING REMARKS

The proposed new nondestructive procedure can determine all the elastic constants (and thickness) of unidirectional laminates. The results show that the proposed method is efficient and accurate in characterizing all five stiffness constants of a unidirectional fiber-reinforced composite laminate. An error analysis has been carried out showing that the determined constants are insensitive to small errors in the data. Extension of this work will provide a nondestructive procedure that can determine the porosity content and fiber/matrix volume fraction of unidirectional as well as multilayered composite systems.

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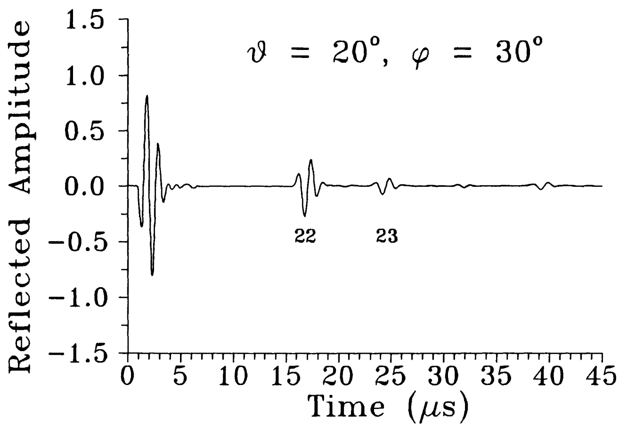

Fig. 6. Reflected signal from a composite laminate for $\theta = 20°$ and $\phi = 30°$.

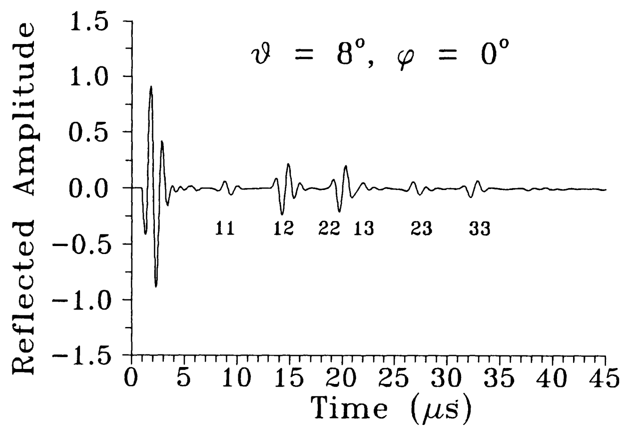

Fig. 7. Reflected signal from a composite laminate for $\theta = 8°$ and $\phi = 0°$.

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