WARD, Ronald Wayne, 1943-
SOME THEORETICAL CONSIDERATIONS FOR FUTURES
TRADING IN COMMODITIES REQUIRING TRANSFORMATION
SERVICES: THE CASE OF LIVE BEEF FUTURES.

Iowa State University, Ph.D., 1970
Economics, general

University Microfilms, Inc., Ann Arbor, Michigan
SOME THEORETICAL CONSIDERATIONS FOR FUTURES TRADING
IN COMMODITIES REQUIRING TRANSFORMATION SERVICES:
THE CASE OF LIVE BEEF FUTURES

by

Ronald Wayne Ward

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Ames, Iowa

1970
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2: MICRO DECISION MAKING MODEL</td>
<td>17</td>
</tr>
<tr>
<td>3: THE MACRO MODEL</td>
<td>54</td>
</tr>
<tr>
<td>4: PRICE DISCOVERY THROUGH COMPARATIVE STATICS</td>
<td>77</td>
</tr>
<tr>
<td>5: FUTURES TRADING AND THE EXISTENCE OF EQUILIBRIUMS</td>
<td>105</td>
</tr>
<tr>
<td>6: LIVE BEEF FUTURES PRICE MOVEMENTS</td>
<td>142</td>
</tr>
<tr>
<td>7: SUMMARY AND CONCLUSION</td>
<td>175</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>186</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>197</td>
</tr>
<tr>
<td>APPENDIX A: SIMPLIFICATION AND PLOTTING OF ISO-VARIANCES</td>
<td>198</td>
</tr>
<tr>
<td>APPENDIX B: CONCAVITY AND THE ISO-VARIANCES</td>
<td>204</td>
</tr>
<tr>
<td>APPENDIX C: MATRIX SOLUTIONS FOR INTERTEMPORAL PRICE DISCOVERY</td>
<td>207</td>
</tr>
<tr>
<td>APPENDIX D: EMPIRICAL REFERENCE FOR LIVE BEEF FUTURES TRADING ACTIVITIES</td>
<td>212</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1.</td>
<td>Trading floor—Chicago Mercantile Exchange</td>
<td>11</td>
</tr>
<tr>
<td>Figure 2.1.</td>
<td>Risk, income, and trade-off maps when net income is independent of hedging</td>
<td>26</td>
</tr>
<tr>
<td>Figure 2.2.</td>
<td>Risk, income, and trade-off maps when hedging is potentially profitable</td>
<td>33</td>
</tr>
<tr>
<td>Figure 2.3.</td>
<td>Risk, income, and trade-off maps with limited feedlot capacity</td>
<td>38</td>
</tr>
<tr>
<td>Figure 2.4.</td>
<td>Risk, income, and trade-off maps for long hedges</td>
<td>43</td>
</tr>
<tr>
<td>Figure 2.5.</td>
<td>Transition from hedging to pure speculation</td>
<td>48</td>
</tr>
<tr>
<td>Figure 2.6.</td>
<td>Forward sales and speculation with short futures</td>
<td>52</td>
</tr>
<tr>
<td>Figure 3.1.</td>
<td>Geometric framework for intertemporal futures and input price discovery</td>
<td>68</td>
</tr>
<tr>
<td>Figure 3.2.</td>
<td>Coordinates from equilibrium with slope $EE &gt; slope FF$</td>
<td>75</td>
</tr>
<tr>
<td>Figure 3.3.</td>
<td>Coordinates for equilibrium with slope $FF &gt; slope EE$</td>
<td>75</td>
</tr>
<tr>
<td>Figure 4.1.</td>
<td>Changes in transformation cost</td>
<td>82</td>
</tr>
<tr>
<td>Figure 4.2.</td>
<td>Market equilibrium with $P^0$ and changing $P_f$</td>
<td>84</td>
</tr>
<tr>
<td>Figure 4.3.</td>
<td>Market equilibrium with $\Delta P_L &gt; 0$ and $\frac{\partial P_f}{\partial P_L} &gt; 0$</td>
<td>86</td>
</tr>
<tr>
<td>Figure 4.4.</td>
<td>Increased price expectations and the input market</td>
<td>91</td>
</tr>
<tr>
<td>Figure 4.5.</td>
<td>Futures and increasing cash price expectations</td>
<td>93</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.6</td>
<td>Increasing expectation with stable futures price</td>
<td>94</td>
</tr>
<tr>
<td>4.7</td>
<td>Increasing price expectations</td>
<td>96</td>
</tr>
<tr>
<td>4.8</td>
<td>Increases in speculation</td>
<td>101</td>
</tr>
<tr>
<td>4.9</td>
<td>Exogenous shifts in supplies of inputs</td>
<td>103</td>
</tr>
<tr>
<td>5.1</td>
<td>The linear space for market equilibriums</td>
<td>106</td>
</tr>
<tr>
<td>5.2</td>
<td>Exogenous inputs and equilibrium where EE = FF</td>
<td>111</td>
</tr>
<tr>
<td>5.3</td>
<td>Market and industrial equilibriums and input changes where slope EE &gt; FF</td>
<td>116</td>
</tr>
<tr>
<td>5.4</td>
<td>Market and industrial equilibriums and input changes where slope FF &gt; EE</td>
<td>116</td>
</tr>
<tr>
<td>5.5</td>
<td>Market and industrial equilibriums and exogenous increases in price expect-</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>ations where slope EE &gt; slope FF</td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>Market and industrial equilibriums and exogenous increases in price expect-</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>ations where slope FF &gt; slope EE</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>Market and industrial equilibriums and increases in speculation where</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>slope EE &gt; slope FF</td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>Market and industrial equilibriums and increases in speculation where</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>slope FF &gt; slope EE</td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td>Market and industrial equilibriums and increases in transformation cost</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>where slope EE &gt; slope FF</td>
<td></td>
</tr>
<tr>
<td>5.10</td>
<td>Market and industrial equilibriums and increases in transformation cost</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>where slope FF &gt; slope EE</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.11. Sets $D_j$ and $E_j$  134

Figure 6.1. The structural relationship between price discovery and the contract life  144
LIST OF TABLES

Table 5.1. Example of movement from one point A to another point B 136

Table 5.2. Example of movement from point A to another point X 139

Table 6.1. Estimates of relationship for path of market equilibrium corresponding to Figure 6.1, before and after correction for autocorrelation 148

Table 6.2. The path of market equilibrium for t=6 before and after correction for positive autocorrelation 151

Table 6.3. Estimates for the Variance of $P_f$, the Variance of $(P_T - P_f)$, and the Mean of $(P_T - P_f)$ over the futures contract life 156

Table 6.4. Test for price difference in the last trading days of each contract, $H_0: \lim_{t+0} [P_x(t) - P_f(t)] = 0$ 166

Table 6.5. Percentage distribution of trader composition of total futures open contracts on the last trading day of the months specified 171

Table 6.6. Mean and standard deviation of futures trading composition 172
CHAPTER 1: INTRODUCTION

Much of the economic literature on futures markets view the cash-futures price spread as the price for carrying a commodity from one period to another. Thus, the difference between the current cash price for grain and the price for delivery at some future date is interpreted as the market price of storage. When processing is involved, as for soybeans, the price spread between spot soybeans and forward soybean products is seen as the market price for the required storage and processing services [97]. In the case of cattle, spread between cash prices for feeders and feed and future prices for fed cattle represents the market price for feedlot services [96]. In each case, a product transformation (i.e., storage and/or processing services) is involved and the price of these transformation services is the linkage leading to the simultaneous determination of spot and futures prices in the cash and futures markets. The existing theory of futures trading in relationship to cash markets has been incomplete at both the individual decision maker level and at the aggregate market level. Within this context, this research will attempt to clarify the role of futures markets in production and marketing decisions under risk and establish a theoretical framework for derivation of cash and futures prices at the aggregate
market level. Specifically, interests will be limited to an evaluation of futures trading in commodities that are in some stage of product transformation; hence, such products will be defined to be not yet marketable. Live beef is such a commodity (e.g., feeder cattle are not marketable until all transformation services are completed).

Conceptual Framework

Futures trading has traditionally consisted of trading in commodities that could be marketed immediately, but may be stored. More recently new commodities on the futures exchanges have not met all the traditional characteristics thought to be essential for a successful market. Many commodities now traded through futures are not storable. Futures trading in such products occurs prior to product maturity. These commodities are in the process of transformation where both quality and quantity changes are occurring. The input will result in an entirely different product after transformation. Storage may be thought of as one form of transformation where at that stage only the quality of the commodity is assumed changed. Within the following chapters analyses will attempt to incorporate this transformation concept into the theory of futures trading. The conceptual framework will be limited to considerations for trading in live beef futures. Yet much of the discussion will directly apply to other commodities listed on the exchanges.
The concept of time will be of utmost importance to the framework. This may best be explained with an example. Let feeder cattle be placed on a feedlot in some period. These inputs will require \( n \) periods (months) of transformation services before they are marketable as fed cattle. Simultaneously, there exists a futures market in live cattle with six possible contracts. At the point of initial inputs there exists some contract \( t \) periods (months) from maturity. The nearer the terminating date of a contract, the smaller \( t \) will be. Transformed goods must be marketed at \( t-n \) where \( n < t \).

Therefore, the theory of decision making must be applicable to the period \( t \) and the interpretation of aggregate market responses must be relevant to this point. Later considerations for movements in \( t \) will be presented. Hence two time concepts shall be noted. First, the cash and futures theory at some point \( t \) for all contracts must be deduced. Then concepts as \( t \) varies must be evaluated.

Objectives

Given the conceptual framework and the elements of time, then the following objectives can be specified:

A) Develop a basic understanding of the organization and operation of the live beef futures market.

B) Obtain information on the performance and characteristics of live beef futures and on the current and past use of such futures.
C) Analyze the relationship between the existence of futures markets and the structure of the related industry and the degree to which the prevailing market structures influence market responses.

Futures markets are advanced and complex systems of trading in contracts for deferred delivery, and the degree of effective utilization of this marketing tool is directly related to the extent of knowledge and understanding of these systems. Without this ingredient of understanding those who could most benefit through futures trading will most likely forego the opportunity. Hence one specific objective is to broaden the present knowledge and understanding of trading in live beef futures. Emphasis is placed on the theoretical and structural relationships between live beef futures and the live beef industry. Therefore, another objective is to establish germane models (which may or may not be operational) that can be utilized in the explanation of the trading activities among producers, processors, and futures traders. Given the development of theoretical models relating activities within the futures market and the beef industry, then these models are to be manipulated to show various theoretical impacts (empirical impacts when the system can be made operational). In essence the objective is to vary a particular system through changes in input and/or status variables and determine the alternative changes in output variables.
The above objectives are addressed through the use of both "micro" and "macro" concepts. Micro is with reference to the individual decision maker, while macro refers to the aggregate market behavior. The theoretical framework of individual decision making (micro) will explicitly show: (1) the alternative futures and cash market positions that can theoretically be established by producers and marketing firms, (2) the role of income, cost, risk, and expectations in the decision processes of market participants, (3) optimal commitments in the futures and cash markets, and (4) hedging and speculative positions in the futures and cash market. Whereas, the macro framework reveals: (1) relationships in implicit forms necessary for input and futures market equilibrium, (2) conditions necessary for relating futures to the cash market via price spreads, (3) procedures for intertemporal price discovery, (4) implications from types of equilibriums derived under alternative circumstances. Specifically, the macro models establish the procedures for deriving simultaneously the market equilibrium input price (feeder cattle) and the futures price. Then the alternative price movements under change will be specified. Finally, the role of the prevailing market structure as it relates to the total macro discussion will be considered. The complete framework of equilibrium will be in static terms, yet movements of equilibrium coordinates will be considered over each contract life (e.g., price movements over the trading period for a futures contract.)
A note on methodology

The subject content of the forthcoming discussion will be primarily that of price theory and its role in allocation and equilibrium. As will become evident in later chapters, much economic theory must be put aside in order that the core analysis be of manageable proportion. The depth and breadth of the economic theory of futures trading is vast, yet incomplete. Hence, the existing theory of futures will be altered and supplemented through the economic considerations to be presented [78, p. 12-31].

Three methods of inquiry are available to economists: deductive method, statistical method, and historical method. The deduction approach will be emphasized where simple economic logic is applied to futures trading. Given a set of assumptions and the market participants to which they apply, then responses or conclusions can be deduced. Arguments then are addressed to both the assumptions and the conclusions deduced from the assumptions. Some applications of the statistical method will be used in later discussion of price movements.

The Concept of Futures

The creation of futures markets like that of the banking system, stock exchange, and other economic systems was the outgrowth from the existing needs of an economic community
functioning in an inadequate market structure. The specific needs of all participants in any economic system should facilitate adjustments leading to the satisfaction of such needs.1

Futures markets

The nature of futures trading constitutes a part of the broad field of risk and uncertainty. The activities of futures trading consist with either assuming risk as speculators or in shifting risk as hedgers (e.g., such classifications will be shown to not necessarily be mutually exclusive). Risk can be thought of as the predictable changes that could either increase or decrease the total satisfaction of a market participant.

Concepts of futures trading draw a parallel to that of the cash market for each specific commodity. Yet differences exist. With cash sales the receipts of goods change ownership. Whereas, futures trading deals only with contracts or promises for deferred delivery or acceptance. The physical good does not immediately change ownership. Cash transactions are completed in the present time period, while futures constitutes a binding commitment to fulfill the conditions of a contract at some indicated time in the future.

1In the present state of the arts, needs may induce adjustments; but such needs are often not independent of the adjustment process, i.e., needs are often manipulated.
Elements of risk are present in every phase of economic activity and all participants must share in assuming it. Yet it varies widely in amount and incidence among groups [61 p.2]. Some groups are willing to accept greater risk while others are not. Therefore, the transactions between the spot and futures markets are indicative of the process of finding an equilibrium point between risk-bearers and risk-aversers. Ultimately, the legality of a futures market lies in the fact that the potentiality for shifting risk through this market exists. Without this risk shifting capacity a futures market would differ little from any organized gambling system.

Historical progression

Futures exchanges in the United States began with the Chicago Board of Trade organized in 1848 [45 p. 11]. Development of this and later exchanges in New York, New Orleans, Minneapolis, Kansas City, and others can be traced back to the westward development and specifically to the changing nature of agriculture in the midwest.

Grains were the first commodities readily adaptable to futures trading. Progression from early methods of trading in the physical product to the transactions involving futures contracts directly resulted from the erratic pricing system. There existed a high degree of uncertainty involving receipts that satisfied time requirements, quality needs, and volumes. Rail and water transportation lacked the desired level of
dependability necessary for increasing demands. Such complications lead to both buyers and sellers frequently contracting forward in an effort to provide for their needs. These "to arrive" contracts (contracts to deliver physical goods at some date in the future) were the natural forerunner of the futures contract. Ultimately through a process of standardization of the "to arrive" contracts, the futures emerged as a highly mobile tool for facilitating a more orderly marketing process.

Growth of the futures from this early beginning has shown vast expansions. New commodities such as coffee, sugar, silk, eggs, potatoes, and products such as zinc, lumber and others have been added to the list of contracts traded. Of these, many have proven successful; some erratic; while others have been complete failures.

**Futures contracts**

Transactions within a futures market are for commodities meeting rigid requirements and subjected to strict control through governmental agencies. Those restrictions placed on the commodity traded are explicitly stated for the actual contract bought or sold. Ambiguity with the term contract may result when the reader considers a spot sell or spot contract, forward contract, and futures contract. Each contract specifies a certain binding condition to which both buyers and sellers are committed. The relationship among these
Concepts of contracts are shown below:

I. Cash Contract
   (1) Spot contract: used for immediate delivery of the actual commodity.
   (2) Forward contract: used for deferred delivery of the actual commodity.

II. Future contract
   (3) Futures contract: used for speculative and hedging purposes.

Cash and futures contracts differ in many other ways. Cash contracts are used to merchandise the given commodity, while futures enable hedging against or speculation on commodity price changes. Cash contracts are executed at exchange tables or privately; whereas, almost all futures transactions are executed in the exchange pit, see Figure 1.1. Futures trading are nearly always in round lots (an exception may be when 1/2 lots etc. are traded) and future months are specified for delivery. Cash contracts consist of trades in irregular amounts where the time used for delivery is variable. Futures contracts provide the sellers with options for determining delivery day within the specific month as well as some range for quality requirements. Cash contracts usually call for specific quality and quantity requirements and the optional period for delivery may or may not exist [62, p. 109]. From this point and throughout the remaining discussion the term contract will be reserved to refer only to futures trading.
Figure 1.1. Trading floor--Chicago Mercantile Exchange
For adequate commitments in a contract, the market must possess a high degree of perfection, liquidity, and security. Liquidity guarantees instant convertibility of commitments, while security assures adequate fulfillment of contract obligations. The hedger also requires the exclusion of all risks other than value risk from the futures market.

These essentials for an active futures market are fulfilled through the futures contract. Through the standardizing of quantity and quality of commodities traded and the strict administrative control over the participants, the exchange has established a high degree of perfection and liquidity. The many specifications of the contract then leave only the contract price, number of contracts, and delivery months to be negotiated.

The high degree of contract standardization must be somewhat offset to provide the flexibility necessary for the market to function during periods of economic and natural vagarities. Flexibility exists in that a range for delivery and quality are left to the discretion of sellers. Blau has summarized this as, "the futures contract can thus be characterized as a compromise between the conflicting principles of standardization and flexibility, both aiming at increasing the perfection and liquidity of the market in different ways"[7, pp. 3-4].
Live beef futures

Inauguration of cattle futures trading in November, 1964, was characterized by much controversy over the economic usefulness of such activities. This contract grew out of a need within the beef industry, yet it lacks the full support of highly organized packinghouses. Ranchers, feeders, and other groups expressed their reservations about this commodity. In essence trading in a new commodity that did not fit the initial concept of storability (and marketability) aroused much hostility. Much of the industry was hidebound by customs, traditions, and the past. Hence there always exist some lag responses to new marketing ideas. The fact that the live beef industry was the largest single source of income in the agricultural sector definitely influenced these initial reactions. Fear of governmental intervention via the futures market existed. Beef prices have continued to be established independently of direct governmental regulations. At the same time the beef industry had experienced a disastrous decline in prices, thus adding to the reservations about new marketing innovations. Finally the prevailing market structure was such that a large group of sellers (producers) faced a small group of buyers (processors), hence further adding to the lack of enthusiasm [93, p. 19].

Introduction of any new commodity into the futures market requires adjustments conforming with the specific nature of
the good traded. The nature of beef futures necessitated many changes from the traditional concepts of futures trading. Trading in nonstorables and the difficulty with inconsistencies in grading compounded the problems with cattle futures. Many traders also felt that any futures should have the following attributes:

a) accepted grade standard
b) commodity must be homogenous
c) minimum degree of perishability
d) ability to make and/or accept delivery

The beef futures like many nonstorable commodities with futures contracts necessitates adjustments in theory to account for such changes. Hence given this brief review of the concepts of futures, the forthcoming chapters will provide a general theory useful in explaining activities of these commodities in some stage of product transformation. Chapter two will evaluate the situation of the decision maker and his cash and futures positions. Chapters three, four and five consider the aggregate futures market in relationship to the prevailing market structure. Finally chapter six looks directly at some pricing phenomena of the live beef futures.

A Note on Futures Literature

An extensive bibliography covering the most recent literature on futures trading has been included. For this
reason a comprehensive review of the literature will not be discussed. This literature can be classified in one of two ways: (1) a general discussion of all futures and (2) futures as applied to a particular commodity group. Studies relevant directly to the live beef futures are becoming increasingly more available. Most of these are applied problems such as in [2, 11, 32, 34, 38, 44, 53, 76, 120, 124], while others deal primarily with conceptual problems [33, 59, 93, 95, 96, 106].

Six studies have been singled out by this author as first priority readings for those with a working knowledge of futures [7, 107, 71, 115, 85, 101]. Blau reviews the general concept of futures trading and its relevance to an economic community [7]. Although this discussion needs some revision due to changes over time, it does provide an interpretation of the role of futures. Stein in a somewhat more technical and abstract discussion presents a theory of futures for the decision maker and considers some implications for trading positions at the aggregate market level [107]. Johnson geometrically illustrates some aspects of futures trading by decision makers [71]. Telser: approaches the theory of futures trading through an intermediate level of mathematics. His assumptions and methodology are somewhat unique to the theory of futures [115]. These first four studies are similar in that each evaluates the futures with reference to storable
goods only. In McKinnon's recent article aspects of futures relevant to products not yet marketable are considered. A theoretical model relating forward contracting to production risk is illustrated [85]. Finally, Preston and Yamey discuss intertemporal price relationships with forward markets [101]. These references do not begin to cover all the aspects of futures trading, but they do provide a general review of the present state of the theory of futures markets.
CHAPTER 2: MICRO DECISION MAKING MODEL

The introduction of a contract in live beef futures has provided a new marketing tool to the live beef industry. This tool can be effectively utilized by both processors and producers. It is then essential that a more comprehensive theory of futures trading, applicable to this commodity, be derived; thus establishing the theoretical role of beef futures in the marketing process. Existing futures theory lacks sufficient detail and applicability at the decision maker level.

A micro model derived within this chapter attempts to illustrate the decision process completed by each market participant establishing both cash and futures positions. Acting within the limits of the postulated assumptions of the model, the immediate consequences of alternative decision policies will be suggested. Each decision policy is made within the time horizon of \( t \) months from contract maturity.

Theoretical Framework

It is assumed that there exist a finite number of decision makers operating in a purely competitive environment. Each market participant has chosen a given occupation and has made the necessary capital investment for functioning in the cash market. For example, the decision maker may be a
feedlot operator who has previously made all necessary capital investments in feedlot facilities. Hence his decision process now consists of making the appropriate decisions in period $t (t>0)$ for the optimal number of feeder cattle to purchase and the optimal futures commitments. This commitment in futures would be expected to be terminated $n$ periods from $t$ where $n$ is the time required for transformation of feeders into slaughter animals. In essence the present decision theory will encompass only the short run since the production capacity of feeders and processors is taken as fixed.

**Income and cost considerations**

Each market participant can establish one or any combination of the following market positions in period $t (t>0)$ where $x$ and $x_F$ are the two choice variables representing the *cash* (spot) and *futures markets*, respectively. $x$ is measured in *live animal units* of given weight and quality and $x_F$ is expressed in *equivalent live animal units*. Thus the market positions can be summarized:

\[
x_F > 0 \quad \begin{cases} 
\text{the holding of short futures commitments} \\
\text{from period } t>0 \text{ to } t-n \text{ for contracts maturing in period } t=0, \ n < t, \ \text{(the decision maker's selling commitments in live beef futures expressed as a given number of steers)}, \\
\text{the holding of long futures commitments} \\
\text{from period } t>0 \text{ to } t-n \text{ for contracts}
\end{cases}
\]
maturing in period $t=0$, $n \leq t$, (the decision maker's buying commitments in live beef futures expressed as a given number of slaughter steers),
cash market purchases in period $t>0$ for commodity requiring $n$ periods for product transformation (the number of feeder cattle purchased and placed in feedlot in period $t>0$),
cash market quantities contracted in period $t>0$ for delivery in period $t-n$ or cash market purchases planned in period $t>0$ for period $t-n$ (the number of slaughter steers contracted for deferred delivery or anticipated purchases of slaughter steers at $t-n$, $n \leq t$).

Current and expected prices are expressed as follows:

$$P(t) = \begin{cases} 
\text{existing cash (spot) price per unit of } x \\
\text{where } x \geq 0 \text{ in period } t>0 \text{ (known price of feeder cattle per animal prior to feedlot transformation)},
\end{cases}$$

$$EP(t-n) = \begin{cases} 
\text{expected cash price per unit of transformed } x \text{ for period } t-n \text{ (expected price of slaughter steers per animal in period } t>0 \text{ for delivery in period } t-n, n \leq t),
\end{cases}$$
\( P'(t) = \begin{cases} 
\text{cash market contracting price per unit of x} \\
(\text{where } x<0) \text{ in period } t>0 \text{ for delivery in} \\
\text{period } t-n, n \leq t,
\end{cases} \)

\( F(t) = \begin{cases} 
\text{existing futures price per unit of } x_F \text{ in} \\
\text{period } t>0 \text{ for contract maturing in period} \\
t=0,
\end{cases} \)

\( EF(t-n) = \begin{cases} 
\text{expected futures price per unit of } x_F \text{ in} \\
\text{period } t>0 \text{ for period } t-n \text{ for contract} \\
\text{maturing in period } t=0.
\end{cases} \)

Each decision maker also incurs costs from his participation in the cash and futures markets. Feedlot operators and other cash market participants have transformation and marketing costs. Similarly, futures market participants must pay a commission charge for each futures commitment established.

Define:

\( C_T'(x) = \text{average cost of product transformation}, \)

\( C_M'(x) = \text{average cost of marketing}, \)

\( C_F'(x_F) = \text{average cost of futures commitments, and} \)

\( C'(x) = C_T'(x) + C_M'(x). \)

A set of expected net income equations for decision makers can now be written. It has been assumed that a linear transformation cost function exists and that all other costs are linearly related to the quantity variables. Costs
(C' and C'ₚ) and prices at period t are known, x and xₚ are the choice variables, and prices for period t-n (P(t-n)), (F(t-n)) are unknown stochastic variables. In addition the decision makers know the probability distribution (density function) of the stochastic variables.

\[
E \pi(x > 0, x_p > 0) = x[EP(t-n) - P(t) - C']
+ x_p[P(t) - EF(t-n) - C'ₚ]
\]  
(2.1)

\[
E \pi(x > 0, x_p < 0) = x[EP(t-n) - P(t) - C']
+ x_p[P(t) - EF(t-n) + C'ₚ]
\]  
(2.2)

\[
E \pi(x < 0, x_p < 0) = x[EP(t-n) - P'(t) + C']
+ x_p[F(t) - EF(t-n) + C'ₚ]
\]  
(2.3)

\[
E \pi(x < 0, x_p > 0) = x[EP(t-n) - P'(t) + C']
+ x_p[F(t) - EF(t-n) - C'ₚ]
\]  
(2.4)

Given the alternative market positions and the associated cost functions, then one of the four expected net income equations will hold uniquely for a decision maker at period t>0 (e.g., some point prior to maturity). Equation 2.1 represents a decision maker's expected net income with both cash and short futures market positions. Equation 2.2 holds for firms with cash and long futures positions. Equation 2.3...
applies when forward contracting and/or anticipated purchases in the cash market exist in conjunction with a long futures position. Equation 2.4 involves the same cash position but in conjunction with a short futures commitment. These equations will be utilized in a micro decision model, but only one equation will be applicable to a decision maker at a given point, i.e., the same decision maker cannot be both short \( (x_p > 0) \) and long \( (x_p < 0) \) at the same time. The allowable combinations of market positions follow from definitions and assumptions in the next section regarding risk and utility maps relating risk to expected net income.

**Decision making risk**

In this framework risk to the decision maker results in variations in realized net income when price expectations do not materialize. Risk aversion is assumed in the sense that each individual chooses those market positions which will minimize his risk for a given expected net income. Prices in future periods are assumed to be random variables where the mean and variance of the distribution is known. Risk is then defined as the variance of net income.

McKinnon states the output at harvest time can be viewed at planting time as a random variable [85, p. 846]. Within the present framework this would imply that at period \( t \) the transformed \( x \) of period \( t-n \) would be viewed as a random variable. In contrast, the model has accounted for production
risks without assuming that the feeder's output of slaughter steers in period t-n is a random variable.

Transformation risk in our framework may exist through changes in quality and weights of the initial cash stock (feeder cattle). It has been assumed (and quite realistically) that quality and weight risk are reflected directly in the prices received, i.e., the value of x changes directly when quality and weight differences occur. Price risks then absorb differences due to variations in quality and weights. This is why it was chosen to measure quantity variables in live animal units.

It has been assumed that the remaining transformation risk is negligible, i.e., the same number of x units of input in period t yields the equivalent number of output units in t-n. Realistically, the physical number of input units may exceed output units due to deaths. An additional stochastic variable η could be introduced where η ∼ ID(μ, σ²). η measures the proportion of x inputs that were lost in the transformation period, thus reducing the output in period t-n to x(1-η). The expected net income equation would then appear as

\[ E\pi = x_p[F(t) - EF(t-n) - C'P] + x[EP(t-n) - P(t) - C'] - x E\eta EP(t-n). \]

η has been ignored in the present framework, although its introduction would not drastically change the optimization format. All other transformation risks are assumed to be reflected in the price movements as explained.
Given the combinations of expected net income and risk and the shape of the decision maker's preference function relating risk to expected net income, the format for determination of optimal market positions is set. The second moment about the mean of the probability distribution of net income has been utilized as a measure of risk. Alternative criteria for risk measurement exist, but none are without their limitations. References [7], [70], [85], [107], and [115] provide useful discussions of some risk measurement criteria.

Optimal Market Positions for the Short Hedger

It is assumed that a market participant makes his cash commodity decision and chooses his position in the futures market at period t>0 based upon his price expectations. The goal of the decision maker is to maximize expected net income. However, since expected prices may not materialize, risk prevails and must be considered when establishing cash and futures positions. Thus the forthcoming models will utilize a tradeoff approach between expected net income and risk as elements in the decision maker's utility function leading to the simultaneous determination of optimal cash and futures market commitments (see Figure 2.1b).

If the variance of net income is taken as a measure of risk, a theoretical risk function for net income as defined in 2.1 can be written:
Figure 2.1. Risk, income, and trade-off maps when net income is independent of hedging.
Short futures hedging and speculation

100% hedging

Cosh position

Expected Net Income

Eff_0 Eff_1 Eff_2 Eff_3

Cash position

a. Iso-variance and iso-income map

Risk

\sqrt{V_2}

\sqrt{V_0}

Expected Net Income

b. Trade-off map
This function assumes that cash and futures prices are random variables for period t-n and that $\sigma_p^2$, $\sigma_F^2$, and $\rho_{PF}$ are known and constant for the given individual decision maker. The greater (smaller) the positive correlation between cash and futures prices the smaller (larger) risk will be. Risk also decreases as the variances of cash and futures prices decrease. Any improvement in the individual's ability to predict prices (e.g., improvements over price expectations of equation 2.1 through 2.4) will change the risk function via changes in the mean square error for cash and futures prices. The simplification and plotting of Equation 2.5 in Figure 2.1 is explained in Appendix A.

This equation holds for any decision maker who establishes a short futures position against a current commitment in the cash market. It can be presented in an iso-variance map where each iso-variance curve includes all combinations of cash and futures positions yielding a constant risk value (Figure 2.1a). In Figure 2.1a the horizontal axis shows the cash commitment ($x$) while the vertical axis indicates the futures commitment ($x_F$). Market positions can be purely speculative, purely hedging, or some combination of the two. All commitments established on the vertical axis only are purely short speculative positions; those on the horizontal
axis involve no futures positions. Any point lying between the cash position axis and the 100 percent hedging line (the 45° line in Figure 2.1a) represents a cash position with a short hedger where this hedge is less than the cash position. All points lying on the 45° line represent 100 percent hedges while points to the left of this line correspond to a combination of hedging and speculation.

Curve $V_0$ in Figure 2.1a gives the alternative values of $x$ and $x_P$ necessary for $2.5$ to sum to $V_0$. In Figure 2.1a it is assumed that $\sigma_p^2$, $\sigma_F^2$, and $\sigma_{PF}$ are such that a section of the iso-variances is concave downward (to the origin). Otherwise, the decision maker could not minimize his variance for a given expected net income through some hedged position. Concave upward iso-variance curves imply that the minimum variance for a given net return can occur only with a zero hedge, i.e., all positions are established on only one axis of Figure 2.1a. The cases considered here will be those leading to use of the futures market (see Appendix B).

To determine the optimal amount of hedge income equation 2.1 must be considered. If the decision maker's expectations are such that $[F(t) - EF(t-n) - C'_F] = 0$, then the expected net income equation is independent of the amount of futures commitments. In this case an iso-net income line (expected net income line) can be drawn parallel to the vertical axis as in Figure 2.1a. This individual will for each iso-net
income choose that level of short futures which will minimize his variance. Graphically, where an iso-net income $E\pi_i$ is tangent to an iso-variance curve $V_i$ yields this minimum point. The combination of all possible tangency points of iso-net income and iso-variance gives the line ZZ and this tangency locus must be linear since all iso-variance curves are completely symmetrical. All points along ZZ can then be superimposed on a tradeoff map between expected net income and risk (Figure 2.1b). Tangency locus ZZ in the iso-variance map must always be linear as indicated. This does not necessarily hold for the same ZZ plotted over the tradeoff map. Z'Z' on the tradeoff map will be linear if the incremental change in both iso-variance and iso-net income remains constant. If increases in iso-net income are accompanied by increasingly larger incremental changes in iso-variances, then Z'Z' will be curvilinear upward. This is economically meaningful in that increases in net income opportunities must be accompanied by proportionally larger risk taking. The alternative of greater incremental increases in expected net income over iso-variances is not meaningful to the above analysis. Therefore it will not be considered. Further, all examples to be presented will be graphically interpreted on the assumption of constant incremental changes in both the iso-variance and iso-net income contours.
Any combination along Z'Z' can be chosen, but the rational decision maker maximizes his utility under the constrained conditions. The market participant will choose that combination giving the expected net income and risk along Z'Z' that will place him on his highest indifference curve. Any point other than the tangency of Z'Z' and I in Figure 2.1b must necessarily yield a lower degree of satisfaction since Z'Z' crosses only lower indifference curves when off the tangency point. In the example from Figures 2.1a and 2.1b, E\text{w}_2 and V_2 give the constrained utility maximization.

Referring back to the iso-variance map, the combination of E\text{w}_2 and V_2 indicates that x^{\#}_F is the optimal short hedge and x^{\#}_c the optimal cash position. Optimization has resulted in a futures commitment where the cash market position was not completely hedged. For this case, optimal output is not influenced by the futures market. Moreover, the optimal futures position can never exceed the optimal cash position, i.e., hedging but not speculation is possible. As shown in Figure 2.1a, however, hedging at less than the 100 percent level may be optimal. If a 100 percent hedging rule were applied, the decision maker would either have to accept a higher level of risk to maintain the same expected income, or reduce his output and expected income to maintain the same level of risk.
Probably a more realistic alternative to the above model is when the expected net returns from futures activities are greater than zero, i.e., \([P(t) - EP(t-n) - C_F'] > 0\). Now expected net income is not independent of the hedging level. Hedging will be utilized not only for its risk shifting capacity, but also for its profit potential. The iso-net income lines may have a slope like that shown in Figure 2.2a. This slope will vary as the expected net return from futures changes. The greater (smaller) the increase in expected futures return, the smaller (larger) will be the slope of each iso-net income line.

For the iso-net revenue contours to be linear as shown in the graphs, it must be assumed that all marginal costs are fixed for all values of \(x\) and \(x_F\) given the decision maker's transformation period. Equation 2.1 through 2.4 thus contain all fixed values once the decision maker establishes his expectations. With this assumption it can then be shown that the slope of the iso-net income lines is always constant for the given transformation period. Introduction of non-linear transformation functions into the income equation could yield nonlinear iso-net income contours, i.e., the marginal productivity of transformation may not be constant and hence costs would change as cash market commitments vary.

Tangency locus ZZ (Figure 2.2a) is again superimposed over the tradeoff map and the same tangency of Z'Z' and I_1
Figure 2.2. Risk, income, and trade-off maps when hedging is potentially profitable.
Short futures hedging and speculation

100% hedging

Cosh position

Risk

Expected Net Income

ETT₀

ETT₁

ETT₂

ETT₃

a. Iso-variance and iso-income map

b. Trade-off map
exist (Figure 2.2b). The constrained utility maximization occurs with $\bar{E}w_2$ and $V_2$. This level of expected net income and risk is utilized to determine the optimal short futures position and the level of total stocks as shown by $x^*_p$ and $x^*$ in Figure 2.2a. Figures 2.2a and 2.2b lead to the conclusion that the decision maker establishes a short hedge position at a higher level than when expected net income is independent of hedging. Indeed, his price expectations may be such that he chooses his optimal positions to the left of the 100 percent hedging line. If this happens, and the futures market position were limited to 100 percent hedging, the firm would find it necessary to raise output to maintain its expected income and risk levels. A smaller increase would be needed to keep risk at the same level but a larger increment in output as well as a higher level of risk would be required to maintain expected income. If output were held constant, risk would decrease but at the cost of a reduction in expected net income.

A third and final example for short hedging can be considered to illustrate the use of the iso-variance and

---

1Tangency locus ZZ coincided in Figures 2.1a and 2.2a only because of the way the expected net income equations were drawn. In both cases, $\bar{E}m$ was tangent to iso-variance $V_j$, hence giving the same ZZ for both situations. Expected net income could have been such that $\bar{E}m_1$ was tangent to the iso-variance $V_j$. This could have given a different ZZ and a different tangency point on the tradeoff map.
tradeoff maps. In the two cases considered, the optimal situation dictated the levels of hedging and production. Since the decision maker is making his decision in period \( t \) for expected results in period \( t-n, n\leq t \), time limitations and other limiting factors may prevent his obtaining and handling the optimal level of total production stocks \( x^* \) shown in Figures 2.1a and 2.2a. For example, a livestock feeder may not have the feedlot capacity to handle the optimal production level. The problem is to determine the optimal hedge given his fixed (maximum) capacity.

Due to the capacity limitation, the decision maker must operate within the output limit shown in Figure 2.3a as \( \bar{x} \). All possible combinations of expected net income and iso-variance must be determined by moving along ZZ up to the limit of line \( \bar{xx} \) and then up along \( \bar{xx} \). Net income-risk combinations along the \( \bar{xx} \) consist of \( EII_i \) and \( V_j \) where \( i \neq j \) except when the line \( \bar{xx} \) intersects a tangency point between an iso-net income and iso-variance curve. Then \( i \) equals \( j \) as shown in the example (Figure 2.3a) where \( \bar{xx} \) intersects \( EII_1 \) and \( V_1 \). At no other point will \( i \) and \( j \) be equal along this line. The implication of this is that an iso-net income line must be evaluated at some point along the line which does not yield the minimum variance. Only when \( i \) equals \( j \) will the minimum iso-variance correspond with the intersection of \( \bar{xx} \) and the relevant iso-net income and iso-variance.
The various intersections of iso-net revenues and iso-variances along \( \overline{xx} \) are then plotted on the tradeoff map of Figure 2.3b, giving the curve \( Z'Z'' \). \( Z'Z' \) of Figure 2.3b shows those minimum risk values for all levels of expected net income. With fixed capacity the expected net income \( E\Pi_2 \) is attainable only with the higher risk \( V'_2 \). The combination of risk and expected net income that maximizes constrained utility occurs at the tangency \( Z'Z'' \) and indifference curve \( I'_o \) with the coordinates \( E\Pi'_1 \) and \( V'_1 \). This tangency can never occur on a higher indifference curve than \( I'_1 \) given the present \( Z'Z' \) line, i.e., \( Z'Z'' \) will always be either tangent or to the left of \( Z'Z' \). The tangent coordinates \( (E\Pi'_1, V'_1) \) from Figure 2.3b are then determined along the line \( \overline{xx} \) of Figure 2.3a, thus giving the optimal short hedge \( \overline{x}_F \) for the fixed capacity \( \overline{x} \). The desired hedging level can be determined for any capacity, but when this level is less than the optimal \( x^* \) there must be a loss in utility to the decision maker.

**Determination of the Optimal Market Position for the Long Hedger**

A long hedge (buying futures) can occur only if a short cash position has been assumed or a long cash position is to be taken in period \( t-n \), i.e., the market participant has established some deferred delivery contractual agreement at price \( P'(t) \) in period \( t>0 \) for delivery in \( t-n \) or he expects to purchase the commodity in period \( t-n \). In most situations
Figure 2.3. Risk, income, and trade-off maps with limited feedlot capacity
a. Iso-variance and iso-income map

b. Trade-off map
of deferred delivery contracting with a long hedge, the decision maker does not have a cash position at the time hedging is undertaken. The intent is usually to take a cash position in period t-n. This market position is somewhat atypical for primary producers who have a cash market commitment. If such a commitment exists the decision maker has established the price for the commodity which he owns in period t>0 for deferred delivery. This involves transforming the product over the period from t>0 to t-n and then making delivery. Given price expectations, the contractual agreement serves to lock in a particular price for his product. The only price risk then within the cash market is that the wrong decision was made. If the price of the contractual commitment for example happens to be less than cash prices in t-n, then the decision maker has given up net income through the forward sale.

Participation in the futures market by individuals owning stocks with a deferred delivery commitment cannot in the traditional sense be hedging. Nevertheless a long futures position taken by such a decision maker possibly could be a hedge against the opportunity cost arising from making the delivery commitment when in fact actual price would have indicated a long cash market position.

The more meaningful evaluation of deferred delivery contracting and long hedging occurs for those market participants not directly involved in the production process at the
initial period \( t > 0 \). Marketing agencies such as brokers, dealers, and processors, may fall into that group of traders utilizing a cash and long futures position.

Two distinct situations could occur. First, there may be a group of buyers, e.g., livestock beef dealers, who contract to deliver live beef at some future period \( t-n \). When the initial contractual agreement is established such participants do not own stocks and they are uncertain as to the price they must pay for such stocks during the delivery period. Hence establishment of a long futures commitment can serve as a hedge against variations in cash prices at \( t=0 \).

The second and possibly more frequent situation would be where a packer has contracted to deliver carcass beef at some future date and must have an ample supply of meat to meet his commitments. Therefore the packer must have a given supply of live beef during period \( t-n \), but he is presently operating in period \( t>0 \). Operating as a risk averter, the packer can establish a long futures commitment to offset an anticipated purchase of live beef during period \( t-n \). Through these transactions the packer will establish the value of his processed product and at the same time possibly reduce his risk from changes in live animal prices.

Equation 2.3 expresses the relevant net income for these situations. For the first situation \( P'(t) \) is the contracted price of live beef to be delivered in period \( t-n \). All other
notation is as previously defined. In the second situation
\( P'(t) \) represents the contracted value of an animal unit after
processing. Therefore in both cases the cash selling price of
output has been established, but the purchase price of live
animal inputs is subject to variations. Given these alterna-
tives, income equation 2.3 along with the appropriate risk
function can be utilized to determine the optimal market
positions. Only the cash and futures prices of period \( t-n \)
are unknown in equation 2.3; therefore the variance of 2.3 can
be written as:

\[
\text{Var}(\Pi) = x^2\sigma^2_F + x_F^2\sigma^2_F - 2xx_F\rho_{FF}\sigma_F\sigma_F. \tag{2.6}
\]

Determination of the optimal market position follows in
the same manner as did the procedure for determining cash
and short hedging positions. The iso-variance and iso-net
income curve are drawn in Figure 2.4a. The third quadrant
is the relevant one since \( x<0 \) and \( x_F<0 \) when a deferred
delivery or anticipated cash market position exists and a
long hedge is established. It is assumed that the iso-variance
for this section contains a concave section. The difference
now is that the origin is in the upper right hand corner of
Figure 2.4a. See the Appendix A for plotting of this
quadrant also.

Movements from \( V_0 \) to \( V_3 \) and from \( E\Pi_0 \) and \( E\Pi_3 \) illustrate
increases in variances and net income respectively in Figure
Figure 2.4. Risk, income, and trade-off maps for long hedges
Cash position

hedge

100% hedging

hedging and speculation

Long futures

a. Iso-variance and iso-income map

Risk

$\sqrt{v_3}$

$\sqrt{v_1}$

b. Trade-off map

Expected Net Income
2.4a. It is assumed that if a decision maker can establish a deferred-delivery sale in period \( t>0 \) (e.g., dealer sells slaughter steers for later delivery) or anticipates a cash purchase in period \( t-n \) (e.g., packer plans to buy slaughter steers in period \( t-n \)), then his optimal futures position must be considered simultaneously with his optimal cash market decision. For every possible expected net income level the decision maker will minimize his risk, i.e., he will choose that position where the iso-net income is just tangent to an iso-variance. Again the mechanism for expressing utility for the decision maker is through the tradeoff map (Figure 2.4b).

The optimal position may correspond to a partial hedge, a 100 percent hedge, or 100 percent hedging along with some purely speculative trading. The same type of effects of non-optimal 100 percent hedging on expected net income, risk, and cash positions as explained in the prior section also exist in this case.

The cash depicted in Figure 2.4a occurs when the decision maker expects the futures price in \( t-n \) to exceed the futures price and average cost of futures transactions in period \( t>0 \). Hence, given that \([F(t) - EF(t-n) + C_F']<0\), not only can long hedging shift some risk, it can prove profitable. As this difference approaches zero, the slope of the iso-net income lines approaches infinity, i.e., \( EN_i \) becomes parallel to the
axis. The tangency locus ZZ retains the same meaning as before. All possible combinations of risk to expected net income giving the minimum variance are then plotted over the tradeoff map on Figure 2.4b. The tangency point between Z'Z' and an indifference curve gives that combination of risk and expected net income leading to the optimal market position. The coordinates of this tangency (EII^, V^) of Figure 2.4b then indicate the optimal long futures x_F^* and cash commitment (or anticipated commitment) x^* as in Figure 2.4a. Through the established position of x_F^* and x^* the decision maker has minimized his risk for a given expected net income and maximized his utility given all other constraints.

**Transition from Hedging to Pure Speculation**

Price expectations could be such that futures positions other than a hedge or a hedge combined with speculation should be established. For example, in Equation 2.1 \([EP(t-n) - P(t) - C'] > 0\) then a spot holding should occur. But if expected futures price movements give \([EF(t-n) + C_F' > P(t)]\) then the short futures position would not be potentially profitable. Similarly for the contracted (or anticipated) cash market position, if \([EF(t-n) < F(t) + C_F']\) then the long position would not be profitable. Hence when price expectations are such that holding cash and long futures or forward cash and short futures provide profit expectations from both markets,
then the futures position is defined as **speculation**. These two cases are represented by equations 2.2 and 2.4.

This definition of speculation in conjunction with that considered earlier reveals two forms of speculation. First, speculation in the futures market occurs when futures commitments (short or long) exceed the 100 percent hedging level. And second, speculation exists when the established futures commitment does not provide hedging possibilities in conjunction with the cash market position. Speculation has been defined only for those decision makers dealing in the cash market. Another group of individuals trading in futures only could be discussed where they are always speculators. For this group, the optimal position always lies on the vertical axis (either short or long) of the iso-variance and iso-net income map.

Figure 2.5a illustrates the transition from a hedging to a purely speculative futures position. The original short position for the expected net income occurred at the tangency $T$ thus minimizing the risk for the given iso-net income. The slope of $EW_2$ in Figure 2.5a is directly related to price expectation for the futures, i.e., $[F(t) - EF(t-n) - C'_p]>0$. As this expectation changes, the slope of the relevant iso-net income also changes. At some point the decision maker may view the movement of futures prices to be such that $[F(t) - EF(t-n) - C'_p]<0$. Iso-net income $EW'_2$ follows directly from
Figure 2.5. Transition from hedging to pure speculation
Short futures

100% hedging

Cash position

Speculation

Long futures

a. Iso-variance and iso-income map

Risk

\( \sqrt{V_2} \)

Expected Net Income

b. Trade-off map

\( E_{T12} \)

\( Z' \)

\( Z \)

\( I_0 \)

\( I_1 \)

\( I_2 \)

\( I_3 \)

\( I_4 \)
this change in expectation. Although expected futures prices are now such that a loss is expected to occur with a short position, this point with a short contract will still be chosen (tangency T'). A short position is taken because the expected futures prices for period t-n does not exceed the present futures price by a significant enough amount to warrant accepting any additional risk from a speculative position. Therefore the decision maker will choose a short futures position which minimizes risk even though a small loss is expected from the short contract (selling futures). For example, futures have been sold for P(t) and are expected to be purchased for EF(t-n) when in fact \[ EF(t-n) + C'_F > P(t) \]. The decision maker can be said to be paying some premium for minimizing risk.

As the decision maker's expectations change, his market positions change accordingly. Given that the expected futures price for period t-n now exceeds futures at period t by a large enough amount, then the decision maker may become a market speculator. This would mean that in Figure 2.5a the iso-net income lines are tangent to the iso-variance only in the lower right quadrant. A cash position corresponds but the trader goes long (buys) in futures as well. The profit potential from speculating is now sufficiently large to warrant accepting any additional risk. The maximum indifference curve possible given the tangency locus of Figure 2.5b then
shows that combination of risk and expected net income $(E_2, V_2)$ leading to the optimal speculative level $x^*_F$ and cash position $x^*$. Any hedging position would greatly reduce utility given the firm's set of expectations.

Another speculative position is that of deferred-delivery contracts or anticipated cash purchases and short futures. The decision maker views cash price movements as yielding a profitable deferred delivery commitment (e.g., for dealers in live beef) or the decision maker must make purchases in period $t-n$ to meet sale commitments. Yet he also views the expected futures prices to be such that a short position is profitable. In period $t$ futures are sold and in $t-n$, $n<t$ all commitments are terminated through offsetting purchases. Profit potentials from the short futures are expected to be large enough to warrant the speculative position.

This speculative commitment is illustrated in Figures 2.6a and 2.6b. Tangency locus ZZ of Figure 2.6a gives the maximum indifference curve at coordinates $(E_3, V_3)$. From these points the optimal positions are chosen where $x^*_F$ is the optimal speculative contract and $x^*$ is the optimal contractual or anticipated cash commitment.

This model extends the analysis for the relation between market output decisions and futures positions beyond the existing available literature. Concepts of decision making are directly applicable to the livestock market, yet the
Figure 2.6. Forward sales and speculation with short futures
Short futures

Speculation

Cash position

100% hedging

Long futures

a. Iso-variance and iso-income maps

b. Trade-off map
same framework is readily applied to other commodities. The implication of the micro model for aggregate market determination of price, output, and returns to all market participants remains to be explored.
CHAPTER 3: THE MACRO MODEL

The decision maker (in the micro model) was assumed to have sufficient knowledge to establish both optimal futures and cash market positions. Optimal market positions were established by determining points in one of the four quadrants given that the relevant iso-variances, iso-net incomes, and tradeoff maps existed (see the last chapter). Thus for the $k^{th}$ decision maker, the necessary elements were available to provide and determine his contribution to supply and demand for futures contracts. Further the optimal cash market positions encompassed the demands for initial inputs and the demand and supply for input transformation services. Some techniques for aggregation over these individual commitments provide the mechanism for movement from the micro to a macro analysis of the two markets. Given that the theory of optimal hedging, optimal speculation, and optimal cash market positions are theoretically established for the assumed framework, then some aggregation process will lead to a macro model giving those levels of variables necessary for the existence of market equilibrium. Hence the procedure at hand is to look at each portion of the total futures and cash market relationships at the aggregate level. From these aggregated relationships some meaningful propositions can be derived that offer
clues to intertemporal price discovery. As previously emphasized, the notation of the micro model is completely independent of the macro model. This has occurred to give emphasis to the concepts of individual versus aggregate trading responses. Specifically the micro theory used total live animal units as one choice variable. Whereas, in the macro model concepts are related through pounds of inputs and transformation services. Hence it is cautioned that the concepts, not the notation, of Chapter two be related to the remaining chapters.

The Transformation Function

The introduction of a transformation function is the first factor entering into the theory of futures-cash trading in nonstorable products. Nonstorables must go through a transformation process, whereas storables are changed through a storage function. The concepts of transformation versus storage are similar in nature. Yet due to the difference in the physical requirements necessary for the completion of either process, it is convenient to separate them. Stored goods are actually transformed simply by the variable of time. But with reference to live beef, the transformation process must be implemented through physical inducements such as feeding, management services, and other facilities necessary for the continued growth of the live animal. Hence transformation within the present contexts refers to changes in both quality
and quantity. Whereas, a storage function would result primarily in quality changes.

If there exists a supply of some storable good, Z, then the amount of hedging is in terms of Z and is measured in the same units at the aggregate level. Some quantity of Z may be consumed this period, hence the amount of Z that must be stored is the difference between total supply and present consumption. This situation is illustrated below:

\[ Z = \text{total supply of some storable good,} \]

\[ C = \text{present consumption of Z,} \]

\[ K = Z - C = \text{the quantity of Z that must be stored,} \]

(demand for storage)

\[ K = K_u + K_H = \text{storage unhedged + storage hedged.} \]

If present consumption of Z is just equal to supply, then there is no supply of futures (from hedgers) nor demand for storage (e.g., \( K_H = K_u = K = 0 \)).

The theory of futures trading in nonstorable goods differs from the above situation. Once a product that is nonstorable reaches a level such as Z above, then it must be completely consumed in that period. For example, feeder cattle are fed out to a finished product. At this time the total supply must be consumed; hence the supply of futures and demand for storage must be zero. A very short interval of storage may occur, but the product changes so drastically in quality and quantity that it must be nonstorable.
Therefore the theory of futures trading in nonstorable products must be evaluated at some point before the nonstorable good reaches maturity (e.g., before it can be consumed or processed). It then must be that futures trading occurs while the product is still in some stage of transformation via changes in both quality and quantity (e.g., recall that a storage function primarily reflects only quality changes). Trading in storables may also occur prior to complete transformation of the product. For example corn production may be hedged and corn may be stored. Hence one primary difference between futures trading in storable versus nonstorable products lies in the ability of hedging beyond the date of product maturity in the first but not in the latter case. Define:

\[ X(t) = \text{initial input of product to be transformed} \]
\[ \text{ (feeder cattle or other nonstorable products measured in pounds),} \]
\[ L(t) = \text{the required transformation services (feedlot services for example, measured in pounds),} \]
\[ T(X) = \text{the final output measured in pounds, after transformation of all } X, \]
\[ T(X(t)) = \text{the final output measured in pounds, after transformation of } X(t). \]

The final output \( T(X) \) must be directly related to the initial levels of inputs \( X \) and the available supplies of transformation services. In addition the final output consists of
inputs that require various degrees of transformation. For example the input of feeder cattle may be with 300 lb., 500 lb., or other weights of feeders. Hence greater transformation services are required for lighter weights of inputs. Finally if total output of nonstorable products can be expressed in terms of initial inputs and transformation services, then one of the first essential steps for the theoretical analysis of futures trading is established. Define the functional relationship as shown:

\[ T(X(t)) = \bar{G}(X(t), L(t)) \ldots \ldots \text{Transformation function.} \]

It follows that for any one period \( t \), inputs and transformations are in fixed proportion. Yet over time this proportion must be a variable. If the transformation function (used in the macro sense) is assumed linearly homogeneous (constant returns to scale), then the initial input can be separated from the transformation function above \( [1, p. 335] \). Therefore

\[ T(X) = G(X, L) = X^{-\alpha} L^\alpha \]
\[ = X \frac{L}{X} = XL^\alpha \]
\[ T(X) = XG(L^*) \text{ since } G(L^*) = L^{\ast \alpha}. \]

This is a very restrictive functional assumption. But given that only the implicit form \( G(X, L) \) is known, then this procedure allows for separating the initial input from the implicit function. All equilibriums, including those of \( X \) and \( L \), will follow later in this chapter.
the transformation function expressed in the form to be utilized exists as in equation 3.1. If there is only one possible time for implementing inputs to produce the final output, then equation 3.1 is expressed in the simple form of equation 3.2. In contrast, if the final output consists of numerous points of input, then the total output \( T(X) \) must be associated with all degrees of inputs and transformation. Hence \( T(X) \) can be derived by equation 3.3. This can be illustrated where the total supply of fed cattle in say

\[
T(X) = \frac{X}{L'X} \]

February consists of feeder cattle placed on feedlots up to \( n \) months prior to February. Again the time \( t \) represents the time from maturity of input \( X \) (e.g., \( t \) units before consumption or processing of inputs).

To simplify the analysis, equation 3.2 will be used as the transformation function linking the futures and cash markets for these products not yet marketable. This equation implicitly assumes that at the time of product inputs begin transformation the final output is known exactly. Some error term could be added to the transformation function, but it
will be ignored here to reduce the complications that are later encountered.

It was stated that \( Z \) (storable goods) can be hedged up to the difference between consumption and supply.\(^1\) For the nonstorable product hedging can account for the total supply of finished goods \( T(X) \) or any level less than \( T(X) \). But such hedges must occur at the time of initial input as was shown through the micro model. If \( T(X) \) is expressed in pounds, for example, then hedging supply can occur up to the maximum pounds of \( T(X) \). Define:

\[
T(X)_H = \text{units (supply of futures) of transformed goods hedged short,}
\]

\[
T(X)_u = \text{units of transformed goods unheded.}
\]

The total supply of transformed goods to be forthcoming at maturity then consists of two components, i.e., that of hedged and unhedged commitments. Through equation 3.4 the relationship is set forth to directly relate the futures market to the

\[
T(X) = T(X)_u + T(X)_H
\]

or

\[
X G(L^*) = T(X)_u + T(X)_H. \tag{3.4}
\]

\(^1\) Most storables must go through some transformation process. For example, corn production may be considered analogous to beef production and hence the transformation function would be relevant to both. Yet at harvest time corn may be subject to a storage function, whereas nonstorables such as beef are not. Again the time framework would be \( t>0 \), when corn production began, through \( t-n \) when harvest occurs. Hence the present theory is applicable to commodities other than beef.
initial input market X. Any change in the input of X must always be accounted for by corresponding changes in the levels of hedged and unhedged stocks. Input market equilibrium can now be derived.

Input Market Equilibrium

The final level of output depends directly on the level of input X. Hence equilibrium within the input market exactly determined the final output of transformed inputs. This must be conditioned on the assumption that transformation services are not a limiting resource within the given transformation period. There is no scarcity of transformation resources or at least the limits to transformation capacity are far above that level needed for any given input equilibrium level. The supply of inputs can be expressed as a function of the input price existing and as well some exogenous variable. At the same time the supply of transformation services will be assumed independent of the transformation price (e.g., the assumption of no scarcity of transformation resources is consistent with this). Define:

\[ \begin{align*}
    P_X &= \text{input price of nonstorable product X}, \\
    P_L &= \text{cost of transformation services per unit of contribution,}\;^{1}
\end{align*} \]

\(^1\)It is recognized that \( P_L \) probably should be discounted since transformations are not instantaneous.
a = some exogenous supply of input variable.

The supply of inputs vary directly with $P_X$ and $a$ as in equation 3.5. Postulated responses for this implicit function

$$X = X(P_X, a) \quad \ldots \ldots \text{Supply of input}$$

and

$$X = \bar{X} \quad \ldots \ldots \ldots \text{Demand equals supply}$$

will be stated later. The inputs of equation 3.4 must always be equal to that of 3.5 for input market equilibrium to exist. The demand for input $X$ is implicit to the relationships of equations 3.6 and 3.7.

At the same time the two components of the transformation function are related to the price sets through some implicit relationship. The responses within the micro model give clues as to the aggregate responses. Firms (market participants) supplying and demanding transformation services are related to observed prices and they also have expectations about forthcoming prices. These positions of transformation unhedged are completely independent of the futures price $P_F$, while hedging commitments are not. Both $T(X)_u$ and $T(X)_H$ vary with transformation cost $P_L$ and some price expectation. Also they must be related to the initial input cost $P_X$. Equations 3.6 and 3.7 expressed these implicit relationships. Short hedgers

$$T(X)_u = T_u(P_X, P_L, e) \quad (3.6)$$

$$T(X)_H = T_H(P_F - P_X, P_L, e) \quad (3.7)$$
of futures are functionally related to the spread of observed futures and input prices. This follows directly for those conditions illustrated in the micro model. Other exogenous variables could possibly be included, but the restrictive functions above will be sufficient for the present framework. The input market equilibrium then is derived as in 3.8. Define:

\[ P_f = \text{futures price of contract traded at time of initial inputs}, \]
\[ e = \text{market price expectations for transformed inputs}, \]
\[ \Delta = P_f - P_x = \text{futures-cash price spread}. \]

\[ X(P_x, a) G(L^*) = T_u(P_x, P_L, e) + T_H(\Delta, P_L, e) \]

Equilibrium

Equation 3.8 must always be satisfied within the framework of price discovery.

Futures Market Equilibrium

Trading activities with the futures market are such that those supplying futures contracts will just equal the demand for futures. The micro model established the situations where futures may be supplied (short futures) or demanded (long futures). In other words the trading exchange always facilitates the adjustment where aggregate short positions just equal the long commitments. This must hold irregardless of the type of futures traded. Further for contracts in nonstorables such as livestock, futures commitments are
usually supplied by the same firms or participants that demand and supply transformation services, while the net long positions are either by speculators or processors (see micro model for further explanation). Define:

- \( F_L \) = long futures positions by hedgers,
- \( F_S \) = net long futures positions by speculators (e.g., long minus short speculation),
- \( b \) = exogenous speculative variable.

The futures market will be in equilibrium as long as equation 3.9 holds. Each commitment is expressed as an implicit function of prices. Long traders respond to present futures

\[
T_H = F_L + F_S \tag{3.9}
\]

\[
T_H(\Delta, P_L, e) = F_L(P_f, e) + F_S(P_f, b) \quad \text{Supply=Demand for Futures} \tag{3.10}
\]

prices and some exogenous component \( b \).

Industry Equilibrium

Industry equilibrium implies that the spread between the futures price and the initial input price just reflects the cost of input transformation. This observation will be shown below. The concept of spread between futures and cash has usually been expressed as \( P_f - P_z \) for those products that are now marketable, where the product at the initial point of storage differs from the final consumption only by the amount of storage. Futures trading for nonstorables
always begins prior to the complete product transformation of the inputs. At the start of the transformation process the futures price reflects prices in terms of finished (transformed) commodities, while the input cash price $P_X$ is in terms of inputs requiring transforming services. These prices must be weighted or expressed in equivalent units for meaningful conclusions to follow. There exist a cash price for the nonstorable commodity at maturity (e.g., there is always a vector of fed cattle prices), but the futures-cash spread at some point $t > 0$ cannot apply to this price since the finished commodity cannot be transformed through storage.

The assumed framework functions around a perfectly competitive market structure. Given the existence of an open market and sufficient time for adjustment (e.g., no barriers to entry or exit) then market profits from product transformation should be zero (e.g., the total revenue from the transformed goods will just equal the cost). This adjustment occurs within a long run equilibrium framework. Let some $P_T$ be the price of the transformed inputs, then equation 3.11 can be determined. Entry and exit from the industry should reduce or increase profits just to zero as with 3.11. At the time these inputs $X$ are demanded, the final output price $P_T$ is unknown. But suppliers of futures (e.g., the short
hedgers) sell futures up to the aggregate amount of \( T(X) \) for price \( P_f \). Substituting \( P_f \) for \( P_T \) in equation 3.11 can be used for arriving at minimum pricing conditions for the short hedger. If 3.11 were consistently less than zero after this substitution, then the level of short hedging may decrease. If 3.11 after the substitution were consistently greater than zero then short hedging may increase since profits are greater. This increase or decrease in hedging will force 3.12 to zero. Hence, from this conclusion the relationship of futures-cash spread can be related directly to the cost of transformation. Adding \( P_f \) to both sides of equation 3.12 gives equation 3.13, the desired equation. The price spread

\[
\begin{align*}
P_f T(X) - P_X X - P_L [T(X) - X] &= 0 \quad (3.12) \\
P_f X G(L^*) - P_X X - P_L X [G(L^*) - 1] &= 0 \\
-P_X &= -P_f G(L^*) + P_L [G(L^*) - 1] \\
P_f - P_X &= [P_f - P_L] [1 - G(L^*)] \quad (3.13)
\end{align*}
\]

directly responds to the cost of input transformation \( P_L \). Equation 3.13 must also be evaluated at different times from maturity. As \( t \) approaches maturity or as \( X \) is completely transformed, the spread of equation 3.13 must approach zero.\(^1\)

\(^1\)The futures price \( P_f \) must be for the futures contract with maturity date nearest to that of input maturity date. If \( T(X) \) occurs in February, the \( P_f \) is in terms of February futures contracts.
At \( t = 0 \), \( X(0) \) requires no transformation. Express:

\[
\lim_{t \to 0} X(t) G(L^*(t)) = X(0), \text{ input = output}
\]

\( G(L^*(0)) = 1 \),

\[
\lim_{t \to 0} [P_f - P_x] = (P_f - P_L) [1 - \lim_{t \to 0} G(L^*(t))],
\]

\[
\lim_{t \to 0} [P_f - P_x] = [P_f - P_L][0] = 0,
\]

or

\[
\lim_{t \to 0} [P_f(t) - P_x(t)] = 0.
\]

These limits show that when the \( X \) input requires no transformation, then the futures-cash price spread should theoretically be zero.

The futures and input prices may be such that \( P_f > P_x \). Hence the price spread of equation 3.13 may be positive, zero, or negative and still satisfy the relationship of 3.13. This will depend directly on the value of \( P_L \). Later delineation of axes for equilibrium analysis will follow from this observation.

Initial Equilibrium

The entire discussion can be presented geometrically if the appropriate market responses are postulated. Figures 3.1a through h provide this graphic aid. An equilibrium in all sections of the framework is illustrated.
Figure 3.1. Geometric framework for intertemporal futures and input price discovery
The first set of postulates are made in conjunction with the brief interpretation of Figure 3.1. It is postulated that:

(1) \( \frac{\partial TH}{\partial \Delta} = T_H^\Delta > 0 \) ... Hedging Response Fig. 3.1b

(2) \( \frac{\partial F_L}{\partial F_p} = F^p_L f < 0 \) ... Long Hedge Fig. 3.1c

(3) \( \frac{\partial F_S}{\partial F_p} = F^p_S f < 0 \) ... Speculation Fig. 3.1h

(4) \( \frac{\partial L}{\partial X} = L^* \) ... Transformation Services Fig. 3.1e

(5) \( \frac{\partial T(X)}{\partial X} = G(L^*) \) ... Transformation Fig. 3.1a

(6) \( \frac{\partial X}{\partial P_x} > 0 \) ... Supply of inputs Fig. 3.1f

Starting with Fig. 3.1a, the output increases directly with increases in inputs via postulate (5). Short hedging increases with larger price spreads, but long hedges decrease as futures prices increase (postulates (1) and (2) and Figures 3.1b and 3.1c). Net speculative activities vary inversely with futures prices as in postulate (3) and Figure 3.1h. The ratio of transformation services to initial inputs (Figure 3.1e) is constant at L*, while the supply of input (Figure 3.1f) changes directly with input price via postulates (4) and (6). Finally, 3.1i illustrates the spread from equation 3.13. With these postulates and the market
structure framework established, the preparatory step for intertemporal price discovery is set.

Intertemporal Price Discovery

At the times futures commitments and transformation commitments are established only two prices relevant to the assumed framework are directly observable, i.e., input and futures prices. In addition transformation costs exist but are assumed exogenously determined. Through equations 3.8, 3.10, and 3.13 these prices can be determined simultaneously. For convenience, the values of $\Delta$(price spread) and the futures price $P_f$ are used as the coordinates for price discovery at equilibrium.¹

Input market price coordinates

Taking equation 3.8 with $P_L$ and $e$ fixed ($P_L^0$, $e^0$), then all possible combinations of price spreads and futures prices can be calculated that satisfies equation 3.14. Differentiating equation 3.14 and solving for the relative response of the spread to changing futures prices gives the desired solution. Equation 3.16 reveals that as futures prices increase, the spread $\Delta$ must also increase if input market equilibrium is

¹The concept of economic equilibrium is used within the context of this study as a state of balance between opposing forces acting upon economic variables. An equilibrium model is then the end product from the construction of a system based on solutions derived from identifiable opposing forces [78, p. 18].
to be maintained. The input price \( P_x \) and futures \( P_f \) are related through \( \Delta \), but through the manipulative procedure the actual \((P_x, P_f)\) relation was not revealed. Any increase in \( P_f \) will increase the desired level of short hedging. Hence, for equilibrium to again follow, the supply of inputs must increase, the level of unhedged stocks decrease, or some combination of both (see equation 3.4). Excess input demands raise \( P_x \); input supplies increase and unhedged stocks decrease. Since a change in \( P_x \) produced both desired effects (e.g., \( T_u \) decreases and \( X \) increases), the needed change in \( P_x \) is somewhat dampened. Both \( P_f \) and \( P_x \) have increased, but the change in \( P_x \) will be less than that of \( P_f \) due to the twofold effect outlined. Therefore \( \frac{\partial \Delta}{\partial P_f} \) is positive. Contour \( T T \) of Figure 3.2 gives all equilibrium coordinates \((\Delta, P_f)\) necessary for input equilibrium.
Futures market price coordinates

Equation 3.10 with $P_L^0$ and $e^0$ is differentiated in a manner similar to that completed above.

$$\frac{\partial \Delta}{\partial P_f} = \frac{P^f_L + P^f_S}{T_H}$$

Both net speculative and long hedges respond negatively to increasing futures prices. The demand for futures has been decreased. Yet the supply of futures is directly related to the futures-cash spread. For equilibrium to continue, the supply of futures must decrease via a decrease in price spread. As $P_f$ increases the spread $\Delta$ must decrease for futures market equilibrium to exist. Equation 3.18 must hold. The locus of all equilibrium coordinates for the futures are drawn in Figure 3.2 as line FF.

Industry equilibrium price coordinates

Differentiating equation 3.13 and solving for $\frac{\partial \Delta}{\partial P_f}$ is sufficient for determining the final set of price coordinates. The difference between final transformation $T(X)$ and initial inputs must be due to transformation services; $G(L^*)>1$ since transformation services are positive.

$$\Delta_E = (P_f - P_x) = [P_f - P_L^0][1 - G(L^*)]$$
\[ \frac{\partial \Delta E}{\partial P_f} = 1 - G(L^*) < 0 \]  

Equation 3.20 is then negative. As futures prices increase, the spread must decrease if the futures-cash spread just reflects the cost of transformation. These coordinates lie along the line $EE$ of Figure 3.2.

\[ X[G(L^*) - 1] > 0 \] or \[ G(L^*) - 1 > 0 \] 

\[ G(L^*) > 1. \]

**Equilibrium**

In Figures 3.2 and 3.3 a graphical representation of the existence of an equilibrium is derived. When there exist a set of coordinates $(\Delta, P_f)$ where the input market is in equilibrium, the futures market is in equilibrium, and the price spread just reflects the cost of transformation; then those coordinates will define an industry equilibrium point. Whereas, when only the input and futures market equilibriums are satisfied simultaneously, then a market equilibrium exist. Hence industry will denote a situation where equations 3.8, 3.10, and 3.13 are simultaneously satisfied. If only 3.8 and 3.10 are satisfied, then market denotes the equilibrium coordinates.

Given the transformation function $G(L^*)$ and cost $P_L$, then $EE$ traces a path of coordinates that must exist for an industry equilibrium. The intersection of $TT$ and $FF$ must be on $EE$ for industry equilibrium. The realized spread will
Figure 3.2. Coordinates from equilibrium with slope EE > slope FF

Figure 3.3. Coordinates for equilibrium with slope FF > slope EE

TT = Input market equilibrium
FF = Futures market equilibrium
EE = Δ reflects transformation cost
L'prcdd = A
sloDe = l-6<0
TT = input market equilibrium
FF = futures market equilibrium
EE = Δ reflects transformation cost

spread = Δ

futures price = $P_f$
slope = 1-G<0

O

E

F

T

P_f

E
not just reflect the cost of transformation if market equilibrium is off E E (e.g., T T and F F intersect off locus E E). The next two chapters will delve into the implication of this condition as well as deriving some meaningful propositions through comparative static procedures.
CHAPTER 4: PRICE DISCOVERY THROUGH COMPARATIVE STATICS

Comparative statics is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogenous variables. The system of equilibrium equations established coordinates between the price spread \( \Delta \) and the futures price \( P_f \). As exogenous variables in the assumed framework change, there must be corresponding changes in equilibrium. This and the next chapter will evaluate two forms of equilibrium: market versus industry equilibrium. Presently, concern is with deriving some meaningful propositions regarding comparative statics and the existence of market equilibriums. Market equilibrium implies that both equations 3.8 and 3.10 are satisfied simultaneously.

Comparative statics can be either qualitative or quantitative in nature. In this chapter interest is limited to the qualitative analysis where the direction of change is considered. It will follow that in the process of determining the qualitative character of a system, some quantitative restrictions must be assumed. Also as was utilized in previous sections, all analysis is put in a continuous framework while in reality the framework is discrete in form (this enables the use of differentiable calculus).
Four distinctive exogenous variables were included in the model: cost of transformation, cash price expectations, degree of speculative interest, and changing supply of inputs requiring transformation. Through independent changes in these four variables some meaningful insights about the functioning of cash and futures trading in products requiring transformation can be gained. These four variables do not exhaust the complete listing of all exogenous elements entering the system, but they may be considered to be among the most significant ones and may be indicative of other forces. Other exogenous forces such as weather, politics, etc. may be thought to be influential. Such forces may result with some impact on those exogenous variables included in this comparative static model. The methodology for theoretical solutions through these four variables can be directly applied to any other variable entering the present framework.

Rewriting the framework of relationships again may be useful prior to the comparative static conclusions (also see Chapter 3):

\[ T(X) = X G(L^*) = T_u + T_H \] ... Transformation function,

\[ T_u = T_u(P_x, P_L, e) \] .............. Unhedged stocks,

\[ T_H = T_H(\Delta, P_L, e) \] .............. Short hedged stocks,

\[ F_L = F_L(P_f, e) \] .............. Long hedges,

\[ F_S = F_S(P_f, b) \] .............. Net speculation,

\[ \Delta = P_f - P_x \] .............. Price spread,
\[ \Delta E = [P_f - P_x] = [P_f - P_L] [1 - G(L_*)] \] Industry equilibrium,

\[ \bar{X} = X(P_x, a) \] Supply of inputs,

\[ T_u + T_H = X G(L_*) \] Input-transformation equilibrium,

\[ T_H = F_L + F_S \] Futures equilibrium.

The postulated responses assumed are:

\[ P_x < 0, P_L < 0 \] Unhedged responses,

\[ T_u < 0, T_H < 0 \] Hedged responses,

\[ F_L < 0 \] Long hedge response,

\[ F_S < 0 \] Speculative response,

\[ X^P > 0 \] Supply of input response.

With these definitions and assumptions some useful observations about futures trading in nonstorable goods such as livestock can be cited. Again nonstorable implies that such goods must be consumed or processed at maturity and cannot be prior to maturity.

**Changing Transformation Costs**

Increasing the cost of transformation directly alters the entire market equilibrium, and as well the industry equilibrium function must change. Differentiating each equilibrium function and solving for the relative price changes provide
useful results. The differentiations of the input market, futures market, and industrial equilibrium relationships are shown in equations 4.1, 4.2, and 4.3.

\[
\frac{\partial P_L}{\partial P_L} + T_{\text{u}}^{\text{P}} + T_{\text{H}}^{\text{A}} \frac{\partial \Delta}{\partial P_L} + T_{\text{H}}^{\text{P}} = \frac{\partial P_f}{\partial P_L} \frac{\partial^2 P_f}{\partial^2 P_L} + \left( \frac{\partial P_f}{\partial P_L} - \frac{\partial \Delta}{\partial P_L} \right) G(L^*) \tag{4.1}
\]

\[
\frac{\partial \Delta}{\partial P_L} = \left( \frac{\partial P_f}{\partial P_L} - 1 \right) \left[ 1 - G(L^*) \right] . \tag{4.3}
\]

The direct effect of \( P_L \) and \( \Delta \) can be determined if the futures price is temporarily held fixed at the initial level (e.g., \( P_f^0 \)). Hence the solutions for \( \frac{\partial \Delta}{\partial P_L} \) in equations 4.1, 4.2, and 4.3 are shown in equation 4.4, 4.5, and 4.6. If \( P_f \) is fixed then \( \frac{\partial P_f}{\partial P_L} = 0 \).

\[
\frac{\partial \Delta_T}{\partial P_L} = \frac{-\left[ T_{\text{u}}^{\text{P}} + T_{\text{L}}^{\text{P}} \right]}{\left[ T_{\text{H}}^{\Delta} - T_{\text{u}}^{\text{P}} + X^P x G(L^*) \right]} = \frac{-[(-) + (-)]}{+} > 0 \tag{4.4}
\]
\[
\frac{\partial \Delta_F}{\partial P_L} = -\left[ \frac{T_{NH}^{PL}}{T_H^{\Delta}} \right] > 0 \quad (4.5)
\]

\[
\frac{\partial \Delta_E}{\partial P_L} = [G-1] > 0
\]

Where \( \Delta_T \) is \( \Delta \) with TT Shift

\( \Delta_F \) is \( \Delta \) with FF Shift

\( \Delta_E \) is \( \Delta \) with EE Shift

(4.6)

Graphically these responses are shown with Figure 4.1 a,b,c where changing \( P_L \) upward shifts TT to T'T', FF to F'F', and EE to E'E'. The futures-cash spread has increased in all cases, but what about the futures price? Will a final equilibrium (intersection T'T' and F'F') result at \( P_f^0 \) or must \( P_f \) also change? In this chapter concern is only with market equilibrium (e.g. Figures 4.1 a and b). It is again noted that TT is the locus of cash market equilibrium coordinates, FF are equilibrium coordinates for the futures market, and EE are coordinates where the cost of transformation is just reflected by the price spread.

Increases in the transformation cost per unit have the first and most direct impact on the transformation side of the markets. Total placements of inputs and transformation procedures are depressed since to produce the same final
product now costs more. To the extent that hedged and unhedged transformation demands are reduced, then supplies of initial inputs X must decrease. For supplies to be reduced, \( P_x \) must decrease. Hence the spread \( (P_f - P_X) \) increases and \( T'T' \) results with the initial futures price, \( P^0_f \) as in Figure 4.1a.

Supplies of futures contracts have been dampened via the reduction in \( T_H \) (e.g., there is a reduction in short hedging). Either or both long hedges and net speculative positions must decrease in response to changing \( T_H \). This occurs only with some change in the futures price \( P_f \). Changing transformation costs have indirectly influenced the long side of the futures market through an equilibrium adjustment between net long and net short positions. Given \( P^0_f \), the shift in futures exists with \( F'F' \), Figure 4.1b. Initially \( P_f \) was fixed while \( P_L \) increased. If futures prices are constant then the long or demand for futures will not respond (under the assumed framework), hence increases in the spread between cash and futures.
must occur. Increases in \( P_L \) dampens supplies of futures \( T_H \), but a increase will offset this movement of futures if equilibrium continues. Through this rationale the shift to \( F'F' \) follows.

Following some exogenous increase in transformation cost, it can be established that a new market equilibrium may exist with the initial futures price. Cost \( P_L \) increases and the demand for transformation services decreases. But if the futures price remains at the initial level, the demand for futures cannot respond to decreased short hedging desires. If so, then all reductions in transformation must be with unhedged stocks (e.g., \( T_u \) decreases and absorbs the full impact of increasing transformation costs). Speculation and long hedging will change via the endogenous variable \( P_F \). It then follows that if market equilibrium occurs, the change in spread between input and futures markets are just equal for the given futures price as in equation 4.7 (e.g., equations 4.4 and 4.5 have been equated). Ultimately, the exact response between \( P_F \) and \( P_X \) can be derived that will always be consistent with the new market equilibrium. Through equations 4.7, 4.8, and 4.9 this response is derived. Figure 4.2 illustrates the situation where a new market equilibrium is consistent with the initial futures price. Hence the following proposition can be stated.
$$-\left[\frac{T^{PL} + T^{PL}}{T^H - T^P + X^P G(L^#)}\right] = -\frac{T^P}{T^H}$$  \hspace{1cm} (4.7)$$

$$\frac{X^P G(L^#) - T^P}{T^H} = \frac{T^P}{T^H}, \quad T^P_x = T^H \frac{\partial T^H}{\partial P_x}$$  \hspace{1cm} (4.8)$$

$$\frac{\partial P_f}{\partial P_x} = 1 + \frac{T^P}{T^H}$$  \hspace{1cm} (4.9)$$

---

Figure 4.2. Market equilibrium with \( P_f^0 \) and changing \( P_L \)

**Proposition 1:** Given an exogenous increase in transformation cost within the meaning of the assumed framework, then an equilibrium price spread change can exist at the initial futures price and still be consistent with both futures and input markets.
The major implication of this is that increasing (or for that matter, decreasing) transformation costs are not necessarily indicative of changes within the futures market. Unhedged activities can absorb the full impact of increasing transformation costs.

In contrast to the above proposition where unhedged stocks absorb the impact of increasing service costs, hedged stock commitments now absorb some of the impact of increasing transformation cost. If this situation arises, responses have been such that short hedges may have been reduced. Thus both $T_u$ and $T_H$ decrease resulting in decreases in supply of inputs via decreases in $P_x$. If supplies of futures $T_H$ decrease then the net long positions or demand for futures must adjust downward. Finally, a new market equilibrium must be situated to the right of the initial futures price and above the initial price spread. With these responses it follows that $\Delta_{PL} > \Delta_{PL}$ or that the determinant of $M_{1.1}$ in Appendix C.1 must be less than zero. Equation 4.10 and 4.11 along with Figures 4.3 suffice to explain this situation. Both $TT$ and $FF$ have shifted upward in Figure 4.3 and they intersect above and to the right of the old equilibrium point $A$. This result leads to proposition 2.

$$\frac{T_{PL}^u \cdot P_x \cdot G(L^*) - T_{PL}^x}{T_{PL}^H} = \frac{T_{PL}^x}{T_{PL}^H} = \frac{T_{HL}^\Delta}{T_{HL}^H} = \frac{\Delta P}{\Delta P_x} = \frac{\partial \Delta}{\partial P_x}$$

(4.10)
\[ \frac{\partial P_f}{\partial P_x} < 1 + \frac{T_u^{PL}}{T_H^{PL}} \]  

(4.11)

Figure 4.3. Market equilibrium with \( \Delta P_L > 0 \) and

\[ \frac{\partial P_f}{\partial P_L} > 0 \]
Proposition 2: Within the meaning of the framework of market equilibrium where both the futures-cash price spread and the futures price can change, then some exogenous increase in transformation costs $P_L$ will produce market adjustments where both the price spread $\Delta$ and the futures price $P_f$ increase if in fact the ratio of unhedged to hedged responses to increase costs exceed the response of price spread to changing input cost $P_X$ (e.g., both $\frac{\partial \Delta}{\partial P_L}$ and $\frac{\partial P_f}{\partial P_L}$ are positive if equation 4.11 is satisfied).

Proposition 2 can be very useful from both a qualitative and quantitative analysis. It first emphasizes the role of market structure in the discovery of intertemporal price movements. Possible movements in both markets can be extrapolated. Also using this proposition may give some clues as to the proper operations of the market. Response $\frac{\partial P_f}{\partial P_X}$ cannot be directly calculated, but may be approximated by using discrete data. Similarly hedging to unhedging commitment responses may be estimated possibly by some form of linear analysis. Taking these estimates:

$$\beta_1 \sim T_u^p L$$

$$\beta_2 \sim T_H^p L$$

$$\beta_3 \sim \frac{\partial P_f}{\partial P_X}$$
Then if \(1 + \frac{\beta_1}{\beta_2} > \beta_3\) futures prices and cash-futures spread should be increasing as \(P_L\) increases, assuming other factors remain the same. If \(1 + \frac{\beta_1}{\beta_2} = \beta_3\) then the spread increases with larger \(P_L\) and futures price should remain constant. In addition, if information is available, \(\beta_1\) could be expressed in a dynamic sense, say \(\beta_1(t)\). The application of proposition 2 is then directly applicable to any of the points of inputs necessary for the final output \(T(X)\). It is recalled that initially the assumption that inputs necessary for final outputs \(T(X)\) could occur at one input time only.

Finally the alternative of a decrease in futures prices as cost of transformation per unit increases has yet to be considered. Reference to the previous graph reveals that if the intersection \(T'T'\) and \(F'F'\) exist to the left of the initial futures price, it must be that the response condition

\[
1 + \frac{T^L_{uP}}{T^L_{uH}} \text{ is less than } \frac{\partial P_f}{\partial P_x}.
\]

This follows in that with the initial \(P_f\) the \(\Delta^PL > \Delta^PL\) (e.g., the spread change due to changes in the input market exceeds that of the futures market). This in turn implies directly that determinant \(|M_{1,1}|\) (see Appendix C.1) is greater than zero, hence \(\frac{\partial P_f}{\partial P_L} = \frac{|M_{1,1}|}{|M_{1,1}|} < 0\). Both the supply and demand for contracts in an equilibrium state must increase if \(P_f\) ultimately decreases.
But increasing $P_L$ reduced unhedging activities, thus leading to the observation that the ratio of unhedged to hedged transformation services must decrease under these circumstances.

Price discovery with moving market equilibriums via changing transformation cost has been discussed. The price spread will always respond positively with increasing transformation costs, but the futures price movements must be determined in light of the elements of the market structure, specifically the responses of hedged and unhedged transformation services. Once the prices are discovered it is only one step back to determining equilibrium volume commitments.

Changing Cash Price Expectations

Generally all participants in the markets have some expectation about forthcoming prices. As was discussed in the micro model, the optimal decision for both markets (cash and futures) resulted directly from the individual expectations. The transformation of $X$ results in the output where the final market price is unknown. Similarly the terminating price of futures is unknown. Each decision maker's set of considerations leading to his expectations may be unique. Their aggregation over expectations may not be meaningful. Stein suggests an aggregative procedure weighting each expectation by some probability distribution [107, p. 16]. Telser associated an averaging technique of expectations of representative of the industry. He reasons that the firm's
expectations tend to be correct on the average. If firms consistently miscalculated the price change the firms clearly make abnormal gains or constantly suffer losses. In the latter those firms are eliminated from the industry while consistent abnormal profits induce greater entry. Hence...

"the persistence of firms in the industry, on the one hand, and the stability in the number of firms in the industry, on the other hand, create the presumption that, on the average, neither large losses nor large profits occur. Therefore on the average, the expected change in price equals the realized price change" [112, pp. 237-238]. Without providing an exact technique for arriving at the representative industrial cash-price expectation, it has been assumed to be e, (e.g., e = the expected price of \(X G(L^*)\)). As e changes how must the prices respond for the nonstorable market with futures transactions? Considerations here are only for changes in the price expectations in the cash market. The same methodology is directly applicable to futures price expectations given the necessary postulated conditions.

Given an increase in the expected price of the transformed product whose price is presently unknown, then both the input-transformation and futures markets will be affected directly, as well as indirectly. Utilizing the previous technique where futures prices are constant, then any increase in expectations must produce downward shifts in the equilibrium locus TT, see
Figure 4.4. Increases in $e$ make both unhedged and hedged demand more profitable (subjectively in the eyes of the decision makers). Increase demand for inputs produce increases in the input price $P_x$. Implicit to this is the assumption that the supply of transformation services are unlimited and can adjust to changing inputs without influencing the transformation cost $P_L$.\footnote{This may be rationalized for feeder cattle in that aggregate feedlot facilities, storage, and other services usually consist of fixed facilities that do not readily vary. But the number of inputs into such facilities can vary up to its capacity. Feed cost may differ somewhat.} There is no scarcity of transformation services. As $P_x$ rises, the supply of inputs increase since $X^P_x > 0$. Therefore the spread must decrease at the given $P_f$ if the input market adjustment remains in equilibrium. TT will shift to $T'T'$ in Figure 4.4. Also equation 4.12 substantiates this relationship.

![Diagram showing the relationship between input and output prices with a shift from T to T']
Higher expectations regarding the price of the transformed inputs produce conflicting responses in the futures market. Long hedgers, who primarily are the purchasers of forthcoming transformed inputs, view the increase in e as rising cost for needed inventories. Yet short hedgers see this as opportunity for abnormally higher profits. Hence, an unusually high expected cash price produces adjustments which ultimately reduce the planned inventory purchases and correspondingly long hedges are reduced (e.g., the demand for futures falls). This same high expected cash price strengthens the supply for futures, thus giving an excess supply of futures contracts where supplies increase and demands decrease. With a constant futures price, futures equilibrium can exist only if short hedging decreases via a decrease in the price spread. Therefore the shift of FF to F'F' occurs with the initial futures price. See Figure 4.5 and equation 4.13.

The price spread must always decrease as expectations increase (e.g., the spread varies inversely with expectations), but conclusive statements about the final futures price must be evaluated under alternative circumstances.

Can adjustments from one market equilibrium to another as expectations increase occur without changes in activities
of speculators? Specifically when can market equilibrium be maintained without movements in the futures price? Market equilibrium with unchanged futures $P_f^0$ implies that some set of coordinates $(\Delta, P_f^0)$ just satisfies both input and futures market equilibrium equations. Given the assumed framework where only expectations are exogenously varied, then the fixed futures directly implies no change in speculative commitments (e.g., $F_S = F_S(P_f, b)$).

If contours $T'T'$ and $F'F'$ of the previous two figures just intersect at the price $P_f^0$ (the initial equil. price $P_f$); then both the input and futures should respond equally to changing expectations, see Figure 4.6. In essence the shift
in the cash market equilibrium locus must equal the shift in futures locus (e.g., shifts in TT and FF). Futures price has not changed, then \( \Delta_T^e = \Delta_T^e \). \( \Delta_T^e \) and \( \Delta_F^e \) follow in equation 4.14 where equation 4.12 is equated to 4.13.

![Diagram showing cash and futures markets](image)

Figure 4.6. Increasing expectation with stable futures price

\[
\frac{-(T_u^e + T_H^e)}{[T_H^\Delta - T_u^p + p^p x G(L^*)]} = \frac{F_L^e - T_H^e}{T_H^\Delta}
\]

(4.14)

or

\[
-T_u^e T_H^\Delta = [F_L^e - T_H^e] [p^p x G(L^*) - T_u^p x] + F_L^e T_H^\Delta
\]

It was shown that when the input market is in equilibrium, the \([p^p x G(L^*) - T_u^p x] = T_H^p x\) and \(T_H^p x = T_H^\Delta \frac{\partial \Delta}{\partial p} \). Substituting this into equation 4.14 gives the alternative relationship of equation 4.15. In addition, speculative activities do
not vary since the final futures price $P_f$ remains unchanged (e.g., $P^0_f$).

$$\frac{\partial \Delta}{\partial P_x} = \frac{-(P^e_L + T^e_u)}{(P^e_L - T^e_H)}$$ (4.15)

Equation 4.15 sets forth the restriction that must exist if market equilibrium continues without changes in the futures price. The basic assumption here is that futures prices can vary in the adjustment process as expectations change, but the final equilibrium point after such adjustments can occur with a zero net change in futures prices. The direct implication is that given the necessary responses by hedgers to changing expectations, then it may not be necessary for speculative adjustments to occur (e.g., speculation will change here only if the net change in futures price differs from zero). Finally 4.15 can be expressed with the alternative form as in equation 4.16 where the relationship between input price and futures price is derived. If 4.16 is satisfied then market equilibrium can exist following the adjustment process to greater cash price expectations.

$$\frac{\partial P_f}{\partial P_x} = \frac{-(T^e_H + T^e_u)}{(P^e_L - T^e_H)}$$ (4.16)

The above conditions are summarized with proposition 3.

**Proposition 3:** If a zero net change in futures prices is associated with increasing price expectations, it then
follows that a new set of equilibrium coordinates consistent with market equilibrium exist if the restriction of equation 4.16 is satisfied.

Futures price may change as expected cash prices increase. The realized direction of this price shift depends on the relative responses of demand for hedged and unhedged transformation services to these expectations. The final response of $P_f$ can be derived after some limits or conditions about market responses are determined. If market equilibrium adjustments result in decreases in futures prices (e.g., $\frac{\partial P_f}{\partial e} < 0$), then the response of $\Delta P_f$ in the futures market must exceed that of the input market at the original futures price. Equivalently, from Appendix C.2, matrix determinant $|M_{2,1}| > 0$ indicates a final intersection of $T'T'$ and $F'F'$ where both the price spread and futures price decrease. Graphically, this situation is illustrated in Figure 4.7.

![Figure 4.7. Increasing price expectations](image)
Given $|M_{2.1}| > 0$, then the following restrictions are apparent within the response system. If $|M_{2.1}| > 0$ and $|M_2| < 0$, then the derivative $\frac{\partial \bar{p}_f}{\partial e} = \frac{|M_{2.1}|}{|M_2|} < 0$; hence equilibrium must occur with lower futures prices. A solution from solving $|M_{2.1}| > 0$ follows in equations 4.17 and 4.18.

\[
\frac{\partial \Delta}{\partial \bar{p}_x} > \frac{[T_u^e + F_L^e]}{[T_H^e - F_L^e]} \quad \text{(see Appendix C.2)} \quad (4.17)
\]

\[
\frac{\partial \bar{p}_f}{\partial \bar{p}_x} > \frac{[T_H^e + T_u^e]}{[T_H^e - F_L^e]} = K > 0 \quad (4.18)
\]

Finally the condition to assure that the futures price will fall given some price expectation increase can be expressed in terms of observed futures and initial input prices. Equation 4.18 establishes that if the response of futures prices to input prices exceeds some positive constant $K$, then market equilibrium adjustments to expectations result with decreases in the price spread and futures price. Further it was shown that the spread must always fall regardless of the direction of movement of $P_f$. This restriction can be defined:

\[0 < K < \frac{\partial \bar{p}_f}{\partial \bar{p}_x} \]
Proposition 4: Exogenous raises in the expected price of the transformed product may produce decreasing futures-input price spreads and a decreasing futures price as both markets adjust to new equilibrium points. If the relative response of futures price to input price is greater than some positive constant K, the above change in equilibrium coordinates (Δ, Pf) will result.

Greater price expectations can have the opposite effect to that of proposition 4. Equilibrium in the futures and input markets must exist with decreasing spreads (Δ) when expectations increase, but the futures price may increase (e.g., ∂Pf/∂e > 0). For this to hold |M2.1| < 0 since |M2| < 0, see Appendix C.2. Solving these matrices gives the necessary restrictions for increasing futures. Therefore, if

\[
\frac{\partial \Delta}{\partial P_x} < \frac{T^e_u + F^e_L}{T^e_H - F^e_L} \tag{4.19}
\]

\[
\frac{\partial P_f}{\partial P_x} < \frac{T^e_H + T^e_u}{T^e_H - F^e_L} = K \tag{4.20}
\]

the market structure is such that ∂Pf/∂Px is less than the constant K, then an increase in expected price will produce decreases in the price spread and increases in the futures price at the new equilibrium coordinates. Equilibrium coordinates at
point A, Figure 4.7, now move to a new point B' where B' would be to the right of A.

It has been shown that the theoretical role of price expectations can have significant influence within both the futures and input markets. Further if knowledge of the market structure is even partially known, then meaningful propositions can be derived. Specifically, in a period of general increases in prices of transformed goods, the futures-input price spread should always decrease and futures price will decrease if

\[ \frac{\partial P_f}{\partial P_x} > K \] and increase if \[ \frac{\partial P_f}{\partial P_x} < K. \] As always comparative static conclusions must be limited to interpretation for those exogenous changes occurring with everything else fixed.

Exogenous Chance in Speculation

Speculative activities can increase due to exogenous forces, hence with the present framework such forces are directly evident through the function \( F_g \). Increasing the net long futures commitments will necessitate adjustments in both futures and input markets. Therefore the intertemporal movements of \( \Delta \) and \( P_f \) must be evaluated under these assumed circumstances.

Increased speculation in futures trading results with direct responses in the futures market and indirect effects on the input and transformation market. Futures demand respond as speculative interest increase via the exogenous
variable \( b \). For continued existence of an equilibrium in the futures market it follows that short hedging or supplies of contracts must increase accordingly. This can occur if the price spread \( \Delta \) increases. Excessive demand pressures for speculative futures contracts will force the futures price upward if equilibrium is attained, thus increasing \( \Delta \). With this shift in \( \Delta \), then a new futures equilibrium exists along the new locus of equilibrium points at \( F'F' \) (see Figure 4.8). Solutions from Appendix C.3 reveal that both the price spread and futures must increase.

\[
\frac{\partial \Delta_F}{\partial b} = \frac{F_S^b}{T_H^\Delta} > 0 \text{ at the initial } F_f, \text{ where } F_S^b > 0 \quad (4.21)
\]

\[
\frac{\partial F_f}{\partial b} = \frac{|M_{3.1}|}{|M_3|} > 0 \quad \frac{\partial \Delta}{\partial b} = \frac{|M_{3.2}|}{|M_3|} > 0
\]

(see Appendix C.3) \( (4.22) \)

Pressures within the futures market via \( b \) will not directly influence the input market at the initial futures price. But in fact it was shown that \( F_f \) must increase, hence hedging short (supplying contracts) increases. Increased hedging \( T_H \) requires adjustments in the input transformation market if equilibrium continues. As \( T_H \) increases there must be a simultaneous rise in input transformation or a decrease in the level of unhedged stocks. Input price \( P_x \) will increase due to increased input demands, thus facilitating the equilib-
rium adjustment by increasing the supply of inputs since $X^P \Delta x > 0$. In addition the level of $T_u$ will decrease since $T^P_u \Delta x < 0$. From these responses toward equilibrium it must be that at least all added inputs and transformation services, resulting from the effect of greater speculation, are completely hedged. Again the price spread has increased while both futures and input price rose. Hence the increase in input price must be less than the futures price increase. Otherwise larger supplies of futures in response to increased speculation could not have occurred. Finally at the new market equilibrium the coordinates of Figure 4.8 have increased, but the rising spread is somewhat dampened in magnitude since input price also increased.

**Proposition 5:** Within the assumed framework an exogenous increase in net speculative activities must result with increases in both the price spread and futures price if a new market equilibrium exists. Further all additional inputs as a result of this change will be completely hedged.

![Figure 4.8. Increases in speculation](image)
Exogenous Input Supply Changes

Exogenous shifts in the supply of inputs requiring transformation services necessitates adjustments by those demanding transformation services. Let the exogenous supply variable "a" increase and $X_a > 0$. Initially the excess supply of inputs dampens the input price. Hence both hedged and unhedged transformation procedures are now more profitable. $T_u$ increases since $T_u^p < 0$ and $T_H$ changes with $T_H \frac{\partial \Delta}{\partial F_x}$. The price spread $\Delta$ has increased at the initial futures price $P_f$. Therefore $TT$ must shift upward to $T'T'$, Figure 4.9 and Appendix C.4.

Short hedges have increased at the initial futures price. But long hedging and speculation will respond to greater supplies only through the indirect effect of "a" on $P_f$, i.e., $\frac{\partial P_f}{\partial a}$. Then it must be that locus FF will not change in response to "a" unless $P_f$ also varies. Finally short hedges $T_H$ have increased and the spread $\Delta$ increases. Long commitments must increase via decreases in $P_f$. This is expressed in the proposition below and with Figure 4.9.

**Proposition 6:** Exogenous input supplies shift upward in the present framework and simultaneously the futures price decreases while the spread increases. This will occur regardless of the magnitude of response to this exogenous variable (assuming sign of slopes remain the same).
Implicit to this proposition is the restriction that the relative response of futures to input prices must be such that $0 < \frac{\partial P_f}{\partial P_X} < 1$. Both prices $(P_f, P_X)$ decrease, but futures must decrease less since the spread must increase. If $\frac{\partial P_f}{\partial P_X}$ again estimated by $\beta_3$ and $0 < \beta_3 < 1$, then in a period of excess input supplies the predicted movement in both the futures market and the price spread can be stated.

![Figure 4.9. Exogenous shifts in supplies of inputs](image)

Ultimately through this chapter a method of explaining the intertemporal relationship of prices and commitments to independent exogenous forces has been set forth. Market equilibriums can be established at numerous coordinates of observable prices $(P_f, P_X)$, but they have been shown to be dependent on the postulated market structure. In all cases considered there did exist alternatives for reaching a market equilibrium. But in the process of evaluating the economic interrelationships of futures to nonstorable products, the
concepts of market equilibrium to industry equilibrium (defined in next chapter) is of utmost importance. Specifically, movements in the price spread $\Delta$ have been derived without regard to the theory of industrial equilibrium as discussed in chapter three. Intertemporal price discovery can be established given the knowledge of the prevailing market structures. Yet these prices must be related to the concept of transformation cost. Therefore these comparative static conclusions will be evaluated with respect to the cost of transformation in the next chapter.
CHAPTER 5: FUTURES TRADING AND THE EXISTENCE OF EQUILIBRIUMS

The essence of the last two chapters is that an equilibrium can and has been derived. All possible combinations of coordinates $A$ and $P_f$ satisfying market equilibrium were determined. These point sets (coordinates) are in fact in a linear space separated by the two linear lines, i.e., the price spread and futures axes. As was stated the price spread could be either positive or negative, but the futures price must always be positive. The concept of non-positive prices carry no meaning in the discovery of prices, output, and equilibrium analysis with futures trading. Hence, all intersections of futures market equilibriums and input-transformation equilibriums are then limited to a linear space as in Figure 5.1 (e.g., the price spread axis and the futures price axis define two sets $A$ and $B$, Figure 5.1). In addition it was established that the price spread may be zero, hence market equilibrium could occur on the futures price axis. The contrary assumption holds for futures prices. It is assumed that futures prices will always be positive, thus implying that market equilibrium cannot exist on the price spread axis. It follows that the futures price axis is defined as a supporting plane to both equilibrium sets $A$ and $B$ since points in
either A or B (Figure 5.1) can be contained in the plane. But the bounding line, Δ axis, is a separating plane to both A and B since no point sets of A or B can be in common with Δ axis. Through the combined interpretations of the simple mathematics already used and the notion of linear spaces, some additional propositions regarding equilibriums between futures and input markets can be determined.

![Diagram of linear space for market equilibriums](image)

**Figure 5.1.** The linear space for market equilibriums

Sets A and B contain all feasible market equilibriums prior to and after the impact of changing exogenous forces. At each point in either of these sets there exists a unique set of coordinates \((Δ, P_f)\). Or expressed otherwise, the discovery of observable prices \(P_f\) and \(P_x\) depend on finding those coordinates that are just consistent with the prevailing market structure and the values of included variables. Although the coordinates are unique, this does not imply that the combinations of variables necessary for a given coordinate is unique. For example some move to coordinates
(\Delta', P_f') could be through increasing price expectations, decreasing transformation costs, or numerous other combinations of forces.

In addition to market equilibrium coordinates, there is theoretically some price spread that just reflects the cost of transformation. Equation 3.13 derived that price spread which just reflects the cost of transformation. It then follows that all \( \Delta E \) are in fact some subset of sets A and B. In essence within the sets of market equilibrium coordinates there exists a set of price coordinates that will always exactly express the cost of input transformation. Define:

Set A: market equilibrium set with \( \Delta > 0 \),
Set B: market equilibrium set with \( \Delta \leq 0 \),
Set E: industry equilibrium set with \( \Delta \geq 0 \),
Set C: union of sets A and B.

Given the union of sets A and B and the interpretation of set E, then the major problem of this chapter is to evaluate the implications and adjustments that follow when the realized point sets of C are not point sets of E. This point may need clarification. The set E defines all price coordinates
that satisfy the industry equilibrium constraint of transformation cost, while set C contains all coordinates of \((\Delta, P_f)\) for market equilibrium. Then if market equilibrium is realized with coordinates nowhere in set E, the realized price spread may not reflect the cost of transformation. All market equilibrium point sets (coordinates) found also in set E will be defined as industrial equilibrium points. Whereas those point sets everywhere outside of E are not industrial equilibrium points. Industrial equilibriums imply market equilibriums, but not the converse. Some qualitative evaluations of the success of the futures market in relation to the cash market can follow from considerations of sets C and E. The reduction of the area of set C outside set E may be one element for successful futures operations.

The procedure from here will be to first evaluate the possibilities of movement from one point in set E to another point in the same set i.e., moving from one industrial equilibrium to another. The alternative situation may exist with movements from points outside E but in set C to points within E, i.e., moving from market to industry equilibriums. These movements result from changes in the exogenous forces presented in the last chapter. Specifically, exogenous changes were considered that influenced only the input transformation market, others just affected the futures market, and finally situations of simultaneous changes in coordinates of both
markets were stated. Again exerting the influence of the exogenous variable can yield meaningful propositions to the analysis.

Industrial Equilibrium Movements

Four exogenous variables were considered under the comparative static manipulations. Using these variables to induce market adjustment or rather these variables induce market adjustment, then if initially in set E (industry equilibrium) when will the stated adjustments remain within the set E? For example can industry equilibrium and increasing input supplies be consistent? The existence of an initial industry equilibrium as derived in previous chapters occurred where the coordinates of locus TT, FF, and EE were consistent for each locus. Recalling that TT is a locus of coordinates satisfying input market equilibrium, FF is a locus of coordinates satisfying futures market equilibrium, and EE is simply the set E. TT is then a subset of set C and so is FF. Intersection of TT and FF yields one point set of set C (e.g., a point of market equilibrium). It was established that after the influence of some exogenous variables either or both TT and FF shifted and new intersection points occurred. Some subset D is defined to include all points that may occur after the influence of an exogenous force at some initial stable equilibrium point. Set D is those coordinates of
equilibrium associated with some exogenous factor. Define:

Set D: subset of C resulting from exogenous forces applied at some industrial equilibrium point.

Exogenous shifts in input supplies

Shifts in the amount of inputs X produced adjustments where TT must shift if the input market remained in equilibrium. But the futures market did not respond directly to shifts in the exogenous supply variable (see chapter four). If an industry equilibrium initially existed at some point A (see Figure 4.9), then it can be shown that this initial equilibrium may be unique when the only exogenous change is due to the variable "a". In other words there may be only one price spread and futures price that exactly reflects the cost of transformation and satisfies the market equilibrium condition. Three situations are to be considered.

Set D defines all of those market equilibrium points resulting from an exogenous change in supplies. In Figure 5.2 this set is limited to coordinates on the locus FF only (set is indicated by hash marks on FF). All new points occur with the intersection of T'T' with FF. Starting with the most restrictive situation where the coordinates (A, P^r) satisfying the locus EE will always just satisfy locus FF (e.g., the line EE and FF are the same), then it follows that market adjustments in response to exogenous input changes will
always yield a new industry equilibrium point. In Figure 5.2 there are an finite number of market equilibrium points satisfying the conditions for industry equilibrium. The set D is a subset of set E. Hence with this market structure (where FF and EE correspond) a change in inputs through exogenous factors will only change the coordinates of an equilibrium; the conditions reflecting transformation costs are satisfied. The implication from this conclusion is that irregardless of the changes in the supply of inputs there will not exist an incentive for changing participation by new firms within the input or futures markets. In those situations where realized spreads differ substantially from the cost of transformation, then some incentive for additional entry to or exit from the markets exist. The incentive for entry and exit is a direct function of whether equation 3.13, chapter three, is satisfied.

Figure 5.2. Exogenous inputs and equilibrium where EE=FF.
In many situations coordinates \((\Delta, P_f)\) satisfying FF will differ from those of EE. This implies that the slope of FF will differ from the slope of EE (e.g., \(\Delta_{FF} < \Delta_{EE}\)), but it has been established that both slopes are negative (see chapter three). Initially equilibrium exists in Figures 5.3 and 5.4 at some point A. It then follows that an exogenous supply shift cannot result in a new industry equilibrium if all other exogenous factors are constant. For the market structure where \(\Delta_{FF} > \Delta_{EE}\) (e.g., slope of EE exceeds that of FF) any exogenous supply increase will result in an actual futures-cash price spread that is less than the cost of transformation relationship. In Figure 5.3 the newly established market equilibrium falls below the line EE. Hence the spread fails to reflect the cost of transformation by the amount shown in the shadowed area. The alternative market structure where \(\Delta_{EE} > \Delta_{FF}\) yields price spreads greater than transformation cost as in Figure 5.4. Industry and market equilibriums are consistent only at the initial starting point A. Any change in supplies produces equilibriums where the realized price spread either exceeds or is less than the cost of input transformation. In terms of linear spaces, sets E and D have only one point in common. Any exogenous supply increase results in movement from this common point to some point only in set D. This is shown where:
Proposition 7: Within the meaning of the assumed framework of sets C, D, and E it follows that exogenously changing input supplies can result in movements from one to another industry equilibrium if the market structure is such that those coordinates \((\Delta, P^*_f)\) satisfying equilibrium in the futures market will just reflect the cost of input transformation. Otherwise the movement will be from the industry equilibrium to simply a market equilibrium.

Some useful observations can be gained from this proposition. Livestock producers can and do make up much of the market for transformation services and futures trading in live beef. If producers as a whole anticipate increasing input supplies; they can, given this assumed framework, draw some meaningful conclusions about forthcoming price spreads. When the prevailing market structure is such that input adjustments cause movement from industry equilibrium, then entry and exit occur. In the situation of Figure 5.4, the realized price spread exceeds the cost of transformation. Hence greater market entry may occur since buying inputs \(X\), selling futures,
and transforming these inputs is definitely profitable. When some exogenous input change has occurred, some meaningful evaluation of futures for forward pricing can be determined (this is shown later).

**Exogenous changes in expectations**

Exogenous changes in the expected price of the transformed input at maturity was shown to have a simultaneous influence on both the futures and cash markets. When expectations increased the price spread decreased. But the response of the futures price depended on the exact response of some elements of the market structure. Specifically, new market equilibriums occurred as expectations increased and the futures price increased if \( \frac{\partial P_f}{\partial P_x} < \kappa \), (see Proposition 4), or decreased when \( \frac{\partial P_f}{\partial P_x} > \kappa \). These alternatives again define some linear space where new market equilibriums are possible. Those market equilibrium points resulting from cash price expectation changes are defined in some set \( D \). Hence set \( D \) now includes all coordinates of \( A \) and \( P_f \) satisfying market equilibrium after adjustments due to changing expectations.

If initially at some industry equilibrium point \( A \) in Figures 5.5 and 5.6, under what circumstances will adjustments be limited to movements within the set \( E \) (set \( E \) contains industry equilibrium coordinates)? In other words, will
Figure 5.3. Market and industrial equilibriums and input changes where slope EE > slope FF

Figure 5.4. Market and industrial equilibriums and input changes where slope FF > slope EE
future-cash spread = \Delta

futures price = P_f

slope = 1 - G < 0

TT = input market equilibrium
FF = futures market equilibrium
EE = \Delta reflects transformation cost

\Delta
changing expectations be consistent with an industrial equilibrium? Again it can be shown that these movements depend directly on responses within the market structure.

First consider the situation of Figure 5.5. Market responses are such that all coordinates satisfying EE are not consistent with FF. Let the slope of line EE exceed that of FF. Set D then contains all points of intersection of TT and FF as adjustments to expectations are made. Given this market structure it follows that there may be numerous points of market equilibrium that intersect just on the line EE. The set D and set E intersect. Movements from the initial point A to a new point is feasible. In Figure 5.5 both points A and B are in sets E and D. If expectations have increased then it is necessary for futures prices to increase if the movement from points A to B occurred. In contrast, a decrease in expectations must result with decreasing futures price if an industry equilibrium continues to exist. This simply holds due to the fact that the slope of EE is negative. Define:

Reference to Figure 5.5,
Slopes EE > FF.
Figure 5.5. Market and industrial equilibriums and exogenous increases in price expectations where slope EE > slope FF

Figure 5.6. Market and industrial equilibriums and exogenous increases in price expectations where slope FF > slope EE
TT = input market equilibrium
FF = futures market equilibrium
EE = Δ reflects transformation cost

future-cash spread = Δ

futures price = Pf

slope = 1 - G < 0
The intersection of set E and D defines all points of industry equilibrium that may occur as expectations change. Within the intersection the necessary movements of futures prices must have occurred.

With Figure 5.6 the slope of the futures market exceeds that of EE. As expectations increase the spread decreases and futures price movements are indefinite. Set D now consists of those intersecting points with this different slope assumption. Yet irregardless of the new market equilibriums, there cannot be a new industry equilibrium when \( \Delta_{F} > \Delta_{E} \) (slope FF greater than slope EE). Set D and set E have only the initial point set A in common. Within this assumed framework changing price expectations are not sufficient for continued industrial equilibrium. The price spread after adjustments to price expectations will not reflect the cost of input transformation. This situation is defined where:

Reference to Figure 5.6, Slope FF > Slope EE.
Proposition 8: Within the meaning of the assumed framework, exogenous changes in expected output prices can result in movements from one industry equilibrium to another if the prevailing market structure is such that the slope of EE exceeds that of FF (e.g., set D and set E have more than one point set in the intersecting set). Otherwise there is only one unique industrial equilibrium point.

These conclusions again can be very useful. If the market structure is even partially understood, then the impact of a general increase in expected prices can be deduced. Taking the latter case where $\Delta_{F}^{P} > \Delta_{E}^{P}$ is known (Figure 5.6), then adjustments to greater expectations produce price spreads that are less than necessary to cover transformation costs (e.g., market equilibrium of set D is everywhere below the locus EE except at point A). The realized spread will always be below that spread necessary to just reflect the cost of input transformation. This difference may produce exits from the futures market. The alternative where realized spread exceed transformation cost (the case of decreasing expectations in Figure 5.6) may result in greater futures participation, especially by shorter hedgers, since the profitability for increasing inputs, selling futures, and transforming inputs is greatly increased with little additional risk. Exit may not be as straight forward since futures commitments depend on the level of risk aversion in conjunction with relative prices.
Aggregate exits may occur, but they will not fall below the aggregate subjective level necessary for risk aversion.

The case of Figure 5.5 also provides insights for market participants. The area of industrial equilibrium is much greater than the case of Figure 5.6. Hence the degree of entry and exit may be lessened. In a period of changing expectations the use of futures for forward pricing may be greatly increased since the number of industry equilibrium points have increased i.e., the futures and input price spread tend to remain related to transformation cost. The impact of changing expectation for this situation (Figure 5.5) is somewhat dampened to the extent the industry equilibrium continues to exist.

**Exogenous speculative interest**

The fact that those coordinates satisfying futures market equilibrium must shift with changing speculative interest was previously set forth. In addition the input market did not shift (see chapter four). The set D now is defined as all those market equilibriums where the shifting FF just intersect the locus TT. As FF shifted upward due to increased speculative activities both the price spread and futures price increased. The converse holds for decreasing speculative interest.

Starting at some equilibrium point A, then any change in speculation cannot result with a new industry equilibrium. Reference to Figures 5.7 and 5.8 substantiates this observation.
Increased speculation produces market equilibriums along the path of TT. But the set E has only one coordinate in common with TT (e.g., EE and TT intersect only at point A). Further all coordinates or point sets of set E are on the line EE, thus implying that EE does not shift as speculative interests change. It then must be that the set D and set E have only one point in common. Hence any change in speculative activities can only result in movements away from a industrial equilibrium if initially at industry equilibrium. Define the set relationship as:

Reference to Figures 5.7 and 5.8.

This failure to meet the necessary equilibrium requirements produces a price spread that differs from that necessary to reflect the cost of transformation. Again the incentive for entry or exit of new participants has been established via changing speculation. Comparative statics and linear spaces have been sufficient to arrive at both market and industry equilibriums, but the actual mechanisms for entry and exit have not been derived. The present framework then is useful up to the point of providing incentive for other exogenous
changes not directly included in the model. Finally this situation as referred to in Figures 5.7 and 5.8 is expressed through proposition 9.

**Proposition 9:** Within the meaning of the assumed framework it follows that exogenous changes in speculative activities in the futures market cannot result in movements from one industrial equilibrium to another.

**Exogenous change in cost of transformation**

In all previous situations the set E and the locus EE had all coordinates in common. But with exogenous changes in the cost of input transformation this no longer holds. The set E contains numerous point sets not on the initial locus EE (e.g., it was shown that EE will shift as the cost of transformation changes, see chapter three). Further it was shown that both input market equilibrium locus TT and futures locus FF shifted directly with changing $P_L$. The intersection of these market shifts are then contained in the set D.

All three equilibrium conditions shift in the same direction as cost are exogenously changed. It then is evident that there are numerous points that could satisfy an industrial equilibrium. In essence the intersection of sets E and D
Figure 5.7. Market and industrial equilibriums and increases in speculation where slope EE > slope FF

Figure 5.8. Market and industrial equilibriums and increases in speculation where slope FF > slope EE
TT = input market equilibrium
FF = futures market equilibrium
EE = Δ reflects transformation cost

future-cash spread = Δ

slope = 1 - G < 0
have many common point sets. In Figures 5.9 and 5.10 the initial point A exists. But as cost increases the input locus TT shifts to some T'T', futures shift to F'F', and EE shifts to E'E', thus giving a new point B. Of course point B may not occur if a specific response in EE or other loci do not occur. This is summarized in proposition 10.

**Proposition 10:** Within the meaning of the assumed framework, an exogenous change in transformation cost can result in new industry equilibrium where the price spread continues to reflect the cost of transformation. This does not guarantee that it will occur.

In essence there exists some degree of price adjustments to changing costs that will not necessitate either entry or exit of new firms into the market. Prices can adjust and industrial equilibrium continue even in a closed market since there are numerous points of industry equilibrium. No incentives for added entries or exits have been derived. But if the market equilibriums and EE are not consistent then the incentive for entry and exit again occurs. Define:

Reference to Figures 5.9 and 5.10.
Figure 5.9. Market and industrial equilibriums and increases in transformation cost where slope EE > slope FF

Figure 5.10. Market and industrial equilibriums and increases in transformation cost where slope FF > slope EE
TT = input market equilibrium
FF = futures market equilibrium
EE = \Delta reflects transformation cost

Future-cash spread = \Delta

futures price = P_f

slope = 1 - G < 0
If movements from point set A to point set B exist, then industrial equilibrium continues. But if from A to some point set X, then market equilibrium and industry equilibrium are no longer consistent.

In each of the four situations derived from exogenous forces the case is summarized with set E, set D, and their intersection. Movements within the coordinates of the intersection always provide a price spread consistent with transformation cost. Movements to coordinates outside of this intersection yields spreads that are considered somewhat indicative of changes in market participation via entries and exits. From these conclusions some implications of futures for forward pricing will be deduced.

**Equilibrium and Forward Pricing**

Through the tools of simple mathematics and linear spaces, many observations and propositions related to futures trading in products requiring transformation services were sighted. It was established that under specific exogenous forces the possibilities for continued industry equilibrium were slight, yet with alternative exogenous forces the same equilibrium may have a greater chance for continued existence. Hence to draw the conclusion that one particular futures will under all circumstances maintain a industry equilibrium must be an overstatement of the facts. The history or historical
occurrence of exogenous forces must be considered. Further all forces (exogenous variables) have been evaluated with the assumption that all others were fixed. In reality there may be a continuous and simultaneous adjustment to many exogenous effects; although, it may be that particular markets such as livestock have a greater frequency of occurrence of some exogenous force such as changing input supplies. Again to keep the discussion within a framework adaptable to the tools being used, only one exogenous change is assumed to occur at a time.

Initially through the mathematics of comparative statics both futures and input prices were discovered. Then given exogenously induced changes, the relative response of these prices were determined via \((\Delta, P_f)\). With each exogenous event there resulted a set D that contained all coordinates of \((\Delta, P_f)\) consistent with market equilibrium. In addition a set E and its intersection with set D led to some useful interpretations. It is now only one step further to evaluate the role of futures prices for nonstorables and forward pricing.

Forward pricing implies the ability to establish the selling prices of goods that are to be delivered at a later date. Such prices are often pegged by governmental agencies through commodity price floors and other techniques. Futures prices may serve as forward prices. For example, trading in live beef futures today may be indicative of the cash market price at some future data. Using futures for forward pricing
may be considered in two ways: 1) if futures prices are related to the costs of transformation as previously discussed then they may be useful forward prices, 2) futures as historically related to the final output price may give guidelines to decision making. The first of these alternatives will be considered in the remainder of this chapter, and the latter will be considered in the next chapter.

Forward price evaluation

The premise of this section is simply that if the probability of the occurrence of an industry equilibrium can be estimated, then futures for forward pricing can be evaluated. To do this the entire set of exogenous forces must be included. Four considerations of exogenous changes were previously presented. As stated these may be indicative of many other forces not discussed. If there exists a finite number of such exogenous forces entering the system, then they can be expressed in a linear space notation. Define:

Set $J$: a set of all possible exogenous variables that could influence the framework assumed.

Set $J$ has $j$ point sets (exogenous variables) and for each $j$ there exists a set $D_j$ that expresses market equilibrium coordinates. Some $j$ implies a set $D_j$. The set $D$ was used earlier in this chapter without adding the subscript. It is now added since all exogenous effects are being considered simultaneously. There exists a frequency distribution of these
exogenous events. For example trading in live beef futures and in feeder inputs are subjected to many exogenous changes; but through historical occurrences, the frequency of each event may be determined. Hence define Prob (j) as the probability of event j. The equality of equation 5.1 then holds (e.g., sets D_j and E_j occur with j, hence they have the same probability of occurrence).

\[
\text{Prob (j)} = \text{Prob (D_j)} = \text{Prob (E_j)} \quad (5.1)
\]

Each set D_j consists of C_{ij} point sets where each C_{ij} is a coordinate of (A, P_f) that satisfies a market equilibrium. For example it was shown that changing expectations produced new market equilibrium points. Hence expectations are one point set of set J and all results from market adjustments are point sets C_{ij} of set D_j. Each C_{ij} follows some frequency distribution \( f(C_{ij}) \), (e.g., some adjustments may be more likely than other ones). Set D_j and E_j intersected as shown in Figures 5.2 through 5.10. Hence some point sets C_{ij} are common to both D_j and E_j. Assume the first \( m_j \) of the \( n_j \) point sets are in both set D_j and E_j. Define:

\[
\int_{i=1}^{n_j} f(C_{ij}) \, dC_{ij} = 1 \quad i = 1, 2, \ldots, m_j, m_j + 1, \ldots, n_j
\]

\[
\sum_{i=m_j}^{n_j} \int_{i=1}^{c} f(C_{ij}) \, dC_{ij} \leq 1 \quad \text{Probability of intersection of set } D_j \text{ and set } E_j.
\]
If again the initial point of industry equilibrium is some point A (see Figures 5.2 through 5.8), then what is the probability of moving to some new equilibrium point B, see Figure 5.11. There are \( m_j \) points in the intersection, thus there exists \( m_j - 1 \) alternatives from A (let \( A = C_{ij} \)) satisfying industry equilibrium. The probability of moving from point A to another point within the intersection follows in equation 5.2.

\[
\text{Prob}(A \rightarrow B) = \int_{C_{ij}} f(C_{ij}) \, dC_{ij} < 1
\]

(see Figure 5.11) \hspace{1cm} (5.2)

![Figure 5.11. Sets D and E](image)

Through equations 5.1 and 5.2 the desired form is derived where the probability of some exogenous event exists and the probability of a industry equilibrium given each event is expressible. These probabilities are not independent in
that Prob(A→ B) will occur is the product of the probability that the one of them will occur and the probability that the other will occur given that the first has occurred, occurs, or will occur. See equation 5.3, where the probability of both events (exogenous force j and movement to another equilibrium) is derived. Summing over all possible exogenous events

\[
\text{Prob}(D_j) \text{ Prob}(A→ B) = \text{Prob}(D_j) \int_2^{m_j} f(C_{ij}) dC_{ij} (5.3)
\]

then establishes the probability of getting a new industrial equilibrium point. Equation 5.4 then provides a probability over all exogenous events that indicates the likelihood of a

\[
\text{Prob(industry equilibrium)} = \sum_j \text{Prob}(D_j) \text{ Prob}(A→ B) (5.4)
\]

movement from one point A to another point B. A simple example may aid for interpretation. Assumed j=1,2,3 and the probabilities are assigned as in Table 5.1. In this discrete case the probability of arriving at a new industrial equilibrium point is .58. In other words, .58 shows the probability of moving from one industry equilibrium point to another when the probability of occurrence of each exogenous force is considered.

The essence of equation 5.4 is that there is some probability that the futures-input price spread will just reflect the cost of transforming the inputs X. If this probability is sufficiently large (e.g., if the probability exceeds some subjective minimum level) then utilizing futures prices as
Table 5.1. Example of movement from one point A to another point B

<table>
<thead>
<tr>
<th>j</th>
<th>P(D_j)</th>
<th>P(A→B)</th>
<th>P(D_j n (A→B))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.3</td>
<td>.2</td>
<td>.06</td>
</tr>
<tr>
<td>2</td>
<td>.3</td>
<td>.4</td>
<td>.12</td>
</tr>
<tr>
<td>3</td>
<td>.4</td>
<td>1.0</td>
<td>.40</td>
</tr>
</tbody>
</table>

forward prices is acceptable. In other words, the premise is that futures must exactly reflect the cost of transformation if it serves for forward pricing. If the probability falls below some minimum level, then futures cash price spreads will be sufficiently different from transformation costs too great of a percentage of the time. Thus futures prices will not be used for forward pricing. This is summarized with the proposition 11.

Proposition 11: Within the meaning of the assumed framework, if the probability of a new industrial equilibrium exceeds some minimum level, then futures prices may be used for forward pricing.

Proposition 11 is very restrictive in that an industry equilibrium must exist with some probability if forward pricing via futures is used. A somewhat lesser restriction can be stated, but in the process more information must be known. Specifically, earlier in the chapter it was shown...
that any set D (set \(D_j\)) consists of coordinates where both increases and decreases in the exogenous variable occurred. If in fact set D can be subdivided into three subsets where one includes those industry equilibrium points, next are those coordinates where the spread \(\Delta\) exceeds the transformation cost relationships of equation 3.13, and finally coordinates exist where \(\Delta\) is less than the transformation cost relationship. If the probability of the first two subsets above can be determined, then a less restrictive alternative to proposition 11 exists. The probability of the price spread that equals or exceeds transformation cost must be determined.

Some set \(D_j\) contains \(n_j\) point sets as assumed earlier and \(m_j\) of these point sets are industry equilibrium points (e.g., intersection of set \(D_j\) and \(E_j\)). Now assumed that \(m_{j+1}\) through \(q_j\) point sets have coordinates such that the \(\Delta\)(price spread) is above the transformation cost relationship. Define:

\[
\int_{1}^{m_j} f(C_{ij})dC_{ij} + \int_{m_{j+1}}^{q_j} f(C_{ij})dC_{ij} + \int_{q_{j+1}}^{n_j} f(C_{ij})dC_{ij} = 1.
\]

Hence the probability that the new market equilibrium coordinates will either equal or exceed the transformation cost is derived in equation 5.5. Stated otherwise, it is the probability of moving from an industry equilibrium coordinate to some coordinate where \(\Delta\) at least reflects the transformation cost (e.g., \(A \rightarrow \chi\)). Define:
Then in a similar manner as shown with equation 5.3, the probability of some exogenous event $j$ occurring and of movements to new coordinates to satisfy the transformation cost condition can be calculated.

$$\text{Prob}(A \rightarrow \chi) = \sum_{j<n_j} \int_2 f(C_{ij}) \, dC_{ij}$$  \hspace{1cm} (5.5)$$

Summing over all possible exogenous events of set $J$ then provides the probability of arriving at a set of coordinates $(\Delta, P_f)$ where the price spread is at least as great as necessary for the transformation cost constraints (see Equation 5.7).

Recalling that $\Delta_E = [P_f - P_L][1 - G(L^*)]$ is the relationship between spread and costs of transformation. This situation

$$\text{Prob} \left( \Delta \text{ satisfying transf. cost relationship} \right) = \sum_j \frac{\text{Prob}(D_j)}{\text{Prob}(A \rightarrow \chi)}$$  \hspace{1cm} (5.7)$$
can be expressed by example with Table 5.2 where $P(A\rightarrow x)$ is substituted for $P(A\rightarrow B)$ of Table 5.1. For these assumed frequencies it follows that the probability of getting a price spread $\Delta$ that is equal to or greater than the transformation cost relationship is .73 (e.g., this desired $\Delta$ will occur 73 percent of the time). If this calculated probability is above some subjective minimum, then the futures price may be used as a forward price.

Table 5.2. Example of movement from point A to another point $x$

<table>
<thead>
<tr>
<th>$j$</th>
<th>$P(D_j)$</th>
<th>$P(A\rightarrow x)$</th>
<th>$P(D_j \cap (A\rightarrow x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.3</td>
<td>.5</td>
<td>.15</td>
</tr>
<tr>
<td>2</td>
<td>.3</td>
<td>.6</td>
<td>.18</td>
</tr>
<tr>
<td>3</td>
<td>.4</td>
<td>1.0</td>
<td>.40</td>
</tr>
</tbody>
</table>

What are the consequences of equations 5.4 and 5.7? A spread that will at least cover the cost of transformation (within the meaning of the cost-spread relationship) will occur with some probability. Then the futures may be used as a forward price. In other words, selling futures for $P_f$, buying the inputs and transforming them will be profitable. In addition forward contracting outside of an organized market can occur, but the price derived in the organized futures
exchange serves as the guideline for the forward price. Producers could contract with packing plants for delivery at a later date. Then with the restrictive model developed here, the probability of success from using futures prices for guidelines to forward pricing can be calculated. The success lies in that a forward price for the transformed input assures those providing transformation services a fixed return. Of course the probabilities of equations 5.4 and 5.7 do not assure that the new coordinates \((\Delta, P_f)\) will result with \(\Delta \geq [P_f - P_L][1-G]\). If \(\Delta \leq [P_f - P_L][1-G(L^*)]\) and \(P_f\) is used for forward pricing such commitments in the macro sense will produce losses.

Ultimately, it has been established that futures prices can be used as guidelines for forward pricing. There exist many exogenous events that may occur and they must be used in the evaluation of a futures price movements. Finally if the probability of arriving at some new spread \((\Delta)\), given the possibilities of all exogenous events, exceeds some minimum level; then the futures price in such nonstorable products may be utilized for forward pricing.

**Proposition 12:** Within the meaning of the assumed framework, the success of futures for forward pricing depends directly on the probability of arriving at a spread and futures price coordinate \((\Delta, P_f)\) that equals or exceeds the transformation cost relationship (e.g., \(\Delta \geq [P_f - P_L][1-G(L^*)]\)).
Finally through this chapter and the preceding ones, a framework for the existence and uniqueness of equilibriums has been established. Both the market and industry equilibriums are essential concepts to the understanding of price responses and market positions. Hence in the next chapter some discussion of price and quantity movements will be considered that are directly relevant to these theoretical discussions.
CHAPTER 6: LIVE BEEF FUTURES PRICE MOVEMENTS

Within the contexts of the last few chapters some theoretical considerations were presented for relating futures markets to trading in commodities that required further product transformation. In essence, the commodity was not yet marketable but was still related to a futures market. The case for live beef futures was shown to be such a product. The micro framework set forth the optimal levels of market commitments for both a cash and futures position. The aggregate effect of all traders in a micro sense lead to a format for price discovery in a macro sense. Specifically, chapter two specified the optimal levels of inputs and futures commitments. Through chapters three and four a method for price discovery was illustrated, while chapter five considered the implications of derived prices in relationship to market and industrial equilibriums.

Drawing from these theoretical considerations, some empirical movements in prices can be interpreted. All previous analyses of futures and cash prices were discussed in a framework where the input (the initial placing of feeder cattle on feedlots) always required $t$ units of time for complete product transformation (see chapter three). If $t=6$, then the analysis consisted of all cash and futures prices that were consistent
with the fact that inputs require six months of transforming services. Futures prices must also be evaluated in a different time series format. To this extent interest is with the price movements over the life of each contract. Considerations of both forms of prices will follow in this chapter.

Figure 6.1 provides a graphical summary of the past theoretical discussion and establishes a pictorial reference for some empirically observed price movements. The structural relationship between price discovery and the futures contract life is related in this figure. The lower portion of 6.1 illustrates the time trend of futures prices. Each futures contract is traded up to \( n \) months prior to the terminating date of the contract. There exist a series of futures prices in this contract that may vary systematically over the contract life.\(^1\) As shown on the horizontal axis of 6.1, the contract life is measured in months from the maturity date up to one month prior to contract maturity. The point of interest in this respect is to test if there is some systematic price trend over each contract life. In addition, observations relating price movements in the last 20 trading days will also be of interest.

The upper portion of Figure 6.1 relates the previous discussion of market equilibrium to that of each futures

\(^1\)There are presently six contracts traded in live beef futures: February, April, June, August, October and December.
Figure 6.1. The structural relationship between price discovery and the contract life.
contract. Recalling that TT is the input market equilibrium and FF is the futures market equilibrium locus, it then follows that the derived futures price \( P_f \) of the upper graph (Figure 6.1) corresponds to some point in the time series over each contract life. In the case illustrated here the derived \( P_f \) (futures price) from some intersection of T'T' and F'F' is put in the proper perspective to the futures contract trend path (this situation is only hypothetical). This is shown where an arrow is drawn from the price axis of the upper graph to some price (the circled point) in the lower graph. Hence each futures price of a contract exists as a result from trading activities as illustrated with the upper graph.

Utilization of this figure will enhance in the presentation of some observed price movements. Therefore the procedure at hand is to look at observed activities within the live beef futures as they are related to the upper and lower portions of Figure 6.1.

Market Equilibrium

The economic role of the futures-cash price spread has been established and the various movements in both the spread and futures prices were explained by shifts in TT and FF. Hence there may exist some systematic trend in these market equilibriums (intersection of TT and FF).
Path of market equilibrium

Figures 4.2 through 4.9 delineated shifts in TT and FF that may occur as activities within the futures and cash markets change. It was shown that under some circumstances both $A$ and $P_f$ vary directly, while under other situations they vary inversely. These responses depend directly on the structures of both markets. A path of these shifts can be estimated for live beef futures.\(^1\)

Price spread $A$ and futures price $P_f$ must be made operational. Define:

\[
P_f(t)_j = \begin{cases} 
\text{the Chicago futures price (price on the 15th of each month) for live beef futures in that contract t months from maturity and the jth observation in the time series,} \\
\text{the jth price (average monthly price) of feeder steers 550-750 lbs. at Kansas City plus $1.00 for difference in basing point with respect to Chicago,} 
\end{cases}
\]

\[
P_x(t)_j = t = \text{months from futures contract maturity,}
\]

\[
j = \text{time series measured in months beginning with January, 1966, and ending with December, 1968,}
\]

\[
A(t)_j = P_f(t)_j - P_x(t)_j.
\]

---

\(^1\)The path is calculated only for $t=6$. Prices of feeder steers at weights other than those defined were not available. The scheme for selecting the futures price is presented in Appendix D.1.
For example, \( P_f(6)_1 \), is that live beef futures price in the contract that is nearest to 6 months from maturity on January 1966. While \( P_f(6)_{49} \) would be the price recorded as the 49th observation in the series. Hence prices are recorded among all contracts according to the criteria of time from maturity. Later movements over each contract as \( t \) varies will be considered. All prices were deflated by the consumer price index for meats, thus removing any inflationary trend occurring in the time series. Finally, subtracting the feeder price from the futures price yields a measurable form of the price spread, i.e., \( \Delta(t)_j \) is that price spread as recorded in the \( j \)th observation of the time series. Thus each observed \( \Delta(t)_j \) and \( P_f(t)_j \) in the series represents a set of coordinates \( (\Delta, P_f) \) where a market equilibrium has occurred. These are the same coordinates utilized throughout chapters four and five where subscripts were omitted at times for reading convenience.

The path of market equilibrium consists of relating the price spread to the futures price over this series. The hypothesized relationship is shown in equation 6.1. After testing alternative functional forms, the double-log \( \ln \) appeared

\[
\Delta = \phi(P_f) \tag{6.1}
\]

to be acceptable. Ignoring the time series \( j \) notation for the moment, then the relationship between \( \Delta \) and \( P_f \) can be estimated.
Table 6.1. Estimates of relationship for path of market equilibrium corresponding to Figure 6.1, before and after correction for autocorrelation

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff. 1</th>
<th>t test</th>
<th>Coeff. 2</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-log $\Delta'(t)_j$</td>
<td></td>
<td>-5.0126</td>
<td>+2.2820</td>
<td>5.8040</td>
<td>---</td>
<td>---</td>
<td>.6461</td>
<td>.3560</td>
</tr>
<tr>
<td>Double-log $\Delta'(t)<em>j-r\Delta'(t)</em>{j-1}$</td>
<td>- .4691</td>
<td>---</td>
<td>---</td>
<td>1.5151</td>
<td>4.6041</td>
<td>.5616</td>
<td>1.5913</td>
<td></td>
</tr>
</tbody>
</table>

*aDefine: Coefficient 1 = $P_f(t)_j$  
Coefficient 2 = $[P_f(t)_j-rP_f(t)_{j-1}]$, where $r \approx .8220$.

*bThe coefficient of autocorrelation $r$ is approximated from the Durban Watson statistic via  
\[ r = \left[ 1 - \frac{\text{D.W.}}{2} \right]. \]

*c$\Delta'(t)_j = [\Delta(t)_j + 10]$.  

\[ \log (\Delta + 10) = -5.0126 + 2.282 \log P_f \]

or \( \Delta = \frac{P_f^{2.282}}{e^{5.0126}} - 10 \) \hfill (6.2)

The spread \( \Delta \) is related to the futures price as shown in equation 6.2.\(^1\) This equation corresponds to the path of market equilibrium shown in the upper part of Figure 6.1.

The slope relationship between \( P_x \) and \( P_f \) is directly calculable from the path of market equilibrium, equation 6.3. The value of \( \frac{\partial P_f}{\partial P_x} \) can be shown given the coordinates \((\Delta, P_f)\) are known.

\[
\frac{\partial P_f}{\partial P_x} = \left[ 1 - 2.282 \frac{\Delta + 10}{P_f} \right]^{-1}
\] \hfill (6.3)

Within the chapter on price discovery through comparative statics, it was established that the final movements in the spread and futures price depended on the relationship among market responses to exogenous variable changes and \( \frac{\partial P_f}{\partial P_x} \). Take proposition 2 for example from chapter four, both \( \frac{\partial P_f}{\partial P_x} \) and \( P_f \) increased if \( 1 + \frac{T_u^{PL}}{T_H^{PL}} > \frac{\partial P_f}{\partial P_x} \). Let \( \Delta = -2.00 \) and \( P_f = \frac{T_u^{PL}}{T_H^{PL}} \)

\(^{1}\)The spread can be either positive or negative (see chapter three) but logs are defined only for positive values. All \( \Delta \) were increased to positive values by adding +10 to each. This constant was then subtracted out after deriving the \( \Delta \) estimate equation.
25.00, it then follows that the estimated response \( \frac{\partial P_f}{\partial P_x} \approx 3.703 \).

The situation of proposition 2 then reveals that if a new set of coordinates \((\Delta', P_f')\) are greater than the original set \((\Delta, P_f)\) after some exogenous change in \(P_L\) (transformation service cost), then the relationship of hedged to unhedged responses should be such that \(T^P_L > 2.703 T^P_H\). In essence this exercise illustrates a structural relationship that must exist for certain price movements to have occurred given some particular exogenous variable change. The same procedure can be applied to any of the propositions that were related to \(\partial P_f\).

The original estimates of equation 6.2 ignored the time series. Since this data is a time series, the possibility of autocorrelation should be considered. Table 6.1 reveals that there exist a large positive autocorrelation as is evident by the Durbin-Watson test. A correction for first-order autocorrelation was completed in Table 6.1. Only the estimated values of the parameters were desired in equation 6.2, hence correlated errors are not troublesome since these parameter estimates of 6.2 are unbiased.

Attempts to further explain the variability of the path of market equilibrium of Figure 6.1 were completed as derived in Table 6.2. Two additional variables appeared to be significantly related to \(\Delta\). Define:
Table 6.2. The path of market equilibrium for t=6 before and after correction for positive autocorrelation

<table>
<thead>
<tr>
<th>Function</th>
<th>Dependent</th>
<th>Intercept</th>
<th>Coeff. 1</th>
<th>Coeff. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-log $\Delta'(t)_j$</td>
<td></td>
<td>-3.2289</td>
<td>+2.5708</td>
<td>-0.0633</td>
</tr>
<tr>
<td></td>
<td>n=49</td>
<td></td>
<td>(7.1315)$^b$</td>
<td>(-4.1768)</td>
</tr>
<tr>
<td>Double-log $[\Delta(t)<em>j - r\Delta(t)</em>{j-1}]^c$</td>
<td></td>
<td>-0.0095</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>n=48</td>
<td></td>
<td>$r = 0.7239$</td>
<td></td>
</tr>
<tr>
<td>Double-log $[\Delta(t)<em>j - r\Delta(t)</em>{j-1}]$</td>
<td></td>
<td>-0.0161</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>n=48</td>
<td></td>
<td>$r = 0.7239$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Define: Coeff. 1 = $P_f(t)_j$,
Coeff. 2 = $Vol(t)_j$,
Coeff. 3 = $P_T(t)_j$,
Coeff. 4 = $[P_f(t)_j - rP_f(t)_{j-1}]$,
Coeff. 5 = $[Vol(t)_j - rVol(t)_{j-1}]$,
Coeff. 6 = $[P_T(t)_j - rP_T(t)_{j-1}]$.

$^b$t test are immediately below each coefficient estimate and .05 level of significance was used.

$^c\Delta'(t)_j = [\Delta(t)_j + 10]$. 
<table>
<thead>
<tr>
<th>Independent Variables&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Coeff. 3</th>
<th>Coeff. 4</th>
<th>Coeff. 5</th>
<th>Coeff. 6</th>
<th>R</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-.7621</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.7644</td>
<td>.5504</td>
</tr>
<tr>
<td></td>
<td>(-2.1813)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>+1.7511</td>
<td>+.0008</td>
<td>-1.0493</td>
<td>.6722</td>
<td>1.3425</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.8860)</td>
<td>(.0586)</td>
<td>(-2.9944)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>+1.7612</td>
<td>---</td>
<td>-1.0506</td>
<td>.6722</td>
<td>1.3446</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.6663)</td>
<td></td>
<td>(-3.0376)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vol(t)\_j = volume traded in live beef futures for that contract t months from maturity and the jth observation in the time series,

P\_T(t)\_j = average monthly price of Chicago choice slaughter steers for the jth observation in the time series.

A double-log function again proved to be an acceptable functional form as presented with equation 6.4. Equation 6.4 was corrected for first-order autocorrelation since the correlation coefficient for errors was positive (e.g., r \approx 0.7239). As shown in Table 6.2, the Log Vol. variable became

\[
\log(A + 10) = -3.2287 + 2.5708 \log p^f - 0.0633 \log \text{Vol.} - 0.7621 \log p^T
\]

(6.4)

insignificant after this correction for autocorrelation. Removing this variable and reestimating 6.4 and then again correcting for autocorrelation gave the alternative form in equation 6.5.

\[
\hat{\Delta}(t)\_j = \left[ P_f(t)\_j \frac{1.761 P_f(t)\_j - 1.275 P_T(t)\_j - 1.051 P_T(t)\_j - 0.761 \Delta(t)\_j - 1.016 e^{0.016}}{10^{0.724} - 10} \right]
\]

(6.5)

The path of market equilibrium of Figure 6.1 now is expressed where the price spread of time j for futures contracts t
months (six months) from maturity are significantly related to the futures price and cash stock price of time \( j \) and also related to the previous futures price, cash price, and the price spread of the last time period \( j-1 \). Throughout the earlier chapters the role of expectations was emphasized. For many commodities the influence of cashprice \( P_T \) (e.g., \( P_T = P_x(0) \)) on expectations is great. Hence the significance of \( P_T \) in explaining \( \Delta \) movements is probably indicative of the influence of \( P_T \) in formulizing price expectations. Equation 6.5 is still incomplete if the intent is to completely explain the variability of \( \Delta \). At the present stage of trading in the live beef futures much needed data is not yet published. Classification of hedging and speculative activities are not yet available. In the absence of this information, equations such as 6.5 must suffice to shed light on the price-making activities.

**Intersection of TT and FF**

Any attempt to carry the discussion of TT and FF beyond the theoretical stage of analysis was completely aborted. Estimate of each equilibrium locus shown in Figure 6.1 would contribute to the total understanding of the interrelationship between the futures and cash market. Without sufficient information on contract trading composition, any estimates of TT and FF would be questionable. Some discussion of trading composition will be included later in this chapter.
Pricing Activities Over All Contracts

Price movements over the life of futures contracts must be evaluated. The last section briefly established relationships given a constant $t$ (time from contract maturity). The procedure now consists of observing pricing phenomena as $t$ varies. Specific interests are with the variability of prices, trend or seasonality of prices, and prices in last days of a trading period. This analysis may give insights regarding any statistically significant evidence of structure (anything but randomness) in the movements of beef futures prices.

Variance of futures prices

Futures prices vary over the life of trading contracts as is evident below. Observing $P_f(t)$ as $t$ varies revealed that in the trading period from January, 1966, through December, 1969, there existed a systematic trend in the variability of futures prices. The variance of futures price and time from contract maturity were inversely related, i.e., as contracts approached maturity there existed a significant increase in the variance of futures price, see Appendix D.2. The first set of equations in Table 6.3 establishes this fact for live beef futures prices. Of these the double-log form gave the most desirable results. Hence the estimated variance of futures prices (for all contracts) is related to the contract life as in equation 6.6.
Table 6.3. Estimates for the Variance of $P_f$, the Variance of $(P_T - P_f)$, and the Mean of $(P_T - P_f)$ over the futures contract life

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Var($P_f$)</td>
<td>3.1478</td>
<td>-0.1713</td>
<td>-5.0183</td>
<td>0.8846</td>
<td>1.2646</td>
<td>9</td>
</tr>
<tr>
<td>Exponential</td>
<td>Var($P_f$)</td>
<td>1.1649</td>
<td>-0.0714</td>
<td>-5.8364</td>
<td>0.9108</td>
<td>1.3282</td>
<td>9</td>
</tr>
<tr>
<td>Double-log</td>
<td>Var($P_f$)</td>
<td>1.2075</td>
<td>-0.2812</td>
<td>-7.3621</td>
<td>0.9411</td>
<td>2.3698</td>
<td>9</td>
</tr>
<tr>
<td>Linear</td>
<td>Var($P_T-P_f$)</td>
<td>1.1661</td>
<td>+0.2330</td>
<td>3.0169</td>
<td>0.7518</td>
<td>2.4620</td>
<td>9</td>
</tr>
<tr>
<td>Exponential</td>
<td>Var($P_T-P_f$)</td>
<td>0.2353</td>
<td>+1.086</td>
<td>2.8041</td>
<td>0.7273</td>
<td>2.7995</td>
<td>9</td>
</tr>
<tr>
<td>Double-log</td>
<td>Var($P_T-P_f$)</td>
<td>0.2476</td>
<td>+0.3729</td>
<td>2.3012</td>
<td>0.6563</td>
<td>2.2476</td>
<td>9</td>
</tr>
<tr>
<td>Linear</td>
<td>Mean($P_T-P_f$)</td>
<td>0.1353</td>
<td>+0.3178</td>
<td>7.6302</td>
<td>0.9448</td>
<td>0.8510</td>
<td>9</td>
</tr>
<tr>
<td>Exponential</td>
<td>Mean($P_T-P_f$)</td>
<td>-1.1525</td>
<td>+0.2880</td>
<td>4.5214</td>
<td>0.8631</td>
<td>0.8012</td>
<td>9</td>
</tr>
<tr>
<td>Double-log</td>
<td>Mean($P_T-P_f$)</td>
<td>-1.4344</td>
<td>+1.2105</td>
<td>8.3021</td>
<td>0.9528</td>
<td>1.7641</td>
<td>9</td>
</tr>
</tbody>
</table>

$^a$Variance of cash price are defined where: $\text{Var}(P_T) = 4.821$ for Chicago, $\text{Var}(P_T) = 4.6112$ for 7 markets combined.

$^b$Best estimate according to the combined results from the t test, R, and D.W.
\[
\text{LogVar}(P_f(t)) = 1.2075 - .2812 \text{Log } t
\]

or

\[
\text{Var}(P_f(t)) = 3.345 \cdot t^{-.2812} \tag{6.6}
\]

This significant relationship is important to both the hedger and speculator discussed in the micro framework of earlier chapters. All futures positions established at \( t > 0 \) must be terminated at or near the time of contract maturity. This increase in price variance may be advantageous to the speculator since price differences are his key to gains (or losses). His gains (or losses) should be indicative of his ability to determine such movements. In contrast, the same divergency of prices at or near the date of contract maturity may not be so advantageous to the hedger. Large variances in futures prices may be a deterrent to greater hedging participation to the extent that futures trading introduces a new risk element (e.g., risk from undesired futures price movements). In essence the larger the range of prices that may occur, the greater the probability of some price occurring that is undesirable for the hedger. These hedgers may not be willing to accept this added risk. Equation 6.6 alone may suggest that the live beef futures as presently functioning is a good speculative market, but may not be as desirable for hedgers.
Table 6.3 further reveals that even with this change in pricing variance, at no time within the brief history of live beef futures has the variance of futures prices exceeded the variance of fed cattle prices at either Chicago or 7 markets combined. The futures market for fed cattle (over the contract life) has been a less volatile market than the cash market in terms of price movements.

With reference to Figure 6.1 the variance of $P_f$ is interpreted where those points about the trend line of the lower graph tend to become disperse as futures contracts mature.

**Futures as a forward price**

The same set of futures prices used above may also be evaluated in light of their ability to add to the interpretation (and derivation) of forthcoming fed cattle prices. Each $P_f(t)$ is derived through the aggregate effect of all decision as was theoretically shown through the price discovery and equilibrium chapter. Hence to some extent $P_f(t)$ is the resultant for all present sources of market information and expectations. Given that $P_f(t)$ is a summary response of all trading information, does it provide any reliable information about forthcoming cash prices at maturity? In essence, what has been the historical relationship of futures prices ($t > 0$) to the final cash price ($t=0$) after product transformation is completed?
In retrospect the price difference between the fed cattle price ($P_{T}(t=0)$) at maturity and the futures price ($P_{f}(t>0)$) at some date prior to maturity can be calculated, i.e., the difference $= [P_{T}(t=0) - P_{f}(t>0)]$. The mean difference over the time series from January, 1966, to December, 1969, is calculated for each $0<t<9$ in Appendix D.2. Simple regressions of these means against the variable $t$ are derived in Table 6.3. The double-log function proved most useful primarily because of its failure to indicate autocorrelation. Equation 6.7 indicates that those prices for distance beef futures tend to underestimate the forthcoming cash price on the average overall contracts (e.g., equation 6.7 is always positive). The mean difference showed a significant positive relationship to $t>0$, hence the more distance the futures the poorer the futures price is as a forward pricing indicator.

The variances of these price differences have been calculated and related to the contract life in Table 6.3. Results expressed through equation 6.8 establishes that the variance of price differences is greatly increased with those more distance contracts. This is also indicative of the poor reliability of distance futures as a forward indicator.

The brief historical movements of futures prices for all contracts combined (see pricing scheme in Appendix D.1)
suggest that the futures price and final cash price are similar only in the nearer futures. Even though the \( P_f \) represents a good summary of available information at some point \( t>0 \), in retrospect the reliability of this price as a cash price indicator decreases significantly the more distant the futures contract.

This discussion relates to the last chapter where a theoretical construct suggested criteria for evaluating futures for forward pricing. It was required that the spread at least satisfy the conditions for industrial equilibrium. The spread must at least reflect the cost of transformation. In the absence of accurate measurements of transformation cost, the above procedure serves as an alternative test of the usefulness of forward pricing through futures. The cash price of transformed goods should at least reflect the initial input price plus all transforming cost. Therefore a test for the difference of this price and futures as calculated in equation 6.7 serves as a proxy analysis to the last chapter. In essence, a test of the relationship \([P_f(t>0) - P_x(t>0)]\) and \([P_T(t=0) - P_f(t>0)]\) may be construed to give similar conclusions; although, the latter may be more restrictive than the first.
Price trends within each contract

The existence of a price trend for each live beef futures contract as suggested in Figure 6.1 would present significant implications to all traders involved in both markets. Specifically, there are always those viable groups of traders ready to exploit any significant trend pattern within a contract life. An upward seasonal (or trend) implies that adherence to a long position in futures would be profitable on the average and the reverse holds for a downward seasonal. If hedgers are net short and are shifting risk to speculators, there ought to be an upward seasonal of futures prices if speculators are receiving compensation for risk acceptance.

Taking the complete set of deflated futures prices for each live beef futures contract and relating this to the dummy variable t (time from maturity) produced significant trend or seasonal results in three of the six futures contracts. Each of these significant relationships revealed a negative seasonal effect, i.e., futures prices tended to decrease as the contract maturity date neared. The complete regression relationships are derived in Appendix D.3. After testing for alternative functional forms, the linear relationship proved to be a satisfactory representation of any trend. These relationships are expressed in 6.9.
\[
\begin{align*}
\text{Feb. } P_f^*(t) &= 18.4740 + .1629t^* \\
\text{Apr. } P_f^*(t) &= 18.2860 + .1603t^* \\
\text{June } P_f^*(t) &= 18.8900 + .0776t \\
\text{Aug. } P_f^*(t) &= 19.6970 - .0323t \\
\text{Oct. } P_f^*(t) &= 19.9450 - .0519t \\
\text{Dec. } P_f^*(t) &= 19.0960 + .0871t^* \\
\end{align*}
\]

The hypothetical line of Figure 6.1 is somewhat indicative of the relationship for February, April and December contracts. This information suggests that on the average short hedgers experience advantageous futures price movements in these contracts. Whereas long speculators trading on the premise of rising prices experience negative returns. No seasonal effects were evident for June, August and October contracts. These observations must be evaluated in light of the total variability of each set of prices. The trend relationship accounted for a small portion of total price variability in all contracts as is shown with the small coefficient of determination. Hence, trends that appear advantageous to the short hedger may not be so when the high degree of variability of $P_f$ is considered. Likewise, what at first may appear to be undesirable to the long speculator may change given considerations of high price variability.

Nevertheless, the existence of any detectable trend at all implies that some traders can make a positive return.
The present inability to empirically identify the trading composition of live beef futures prevents identification of benefits accruing to various market participants [42, p. 14].

Calculations in Appendix D.3 established that some degree of positive autocorrelation existed over the complete set of prices in each futures contract. Hence there may be a tendency toward some degree of continuity in price movements. A first order autoregressive scheme was sufficient to correct for autocorrelation. The test for autocorrelation after this correction is shown in Appendix D.4. In all corrected equations the Durbin-Watson statistic was above the D.W. level revealing positive autocorrelation. This implies that live beef futures prices within each contract does have some month-to-month price relationship rather than complete randomness.

A test of significance for trends in equations of Appendix D.4 still substantiates that February, April and December are the only contracts with significant trends. Again the linear estimates appear to be a satisfactory functional form. One change is noted in that all trends showed negative seasonal movements after this correction, yet trends in June, August and October futures prices were still insignificant.

Interpretation of price movements must be conditioned in that live beef futures is still in an infant stage of development. Hence, the above findings of some degree of non-randomness within the pricing structure over each contract
life may change significantly as new information becomes available.

Last trading days

Movements of futures and cash prices exist as a result of trading pressures expressed via loci TT and FF. These interactions over each contract produced the trends and variability of all prices. Positions established and expressed through FF must be terminated on or before the last trading days of each contract life. Any open interest not previously terminated must be terminated within the last 20 trading days of each contract as illustrated in Figure 6.1.

It has been theoretically established that the cash price of transformed goods and the futures price must be equal in these last trading days (see chapter three). As \( t \) approaches zero the cash and futures are the same (e.g., \( \lim_{t \to 0} [P_T(0) - P_f(0)] = 0 \)). This hypothesized relationship is tested for the live beef futures.

Data for the difference at \( t=0 \) is in Appendix D.5. The mean, variance, and test for the difference in fed cattle and beef futures prices at \( t=0 \) are shown in Table 6.4. Each contract mean difference was either above or below zero. Yet testing for the hypothesis that this mean is not significantly different from zero revealed that such hypothesis could be rejected only for October futures, i.e., the mean difference
In October futures prices was significantly above zero at the .05 level. Data supported the theoretical lim t→0 for the remaining five contracts. Futures prices in the last trading days of October futures during 1968 was completely out of line with all previous prices. Hence this unusual period tended to raise the mean difference for October futures above the significant level. Some difference would be expected due to trading costs, lags in information received, lack of trading experience, lack of trading participation by both hedgers and speculators and other market imperfections. But for five of the six contracts the theoretical limits were empirically validated. Live beef futures have been functioning properly in the last twenty trading days of these contracts.

Interpretation of these findings are directly applicable to Figure 6.1. Within the segment denoted as Last 20 Trading Days, the cash and futures prices are similar. No further product transformation is required at this point. A set of equilibrium loci exist for this time similar to those shown in the upper graph. But all intersections of FF and TT should be at or near the horizontal axis where Δ=0 since the cash and futures prices come together at this time. It is noted that the fed cattle price and feeder steer price at t=0 are the same, i.e., $P_T(t=0) = P_X(t=0)$ since $P_X$ requires no further transformation.
Table 6.4. Test for price difference in the last trading days of each contract,

\[ H_0: \lim_{t \to 0} [P_x(t) - P_f(t)] = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>FEB(^a)</th>
<th>APR</th>
<th>JUNE</th>
<th>AUG</th>
<th>OCT</th>
<th>DEC</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>+ .0367</td>
<td>+ .1783</td>
<td>- .0408</td>
<td>+ .0342</td>
<td>+ .5200</td>
<td>- .2092</td>
<td>+ .0865</td>
</tr>
<tr>
<td>S.D.</td>
<td>.5342</td>
<td>.6873</td>
<td>.4336</td>
<td>.5217</td>
<td>.5957</td>
<td>.4894</td>
<td></td>
</tr>
<tr>
<td>VAR.</td>
<td>.2854</td>
<td>.4724</td>
<td>.1880</td>
<td>.2722</td>
<td>.3546</td>
<td>.2395</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>VAR/n</td>
<td>.0238</td>
<td>.0394</td>
<td>.0157</td>
<td>.0227</td>
<td>.0296</td>
<td>.0200</td>
<td></td>
</tr>
<tr>
<td>(\sqrt{\text{VAR}/n})</td>
<td>.1543</td>
<td>.1985</td>
<td>.1253</td>
<td>.1507</td>
<td>.1718</td>
<td>.1414</td>
<td></td>
</tr>
<tr>
<td>t test</td>
<td>+ .2378</td>
<td>+ .8982</td>
<td>- .3256</td>
<td>+ .2269</td>
<td>+ 3.0267(^b)</td>
<td>+ 1.4795</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)See Appendix D.5 for data.

\(^b\)Significant at the .05 level.
Contract Trading and Composition

In the last two sections price movements were evaluated with reference to Figure 6.1. Although not a measurement of prices, the trading levels and market compositions contribute to the total framework. As emphasized in prior discussion, the lack of composition data prevents obtaining an accurate estimate of the interrelationship between the cash and futures markets. Trading levels and composition will be briefly discussed below.

Trading volume and open interest

Appendix D.6 establishes the live beef futures rate of growth over the brief history of trading. In terms of volume of contracts traded from 1965 through 1969, live beef futures has been a fairly active market relative to the history of trading in livestock contracts. Trading levels over each contract within the 1965-69 period revealed that the mean trading levels of February, June, and December contracts were the largest with February contracts having the highest mean volume and open interest levels. Although trading participation did vary among contracts, each did appear to have sufficient participation to assure some degree of liquidity over the contract life. This participation is essential for the existence of a liquid market, i.e., there must always be sufficient trading interest (subjective level) to assure continuous ease of buying and selling.
All futures contracts are similar in that trading levels begin at some point \( n \) months from maturity and increase up to a period prior to the maturity date after which time offsetting positions are taken. Trends in the volumes and open interest levels have been estimates for the six live beef futures contracts. Numerous functional forms were tried, but in most cases the exponential relationship best expressed the trading trends. Most contract trading levels became inactive in periods exceeding ten months from maturity. These levels continued to increase up to \( t=1 \). The linear, exponential, and double-log forms are calculated in Appendices D.7 and D.9 and the exponential set of equations are derived in 6.10 and 6.11.

It is noted that a linear open interest trend for June contracts was favored over the exponential. All other contracts were best expressed by the exponential relationship.

\[
\begin{align*}
\text{Feb. } \text{Vol}(t) &= 350e^{-0.2571t} \\
\text{April } \text{Vol}(t) &= 325e^{-0.2727t} \\
\text{June } \text{Vol}(t) &= 325e^{-0.1820t} \\
\text{Aug. } \text{Vol}(t) &= 314e^{-0.2193t} \\
\text{Oct. } \text{Vol}(t) &= 314e^{-0.2890t} \\
\text{Dec. } \text{Vol}(t) &= 248e^{-0.1955t}
\end{align*}
\]

(6.10)
Feb. $0.\hat{l}(t) = 6003e^{-0.2624t}$

April $0.\hat{l}(t) = 4116e^{-0.2419t}$

June $0.\hat{l}(t) = 4169e^{-0.1884t}$ See Appendix D.9 for linear estimate.

Aug. $0.\hat{l}(t) = 6634e^{-0.3124t}$

Oct. $0.\hat{l}(t) = 6770e^{-0.3496t}$ (6.11)

Dec. $0.\hat{l}(t) = 5886e^{-0.2531t}$

Both series of trading level indicators (volume and open interest) showed some nonrandom movement, thus suggesting that there exist some systematic degree of participation by all traders combined over each contract life. A trend in trading levels would be expected since contracts must be purchased and sold within a specified period. Again it is essential that the classification of traders be known for a complete understanding and explanation of any observed trend. Past data does not reveal the degree to which contracted positions were speculative or hedges.

The initial estimates (including those of 6.10 and 6.11) did not give a statistically valid test of trend since all equations had a degree of autocorrelation. All equations were corrected for a first-order autocorrelation in Appendix D.8 and D.10. In each case the exponential relationship was still chosen. Autocorrelation may indicate a lagged relationship among past dependent variables, yet it is suspected that much autocorrelation in volume trading is due to omission of variables.
Market composition

In this final section a brief discussion of the market composition follows. Market composition data for all contracts combined for the months from June, 1968, through February, 1969, are shown in Table 6.5. This data is from unpublished sources and its reliability or accuracy is not validated. Table 6.5 is best used to emphasize the type of information eventually needed to quantify that theoretical framework for market and industrial equilibriums previously presented.

Tables 6.5 and 6.6 set forth the percentage distribution of open interest according to trader classification. On the average larger traders accounted for a greater percent of short than for long positions with 59.6 and 42.7 percents, respectively. Large speculative positions were more prevalent on the long side while hedgers were the predominate large short traders. Larger short commitments tended to vary more than did the large long. Yet the variance of long speculative positions exceeded that of short hedgers. Finally, in both long and short commitments a substantial share of positions were not classified according to intent of trade.

The distribution of hedge and speculative positions on both sides of the market is indicative of some balance in trading activities, i.e., this preliminary data suggest that this futures is an active hedging market. As new and more accurate data become available this observation may be
Table 6.5. Percentage distribution of trader composition of total futures open contracts on the last trading day of the months specified

<table>
<thead>
<tr>
<th>COMMITMENT DESCRIPTION</th>
<th>6/68</th>
<th>7/68</th>
<th>8/68</th>
<th>9/68</th>
<th>10/68</th>
</tr>
</thead>
<tbody>
<tr>
<td>LONG</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. LARGE TRADERS</td>
<td>43.04</td>
<td>41.37</td>
<td>44.60</td>
<td>42.05</td>
<td>37.78</td>
</tr>
<tr>
<td>Speculators long</td>
<td>64.21</td>
<td>69.57</td>
<td>45.46</td>
<td>44.66</td>
<td>41.92</td>
</tr>
<tr>
<td>Hedging long</td>
<td>35.79</td>
<td>30.43</td>
<td>54.54</td>
<td>55.34</td>
<td>58.08</td>
</tr>
<tr>
<td>b. SMALL TRADERS</td>
<td>56.96</td>
<td>58.63</td>
<td>55.40</td>
<td>57.95</td>
<td>62.22</td>
</tr>
<tr>
<td>TOTAL (a. + b.)</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>SHORT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. LARGE TRADERS</td>
<td>69.96</td>
<td>70.34</td>
<td>64.98</td>
<td>67.35</td>
<td>60.76</td>
</tr>
<tr>
<td>Speculators long</td>
<td>38.68</td>
<td>18.85</td>
<td>43.07</td>
<td>19.90</td>
<td>14.96</td>
</tr>
<tr>
<td>Hedging short</td>
<td>61.32</td>
<td>81.15</td>
<td>56.93</td>
<td>80.10</td>
<td>85.04</td>
</tr>
<tr>
<td>d. SMALL TRADERS</td>
<td>30.04</td>
<td>29.66</td>
<td>35.02</td>
<td>32.65</td>
<td>39.24</td>
</tr>
<tr>
<td>TOTAL (c. + d.)</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>TOTAL OPEN CONTRACTS</td>
<td>9,966</td>
<td>10,678</td>
<td>10,737</td>
<td>12,035</td>
<td>13,037</td>
</tr>
</tbody>
</table>

Unpublished data provided by the Commodity Exchange Authority, Chicago.
<table>
<thead>
<tr>
<th></th>
<th>11/68</th>
<th>12/68</th>
<th>1/69</th>
<th>2/69</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>42.61</td>
<td>43.51</td>
<td>40.57</td>
<td>49.14</td>
</tr>
<tr>
<td></td>
<td>46.15</td>
<td>52.12</td>
<td>54.89</td>
<td>71.83</td>
</tr>
<tr>
<td></td>
<td>53.85</td>
<td>47.88</td>
<td>45.11</td>
<td>28.17</td>
</tr>
<tr>
<td></td>
<td>57.39</td>
<td>56.49</td>
<td>59.43</td>
<td>50.86</td>
</tr>
<tr>
<td></td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td></td>
<td>59.31</td>
<td>50.02</td>
<td>47.07</td>
<td>46.37</td>
</tr>
<tr>
<td></td>
<td>16.72</td>
<td>21.99</td>
<td>21.94</td>
<td>29.06</td>
</tr>
<tr>
<td></td>
<td>83.28</td>
<td>78.01</td>
<td>78.06</td>
<td>70.94</td>
</tr>
<tr>
<td></td>
<td>40.69</td>
<td>49.98</td>
<td>52.93</td>
<td>53.63</td>
</tr>
<tr>
<td></td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td></td>
<td>14,250</td>
<td>15,303</td>
<td>18,354</td>
<td>18,708</td>
</tr>
</tbody>
</table>
Table 6.6. Mean and standard deviation of futures trading composition

<table>
<thead>
<tr>
<th>Trader</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Long Traders(^a)</td>
<td>42.74%</td>
<td>3.098%</td>
</tr>
<tr>
<td>Large Short Traders(^a)</td>
<td>59.57%</td>
<td>9.598%</td>
</tr>
<tr>
<td>Long Speculators(^b)</td>
<td>54.53%</td>
<td>11.369%</td>
</tr>
<tr>
<td>Short Hedgers(^c)</td>
<td>74.98%</td>
<td>9.885%</td>
</tr>
</tbody>
</table>

\(^a\)Expressed as a percentage of open contracts.

\(^b\)Expressed as a percentage of open contracts by long large traders.

\(^c\)Expressed as a percentage of open contracts by short large traders.
invalidated. One continual problem with such data has been the interpretation of what a hedge position is by those establishing the commitment.

Throughout past chapters the theory relating cash and futures markets exist given the premise that each contract trading composition was identified. The relationships among $T_H$ (short hedgers), $F_L$ (long hedgers), and $F_S$ (net speculative) were essential to price discovery, market equilibrium, and industrial equilibrium. Hence once the information similar in format to that of Table 6.5 is known, much of the previous theory could be made operational. The theoretical format has been established, but the structural parameters were not estimated.

Some preliminary attempts to relate this short interval of data to futures prices have been completed in Appendix D.11. Four functional forms relating $P_L$ to long speculative and short hedging positions were completed. In each case futures prices were significantly related to the long speculative market but not to the short hedging market. This may partially be representative of the relative degree of sensitivity to price movement between speculators and hedgers. Again such interpretations of data must not be weighted heavily due to the lack of information and questionable accuracy.

Within the confounds of this chapter attempts have been made to illustrate how and where the futures theory relates
directly to observed pricing and trading phenomena. Yet no attempt to completely quantify (make operational) this theoretical system was intended. Although, the fact that nonrandom activities do exist has added some information that may be useable in procedures for ultimately specifying the theoretical system. Figure 6.1 has provided a graphical summary of the types of activities occurring in a futures market. The theoretical concepts illustrated in Figure 6.1 could be made operational once all information is recorded in sufficient detail. One approach would be to estimate the relationships of equilibrium via structural equations for TT and FF. Some function for deriving expectations must be included. In essence, a system of simultaneous equations could be derived for each period t. Such a system may have variable parameters over a futures contract life (as t varies).

A simultaneous equation system would provide results for interpreting the responses of both the cash and futures prices to changes within either market. The primary intent of this method would be to explain vargarieties of both markets. Most existing cash market models such as a beef sector model have not included the effects of futures trading. The concept of futures trading has become an integral part of market planning for many commodities, hence the effect of futures trading should be included in many existing forecasting and explanatory models.
CHAPTER 7: SUMMARY AND CONCLUSION

The theory of futures trading in commodities not yet marketable has been lacking in substance and content as was emphasized in the introduction. A basic understanding of the organization and operation of such markets needed more theoretical scrutiny. Obtaining information on the past performance and characteristics of trading in such commodities was essential. Understanding the prevailing market structure as it relates to futures trading was necessary for the development of a theoretical construct representative of such markets. Within the realm of the above statements, this theoretical presentation has filled some missing gaps in the existing economic theory of trading in commodities that are in some stage of product transformation. Specifically, tradings in live beef futures have been evaluated.

The subject content of futures trading was such that a logical division in the nature of the theoretical framework was immediately evident. At one extreme there exist the individual decision maker faced with a new marketing tool (e.g., the futures market) at his disposal. At the other extreme, there exist a new market (futures market) interacting with an established market to perform some aggregate economic function. The concept of "micro" denoted a theoretical
framework for the utilization of this marketing tool. Whereas, "macro" denoted the theoretical framework for analyzing and interpreting the relationship between these two inseparable markets (e.g., the cash and futures markets for live beef). Although live beef futures has been the focal point, much of the derived theory is directly applicable to trading activities for other commodities. Hence much of the discussion was intended to provide a more inclusive theoretical framework indicative of the structure of the many different futures contracts.

Micro Synopsis

A set of price expectations, a probability distribution for these prices and a preference function for risk aversion have provided the essential elements for the generalized futures model developed in the micro framework. Within this framework optimal futures and cash market commitments were established for producers and marketing agencies. For the primary producer buying feeder cattle, the optimal cash market position was derived simultaneously with the optimal futures commitment. Processors, dealers, and other marketing agencies can utilize futures as hedges against contracted or anticipated purchases of live animals. The ratio of futures to cash position could be a) less than one (less than 100 percent hedging), b) equal to one (100 percent hedging), or
c) greater than one (hedging and speculation). In addition, it was shown that cash market participants can be pure speculators when their expectations warrant such commitments. Within the range of these commitments it was established that rigid hedging policies were not necessarily consistent with optimization. Implementing 100 percent hedging policy was shown to yield lower expected incomes and higher risk in many situations. Lack of necessary capital investments may result in loss of utility. Specifically, inadequate transformation services prevent obtaining those optimal cash and futures positions as dictated by the theory. Through the procedures of the micro framework some attempt has been completed to supplement the existing economic theory of uncertainty as it relates to the utilization of futures markets as a decision tool. Although some additions to existing theory have been made, there are further considerations that were not explored. The optimal level of \( x \) was established, yet \( x \) may take many different forms. Feeder cattle can be purchased at many weight ranges each requiring different transformation periods. Optimal timing of futures commitments was ignored. For example, an optimal futures commitment was derived; yet for beef futures there exist six possible contracts which could be used. Hence the existing micro theory needs modification to compensate for the varying nature of both inputs and contracts. Further no attempt to
empirically utilize the theory was intended. But as more information about the individual's decision making activities is available, efforts to partially quantify such theory may be useful.

Costs of inputs were assumed given for the market participants who have cash positions in period t. Without instantaneous product transformation, cost may be uncertain. The theory of long hedging could have been applied directly to the uncertainties of input prices for the participants making cash market sales commitments (e.g., the beef feeder). Hedges in both the input and output markets require two distinct futures markets. The processor (for example) can hedge inputs through live beef futures, yet outputs must be hedged through a futures market for beef carcasses. A beef feeder who completely hedges both his inputs and outputs and has expectations of zero gain or loss in the futures market could be compared to a feedlot operator feeding cattle on contract. Both sell a service at a fixed price. Both give up the possibility of higher income to avoid risk.

The theory was largely limited to discussion of decision making in the cash and futures markets. However, a decision maker could establish many of the hedging positions through forward contracting outside of an organized market. A producer could sell forward directly to a buyer, thus establishing both the selling and buying prices between two decision
makers. If both futures markets and forward contracting are feasible, then a firm must decide which alternative to utilize. Most futures markets are characterized by ease of transactions, high liquidity, and security [7]. Forward contracting may not provide these essential elements for successful hedging; hence direct forward commitments may introduce new risks through an effort to minimize price risks. On the other hand, through forward contracting the decision makers can specify terms most advantageous to the two participating parties; but futures must be traded in specified and rigidly enforced contracts. Such futures contracts are designed to meet a broad spectrum of trading needs; hence it may not be the optimal contract for any one decision maker. The micro framework has not provided a formal decision criteria for evaluating these two hedging alternatives. The similarities and differences of the two have been discussed but some extreme assumptions would be required to compare them in the model presented [115].

The model extended considerably the analysis of the relation between market output decisions and futures positions. Hedging and speculation were given precise meanings and economically related to the behavior of both buyers and sellers of a commodity traded on a futures market. The implications of the micro model for aggregate market determination of price, output, and profits of buyers and sellers followed directly from this framework.
Interpretation of the futures market and its relationship to commodities in the process of transformation (e.g., commodities that are not yet marketable) necessitated the use of both simple mathematical concepts and geometry. Through the use of these elementary tools a format was established for presenting a theory which showed the intrarelationship and interrelationship between futures trading and the cash market at the aggregate level. Specifically, the equilibrium relationships were shown, a method for explaining price discovery was explored, and finally the implications and effects from changes within the system were discussed.

At the outset it was shown that three basic equilibrium components must be satisfied if a general macro (used in the sense of two aggregate markets) model was to be specified. Specifically, the input transformation market must be in equilibrium. Net long and net short futures commitments must always be equal. Finally, the theory of transformation as applied to the problem via transformation services must be reflected in the difference between the futures price and the initial input price. Similarities among all commodities traded on the futures exchange exist, but one unique difference between storable and nonstorables was the fact that futures trading in nonstorables cannot extend beyond the date of maturity for the commodity. Whereas, the theory of storage
and futures trading was pertinent to storable goods after commodity maturity. In essence the equilibrium theory was developed for commodities that were not yet marketable. Emphasis was not on the difference in storable versus non-storable, rather emphasis was on how to specify a more general theory that may be indicative to many situations.

Implicit functional forms were postulated for each relationship. Through these postulates it was shown that the interaction between the input-transformation equilibrium and the futures market equilibrium was sufficient for price discovery in both the cash and futures markets. The equilibrium price coordinates (market equilibrium) was geometrically derived at that point of intersection between futures and cash market equilibrium locus. Therefore, changes within either implicit functional form was sufficient for explaining price movements. Only a few of the shifts resulting in new prices were considered; nevertheless, the methodology was directly applicable to any additional components that may be postulated.

Market equilibrium was sufficient for explaining price discovery, yet it was not sufficient for explaining additional entries and exits within the futures market. The concept of industrial equilibrium must be considered. Within the present construct, industrial equilibrium exist with the situation where the futures-cash price spread just reflected the cost
of transformation through a defined relationship. Given a market equilibrium with a futures-cash price spread either above or below that spread satisfying the transformation relationship, then the incentive for entry or exit is present. Finally at that point where market equilibrium and the transformation relationship coincide, an industrial equilibrium point was established. Through this procedure a method for theoretically looking at the prevailing market structure as it relates to the live beef futures market was presented.

Through theoretical abstraction the futures market for live beef and the cash markets have been related. Yet many problems arise with such procedures. Foremost among these was the problem of simplicity versus realism. Any effort to aggregate and simplify causes reduction in realism. Yet the complexities of most economic systems are such that in the absence of simplification few conclusions can be derived. Of example, the aggregate transformation function was assumed to be linear homogeneous. Initial inputs and transformation services were the only components of the function. Lack of realism was increased since both the function and its components were aggregated. Yet such simplifications were necessary for deriving the desired framework.

A series of propositions were discussed, each relating to some aspect of the theoretical framework developed. Interpretations of these propositions established the
structural relationship that must theoretically exist for a particular pricing response to occur. The basic limitation of such procedures, and in fact comparative statics in general, is that such price response are considered with numerous other factors held fixed. Hence each proposition gave insights into the pricing mechanism through the futures-cash relationship, yet applicability beyond a comparative static framework was limited.

The complete macro framework was derived without directly mentioning the role of consumption. Consumer behavior is a vital link in the market place and any changes in such behaviors will affect all prices and commitments. At the time input decisions are made, the final demand for the transformed product is unknown. This is primarily why price uncertainties exist and why futures trading serves an economic function. Consumer behavior is implicit to both the "micro" and "macro" models. Price expectations reflect consumer behavior. Expectations for most situations are unobserved variables, yet many expectation functions may be specified. With the development of a more sophisticated model it may be desirable to specify some expectation relationship, thus showing the direct influence of consumer behavior. The next synopsis will elaborate on this point.
Empirical Synopsis

The final chapter was intended to basically provide a review of existing trading activities for live beef futures and to suggest what information is needed to further quantify the theoretical framework. The complete framework was summarized through Figure 6.1 where prices were derived and then related to the total contract life. Specifically, at each period \( t > 0 \) the interaction of all elements of the model gave equilibrium prices. Empirical evidence established that some nonrandom movements of these prices existed. There exists a trend in the variability of prices as contracts mature. Some contracts showed a significant negative trend in futures prices. In many relationships a first order autocorrelation was evident. Each of these observations suggest some nonrandomness in futures prices over the contract life.

Many variables were tested for their influence on trading activities. Present cash prices of transformed goods were significant in explaining movements in the price spread. The influence of consumer behavior is reflected through this variable. This further suggests that expectations about forthcoming cash prices \( P_T \) are influenced by present final product prices and hence the present consumers' demand. Therefore any expectation model should include lagged variables representative of consumer behavior.
The role of market trading composition was emphasized throughout the analysis. Yet lack of data prevented adequate treatment of this factor. The format for needed data has been specified and appropriate suggestions made.

In the final analysis it follows that tradings in live beef futures like that of all commodities with corresponding futures contracts can be explained through a theoretical model once the appropriate postulates are made. Useful observations and insights can be gained from such procedures. Yet such models often ignore practical problems that may in fact negate theoretical conclusions. For example, the specification of the contract for live beef futures has not been discussed. The present theory assumes a satisfactory specification for all interested participants. Any deviation from this assumption would greatly alter all conclusions. Assumptions regarding the flow of information and the distribution of knowledge are sometimes unrealistic. For example, a lack of understanding of the economic usefulness of futures markets often negates what theory would suggest as a prevailing situation.

As trading in live beef futures continues and as information sources are increased, it may be desirable to apply the present framework for price discovery, forecasting, and policy decisions. The methodology has been completed and preliminary empirical suggestions made.


47. Gray, Roger W. The attack upon the potato futures market in United States. Food Research Institute Studies 4: 99-121. 1964.


56. Heady, Earl, Hildreth, R. J. and Dean, Gerald W. Uncertainty, expectation and investment decisions for a sample of Central Iowa farmers. Iowa State University of Science and Technology Agricultural Experiment Station Research Bulletin 447. 1957.


ACKNOWLEDGEMENT

The author wishes to express appreciation and gratitude to Dr. Lehman B. Fletcher for his continual guidance, suggestions, and encouragement during the course of these graduate studies. Dr. J. Marvin Skadberg and other committee members have also provided invaluable suggestions for which the author expresses sincere appreciation. Gratitude is expressed to the U.S.D.A. for their financial assistance without which this research would have been impossible.

The author wishes to acknowledge a special note of gratitude to his wife, Geraldine, for her continual moral and physical support during this period of difficult but very enjoyable part of our lives.
APPENDIX A. SIMPLIFICATION AND PLOTTING OF ISO-VARIANCES
The iso-variance curve follows from the general form of an analytic geometry equation such as shown in (1).

\[ Ax^2 + Bxx_p + Cx_p^2 + Dx + Ex_p + G = 0 \]

The general equation can then be shown to yield one form of a conic section, i.e., any plane section of a right circular cone is an ellipse, parabola, hyperbola, or a limiting form of one of these curves. In fact the iso-variance function utilized throughout the text must be shown to be an \textit{ellipse}.

The above general form can be expressed as the specific risk function of equation 2.5 and 2.6 in the text, hence giving (2).

\[ \sigma^2_p x^2 - 2\rho \sigma_p \sigma_F x x_p + \sigma_F^2 x_p^2 = \text{Var} (\pi) \]

where

\[ A = \sigma_p^2 = \text{Var}[P(t-n) - P(t)] = \text{Var}[P(t-n) \mid P(t)], \]
\[ B = -2 \rho \sigma_p \sigma_F, \]
\[ C = \sigma_F^2 = \text{Var}[F(t-n) - F(t)] = \text{Var}[F(t-n) \mid F(t)], \]
\[ D = E = 0, \]
\[ G = - \text{Var} (\pi). \]

Prior to reducing equation (2) to the standard form (removing the \( x x_p \) term) it is desirable to determine the type of conic section. If this section is such that \( B^2 - \)
4AC<0 then it forms an ellipse [128, p. 132]. Taking the
definition from (2) it follows that the above restriction
exists as long as the correction between cash and futures
is not perfect. This follows in (3).

\[ (3) \quad 4p^2 \sigma_p^2 \sigma_F^2 - 4\sigma_p^2 \sigma_F^2 < 0 \]

then

\[ \rho^2 < 1. \]

The objective now is to solve (2) for x and \( x_F \). This
can be done if the cross product term is removed. The process
of rotation of the axes effects the removal of the \( x x_F \) term
from the equation of the second degree if the proper angle
is chosen. Using the rotation formulas of (4) and (5) and
the substituting into (2) the coefficients for the cross pro-
duct \( x' x'_F \) can be determined as in (6).

\[ (4) \quad x = x' \cos \theta - x'_F \sin \theta \]
\[ (5) \quad x'_F = x' \sin \theta + x'_F \cos \theta \]
\[ (6) \quad B' = -A \sin 2\theta + B \cos 2\theta + C \sin 2\theta \]
The \( x' x'_F \) term will vanish if \( B' = 0 \). Then (7) follows
directly.

\[ (7) \quad \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\rho \sigma_p \sigma_F}{\sigma_F^2 - \sigma_p^2} \]
Cos 2θ can be determined directly by the use of the formula
\[ a^2 = b^2 + y^2 \] as in Figure A.1 since \( \cos 2\theta = \frac{y}{a} \).

\[ \gamma = (\sigma_P^2 - \sigma_p^2) \]

Solving for \( \alpha \), the \( \cos 2\theta \) relationship is expressed as in (8).

\[ \text{(8)} \cos 2\theta = \frac{[\sigma_P^2 - \sigma_p^2]}{\sqrt{[\sigma_P^2] + [\sigma_P^2]^2 + 2\sigma_P^2\sigma_p^2(2\rho^2 - 1)}} = \frac{\gamma}{\alpha} \]

Both \( \cos \) and \( \sin \) must be known for solutions to (4) and (5) to be determined. Hence using the half-angle formulas of trigonometry, these angles can be derived.

\[ \text{(9)} \sin \theta = \sqrt{1 - \cos 2\theta} = \sqrt{1 - \frac{[\sigma_P^2 - \sigma_p^2]}{\sqrt{[\sigma_P^2]^2 + [\sigma_P^2]^2 + 2\sigma_P^2\sigma_p^2(2\rho^2 - 1)}}} \]

\[ \text{(10)} \cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}} = \sqrt{\frac{(\sigma_P^2 - \sigma_p^2)}{\sqrt{(\sigma_P^2)^2 + (\sigma_P^2)^2 + 2\sigma_P^2\sigma_p^2(2\rho^2 - 1)}}} \]

The values from (9) and (10) are substituted into (4) and (5) and these solutions are then substituted into the risk function (2). This procedure removes the cross product
term from the iso-variance equation. Solving the new risk function for $x'_F$ given each possible $x'$ can follow after the appropriate level of $\text{Var}(\pi)_{ij}$ is assumed.

This procedure can be shown with the following hypothetical example. Let $\frac{\sigma^2}{p} = 41$, $\frac{\sigma^2}{p} = 34$, and $\rho = .321$, then the iso-variance exist as illustrated in (11). The iso-variance level of 25 was

\[
(11) \quad \text{Var}(\pi) = 41 x^2 - 24 x x'_p + 34 x'_p^2 = 25
\]

chosen so the above pedagogical example would present a simplified solution. The $\tan \theta$ of (7) and the $\cos \theta$ can be calculated.

\[
(12) \quad \tan \theta = -\frac{24}{7}
\]

\[
(13) \quad \cos \theta = -\frac{7}{25}
\]

Using the trigometric formulas (9) and (10) gives the desired values.

\[
(14) \quad \sin \theta = \frac{4}{5}
\]

\[
(15) \quad \cos \theta = \frac{3}{5}
\]

Hence (4) and (5) follow in (16) and (17).

\[
(16) \quad x = \frac{3}{5} x' + \frac{4}{5} x'_p
\]

\[
(17) \quad x'_p = \frac{4}{5} x' + \frac{3}{5} x'_p
\]
Substituting (16) and (17) back into (11) yields the desired relationship where the cross product is removed.

\[
25x'^2 + 50x_p^2 = 25
\]

\[
x'^2 + 2x'_p = 1
\]

From (18) it follows that an ellipse of semi-axes 1 and \( \frac{\sqrt{2}}{2} \) exist for \( x' \) and \( x'_p \) respectively.

For those pairs \((x', x'_p)\) the alternative combinations of futures \((x_p)\) and cash positions \((x)\) can be determined by substituting back into (4) and (5) given the present set of assumed parameters. Of course these values will change as such parameters change, but for the ellipse to exist the coefficients in (18) must always be of the same sign and never equal zero [128].
APPENDIX B: CONCAVITY AND THE ISO-VARIABLES
A sufficient condition for the concave downward function can be determined.

(1) \( \text{Var}(\pi)_j = x^2 \sigma_p^2 + x_F^2 \sigma_F^2 - 2x \sigma_p \sigma_F \rho_p \)

(2) \( d\text{Var}(\pi)_j = Vx dx + Vx_F dx_F = 0 \)

Then the slope at any point on the iso-variance curve is determined as in (3).

\[
\frac{dx_F}{dx} = -\frac{Vx}{Vx_F} = -\frac{[x \sigma_p^2 - x_F \sigma_p \sigma_F]}{[x_F \sigma_F^2 - x \sigma_p \sigma_F]} < 0
\]

For the risk function to be concave downward the derivative of (3) must be negative within some limits of \( x \) and \( x_F \). Taking this derivative the sufficient restraint follows.

\[
\frac{d}{dx} \left( \frac{-Vx}{Vx_F} \right) = \frac{d^2 x_F}{dx^2} = \frac{-1}{(Vx_F)^2} \left\{ Vx_F \frac{dVx}{dx} - Vx \frac{dVx_F}{dx} \right\} < 0
\]

\[
\frac{dVx}{dx} = \sigma_p^2 \frac{dx_F}{dx} \rho_p \sigma_F
\]

\[
\frac{dVx_F}{dx} = \sigma_F^2 \frac{dx_F}{dx} - \rho_p \sigma_p \sigma_F
\]

Utilizing results from (3), (5), and (6) and substituting into (4) leads to the conclusion shown in (7).
\[
\frac{d^2x_F}{dx^2} = \frac{[-(1 - \rho^2)]}{[x_F \sigma_p^2 - x \rho \sigma_p \sigma_F]^2} \left\{ x_F \sigma_p^2 \sigma_F^2 - x \sigma_p^2 \sigma_F^2 \frac{dx_F}{dx} \right\} < 0
\]

For (7) to always be negative it follows that the relationship within the brackets must be positive since both \((1 - \rho^2)\) and \([x_F \sigma_p^2 - x \rho \sigma_p \sigma_F]^2\) are never negative. Therefore this yields (8).

\[
(8) \quad x_F \sigma_p^2 \sigma_F^2 - x \sigma_p^2 \sigma_F^2 \frac{dx_F}{dx} > 0
\]

\[
(9) \quad x_F > x \frac{dx_F}{dx}
\]

Given the specifications that both risk and correlation are positive, then the evaluation of (9) reveals that

a) \(\frac{dx_F}{dx} > 0\) where \(V_x < 0\) and \(V_{x_F} > 0\) and

b) \(\frac{dx_F}{dx} < 0\) where \(V_x > 0\) and \(V_{x_F} > 0\)

define the concave section. The initial risk function follows directly from (9) given these limits.

\[
(10) \quad x_F > -\frac{(x \sigma_p^2 - x_F \rho \sigma_p \sigma_F)}{(x_F \sigma_p^2 - x \rho \sigma_p \sigma_F)} x
\]

\[
(11) \quad x_F \sigma_P^2 - x x_F \rho \sigma_p \sigma_F > -(x^2 \sigma_p^2 - x x_F \rho \sigma_p \sigma_F)
\]

\[
(12) \quad x^2 \sigma_p^2 - 2x x_F \rho \sigma_p \sigma_F + x_F \sigma_F^2 > 0
\]

These procedures establish the fact that the iso-variances must have a concave segment.
APPENDIX C: MATRIX SOLUTIONS FOR INTERTEMPORAL PRICE DISCOVERY
Appendices C.1 through C.4 were derived from the market equilibrium equations 3.8 and 3.10. Equation 3.8 was written slightly different so that all relationships can be expressed as functions of $\Delta$ and $P_f$.

\[
X(P_x, a) G(L^*) = T_u(P_x, P_L, e) + T_H(\Delta, P_L, e) \tag{3.8}
\]

\[
X(P_f - \Delta, a) G(L^*) = T_u(P_f - \Delta, P_L, e) + T_H(\Delta, P_L, e) \tag{3.8'}
\]

where $\Delta = [P_f - P_x]$ or $P_x = [P_f - \Delta]$

\[
T_H(\Delta, P_L, e) = F_L(P_f, e) + F_S(P_f, b) \tag{3.10}
\]

Math Appendix C.1.

Change in Transformation Service Cost.

\[
\begin{bmatrix}
P_x - P_x G \\
T_u - T_u + X^G
\end{bmatrix}
\begin{bmatrix}
\frac{\partial P_f}{\partial P_L} \\
\frac{\partial \Delta}{\partial P_L}
\end{bmatrix}
= 
\begin{bmatrix}
-P_f + P_L \\
-T_H
\end{bmatrix}
\]

\[
|M_1| = T_u [P_x - P_x G] + [T^\Delta - T_u + X^G] = +[(-) - (+)] + [(-) + (-)] = (+) - (-) + (+) = (+) - (+) - (-) = (-) < 0
\]

\[
|M_{1.1}| = - [T_u + T_H][T^\Delta - T_u + X^G] = - [(-) + (-)](+) - (+)(+) = (-)(-) - (+) = (+) - (+) < 0
\]
\[ |M_{1.2}| = [T^p_x - X^p x G][-T^p_H] - [F^p f + F^p_p][T^p_L + T^p_H] \]

\[ = [(-) - (+)] (+) - [(-) + (-)][(-) + (-)] \]

\[ = (-)(+) - (-)(-) = (-) - (+) = (-) < 0 \]

\[ \therefore \frac{\partial P_f}{\partial P^o_L} = \left| M_{1.1} \right| > 0 \text{ and } \frac{\partial \Delta}{\partial P^o_L} = \left| M_{1.2} \right| > 0 \]

Math Appendix C.2.

Change in Expectations

\[
\begin{bmatrix}
[T^p_x - X^p x G(L^*)][T^\Delta - T^p_x + X^p x G]
\end{bmatrix} \begin{bmatrix}
\frac{\partial P_f}{\partial \epsilon}
\frac{\partial \Delta}{\partial \epsilon}
\end{bmatrix} = \begin{bmatrix}
-\left[ T^e_u + T^e_H \right]
F^e_L - T^e_H
\end{bmatrix}
\]

\[ T_u(P_f - \Delta, P^o_L, \epsilon) + T_H(\Delta, P^o_L, \epsilon) = X(P_f - \Delta, a) G(L^*) \]

\[ T_H(\Delta, P^o_L, \epsilon) = F^o_L(P_f, \epsilon) + F^o_S(P_f, b) \]

\[ |M_2| = [T^p_x - X^p x G]T^\Delta + [F^p f + F^p_p][T^\Delta - T^p_x + X^p x G] < 0 \]

\[ = (-)(+) + (-)(+) = (-) \]

\[ |M_{2.1}| = -[T^e_u + T^e_H][T^\Delta] - [F^e_L - T^e_H][T^\Delta - T^p_x + X^p x G] \leq 0 \]

\[ = -[(+) (+)] - [(-) - (+)][(+)] = (-) - (-)(+) \]

\[ = [(-) + (+)] > 0 \]
\[ |M_{2.2}| = [T_u^P - X^P G][P_L^e - T^H] - [P_L^P + P_S^P][T_u^e + T_H^e] > 0 \]

\[ = (-)(-) - (-)(+) = (+) + (+) = (+) \]

\[ \therefore \frac{\partial P}{\partial e} = \frac{|M_{2.1}|}{M_2} < 0 \quad \text{and} \quad \frac{\partial \Delta}{\partial e} = \frac{|M_{2.2}|}{M_2} < 0 \]

Math Appendix C.3

Change in Speculative Interest

\[
\begin{bmatrix}
-F_S^P + F_L^P & T_A^H \\
[T_u^P - X^P G(L^*)][T_H^A - T_u^P + X^P G(L^*)]
\end{bmatrix}
\begin{bmatrix}
\frac{\partial P}{\partial b} \\
\frac{\partial \Delta}{\partial b}
\end{bmatrix}
= \begin{bmatrix}
F_S^b \\
0
\end{bmatrix}
\]

\[ |M_3| = -[F_S^P + F_L^P][T_H^A - T_u^P + X^P G(L^*)] - T_H^A[T_u^P - X^P G(L^*)] \]

\[ = [(-) + (-)] (+) - (+)[(-) - (+)] = (+) - (-) = (+) \]

\[ |M_{3.1}| = F_S^b[T_H^A - T_u^P + X^P G] > 0 \quad \text{since} \quad F_S^b > 0. \]

\[ |M_{3.2}| = -[T_u^P - X^P G(L^*)] F_S^b > 0 \]

\[ \therefore \frac{\partial P}{\partial b} = \frac{|M_{3.1}|}{M_3} > 0 \quad \text{and} \quad \frac{\partial \Delta}{\partial b} = \frac{|M_{3.2}|}{M_3} > 0 \]
Math Appendix C.4.

Changing in Input Supplies

\[
\begin{bmatrix}
[T_{u}^P - X^P G][T_{H}^A - T_{u}^P + X^P G(L^*)] \\
-[F_{L}^P + F_{S}^P]
\end{bmatrix}
\begin{bmatrix}
\frac{\partial P_{l}}{\partial a} \\
\frac{\partial \Delta}{\partial a}
\end{bmatrix}
= \begin{bmatrix}
x^a \\
0
\end{bmatrix}
\]

\[
|M_{4}| = [T_{H}^A T_{u}^P - T_{H}^A X^P G(L^*)] + [F_{L}^P + F_{S}^P][T_{H}^A - T_{u}^P + X^P G(L^*)]
\]

\[
= [(+) (-) - (+)] + [(-) (+)]
\]

\[
= (-) + (-) + (-) = (-)
\]

\[
|M_{4.1}| = x^a T_{H}^A > 0 \text{ since } x^a > 0
\]

\[
|M_{4.2}| = -[F_{L}^P + F_{S}^P] [x^a] (-1) = x^a [F_{L}^P + F_{S}^P] < 0
\]

\[
\therefore \frac{\partial \Delta}{\partial a} = \frac{|M_{4.2}|}{|M_{4}|} > 0 \text{ and } \frac{\partial P_{l}}{\partial a} = \frac{|M_{4.1}|}{|M_{4}|} < 0
\]
Appendix D.1. A scheme for selecting that futures contract with maturity date nearest to that of the maturity of the transformed input

<table>
<thead>
<tr>
<th>Date for Final Output</th>
<th>Near Futures Contract</th>
<th>Months before X(t) is Mature&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t=1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>JAN</td>
<td>PEB JAN DEC NOV OCT SEPT AUG JULY JUNE MAY</td>
<td></td>
</tr>
<tr>
<td>FEB</td>
<td>FEB FEB JAN DEC NOV OCT SEPT AUG JULY JUNE</td>
<td></td>
</tr>
<tr>
<td>MAR</td>
<td>APR MAR FEB JAN DEC NOV OCT SEPT AUG JULY</td>
<td></td>
</tr>
<tr>
<td>APR</td>
<td>APR MAR FEB JAN DEC NOV OCT SEPT AUG JULY</td>
<td></td>
</tr>
<tr>
<td>MAY</td>
<td>JUNE MAY APR MAR PEB JAN DEC NOV OCT SEPT</td>
<td></td>
</tr>
<tr>
<td>JUNE</td>
<td>JUNE JUNE MAY APR MAR FEB JAN DEC NOV OCT</td>
<td></td>
</tr>
<tr>
<td>JULY</td>
<td>AUG JULY JUNE MAY APR MAR FEB JAN DEC NOV</td>
<td></td>
</tr>
<tr>
<td>AUG</td>
<td>AUG AUG JULY JUNE MAY APR MAR FEB JAN DEC</td>
<td></td>
</tr>
<tr>
<td>SEPT</td>
<td>OCT SEPT AUG JULY JUNE MAY APR MAR FEB JAN</td>
<td></td>
</tr>
<tr>
<td>OCT</td>
<td>OCT OCT SEPT AUG JULY JUNE MAY APR MAR FEB</td>
<td></td>
</tr>
<tr>
<td>NOV</td>
<td>DEC NOV OCT SEPT AUG JULY JUNE MAY APR MAR</td>
<td></td>
</tr>
<tr>
<td>DEC</td>
<td>DEC DEC NOV OCT SEPT AUG JULY JUNE MAY APR</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>This column indicates those months of maturity for some product that has just completed a transformation process.

<sup>b</sup>Six futures contracts exist, hence that contract maturing nearest to that maturity date for the product is chosen. For example, livestock mature in Jan., hence futures tradings will be in February contracts.

<sup>c</sup>t is the months from maturity of the futures contract. If feeders are placed on a feedlot and require six months for feeding and if such inputs X(6) mature in January, then any futures hedging will be in February contracts with positions established in August. See the first line above for this example.
Appendix D.2. The variance of futures prices, the variance of cash-futures price difference, and the mean difference in relationship to months from contract maturity for all contracts combined

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>Var((P_f))^a</th>
<th>Var((P_T-P_f))^b</th>
<th>Mean((P_T-P_f))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000</td>
<td>3.16000</td>
<td>1.31000</td>
<td>0.24000</td>
</tr>
<tr>
<td>2.00000</td>
<td>3.20000</td>
<td>2.85000</td>
<td>0.33000</td>
</tr>
<tr>
<td>3.00000</td>
<td>2.32000</td>
<td>1.08000</td>
<td>1.28000</td>
</tr>
<tr>
<td>4.00000</td>
<td>2.11000</td>
<td>1.66000</td>
<td>1.59000</td>
</tr>
<tr>
<td>5.00000</td>
<td>2.09000</td>
<td>2.25000</td>
<td>2.06000</td>
</tr>
<tr>
<td>6.00000</td>
<td>2.09000</td>
<td>2.44000</td>
<td>2.31000</td>
</tr>
<tr>
<td>7.00000</td>
<td>2.03000</td>
<td>2.70000</td>
<td>2.47000</td>
</tr>
<tr>
<td>8.00000</td>
<td>1.92000</td>
<td>3.01000</td>
<td>2.54000</td>
</tr>
<tr>
<td>9.00000</td>
<td>1.70000</td>
<td>3.68000</td>
<td>2.60000</td>
</tr>
</tbody>
</table>

^aVariance for each t is calculated over monthly futures price data from 1965 through 1969, reference [12], [13], [14].

^bSee reference [122] for data sources for \(P_T(0)\).
Appendix D.3. Estimate and test for futures price trend over the life of each live beef futures contract

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Feb</td>
<td>18.4740</td>
<td>+.1629</td>
<td>4.1445</td>
<td>.5530</td>
<td>.5502</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Feb</td>
<td>2.9168</td>
<td>+.0083</td>
<td>4.1757</td>
<td>.5558</td>
<td>.5564</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb</td>
<td>2.9102</td>
<td>+.0354</td>
<td>3.7120</td>
<td>.5110</td>
<td>.5761</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr</td>
<td>18.2860</td>
<td>+.1603</td>
<td>3.4309</td>
<td>.4636</td>
<td>.4610</td>
<td>45</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr</td>
<td>2.9054</td>
<td>+.0024</td>
<td>3.4380</td>
<td>.4644</td>
<td>.4561</td>
<td>45</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr</td>
<td>2.9092</td>
<td>+.0291</td>
<td>2.5136</td>
<td>.3579</td>
<td>.4069</td>
<td>45</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun</td>
<td>18.8900</td>
<td>+.0776</td>
<td>1.3270</td>
<td>.2053</td>
<td>.4605</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun</td>
<td>2.9380</td>
<td>+.0039</td>
<td>1.2613</td>
<td>.1956</td>
<td>.4427</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun</td>
<td>2.9523</td>
<td>+.0051</td>
<td>.3716</td>
<td>.0587</td>
<td>.4297</td>
<td>42</td>
</tr>
<tr>
<td>Linear</td>
<td>Aug</td>
<td>19.6970</td>
<td>-.0323</td>
<td>-.5573</td>
<td>.0879</td>
<td>.5365</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug</td>
<td>2.9794</td>
<td>-.0018</td>
<td>-.5867</td>
<td>.0924</td>
<td>.5310</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug</td>
<td>2.9892</td>
<td>-.0129</td>
<td>-.9857</td>
<td>.1540</td>
<td>.5704</td>
<td>42</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct</td>
<td>19.9450</td>
<td>-.0519</td>
<td>-.9209</td>
<td>.1459</td>
<td>.5745</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct</td>
<td>2.9930</td>
<td>-.2895</td>
<td>-1.0010</td>
<td>.1583</td>
<td>.5683</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Oct</td>
<td>2.9949</td>
<td>-.0119</td>
<td>-.9576</td>
<td>.1516</td>
<td>.5712</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec</td>
<td>19.0960</td>
<td>+.0871</td>
<td>1.9164</td>
<td>.2835</td>
<td>.4413</td>
<td>44</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec</td>
<td>2.9494</td>
<td>+.0043</td>
<td>1.8694</td>
<td>.2772</td>
<td>.4431</td>
<td>44</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec</td>
<td>2.9376</td>
<td>+.0237</td>
<td>2.3404</td>
<td>.3397</td>
<td>.4199</td>
<td>44</td>
</tr>
</tbody>
</table>

a All prices were deflated by using the Dow Jones Futures Index starting with January, 1966, through December, 1969.

b D. W. below approximately 1.44 indicates positive autocorrelation.
Appendix D.4. Estimate and test for futures price trend over the life of each live beef futures contract after the correction for a positive first order autocorrelation in the errors

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t.test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Feb P_f</td>
<td>5.1923</td>
<td>+.1064</td>
<td>3.4535</td>
</tr>
<tr>
<td>Exponential</td>
<td>Feb P_f</td>
<td>.8173</td>
<td>+.0053</td>
<td>3.4114</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb P_f</td>
<td>.8462</td>
<td>+.0201</td>
<td>2.9443</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr P_f</td>
<td>4.3661</td>
<td>+.0811</td>
<td>2.6666</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr P_f</td>
<td>.6707</td>
<td>+.0042</td>
<td>2.6960</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr P_f</td>
<td>.5991</td>
<td>+.0148</td>
<td>2.2587</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun P_f</td>
<td>4.4307</td>
<td>+.0496</td>
<td>1.2189</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun P_f</td>
<td>.6547</td>
<td>+.0024</td>
<td>1.1506</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun P_f</td>
<td>.7367</td>
<td>+.0054</td>
<td>.6508</td>
</tr>
<tr>
<td>Linear</td>
<td>Aug P_f</td>
<td>5.2170</td>
<td>+.0383</td>
<td>.8952</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug P_f</td>
<td>.7876</td>
<td>+.0021</td>
<td>.9115</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug P_f</td>
<td>.8465</td>
<td>+.0060</td>
<td>.6801</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct P_f</td>
<td>5.6908</td>
<td>+.0135</td>
<td>.2862</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct P_f</td>
<td>.8487</td>
<td>+.0005</td>
<td>.1983</td>
</tr>
<tr>
<td>Double-log</td>
<td>Oct P_f</td>
<td>.8530</td>
<td>+.0015</td>
<td>.1639</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec P_f</td>
<td>4.2115</td>
<td>+.1298</td>
<td>3.8938</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec P_f</td>
<td>.6533</td>
<td>+.0066</td>
<td>3.9325</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec P_f</td>
<td>.6179</td>
<td>+.0297</td>
<td>4.4439</td>
</tr>
</tbody>
</table>

*aSample variance before and after the correction for autocorrelation.*
<table>
<thead>
<tr>
<th>R</th>
<th>D.W.</th>
<th>n</th>
<th>Sample Variance (before)</th>
<th>Variance (after)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4888</td>
<td>1.5832</td>
<td>40</td>
<td>.6199</td>
<td>.2769</td>
</tr>
<tr>
<td>.4842</td>
<td>1.5726</td>
<td>40</td>
<td>.0016</td>
<td>.0007</td>
</tr>
<tr>
<td>.4310</td>
<td>1.4983</td>
<td>40</td>
<td>.0017</td>
<td>.0008</td>
</tr>
<tr>
<td>.3805</td>
<td>1.5177</td>
<td>44</td>
<td>.9932</td>
<td>.3374</td>
</tr>
<tr>
<td>.3841</td>
<td>1.5097</td>
<td>44</td>
<td>.0027</td>
<td>.0009</td>
</tr>
<tr>
<td>.3291</td>
<td>1.4792</td>
<td>44</td>
<td>.0030</td>
<td>.0009</td>
</tr>
<tr>
<td>.1956</td>
<td>1.7840</td>
<td>41</td>
<td>1.3633</td>
<td>.5305</td>
</tr>
<tr>
<td>.1812</td>
<td>1.7675</td>
<td>41</td>
<td>.0037</td>
<td>.0014</td>
</tr>
<tr>
<td>.1037</td>
<td>1.7542</td>
<td>41</td>
<td>.0039</td>
<td>.0014</td>
</tr>
<tr>
<td>.1419</td>
<td>1.7836</td>
<td>41</td>
<td>1.3563</td>
<td>.8013</td>
</tr>
<tr>
<td>.1444</td>
<td>1.7816</td>
<td>41</td>
<td>.0037</td>
<td>.0015</td>
</tr>
<tr>
<td>.1083</td>
<td>1.7643</td>
<td>41</td>
<td>.0036</td>
<td>.0015</td>
</tr>
<tr>
<td>.0464</td>
<td>1.7804</td>
<td>40</td>
<td>1.2117</td>
<td>.5487</td>
</tr>
<tr>
<td>.0322</td>
<td>1.7971</td>
<td>40</td>
<td>.0031</td>
<td>.0014</td>
</tr>
<tr>
<td>.0264</td>
<td>1.7961</td>
<td>40</td>
<td>.0032</td>
<td>.0014</td>
</tr>
<tr>
<td>.5196</td>
<td>1.6478</td>
<td>43</td>
<td>.9330</td>
<td>.3363</td>
</tr>
<tr>
<td>.5233</td>
<td>1.6496</td>
<td>43</td>
<td>.0024</td>
<td>.0009</td>
</tr>
<tr>
<td>.5702</td>
<td>1.7304</td>
<td>43</td>
<td>.0023</td>
<td>.0008</td>
</tr>
</tbody>
</table>
Appendix D.5. Data for the last trading days in each contract of live beef futures\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>FEBRUARY(^b)</th>
<th>APRIL</th>
<th>JUNE</th>
<th>AUGUST</th>
<th>OCTOBER</th>
<th>DECEMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.55000(^c)</td>
<td>0.77000</td>
<td>0.37999</td>
<td>-0.59999</td>
<td>-0.50000</td>
<td>-0.64999</td>
</tr>
<tr>
<td></td>
<td>-0.58000</td>
<td>0.95001</td>
<td>-0.09999</td>
<td>-0.43001</td>
<td>-0.30000</td>
<td>-0.70000</td>
</tr>
<tr>
<td></td>
<td>0.20001</td>
<td>0.70000</td>
<td>0.14999</td>
<td>0.01999</td>
<td>0.06001</td>
<td>-0.39000</td>
</tr>
<tr>
<td></td>
<td>-0.43001</td>
<td>-0.45000</td>
<td>0.06999</td>
<td>-0.70001</td>
<td>0.78000</td>
<td>0.71999</td>
</tr>
<tr>
<td>1967</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.31999</td>
<td>-0.48000</td>
<td>-0.75000</td>
<td>-0.43001</td>
<td>0.25000</td>
<td>0.36000</td>
</tr>
<tr>
<td></td>
<td>-0.47000</td>
<td>-1.37001</td>
<td>-0.55000</td>
<td>0.10001</td>
<td>0.75000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.97000</td>
<td>0.75000</td>
<td>0.14999</td>
<td>0.20000</td>
<td>1.62000</td>
<td>0.23000</td>
</tr>
<tr>
<td>1968</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.38000</td>
<td>0.65001</td>
<td>0.02000</td>
<td>0.20001</td>
<td>1.22000</td>
<td>0.03001</td>
</tr>
<tr>
<td></td>
<td>0.61000</td>
<td>0.45000</td>
<td>0.02000</td>
<td>0.34999</td>
<td>0.72000</td>
<td>-0.87000</td>
</tr>
<tr>
<td></td>
<td>0.64999</td>
<td>-0.18001</td>
<td>0.72000</td>
<td>0.18001</td>
<td>0.59999</td>
<td>-0.70001</td>
</tr>
<tr>
<td>1969</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.20000</td>
<td>0.20000</td>
<td>0.09999</td>
<td>1.20001</td>
<td>0.66000</td>
<td>-0.25000</td>
</tr>
<tr>
<td></td>
<td>-0.22000</td>
<td>0.14999</td>
<td>-0.70000</td>
<td>0.31999</td>
<td>0.38000</td>
<td>-0.29001</td>
</tr>
</tbody>
</table>

\(^a\)Data Source: Livestock and Meat Statistics, reference [122].

\(^b\)Difference = [fed cattle price in Chicago for choice (1100 to 1250 lbs.) steers--live beef futures price].

\(^c\)All prices are recorded as the week ending price for the last three weeks of each contract trading period.
Appendix D.6. Monthly trading of live beef futures contracts on the Chicago Mercantile Exchange

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>......</td>
<td>......</td>
<td>......</td>
<td>......</td>
<td>......</td>
<td>......</td>
</tr>
<tr>
<td>1965</td>
<td>1,081</td>
<td>1,943</td>
<td>3,012</td>
<td>3,760</td>
<td>6,059</td>
<td>9,114</td>
</tr>
<tr>
<td>1966</td>
<td>15,327</td>
<td>12,680</td>
<td>15,402</td>
<td>11,306</td>
<td>7,510</td>
<td>12,262</td>
</tr>
<tr>
<td>1967</td>
<td>32,357</td>
<td>27,584</td>
<td>22,596</td>
<td>20,175</td>
<td>27,056</td>
<td>25,497</td>
</tr>
<tr>
<td>1968</td>
<td>25,961</td>
<td>31,906</td>
<td>21,080</td>
<td>25,507</td>
<td>15,812</td>
<td>16,920</td>
</tr>
<tr>
<td>1969</td>
<td>35,316</td>
<td>46,633</td>
<td>109,477</td>
<td>......</td>
<td>......</td>
<td>......</td>
</tr>
</tbody>
</table>

^Reference [14].

This data was for all contracts combined. But the average mid-month (15th of month) volume and open interest varied among contracts, where the mean levels were as calculated:

<table>
<thead>
<tr>
<th>Contracts</th>
<th>Vol.</th>
<th>O.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEB</td>
<td>172</td>
<td>2428</td>
</tr>
<tr>
<td>APR</td>
<td>143</td>
<td>1876</td>
</tr>
<tr>
<td>JUN</td>
<td>183</td>
<td>2268</td>
</tr>
<tr>
<td>AUG</td>
<td>146</td>
<td>1937</td>
</tr>
<tr>
<td>OCT</td>
<td>126</td>
<td>1669</td>
</tr>
<tr>
<td>DEC</td>
<td>163</td>
<td>2236</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>......</td>
<td>......</td>
<td>......</td>
</tr>
<tr>
<td>4,964</td>
<td>5,185</td>
<td>2,866</td>
</tr>
<tr>
<td>10,405</td>
<td>20,636</td>
<td>12,557</td>
</tr>
<tr>
<td>21,544</td>
<td>21,838</td>
<td>31,691</td>
</tr>
<tr>
<td>17,586</td>
<td>16,575</td>
<td>10,838</td>
</tr>
<tr>
<td>......</td>
<td>......</td>
<td>......</td>
</tr>
</tbody>
</table>
Appendix D.7. Estimate and test for futures volume trading trend over the life of each live beef futures contract

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Feb Vol.</td>
<td>346.7100</td>
<td>-29.2944</td>
<td>-2.6978</td>
<td>.3966</td>
<td>1.2418</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Feb Vol.</td>
<td>5.8602</td>
<td>.2571</td>
<td>-4.1054</td>
<td>.5493</td>
<td>.9437</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb Vol.</td>
<td>6.0478</td>
<td>-1.0775</td>
<td>-3.6100</td>
<td>.5004</td>
<td>.8926</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr Vol.</td>
<td>261.2800</td>
<td>-19.7277</td>
<td>-3.2192</td>
<td>.4407</td>
<td>1.0208</td>
<td>45</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr Vol.</td>
<td>5.7829</td>
<td>-.2727</td>
<td>-4.2434</td>
<td>.5433</td>
<td>.8204</td>
<td>45</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr Vol.</td>
<td>5.8090</td>
<td>-1.0425</td>
<td>-3.4053</td>
<td>.4609</td>
<td>.7880</td>
<td>45</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun Vol.</td>
<td>317.3200</td>
<td>-23.4790</td>
<td>-3.7334</td>
<td>.5083</td>
<td>1.0248</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun Vol.</td>
<td>5.7845</td>
<td>-.1820</td>
<td>-3.6437</td>
<td>.4992</td>
<td>.7595</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun Vol.</td>
<td>5.8176</td>
<td>-.6966</td>
<td>-3.0676</td>
<td>.4364</td>
<td>.7635</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug Vol.</td>
<td>5.7504</td>
<td>-.2193</td>
<td>-4.8313</td>
<td>.6071</td>
<td>1.0475</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug Vol.</td>
<td>5.7634</td>
<td>-.8247</td>
<td>-3.8630</td>
<td>.5212</td>
<td>1.0835</td>
<td>42</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct Vol.</td>
<td>236.0800</td>
<td>-19.3190</td>
<td>-3.0105</td>
<td>.4342</td>
<td>1.0762</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct Vol.</td>
<td>5.7049</td>
<td>-.2890</td>
<td>-4.6028</td>
<td>.5933</td>
<td>1.4018</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Oct Vol.</td>
<td>5.6620</td>
<td>-1.0407</td>
<td>-3.5563</td>
<td>.4949</td>
<td>1.3132</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec Vol.</td>
<td>262.2200</td>
<td>-16.4863</td>
<td>-1.8744</td>
<td>.2778</td>
<td>.9736</td>
<td>44</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec Vol.</td>
<td>5.5139</td>
<td>-.1955</td>
<td>-3.0173</td>
<td>.4221</td>
<td>.9784</td>
<td>44</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec Vol.</td>
<td>5.4380</td>
<td>-.6915</td>
<td>-2.2839</td>
<td>.3324</td>
<td>.9311</td>
<td>44</td>
</tr>
</tbody>
</table>
Appendix D.8. Estimate and test for futures volume trend over the life of each live beef futures contract after correction for autocorrelation

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>Feb Vol.</td>
<td>2.6797</td>
<td>- .2104</td>
<td>-3.2768</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb Vol.</td>
<td>2.5857</td>
<td>- .8426</td>
<td>-3.0271</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr Vol.</td>
<td>139.1100</td>
<td>-19.9147</td>
<td>-3.1819</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr Vol.</td>
<td>2.4375</td>
<td>- .2789</td>
<td>-4.6053</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr Vol.</td>
<td>2.3363</td>
<td>- 1.0427</td>
<td>-3.9603</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun Vol.</td>
<td>167.0000</td>
<td>-23.5903</td>
<td>-3.6988</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun Vol.</td>
<td>2.3245</td>
<td>- .2140</td>
<td>-4.8638</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun Vol.</td>
<td>2.3261</td>
<td>- .7780</td>
<td>-4.1886</td>
</tr>
<tr>
<td>Linear</td>
<td>Aug Vol.</td>
<td>183.6100</td>
<td>-20.5027</td>
<td>-3.0724</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug Vol.</td>
<td>3.0554</td>
<td>- .2190</td>
<td>-4.7474</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug Vol.</td>
<td>3.0999</td>
<td>- .7454</td>
<td>-3.6349</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct Vol.</td>
<td>122.1000</td>
<td>-17.0613</td>
<td>-2.3709</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct Vol.</td>
<td>4.0576</td>
<td>- .3042</td>
<td>-4.2107</td>
</tr>
<tr>
<td>Double-log</td>
<td>Oct Vol.</td>
<td>3.7492</td>
<td>- 1.0629</td>
<td>-3.3516</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec Vol.</td>
<td>109.2600</td>
<td>- 9.2411</td>
<td>.9782</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec Vol.</td>
<td>2.5540</td>
<td>- .1145</td>
<td>-1.7595</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec Vol.</td>
<td>2.3775</td>
<td>- .3479</td>
<td>-1.2520</td>
</tr>
<tr>
<td>R</td>
<td>D.W.</td>
<td>Sample Variance (before)</td>
<td>Sample Variance (after)</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>------</td>
<td>--------------------------</td>
<td>--------------------------</td>
<td></td>
</tr>
<tr>
<td>.3155</td>
<td>1.8013</td>
<td>47,280</td>
<td>44,547</td>
<td></td>
</tr>
<tr>
<td>.4694</td>
<td>2.1798</td>
<td>1.5726</td>
<td>1.1867</td>
<td></td>
</tr>
<tr>
<td>.4408</td>
<td>2.1524</td>
<td>1.6881</td>
<td>1.1867</td>
<td></td>
</tr>
<tr>
<td>.4407</td>
<td>2.2003</td>
<td>17,086</td>
<td>13,079</td>
<td></td>
</tr>
<tr>
<td>.5793</td>
<td>2.4118</td>
<td>1.8786</td>
<td>1.2344</td>
<td></td>
</tr>
<tr>
<td>.5214</td>
<td>2.2710</td>
<td>2.0992</td>
<td>1.3584</td>
<td></td>
</tr>
<tr>
<td>.5096</td>
<td>2.2289</td>
<td>15,746</td>
<td>11,946</td>
<td></td>
</tr>
<tr>
<td>.6145</td>
<td>2.4743</td>
<td>2.9942</td>
<td>0.5769</td>
<td></td>
</tr>
<tr>
<td>.5570</td>
<td>2.2707</td>
<td>1.0702</td>
<td>0.6388</td>
<td></td>
</tr>
<tr>
<td>.4414</td>
<td>1.9755</td>
<td>15,164</td>
<td>13,768</td>
<td></td>
</tr>
<tr>
<td>.6052</td>
<td>2.1797</td>
<td>0.8304</td>
<td>0.6277</td>
<td></td>
</tr>
<tr>
<td>.5030</td>
<td>2.0856</td>
<td>0.9577</td>
<td>0.7387</td>
<td></td>
</tr>
<tr>
<td>.3590</td>
<td>2.2920</td>
<td>15,685</td>
<td>12,667</td>
<td></td>
</tr>
<tr>
<td>.5604</td>
<td>2.0582</td>
<td>1.5010</td>
<td>1.4014</td>
<td></td>
</tr>
<tr>
<td>.4777</td>
<td>2.0520</td>
<td>1.7492</td>
<td>1.5808</td>
<td></td>
</tr>
<tr>
<td>.1510</td>
<td>2.1619</td>
<td>34,889</td>
<td>25,854</td>
<td></td>
</tr>
<tr>
<td>.2649</td>
<td>2.6260</td>
<td>1.8936</td>
<td>1.2290</td>
<td></td>
</tr>
<tr>
<td>.1919</td>
<td>2.6138</td>
<td>2.0495</td>
<td>1.2762</td>
<td></td>
</tr>
</tbody>
</table>
Appendix D.9. Estimate and test for Open Interest trend over the life of each live beef futures contract

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Feb O.I.</td>
<td>4863.0000</td>
<td>-407.5570</td>
<td>-4.1735</td>
<td>.5556</td>
<td>.2552</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Feb O.I.</td>
<td>8.7109</td>
<td>-0.2624</td>
<td>-4.6090</td>
<td>.5938</td>
<td>.5108</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb O.I.</td>
<td>8.8370</td>
<td>-1.0587</td>
<td>-3.8108</td>
<td>.5209</td>
<td>.5115</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr O.I.</td>
<td>3380.0000</td>
<td>-251.6370</td>
<td>-3.7558</td>
<td>.4970</td>
<td>.2104</td>
<td>45</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr O.I.</td>
<td>8.3207</td>
<td>-2.2419</td>
<td>-3.9026</td>
<td>.5114</td>
<td>.2737</td>
<td>45</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr O.I.</td>
<td>8.3132</td>
<td>-0.9056</td>
<td>-3.0770</td>
<td>.4248</td>
<td>.3145</td>
<td>45</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun O.I.</td>
<td>4100.1000</td>
<td>-319.1244</td>
<td>-4.7324</td>
<td>.5991</td>
<td>.2938</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun O.I.</td>
<td>8.3445</td>
<td>-1.1884</td>
<td>-3.5372</td>
<td>.4881</td>
<td>.4520</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun O.I.</td>
<td>8.3520</td>
<td>-0.7036</td>
<td>-2.8976</td>
<td>.4165</td>
<td>.5053</td>
<td>42</td>
</tr>
<tr>
<td>Linear</td>
<td>Aug O.I.</td>
<td>3905.1000</td>
<td>-340.2045</td>
<td>-5.9957</td>
<td>.6880</td>
<td>.2615</td>
<td>42</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug O.I.</td>
<td>8.8098</td>
<td>-0.3124</td>
<td>-6.4137</td>
<td>.7120</td>
<td>.5552</td>
<td>42</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug O.I.</td>
<td>8.7804</td>
<td>-1.1441</td>
<td>-4.6865</td>
<td>.5954</td>
<td>.6238</td>
<td>42</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct O.I.</td>
<td>3593.4000</td>
<td>-338.6896</td>
<td>-5.9554</td>
<td>.6901</td>
<td>.3071</td>
<td>41</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct O.I.</td>
<td>8.8254</td>
<td>-0.3496</td>
<td>-8.5728</td>
<td>.8083</td>
<td>.4812</td>
<td>41</td>
</tr>
<tr>
<td>Double-log</td>
<td>Oct O.I.</td>
<td>8.8005</td>
<td>-1.2767</td>
<td>-5.8493</td>
<td>.6836</td>
<td>.5437</td>
<td>41</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec O.I.</td>
<td>4266.7000</td>
<td>-337.1569</td>
<td>-4.5081</td>
<td>.5710</td>
<td>.2375</td>
<td>44</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec O.I.</td>
<td>8.6846</td>
<td>-0.2531</td>
<td>-4.9588</td>
<td>.6077</td>
<td>.3849</td>
<td>44</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec O.I.</td>
<td>8.7313</td>
<td>-0.9863</td>
<td>-4.0205</td>
<td>.5272</td>
<td>.4066</td>
<td>44</td>
</tr>
</tbody>
</table>
Appendix D.10. Estimate and test for Open Interest trend over the life of each live beef futures contract after correction for autocorrelation

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff.</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
<th>Sample Variance (before)</th>
<th>Sample Variance (after)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Feb O.I.</td>
<td>645.2100</td>
<td>-354.5803</td>
<td>-6.6517</td>
<td>.7335</td>
<td>1.6049</td>
<td>3,832,800</td>
<td>907,440</td>
</tr>
<tr>
<td>Exponential</td>
<td>Feb O.I.</td>
<td>2.3333</td>
<td>- .3024</td>
<td>-6.9453</td>
<td>.7479</td>
<td>2.3013</td>
<td>1.2992</td>
<td>.5576</td>
</tr>
<tr>
<td>Double-log</td>
<td>Feb O.I.</td>
<td>2.3674</td>
<td>-1.1952</td>
<td>-6.1242</td>
<td>.7447</td>
<td>2.1007</td>
<td>1.4623</td>
<td>.6367</td>
</tr>
<tr>
<td>Linear</td>
<td>Apr O.I.</td>
<td>424.6100</td>
<td>-253.1235</td>
<td>-7.9815</td>
<td>.7763</td>
<td>1.7149</td>
<td>2,042,300</td>
<td>402,830</td>
</tr>
<tr>
<td>Exponential</td>
<td>Apr O.I.</td>
<td>1.2720</td>
<td>- .3352</td>
<td>-10.9825</td>
<td>.8612</td>
<td>1.5816</td>
<td>1.7482</td>
<td>.3633</td>
</tr>
<tr>
<td>Double-log</td>
<td>Apr O.I.</td>
<td>1.4571</td>
<td>-1.2844</td>
<td>-8.7879</td>
<td>.8048</td>
<td>1.3048</td>
<td>1.9401</td>
<td>.4892</td>
</tr>
<tr>
<td>Linear</td>
<td>Jun O.I.</td>
<td>663.7500</td>
<td>-317.4859</td>
<td>-8.3967</td>
<td>.8024</td>
<td>1.5907</td>
<td>1,810,400</td>
<td>485,500</td>
</tr>
<tr>
<td>Exponential</td>
<td>Jun O.I.</td>
<td>2.0784</td>
<td>- .2728</td>
<td>-8.2213</td>
<td>.7963</td>
<td>2.0265</td>
<td>1.1299</td>
<td>.3532</td>
</tr>
<tr>
<td>Double-log</td>
<td>Jun O.I.</td>
<td>2.2946</td>
<td>- .9658</td>
<td>-6.3282</td>
<td>.7118</td>
<td>1.7947</td>
<td>1.2260</td>
<td>.4670</td>
</tr>
<tr>
<td>Linear</td>
<td>Aug O.I.</td>
<td>539.4900</td>
<td>-340.1941</td>
<td>-11.1524</td>
<td>.8725</td>
<td>1.5170</td>
<td>1,297,700</td>
<td>320,150</td>
</tr>
<tr>
<td>Exponential</td>
<td>Aug O.I.</td>
<td>2.6028</td>
<td>- .3664</td>
<td>-10.1543</td>
<td>.8518</td>
<td>1.6694</td>
<td>.9564</td>
<td>.4049</td>
</tr>
<tr>
<td>Double-log</td>
<td>Aug O.I.</td>
<td>2.8816</td>
<td>-1.2930</td>
<td>-7.3230</td>
<td>.7569</td>
<td>1.5550</td>
<td>1.2523</td>
<td>.6155</td>
</tr>
<tr>
<td>Linear</td>
<td>Oct O.I.</td>
<td>503.4200</td>
<td>-260.3800</td>
<td>-7.5245</td>
<td>.7736</td>
<td>1.1230</td>
<td>1,231,900</td>
<td>315,790</td>
</tr>
<tr>
<td>Exponential</td>
<td>Oct O.I.</td>
<td>2.1921</td>
<td>- .3826</td>
<td>-12.0192</td>
<td>.8898</td>
<td>1.3482</td>
<td>.6333</td>
<td>.2535</td>
</tr>
<tr>
<td>Linear</td>
<td>Dec O.I.</td>
<td>492.2200</td>
<td>-257.4607</td>
<td>-6.3945</td>
<td>.7066</td>
<td>1.2270</td>
<td>2,522,500</td>
<td>524,590</td>
</tr>
<tr>
<td>Exponential</td>
<td>Dec O.I.</td>
<td>1.7205</td>
<td>- .2412</td>
<td>-6.8541</td>
<td>.7307</td>
<td>2.0041</td>
<td>1.1752</td>
<td>.3808</td>
</tr>
<tr>
<td>Double-log</td>
<td>Dec O.I.</td>
<td>1.8135</td>
<td>- .9016</td>
<td>-5.6894</td>
<td>.6642</td>
<td>1.8207</td>
<td>1.3545</td>
<td>.4533</td>
</tr>
</tbody>
</table>
Appendix D.11. Estimate and test for a relationship between the futures price in contracts 6 months from maturity and the futures trading composition

<table>
<thead>
<tr>
<th>Function</th>
<th>Depend.</th>
<th>Intercept</th>
<th>Coeff. 1</th>
<th>t test</th>
<th>Coeff. 2</th>
<th>t test</th>
<th>R</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Pf</td>
<td>22.5600</td>
<td>+ .0591</td>
<td>4.7512</td>
<td>+ .0101</td>
<td>.7069</td>
<td>.8893</td>
<td>1.3029</td>
</tr>
<tr>
<td>Semi-log</td>
<td>Pf</td>
<td>9.7893</td>
<td>+ 3.3236</td>
<td>4.9834</td>
<td>+ .8172</td>
<td>.8504</td>
<td>.8975</td>
<td>1.4669</td>
</tr>
<tr>
<td>Exponential</td>
<td>Pf</td>
<td>3.1284</td>
<td>+ .0022</td>
<td>4.7865</td>
<td>+ .0004</td>
<td>.8907</td>
<td>.8907</td>
<td>1.2969</td>
</tr>
<tr>
<td>Double-log</td>
<td>Pf</td>
<td>2.6473</td>
<td>+ .1248</td>
<td>5.0428</td>
<td>+ .0312</td>
<td>.8996</td>
<td>.8996</td>
<td>1.4614</td>
</tr>
<tr>
<td>Linear</td>
<td>Pf</td>
<td>23.4230</td>
<td>+ .0571</td>
<td>4.8895</td>
<td>+ .0000</td>
<td>.8795</td>
<td>.8795</td>
<td>.9233</td>
</tr>
<tr>
<td>Linear</td>
<td>Pf</td>
<td>26.9070</td>
<td>.0000</td>
<td>.0000</td>
<td>-.0049</td>
<td>.0657</td>
<td>.0657</td>
<td>1.1359</td>
</tr>
</tbody>
</table>

Trading composition consist of the percentage of open commitments classified as large long speculative (coefficient 1) and the percentage classified as large short hedges (coefficient 2).

The functional forms are defined as:

- **Linear** $P_f = \alpha_0 + \alpha_1 \text{[speculation]} + \alpha_2 \text{[hedging]} + \nu$
- **Semi-log** $P_f = \alpha_0 + \alpha_1 \log\text{[speculation]} + \alpha_2 \text{[hedging]} + \nu$
- **Exponential** $\log P_f = \alpha_0 + \alpha_1 \text{[speculation]} + \alpha_2 \text{[hedging]} + \nu$
- **Double-log** $P_f = \alpha_0 + \alpha_1 \log\text{[speculation]} + \alpha_1 \log\text{[hedging]} + \nu$. 