

1           COMPARING THE STRENGTH OF MODULAR  
2           SIGNAL, AND EVALUATING ALTERNATIVE  
3           MODULAR HYPOTHESES, USING COVARIANCE  
4           RATIO EFFECT SIZES WITH MORPHOMETRIC  
5           DATA

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## 25 Introduction

26 Characterizing the extent to which phenotypic traits covary, and deciphering the forces that shape patterns  
27 of trait covariation, are perennial topics in evolutionary biology. Empirical research has demonstrated  
28 that patterns of covariation are unevenly dispersed across traits, with some variables exhibiting high  
29 correlations with one another while other traits are more independent (Olson and Miller 1958; Cheverud 1996;  
30 Klingenberg 2008; Goswami and Polly 2010). Indeed, studies of morphological integration and modularity  
31 are largely concerned with how these trait correlations are distributed, and the extent to which they are  
32 driven by developmental, genetic, or functional linkages among traits (Olson and Miller 1958; Wagner 1984).  
33 In particular, modularity describes patterns where trait correlations are high within subsets of variables,  
34 termed *modules*, and where trait correlations across modules are comparatively weaker (Cheverud 1982;  
35 Goswami 2006; Wagner et al. 2007). Patterns of modularity therefore result in sets of semi-autonomous  
36 traits, which have the potential to respond differentially to natural selection, and thus the capacity to  
37 promote the evolution of novelty (see Wagner and Altenberg 1996; Wagner et al. 2007; Tokita et al. 2007;  
38 Hansen and Houle 2008; Clune et al. 2013).

39  
40 The past several decades have seen resurging interest in understanding the evolution of modularity, and  
41 numerous analytical approaches have been developed for quantifying patterns of modularity in phenotypic  
42 datasets (e.g., Magwene 2001; Mitteroecker and Bookstein 2007; Márquez 2008; Klingenberg 2009; Adams  
43 2016; Goswami and Finarelli 2016). Concomitant with these advances is an increasing number of empirical  
44 studies that characterize patterns of modularity in distinct phenotypic traits, and across a wide variety of  
45 taxa (for recent examples, see: Parsons et al. 2012; Parr et al. 2016; Felice and Goswami 2018; Larouche et  
46 al. 2018; Bardua et al. 2019). Likewise, evolutionary biologists have striven to decipher whether patterns  
47 of modularity are similar among taxa and traits, and across levels of biological organization (Drake and  
48 Klingenberg 2010; Renaud et al. 2012; Sanger et al. 2012; Felice and Goswami 2018; Bardua et al. 2019;  
49 Marshall et al. 2019). Some studies have investigated whether patterns of modularity are conserved across  
50 taxa (e.g., Goswami 2006; Marshall et al. 2019), however, direct quantitative or statistical comparisons of  
51 modularity patterns are generally lacking. We assert that a critical aspect of this endeavor should be to  
52 determine whether the *strength* of modularity is similar across datasets. However, to our knowledge, no  
53 formal statistical procedure has yet been proposed to accomplish this task.

54  
55 Another key challenge in the study of modularity is identifying which sets of traits represent anatomical

56 modules (Zelditch et al. 1990; Hallgrímsson et al. 2007). Biologically, one expects that traits subjected to  
57 common processes may exhibit high correlations with one another, resulting in modular structure (Olson  
58 and Miller 1958; Wagner 1984; Cheverud 1996; Goswami and Polly 2010). However, phenotypic traits are  
59 influenced by an array of genetic, developmental, and functional forces, not all of which act similarly. Thus,  
60 it is reasonable to expect that multiple, competing modular hypotheses may be proposed for the same set of  
61 anatomical traits (e.g., Márquez 2008; Goswami and Finarelli 2016; Felice and Goswami 2018; Bardua et  
62 al. 2019). This then raises the question: which of the alternative modular hypotheses provides the best  
63 description of the observed patterns of trait covariation? Several recent attempts to address this problem  
64 (e.g., Márquez 2008; Goswami and Finarelli 2016) have greatly sharpened our notions of modularity, and  
65 provided insights as to how phenotypic variation may be expected to evolve. For instance, one attempt (MINT:  
66 Márquez 2008) uses goodness of fit measures to evaluate alternative modular hypotheses, while another  
67 approach (EMMLi: Goswami and Finarelli 2016) uses penalized likelihood indices (AICc) to evaluate the fit of  
68 alternative modular hypotheses to the observed trait correlations.

69  
70 Nonetheless, these approaches do not provide a complete analytical toolkit for evaluating alternative modular  
71 hypotheses for the full spectrum of phenotypic datasets evolutionary biologists wish to examine. One  
72 reason for this is that initial investigations into their performance were relatively limited, and suggested  
73 that future improvements or alternatives may be required. For example, simulations revealed that MINT  
74 tended to identify modular patterns when none existed in the data, suggesting that false positives may  
75 be a concern (Márquez 2008: Table 3). Likewise, initial investigations found that EMMLi tended to select  
76 highly parameterized models [Goswami and Finarelli (2016); Goswami pers. comm.], suggesting that it  
77 may display some degree of model misspecification. However, to date, an evaluation of both approaches  
78 under a broad set of plausible scenarios has not been conducted, nor has their performance been directly  
79 compared (see below). As a consequence, it remains possible that neither approach provides a reliable means  
80 of evaluating patterns of modular structure, thereby reducing one's ability to identify evolutionary trends in  
81 trait covariation. Therefore, we suggest that a second pressing analytical need in the study of modularity is  
82 the development of robust statistical methods for comparing the strength of modular signal across alternative  
83 modular hypotheses for the same dataset.

84  
85 In this article, we develop a single statistical tool that accomplishes both of these tasks. Our approach utilizes  
86 a standardized test statistic (an effect size:  $Z_{CR}$ ) for measuring the degree of modularity between sets of  
87 variables, and for comparing these effect sizes statistically. Our procedure is based on the covariance ratio

88 (Adams 2016), and may be used for comparing the strength of modular signal across datasets, as well as for  
89 evaluating patterns of modular signal as defined by alternative modular hypotheses for the same dataset.  
90 Using computer simulations we evaluate the statistical properties of tests based on  $Z_{CR}$ , and find that  
91 they display appropriate type I error rates, high statistical power, and low levels of model misspecification.  
92 Thus, when modular signal is present,  $Z_{CR}$  is unlikely to mis-assign that signal to an incorrect modular  
93 hypothesis. We then compare these results to those found using two alternative methods for comparing  
94 modular hypotheses: MINT (Márquez 2008) and EMMLi (Goswami and Finarelli 2016). We find that both  
95 exhibit poor statistical performance (very high false positive rates, and high model misspecification rates),  
96 making it challenging to arrive at reliable biological inferences when using these approaches. We illustrate  
97 the utility of  $Z_{CR}$  with an empirical example of mandible shape in sigmodontine rodents, where alternative  
98 modular hypotheses, and the strength of modular signal across several species, is compared. Our new method  
99 is implemented in R, and is distributed through the package geomorph (Adams et al. 2019). Finally, the  
100 method can accommodate future analytical developments for characterizing modular signal as they are  
101 developed (see Discussion).

## 102 **Methods**

### 103 *Analytical Development*

104 *Covariance Ratio Effect Sizes ( $Z_{CR}$ ):* One approach to quantifying the degree of modular signal in morpho-  
105 metric datasets is based on the covariance ratio (Adams 2016). This measure is preferable to alternative  
106 estimates, such as the RV coefficient (sensu: Klingenberg 2009), because it is insensitive to the number of  
107 variables ( $p$ ) and the number of specimens ( $n$ ) (Adams 2016). To calculate the covariance ratio ( $CR$ ), one  
108 first concatenates the variables for all modules into an  $n \times p$  matrix ( $\mathbf{Y}$ ). At a minimum, the variables are  
109 represented as mean-centered data, implying that they are residuals from an intercept model. However,  
110 one could also perform additional transformations to the data, such as a phylogenetic transformation (e.g.,  
111 Garland and Ives 2000; Adams 2014; Adams and Collyer 2018a) to account for non-independence among  
112 objects as a result of shared evolutionary history (see Adams and Felice 2014 for a related approach measuring  
113 morphological integration in a phylogenetic context). Using this concatenated dataset, the covariance ratio  
114 for the observed data is found as:

$$CR = \frac{tr(\mathbf{S}_{12}\mathbf{S}_{21})}{tr(\mathbf{S}_{11}^*)tr(\mathbf{S}_{22}^*)} \quad (1)$$

115 where  $\mathbf{S}_{11}^*$  &  $\mathbf{S}_{22}^*$  represent the within-module covariance matrices for each of two modules (with zeroes  
 116 replacing the diagonal elements), and  $\mathbf{S}_{21} = \mathbf{S}_{12}^T$  represents the covariation between modules  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$ .  
 117 For datasets containing more than two modules, the  $CR$  coefficient is calculated for each pair of modules,  
 118 and the average pairwise  $CR$  coefficient is used (see Adams 2016). Empirically, the  $CR$  coefficient has a  
 119 lower limit of 0, an expected value of 1 when there is no modularity, and values greater than 1 are also  
 120 possible if covariances between module variables exceed covariances within modules. Thus,  $CR$  values that  
 121 are closer to 0 describe datasets with a relatively greater degree of modularity, while  $CR$  values closer to  
 122 1 describe datasets with less modular signal. To evaluate  $CR$  statistically, a resampling procedure that  
 123 randomly assigns variables to modules is used, and in each iteration a  $CR_{rand}$  is obtained under the null  
 124 hypothesis of no modularity. The observed value,  $CR_{obs}$ , is then compared to the empirically-generated  
 125 sampling distribution of  $CR_{rand}$  values to evaluate significance (for details see: Adams 2016).

126  
 127 The permutation procedure above is sufficient to statistically determine whether the degree of modular  
 128 signal is greater than expected by chance for a single dataset, but evaluating modular signals across datasets  
 129 requires a standardized effect size to ensure statistical comparability. To accomplish this, we make use  
 130 of prior theoretical developments for effect sizes obtained from statistics from multivariate data and their  
 131 empirical sampling distributions (Collyer et al. 2015; Adams and Collyer 2016). Previous studies used residual  
 132 randomization in permutation procedures (RRPP) to generate sampling distributions. The important insight  
 133 from Collyer et al. (2015) was that a permutation-based effect size may be obtained from any observed test  
 134 statistic, found in relation to the mean and standard deviation of its empirical sampling distribution obtained  
 135 from RRPP. Although the resampling procedure for generating sampling distributions of the  $CR$  statistic is  
 136 different than RRPP, the concept of using the random outcomes to evaluate the size of the observed effect is  
 137 the same. Thus, for the case of the covariance ratio, this effect size is obtained as:

$$Z_{CR} = \frac{CR_{obs} - \hat{\mu}_r}{\hat{\sigma}_r} \quad (2)$$

138 where  $CR_{obs}$  is the observed covariance ratio for the dataset,  $\hat{\mu}_r$  is the expected value of  $CR$  under the

139 null hypothesis of no modularity (found as the mean of the empirical sampling distribution), and  $\hat{\mu}_r$  is the  
140 standard error of the mean, found as the standard deviation of the empirical sampling distribution. Note  
141 that the calculation of  $Z_{CR}$  is the same regardless of how many modules are represented, because  $CR_{obs}$  in  
142 equation 2 represents the average  $CR$  value obtained across all pairs of modules (see Adams 2016). One  
143 should recognize that more negative values of  $Z_{CR}$  represent greater modular signal, because when modular  
144 signal is present,  $CR_{obs}$  will be less than  $\hat{\mu}_r$ .

145

146 Importantly, it can be shown that  $Z_{CR}$  is normally distributed, and thus represents a valid standardized  
147 effect size. To demonstrate this property we performed a simulation experiment, where 500 multivariate  
148 datasets were generated at a given  $n$  and  $p$ , and with all variates drawn from a normal distribution:  $N(0, 1)$ .  
149 Next, variables were randomly assigned to modules, and both the  $CR$  and  $Z_{CR}$  were obtained from each  
150 dataset, with  $Z_{CR}$  obtained using 999 permutations. The mean and standard deviation of  $Z_{CR}$  across the  
151 500 datasets were then calculated, and the sampling experiment was repeated across differing levels of  $n$   
152 and  $p$ . As is clear from Fig. 1, under the null hypothesis of no modularity (i.e., a random association of  
153 variables to modules), the covariance ratio effect size exhibits a constant expected value near zero, and  
154 constant variance (inferred from the constant confidence interval) across the entire spectrum of sample sizes  
155 ( $n$ ) and variable number ( $p$ ). Furthermore, the distribution of  $Z_{CR}$  values from the 500 datasets for any set of  
156 simulation conditions was also found to be normally distributed (Results not shown). Thus,  $Z_{CR}$  represents  
157 a valid standardized effect size that characterizes the strength of modular signal in morphometric datasets.

158

159 *A Two-Sample Z-Score for Comparing Modular Signals:* Once the degree of modular signal has been  
160 characterized, one may wish to evaluate hypotheses that compare multiple effect sizes. For hypotheses  
161 of modularity, this is particularly useful in two biological contexts. First, it may be of interest to  
162 determine whether the *strength* of modular signal is greater in one dataset as compared to another. To  
163 date, no explicit analytical approach has been proposed to statistically compare the strength of modular  
164 signal across datasets, though some studies have evaluated whether general patterns of modularity are  
165 conserved across taxa (e.g., Goswami 2006; Sanger et al. 2012; Marshall et al. 2019). Second, if several  
166 alternative modular hypotheses have been proposed for the same structure (based on developmental,  
167 genetic, or functional grounds), it may be of interest to characterize the degree of modular signal  
168 under each of these hypotheses and determine which provides the best description of the observed pat-  
169 terns of morphological covariation (see Márquez 2008; Goswami and Finarelli 2016; Felice and Goswami 2018).

170









































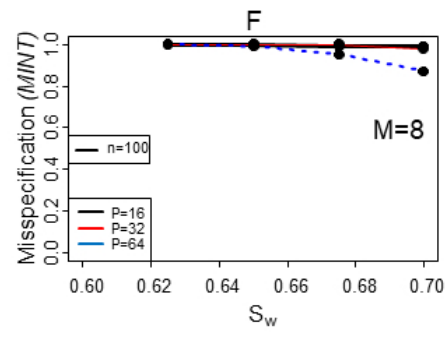
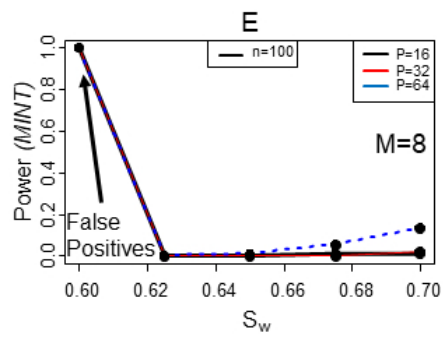
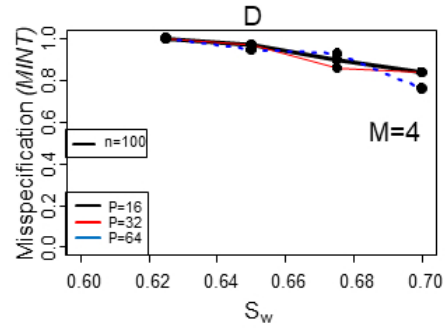
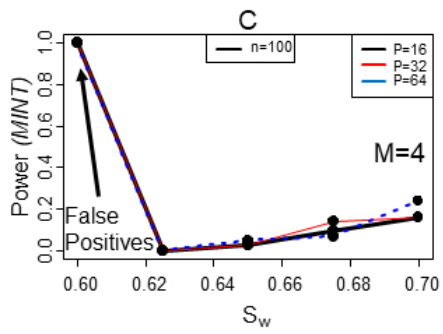
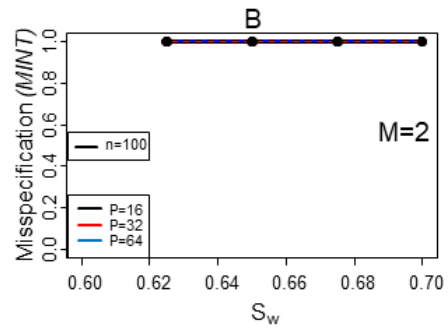
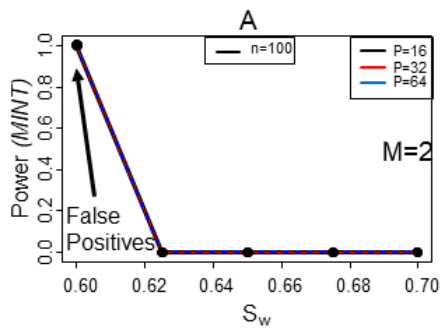




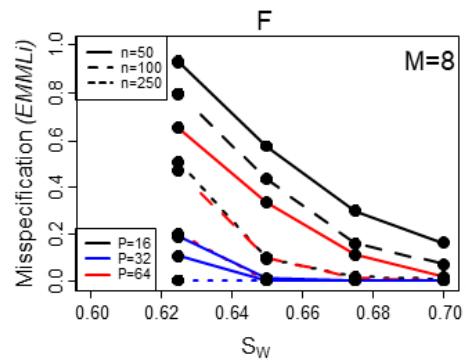
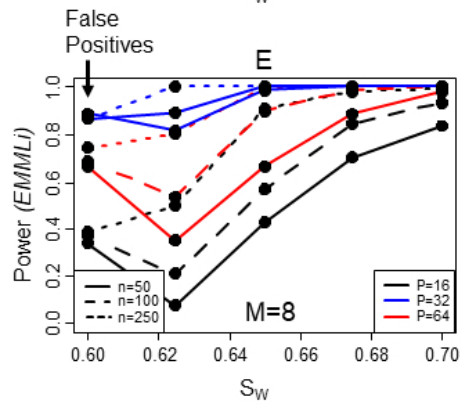
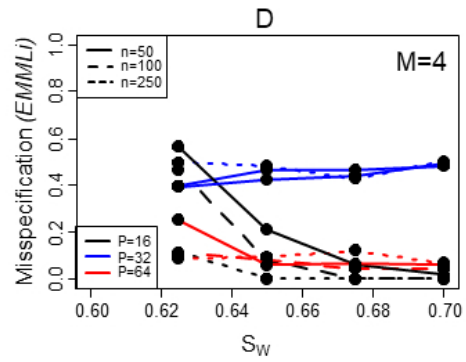
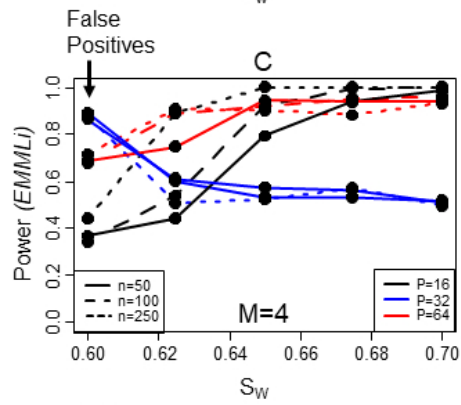
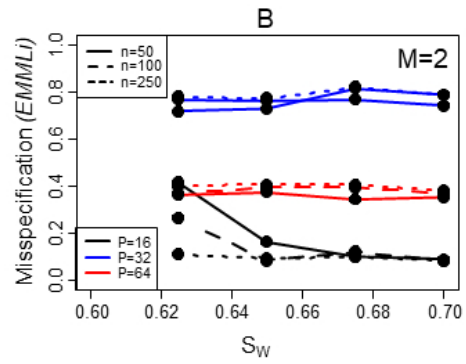
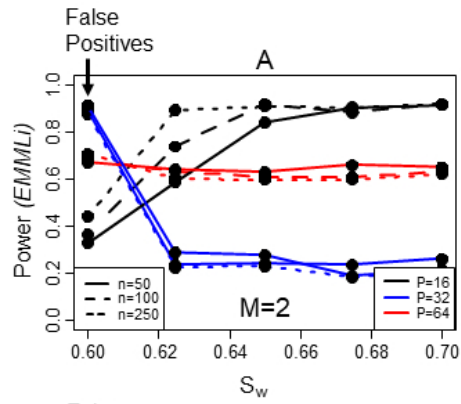








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fig 6

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