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Multicoordination Control Strategy Performance in Hybrid Power Systems

This paper evaluates a state-space methodology of a multi-input multi-output (MIMO) control strategy using a $2 \times 2$ tightly coupled scenario applied to a physical gas turbine fuel cell hybrid power system. A centralized MIMO controller was preferred compared to a decentralized control approach because previous simulation studies showed that the coupling effect identified during the simultaneous control of the turbine speed and cathode airflow was better minimized. The MIMO controller was developed using a state-space dynamic model of the system that was derived using first-order transfer functions empirically obtained through experimental tests. The controller was evaluated in terms of disturbance rejection through perturbations in the gas turbine operation, and set-point tracking maneuver through turbine speed and cathode airflow steps. The experimental results illustrate that a multicoordination control strategy was able to mitigate the coupling of each actuator to each output during the simultaneous control of the system, and improved the overall system performance during transient conditions. On the other hand, the controller showed different performance during validation in simulation environment compared to validation in the physical facility, which will require a better dynamic modeling of the system for the implementation of future multivariable control strategies. [DOI: 10.1115/1.4039356]

Keywords: multi-input multi-output control strategy, MIMO, state-space, centralized control, hybrid power system, fuel cell, gas turbine, fuel cell gas turbine hybrid

1 Introduction

The combination of heat and power in a fuel cell–gas turbine hybrid cycle provides high theoretical efficiency [1,2]. In particular, the ability to recover extra heat produced by the fuel cell into a gas turbine cycle reduces fuel required in the gas turbine system [3,4]. This provides a strong benefit in terms of emissions and fuel cost. Figure 1 shows a fuel cell–gas turbine hybrid layout, where a fuel cell may ideally replace the combustor of a typical Brayton cycle.

One of the primary barriers to commercializing these fuel cell–gas turbine hybrid systems is the simultaneous control of the various parameters that affect the performance of the cycle [5–7]. For instance, within a fuel cell–gas turbine hybrid, a minor deviation in the turbine speed affects the airflow to the cathode of a fuel cell, which affects the generation of the fuel cell waste heat that drives the gas turbine. Although several other parameters affect the fuel cell/turbine performance, the multivariable control strategy developed here was reduced to simultaneously control the turbine speed and the cathode airflow because they were both considered the two most critical parameters in the system.

In general, depending on the level of coupling/decoupling effect existing in the system, multiple parameters can be controlled using decentralized or centralized architectures [8–10]. In this work, a centralized methodology was chosen because in previous studies, Tsai et al. [11] showed that in such a hybrid configuration, a centralized multi-input multi-output (MIMO) controller was more beneficial in the control of the cross-coupling effect. In those simulations, single-input single-output controllers were evaluated against a centralized architecture, and it was found that process instabilities can occur during the simultaneous operation of the actuators when independent controllers were used, rather a centralized approach had to be adopted.

Thus, an experimental validation on the physical plant of a centralized state-space controller is the primary motivation for this work. In particular, the purpose of the study was not an optimization of the state-space controller, but aimed to evaluate the minimization of the dynamic coupling interactions existing in the physical power plant using a centralized architecture to confirm the results of the previous model based studies. In the beginning, the MIMO strategy was tested in simulation environment to prove the stability of the performance, and subsequently it was tested on the physical power plant. For the experimental validation, each control loop was individually tested before the simultaneous operation of the actuators was controlled using the MIMO architecture.

The multivariable control strategy was evaluated under turbine disturbance perturbations, turbine speed, and cathode airflow set-point tracking maneuver. Perturbations in the turbine operation were aimed to reproduce waste heat variations caused by hypothetical fuel cell load transients, which, in general, represent the...
primary challenge in terms of control or disturbance rejection for a gas turbine in this hybrid configuration, whereas the set-point tracking maneuvers evaluated the coupling created by the automated response of each individual actuator.

Results show that a centralized architecture is able to mitigate the coupling effect existing in a $2 \times 2$ control scenario, but different. Similarly, Bhattacharyya and Rengaswamy used empirical nonlinear transfer functions to avoid the extensive part of physical modeling and control design around an operating point is performed. Also, the number of states generated by the differential equations is minimized to reduce the complexity during the control strategy implementation. Using this approach, Fardadi et al. developed a centralized control strategy of a physical-based dynamic model of a single co-flow solid oxide fuel cell (SOFC) to regulate a distributed temperature profile and manage actuator interactions. The centralized controller was designed using a robust methodology and results showed small and smooth monotonous temperature response to rapid and large load perturbations that reduced thermal stresses to the fuel cell material. In contrast, Pohjoranta et al. used empirical models directly identified from experimental data to implement a centralized MIMO advanced control strategy to regulate the SOFC stack temperature in a stand-alone configuration. Empirical transfer functions were chosen to avoid the extensive part of physical modeling and/or model simplification for the development of the control strategy. Results were compared against a traditional single-input single-output controller, and showed that the need for a centralized control method increases especially when several stacks are built in a large SOFC systems. In such multistack systems, it was impractical to implement an optimal compensation by giving set-points to individual stack load controllers while a centralized control does this automatically.

Similarly, Bhattacharyya and Rengaswamy used empirical nonlinear transfer functions to evaluate a centralized nonlinear MIMO controller against a single-input single-output controller [23]. The identification process was based on a detailed isothermal dynamic model of a stand-alone fuel cell that was validated with industrial data. Results showed that a well-tuned single-input single-output controller poorly performed because of the highly interactive dynamics. However, nonlinear models with properly chosen cross terms improved the model performance significantly in a MIMO problem.

Thus, based on those previous studies, empirical transfer functions directly identified from experimental tests were used here to implement a MIMO controller for the physical system. Single perturbations using unit steps to the physical process were considered an appropriate quantification methodology to capture the cross-coupling interactions among hardware components without risking the equipment. Furthermore, the experimental validation of the MIMO controller on the physical hybrid was considered an important evaluation of the state-space methodology compared to all the simulation studies presented here and generally in literature. A $2 \times 2$ highly coupled scenario was preferred compared to a larger case, as often proposed, because it was considered more realistic and less risky to the equipment.

### 3 Hardware

The hybrid performance (Hyper) cycle at the National Energy Technology Laboratory (NETL) is designed to evaluate the dynamic integration of a fuel cell gas turbine system. Figure 2 represents the hardware components that are integrated in the hybrid configuration used in the validation of the MIMO control strategy presented in this work. Such a physical system is only designed to represent the cathode and anode pressure drop existing in the fuel cell gas turbine recuperated hybrid. Specifically, the electrochemical transients of the fuel cell do not affect the turbine operation or in other words the fuel cell is considered electrochemically inactive [24]. However, that architecture still represents a test bed facility for the development and validation of advanced control strategies because of the hardware modifications to the original gas turbine system. As such, the hybrid layout of Fig. 2 is characterized by a larger compressor plenum volume compared to the original gas turbine layout, which creates one of the main control challenges in the system operation.

#### 3.1 Hardware Components

At the Hyper facility, the cathode airflow can be controlled using the cold-air and/or the hot-air bypass valve [25]. The more evident coupling effect of cathode airflow control using the cold-air bypass was preferred because provided a substantial benefit in terms of system operability [26,27]. On the other hand, the electric load was used to control the turbine speed. In general, the electric load is not an actuator that adjusts the turbine speed under perturbation, but is an independent variable that changes with the external demand of electrical power. However, controlling the turbine speed using the electric load was able to examine the shaft power required to keep the turbine speed constant, and to evaluate the relationship between the turbine and the power generator for distributed applications.

##### 3.1.1 Gas Turbine Generator

A 120 kW Garrett Series 85 auxiliary power unit represented the gas turbine compressor system. A single shaft turbine and a two-stage radial compressor were designed to deliver approximately 2 kg/s of compressed airflow at a pressure ratio of about four.

##### 3.1.2 Electric Load Generator

A gear-driven synchronous (400 Hz) generator was directly coupled to the shaft of the gas turbine system, and the electrical generator was loaded by an isolated 120 kW resistor bank. Nine resistors provide the controllability of the electric load for the automated control of the system.

##### 3.1.3 Cold-Air Bypass Valve

The cold-air bypass valve was installed to bypass air from the compressor discharge directly into the turbine inlet using the mixing volume.

##### 3.1.4 Swift Fuel Valve

A Woodward swift elastomeric seat is a 2.54 cm sonic needle and nozzle operated at high speed with a
Here, each input–output relationship was characterized by Eq. (1)

\[ g(s) = \frac{k}{\tau s + 1} e^{-\delta s} \]  

and then assembled into a matrix transfer function

\[
\begin{bmatrix}
y_1 \\ \vdots \\ y_m 
\end{bmatrix} = \begin{bmatrix}
k_{11} e^{-\theta_{11} s} & \cdots & k_{1n} e^{-\theta_{1n} s} \\ \vdots & \ddots & \vdots \\ k_{m1} e^{-\theta_{m1} s} & \cdots & k_{mn} e^{-\theta_{mn} s} 
\end{bmatrix} \begin{bmatrix}
\nu_1 \\ \vdots \\ \nu_n 
\end{bmatrix}
\]

(2)

### 4.1 Mathematical Procedure for Multivariable Control Design

The dynamic cross-coupling interactions among a gas turbine, a fuel cell stack, heat exchangers, and combustors in this fuel cell gas turbine hybrid system makes the use of first principle models for the design of real-time control strategies impractical [28,29]. Because of this, the dynamic characterization of each input/output transient was quantified through empirical transfer functions experimentally derived from the physical process. Thus, unit steps were used to derive first-order plus delay time (FOPDT) transfer functions through experimental tests in the physical system before implementing the MIMO controller [30]. The FOPDT approach reduces the computational burden by catching the dominant plant dynamics just using a simple pole. Specifically, the gain represents the output variation related to a step change of an actuator, \( k = \Delta y / \Delta u \). The time delay \( \theta \) is the dead time of the output when no significant variation is verified after a change in the input. Finally, the pole location determines how fast the output reaches the new set-point; \( \tau \) is defined as the time constant at 63\% of \( \Delta y \). The following equation is a common approximation of the FOPDT model:

\[ g(s) = \frac{k}{\tau s + 1} e^{-\delta s} \]  

(1)

In addition, the RGA number is useful to determine if the system is feasible to control or ill-conditioned. A low RGA number provides adequate controllability. The definition of the RGA number is presented in the below equation:

\[ \text{RGA}_{\text{number}} = ||\text{RGA}(G) - I||_{\text{sum}} \]  

(4)

### 4.1.2 State-Space Design

In traditional control theory, feed-forward compensations are generally added to the control action of a single-input single-output controller to improve the rejection capability of a measurable disturbance. This approach is widely used because it minimizes the coupling interaction around nominal conditions using linear transfer functions that provide a unique insight of the input/outputs properties of the system. However, as shown in simulation studies, feed-forward compensations can cause problems at off-design conditions and especially for this coupled hybrid system [30]. Thus, a state-space model is generally preferred to consider the overall cross-coupling interactions existing in hybrids. Furthermore, a state-space formulation is generally preferred for optimal design or optimal trajectory between operating states [31].

Equation (5) shows the linear state-space representation defined in terms of deviation variables, where \( x \) represents the states vector of the system that is defined as a deviation from some nominal value or nominal trajectory, whereas \( u \) represents the inputs vector. Furthermore, the state matrix \( A \) and the inputs matrix \( B \) include the poles and the gains of each transfer function, respectively.
4.1.3 State-Space Stability Analysis. Once the matrix transfer function was converted into the state-space representation, a stability analysis of the state matrix \((A)\) was conducted. The stability concept is referred to an equilibrium point, that mathematical point in which the state variables remain indefinitely in a function of the time. A generic input/output system is defined “simply stable” if the deviation between the actual state and the equilibrium state is less than an infinitesimal \(\epsilon\) at the initial condition, and remains in the neighborhood of the equilibrium with increasing time to infinity and under perturbations. Whereas the system is “asymptotically stable” if the same deviation lies in the neighborhood of the equilibrium at the initial condition and decays to the equilibrium position with increasing time to infinity. In the state-space representation, the \(A\) matrix and the Laplace variable \((s)\) are taken into account to analyze the stability, and Eq. (6) presents the stability criteria for a time invariant linear system in the continuous time domain:

\[
\det(s \cdot I - A) = 0
\]

A linear dynamic system represented by the state-space formulation is simply stable in open loop if and only if all the poles or all the eigenvalues of the \(A\) matrix are negative. If at least one eigenvalue is equal to zero, the system is asymptotically stable in open loop. On the other hand, the stability analysis of a linear dynamic system in closed-loop configuration is performed by using the \(K\) matrix of the feedback control law, as shown in the below equation:

\[
\det(s \cdot I - (A - B \cdot K)) = 0
\]

In this case, the eigenvalues of the \((A - B \cdot K)\) matrix provide the stability criteria for the closed-loop system. In other words, the effect of the \(K\) matrix controller on the open-loop roots is evaluated by showing the location of the closed-loop roots.

4.1.4 State-Space Controllability Analysis. The controllability of the state vector is generally performed to evaluate whether there are states in the model that cannot be modified by any control design. It is a system algebraic concept that is only important for controller computations and realizations. The controllability of the state-space model is neither a necessary nor a sufficient condition for a system to be controllable in a practicable sense (input–output controllability). This is only a practical concern if the associated model is mathematically unstable and there are uncontrollable states that cannot stabilize the system. The most common definition for testing the state controllability analysis is based on the pair of \((A, B)\) matrices analysis:

\[
P = \begin{bmatrix} B & AB & A^2B & \ldots & A^{n-1}B \end{bmatrix}
\]

The state-space system is completely controllable if and only if the \(P\) controllability matrix defined in Eq. (8) has full rank, or in other words the determinant of \(P\) is equal to the number of states.

4.1.5 Internal Model Control Approach. In terms of control performance, zero steady-state error is an important requirement not only for disturbance rejection but also for reference tracking. In general, the reference input considered can include steps, ramps, and other persistent signals, such as sinusoids. For a step input, it is known that zero steady-state tracking errors can be achieved with a type-one system \([32,33]\). This idea is formalized by introducing an “internal model” of the reference input in the controller as shown in Fig. 3 in order to include an integrator in the feed-forward path between the error comparator and the plant.

The set of gains identified as \(K_c\) and \(K_i\) in Fig. 3 provides the state feedback control action and the integral control action, respectively. Specifically, \(K_c\) is required to ensure zero error at steady-state when a step disturbance or a step reference is applied, and can be designed by performing an augmentation of the state vector as formalized in the internal model control (IMC) concept \([33]\). The design of a controller begins to enable the tracking of a step reference input with zero steady-state error. In this case, the reference input is generated by \(r = x_t, \dot{x}_t = 0\) or equivalently \(\dot{r} = 0\), and the tracking error is defined as \(e = y - r\), or taking the time derivative as \(\dot{e} = \dot{y} - \dot{r} \cdot \dot{x}\). Therefore, the state vector is redefined as \(x_t = \left[ e \quad \dot{e} \right]^T\) and to keep consistency with the derivative of \(x\) \(\dot{x}_t\), the equivalent state-space formulation for the input vector is redefined as \(u_t = \dot{u},\) which yields

\[
\dot{x}_t = \begin{bmatrix} C & 0 \\ 0 & A \end{bmatrix} x_t + \begin{bmatrix} 0 \\ B \end{bmatrix} u_t
\]

The \(A, B,\) and \(C\) matrices in Eq. (9) are the states, inputs, and outputs matrices of the original state-space formulation of Eq. (5).

Hence, the feedback control law that is commonly defined as \(u(t) = -K \cdot x(t)\) becomes \(u_t(t) = -K_t \cdot \dot{x}_t(t)\), in which the state vector augmentation yields \(u_t = -K_e \cdot e - K_i \cdot \dot{x}\). Since the control input in the augmented state-space formulation was defined as \(u_t = \dot{u}\), it can be found by integrating \(u_t\):

\[
u(t) = -K_e \int_0^t e(t) \cdot dt - K_i \cdot x(t)
\]

The corresponding block diagram that includes an internal model, which is basically an integrator of the reference step inputs, is shown in Fig. 3.

4.1.6 Design of the Multivariable Architecture. Because the number of states produced by first-order transfer functions is generally unlimited, the pole placement approach was considered expedient and acceptable. Using this methodology, the root locations of the closed loop system in Eq. (7) are selected through the characteristic equation of the desired response, which is presented in the below equation:

\[
\det(s \cdot I - \tilde{A}) = s^n + \tilde{a}_1 \cdot s^{n-1} + \cdots + \tilde{a}_n \cdot s + \tilde{a}_n
\]

In other words, the roots of the characteristic equation of the closed-loop system in Eq. (7) are placed where the transient performance meets the desired response in Eq. (11). In this case, the control objective is reduced to match the coefficients of the closed loop system in Eq. (7) to those of the desired response in Eq. (11) using the parameters of the \(K\) controller. If the state-space system is completely controllable, each root of the system can be manipulated by a set of parameters or gains presented in the \(K\) matrix of the below equation:

\[
u(t) = \begin{bmatrix} k_{1,1} & k_{1,2} & \cdots & k_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n,1} & k_{n,2} & \cdots & k_{n,n} \end{bmatrix} \cdot \dot{x}_t
\]

The dimension of the \(K\) matrix is related to the dimension of the states matrix \((A)\) and inputs matrix \((B)\) in the state-space formulation. Specifically, the number of columns is equal to the number of states and the number of rows is equal to the number of actuators in the system. The main advantage of using a matrix representation as a control strategy is the mitigation of the decentralized coupling. In particular, the decentralized items in the \(K\) matrix
430 electric load was performed, the turbine speed is affected by more
429 matched the experimental transient, when a step in the turbine
428 kW, %, kg/s, etc. As shown in Fig. 5, the transfer function models
427 any discrepancies between the different engineering units: rpm,
426 speed and cathode airflow. A scaling approach was used to avoid
425 develop single-input single-output transfer functions for turbine
424 A 20 kW electric load variation was performed in Fig. 5 to
422 racy in the dynamic response was verified by fitting each function
420 single step in the linear operating range while the system was run-
418 changes in the Hyper facility affected the reliability of those ear-
416 ysis was performed using Eq. (7).
414 in the Simulink environment. Finally, a closed-loop stability anal-
412 of the step response applied to the desired characteristic equation
411 controller response was quantified by evaluating the settling time
410 or in other words by determining the coefficient in Eq. (11). The
409 quency in the step response of the desired characteristic equation,
408 developed by computing the damping factor and the natural fre-
407 compute a controller ($K_c$).

406 approach was similar to what Tsai et al. mentioned in a previous
405 trol problem designed in this work is illustrated in Fig. 4. Such an
403 influence that adjacent actuators have on each state or output dur-
401 “anticipation” factor in the control action, which diminishes the
400 Such a compensation provides a “communicative effect” or
399 that the turbine speed decreases the turbine inlet temperature by 32 °C, the turbine speed by 1000 rpm, and the cathode airflow by 0.4 kg/s. As illustrated in Eq. (13), each input/output relationship of the system populated a $2 \times 2$ matrix transfer function before a state-space model was derived

\[
\begin{pmatrix}
 y_1 \\
 y_2
\end{pmatrix} =
\begin{bmatrix}
-0.25 \cdot e^{-0.1 s} & -0.065 \cdot e^{-0.5 s} \\
(s + 0.225) & (s + 0.125)
\end{bmatrix}
\begin{pmat}
 u_1 \\
 u_2
\end{pmat}
\] (13)

4.2 Multivariable State-Space Derivation for the Physical Hybrid Cycle. Compared to previous work, new transfer functions were experimentally developed because of hardware changes in the Hyper facility affected the reliability of those earlier models [29]. Each transfer function was developed using a single step in the linear operating range while the system was running at nominal conditions, as shown in Figs. 5 and 6. The accuracy in the dynamic response was verified by fitting each function against experimental data.
A 20 kW electric load variation was performed in Fig. 5 to develop single-input single-output transfer functions for turbine speed and cathode airflow. A scaling approach was used to avoid any discrepancies between the different engineering units: rpm, kW, %, kg/s, etc. As shown in Fig. 5, the transfer function models matched the experimental transient, when a step in the turbine electric load was performed, the turbine speed is affected by more than 2000 rpm, and the coupling on the cathode airflow is about 0.13 kg/s.

Similarly, Fig. 6 presents a step in the cold-air bypass valve. Such a valve can directly control the cathode airflow, but also a strong coupling on turbine speed operation because it affects turbine inlet temperature. An opening action of the cold-air bypass decreases the turbine inlet temperature by 32 °C, the turbine speed by 1000 rpm, and the cathode airflow by 0.4 kg/s. As illustrated in Eq. (13), each input/output relationship of the system populated a

\[
RGA(G(0)) = \begin{bmatrix} 1.15 & -0.15 \\ -0.15 & 1.15 \end{bmatrix} \] (14)

In Eq. (15), the RGA was performed at 1 rad/s (0.16 Hz), and similarly to Eq. (14) showed an acceptable pairing that was confirmed by a low value of 0.2

\[
RGA(G(1)) = \begin{bmatrix} 1.05 & -0.05 \\ -0.05 & 1.05 \end{bmatrix} \] (15)

In Eq. (16), the RGA was performed at 31 rad/s (5 Hz). In this case, the matrix was an identity matrix, which confirmed acceptable pairing, even at high frequency. The RGA number was approximately zero

\[
RGA(G(31)) = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix} \] (16)

4.2.2 State-Space Design. The conversion from a matrix transfer function to a state-space representation is generally not a unique solution because there could be different $A$, $B$, $C$, and $D$ matrices involved. It can be designed by identifying the components of the state vector, and then converting the parameters of a transfer function into the state-space matrices. For this representation, the state vector of the MIMO model was composed by only two measurable states $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, the turbine speed ($x_1$) and the cathode mass flow rate ($x_2$). For the design of this controller, additional poles and zero introduced by the Pade approximation to estimate the time delay were simplified to avoid a nonminimum phase behavior in the model. Such a behavior can introduce instabilities during the design of the controller that are not reproduced.
in the physical process \cite{34}. Thus, the state-space plant model of the system generated by the matrix transfer function of Eq. (13) is

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} -0.225 & 0 \\ -0.69 & -1.43 \end{bmatrix} \cdot x(t) + \begin{bmatrix} -0.25 \\ -0.065 \end{bmatrix} \cdot u(t) \\
y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot x(t)
\end{align*}
\tag{17}
\]

The zero in the position (1, 2) of the states matrix (A) was because changes in cathode airflow do not affect the turbine speed if the fuel cell is electrochemical inactive. Similarly, the zero in the position (2, 1) of the inputs matrix (B) was due to no direct effect from electric load to cathode airflow. Using the definition of the IMC approach, the state-space representation of the MIMO system became as illustrated in the below equation:

\[
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.22 & 0 & -1.43 \\ 0 & -0.69 & -1.43 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.25 & -0.065 \\ 0 & -1.43 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\tag{18}
\]

The time in Eq. (18) was omitted, and \(e_1\) was the error between the turbine speed set-point and the turbine speed feedback, while \(e_2\) was the error between the cathode airflow set-point and the cathode airflow feedback.

4.2.3 Stability Analysis. The stability analysis of the open-loop model in Eq. (17) yielded root locations in \(-0.22\) and \(-1.43\), which means that the system was simply stable in open loop. On the other hand, the stability analysis of the open-loop model using the IMC approach illustrated in Eq. (18) produced two additional roots that were both located in the origin. In this case, the augmented system was asymptotically stable, and presented four roots located in the following positions: \([0 \ 0 \ -1.43 \ -0.22]\).

4.2.4 Controllability Analysis. The size of the \(P\) controllability matrix related to the open-loop state-space model in Eq. (17) was \(2 \times 4\) and the rank was 2, which means that the system was completely controllable. Similarly, the size of the \(P\) controllability matrix of the augmented state-space model presented in Eq. (18) was \(4 \times 8\) and the rank was equal to 4; therefore, even in this case, the system was completely controllable.

4.2.5 Multi-Input Multi-Output Disturbance State-Space Design. A characterization of the disturbances was similarly performed in order to include suitable perturbations to the state-space controller in simulation environment. The matrix transfer function related to the disturbances was defined by assembling each input/output relationship in the system such as for the plant model. Similarly, first-order transfer functions were used to represent the interaction from each disturbance to each output, as shown in the below equation:

\[
\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.25 & 0.065 \\ 0 & 1.43 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\]
4.2.6 Design of the Multivariable Architecture. The $K$ matrix of the state feedback controller using the IMC representation in Eq. (18) was composed by two rows and four columns. The first row of gains was associated with the turbine speed-electric load control loop, whereas the second row of gains was associated with the cold-air bypass-cathode airflow control loop. Considering the individual items in the $K$ matrix, $k_{11}$ represented the integral gain and $k_{31}$ the proportional gain for the turbine speed-electric load control loop, whereas $k_{22}$ and $k_{44}$ represented the integral gain and the proportional gain for the cold-air bypass-cathode airflow control loop. Since a turbine speed change affected the cathode airflow almost instantaneously, $k_{31}$ provided an additional benefit to the control action of the cold-air bypass valve, which yielded an instantaneous change in the cathode airflow as soon as a perturbation in the turbine speed occurred. Similarly, $k_{14}$ provided an additional benefit to the electric load command when the cold-air bypass valve modulated the cathode airflow and affected the turbine speed. On the other hand, $k_{34}$ was not included in the architecture because an additional compensation in the electric load command based on the integral action of the cathode airflow control loop would only have increased the oscillation in the process. Integral actions are very important to keep zero steady-state difference but creates undesirable windup because accumulates the error between the reference value and the feedback measurement over time. Similarly, $k_{23}$ was not included because an additional compensation on the cold-air bypass valve control command based on the integral action of the turbine speed control loop would have increased the oscillation in the process as well. Figure 4 presents the control architecture with the communicative effect involved.

Different set of gains were designed before testing the MIMO control strategy on the hardware system. Equation (20) shows the $K$ matrix using a critically damped design, in which the damping factor and the natural frequency were set equal to 1 and produced 10 s as settling time in the step response of the desired characteristic equation

$$K = \begin{bmatrix} \frac{0.17 \cdot e^{-0.1s}}{s + 0.125} & 0.03 \\ 0.06 \cdot e^{-0.5s} & \frac{1.23 \cdot e^{-0.64}}{s + 1.43} \end{bmatrix} \begin{bmatrix} H_3 \\ H_4 \end{bmatrix} \tag{19}$$

As shown in Eqs. (20) and (21), the main feature of a centralized tuning at least using the place function available in MATLAB CONTROL SYSTEM TOOLBOX is that changing a single parameter during the design process to increase or reduce the controller response, even for a single control loop, affects the overall matrix of gains. As shown in the experimental tests, this can be a limiting aspect especially if the time response in the control loops is substantially different, such as between the turbine speed and the cathode airflow. For instance, changing a single parameter during the design process to increase the response of the cathode airflow control loop due to slow performance could be detrimental for the turbine speed control loop. For instance, it would further increase all the controller parameters in the $K$ matrix, and so the response of the turbine speed, which can further increase the overshooting in the transient.

5 Experimental Methodology

The thermal steady-state provides the required stability in the system that has to be generally guaranteed before connecting any control strategy to the physical plant. This condition is achieved when the turbine speed reaches 40, 500 rpm and the skin temperature of the mixing volume varies less than 0.1 K for a 30 s period. During the startup procedure, a single-input single-output proportional control strategy to the physical plant. This condition is achieved when the turbine speed reaches 40, 500 rpm and the skin temperature of the mixing volume varies less than 0.1 K for a 30 s period. During the startup procedure, a single-input single-output proportional-derivative controller is disconnected. In this case, the turbine operates at constant fuel flow, which is generally considered an open-loop configuration. This represents a temporary operation before connecting the control strategy.

Once the system operated in open-loop configuration, the multivariable control strategy was connected to control the hardware system, and three different set of experiments were designed. For each set of experiment, three different tests evaluated the performance of the strategy. As shown in Table 1, in the first set of experiments, only the electric load–turbine speed control loop was connected to the hardware system, and only two tests were evaluated. In the first test, the turbine speed set-point tracking operation evaluated the coupling on the cathode airflow during the sole automated control of the cold-air valve, whereas in the second test, the fuel valve perturbation simulated the waste heat variation of the fuel cell stack.

In the second set of experiments, the electric load–turbine speed control loop was disconnected and the cold-air bypass–cathode airflow control loop was connected to the hardware. In this case, only a cathode airflow set-point variation was performed, which evaluated the coupling on the turbine speed during the sole automated control of the cold-air bypass valve. Finally, in the third set of experiments, the multicoordination strategy with the communicative effect presented in Fig. 4 was connected to control the hardware system, and all of the three tests evaluated the performance of the strategy.

During each experiment, one test and so one perturbation at the time was performed while all the other actuators were set at constant position. Table 2 summarizes the operating point of the system during each test for all the experiments.
6.1 Evaluation of Load Based Speed Control. The evaluation of each single-input single-output control loop was considered important to quantify the coupling effect when only one actuator at the time was set in automated mode. In these two single-input single-output cases, the cathode airflow or the turbine speed was alternatively set in open-loop configuration, and only the controlled elements from the K matrix of Eq. (21) were considered without involving the communicative effect between control loops. The controller structure was essentially based on a linear combination at discrete time steps (5 ms) that only included the proportional and the integral gains of each loop. Specifically, only the gains $k_{13}$ and $k_{11}$ were used in the first control loop (electric load–turbine speed, experiment 1 in Table 1), and the gains $k_{32}$ and $k_{34}$ were used in the second control loop (cold-air bypass–cathode airflow, experiment 2 in Table 1), which all were designed using a multivariable state-space concept.

6.1.1 Experiment 1—Test 1: Turbine Speed Set-Point Tracking Maneuver. During a change in the electric load, the turbine operates in a variable speed mode affecting the cathode airflow. The dynamic response of the turbine strongly affects the performance of the entire system, and a deviation in the nominal operating point influences the overall mass flow, pressure, and temperature of the fuel cell/gas turbine cycle. For instance, a speed reduction reduces the cathode inlet airflow that causes a decrease in the dissipation of the thermal energy from the fuel cell to the turbine [25]. In this scenario, the turbine speed and the compressor inlet airflow decrease even more than the initial turbine speed set-point change and a stall of the compressor can easily occur, damaging the fuel cell [27].

Considering all these important dynamic effects, the turbine speed setpoint change in Fig. 7 was used to quantify the impact of coupling on the cathode airflow although the fuel cell was electrochemical inactivity. During this test, a turbine speed was perturbed by only 500 rpm, which represents 1.2% of the full range of operation. The cathode airflow was set equal to 0.77 kg/s, slightly below 1.0 kg/s, which represents the typical nominal operating point of fuel cell sized for the Hyper facility.

The comparison between the simulated and the experimental results presented in Fig. 7 shows significant differences in the performance of the controller. For instance, the simulated trend confirms that the overdamped tuning was an adequate design because the turbine speed properly approaches the new setpoint without overshoot. On the other hand, an underdamped behavior characterized the response of the controller in the experimental test, although the original design was still based on an overdamped tuning. Specifically, the rise time between the experimental and the simulated performance of the controller was approximately identical, indeed the time for the output to reach 90% of its final value was in both cases approximately 1.28 s. Instead, the settling time between the experimental and the simulated test showed a difference of about 6–8 s due to the overshooting in the turbine operation. The inconsistency between the controller design and the performance on the hardware system was due to the order of the mathematical model. Although each first-order transfer function matched the experimental transients as shown in Figs. 5 and 6, often the dynamic of a first-order model is very similar to an overdamped second-order transfer function. This can mislead to approximate such a transient behavior with a first-order model in a second-order system. A mismatch in the controller performance between simulation and experimental environment can occur as shown in Fig. 7. A first-order model generally lacks underdamped dynamics.

Furthermore, Fig. 7 shows that the gains $k_{13}$ and $k_{11}$ designed in Eq. (21) cause 200 rpm of overshooting, which is 0.5% of the overall turbine speed range. However, it still represents a significant deviation considering that the amplitude of the step was equal to 500 rpm. The maximum rate of change for the electric load command to manage the turbine speed set-point variation was equal to 1.5 kW in a single time-step (80 ms), and produced a change of 18 kW in 3.2 s. Such a response was difficult to be predicted in simulation environment because the accuracy of the simulated actuator is more precise than the physical one. In other words, the physical actuator has an accuracy of 0.5 kW in a single time-step (5 ms), which limits the precision of the response compared to the simulation study. These effects confirm the challenge of testing control strategies on physical hardware systems.

Similar differences between simulation and experimental trends are presented in the steady-state difference of the cathode inlet airflow. The model quantified a steady-state difference of about 0.1 kg/s due to the sole implementation of the single-input single output controller whereas, the experiment only presents an oscillation of that amount of deviation, and the final steady-state difference between the cathode airflow setpoint (0.77 kg/s) and the plant output is less than half of that. In this case, the oscillation could still damage the fuel cell material, but it confirmed the strong influence that the turbine speed has on the fuel cell airflow.

### Table 1 Experimental testing of the multivariable control strategy

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel valve perturbation</td>
<td>Fuel valve perturbation</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>200 kWth</td>
<td>200 kWth</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Cathode airflow set-point</td>
<td>Cathode airflow set-point</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>(0.2 kg/s)</td>
<td>(0.2 kg/s)</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2 Operating points of the hardware system during each set of experiment

<table>
<thead>
<tr>
<th>Actuator position</th>
<th>Exp. 1, test 1</th>
<th>Exp. 1, test 2</th>
<th>Exp. 2, test 3</th>
<th>Exp. 3, test 1</th>
<th>Exp. 3, test 2</th>
<th>Exp. 3, test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric load</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
</tr>
<tr>
<td>Cold-air bypass</td>
<td>40%</td>
<td>40%</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
<td>Automated</td>
</tr>
<tr>
<td>Bleed-air valve</td>
<td>90% closed</td>
<td>90% closed</td>
<td>90% closed</td>
<td>90% closed</td>
<td>90% closed</td>
<td>90% closed</td>
</tr>
<tr>
<td>Hot-air bypass (%)</td>
<td>1252</td>
<td>1255</td>
<td>1270</td>
<td>1265</td>
<td>1244</td>
<td>1271</td>
</tr>
<tr>
<td>Skin temperature (°F)</td>
<td>51.3</td>
<td>51.2--55.2</td>
<td>51.2</td>
<td>51.3</td>
<td>51.3--55.3</td>
<td>51.3</td>
</tr>
<tr>
<td>Fuel valve (%)</td>
<td>51.3</td>
<td>51.2--55.2</td>
<td>51.2</td>
<td>51.3</td>
<td>51.3--55.3</td>
<td>51.3</td>
</tr>
</tbody>
</table>
6.1.2  Experiment 1—Test 2: Fuel Valve Disturbance Rejection or Fuel Cell Load Transient. In this experiment, the fuel cell was still electrochemically inactive and so a change in the fuel flow of the gas turbine combustor was used to simulate a hypothetical fuel cell waste heat variation. Similarly to the previous case, in the results shown in Fig. 8, the turbine was still controlled through the single-input single-output load-based speed controller designed using the gains \( k_{11} \) and \( k_{13} \) (Eq. (21)). In other words, the cathode airflow was intentionally left in open loop to evaluate the coupling effect under heat perturbations performed through fuel flow variations.

As an example of the magnitude of change, an off-line fuel cell simulation was performed to quantify the heat variation presented in Fig. 8 using a 4% fuel valve step perturbation. This slight change yields a thermal heat deviation from a hypothetical fuel cell to the turbine equal to 200 kWth. Such a deviation represented a fuel cell load turn down of about 30 A over an operating range of 200 A, which means a 15% deviation in the fuel cell current demand around nominal conditions. In terms of fuel cell electric power generation, the fuel valve change reproduced a deviation of about 35 kW of over an operating range of 350 kW, which means a deviation of about 10% around nominal fuel cell power conditions. In open loop, a fuel valve perturbation of 4% caused almost 3000 rpm of turbine speed deviation in about 40 s.

In this case, the controller quickly handled the significant variation of thermal energy by keeping the speed within a safe boundary of nominal operation; indeed, a deviation of just 300 rpm affected the response. Similarly to the previous case, the experiment shows a larger perturbation compared to the simulation results. The turbine speed and the cathode airflow were, respectively, affected by 150 rpm and 6% more in the experiment compared to the simulation results. Also, the electric load control action that keeps the system at steady condition is different; a more consistent electric load change during the experiment to reject the disturbance on the speed operation was maintained compared to the simulation. Specifically, the electric load was changed about 16.5 kW in about 9 s when the step was applied, and the maximum rate was equal to 1 kW in a single time-step (80 ms). Hence, the controller provides stable operation, quick time response, and a pronounced rejection capability when the step was applied to the turbine combustor. In this test, the cathode airflow was not controlled, and a 6% variation was noticed around nominal condition during this perturbation. A pronounced rejection capability for the turbine speed controller provided an important
benefit to maintain the cathode airflow around nominal conditions.

6.2 Evaluation of Cold-Air Bypass Cathode Mass Flow Control. Once the electric load–turbine speed control loop was validated, the single-input single-output cold-air bypass cathode mass flow controller was implemented and tested in the hardware system (experiment 2 in Table 1). In this case, only the centralized items from the second row of the $K$ matrix in Eq. (21) were considered in the structure of the controller. The gains $k_{23}$ and $k_{22}$ represent the proportional and the integral gains, respectively, which were designed using the multivariable state-space procedure.

6.2.1 Experiment 2—Test 3: Cathode Airflow Set-Point Tracking. In a fuel cell/gas turbine hybrid system, a cathode airflow maneuver can be used to control the temperature difference between the inlet and the outlet of a fuel cell stack [25]. In this scenario, a regulator rather than a controller can be preferred to manage the operating point of the solid temperature in the fuel cell material. However, the high coupling between the cold-air bypass and the turbine speed can create control issues, and a quantification is presented in Fig. 9.

In this case, the difference between the simulation and experimental performance was more significant compared to the previous tests especially, in the automation of the valve that handles the airflow setpoint change, and in the coupling of the turbine speed. The simulated trend in the cathode airflow performance follows the step change almost instantaneously due to the very reactive response of the cold-air bypass valve. On the other hand, the experimental trend is significantly delayed due to hardware limitations in the sensor measurement and in the actuation of such a valve. Specifically, during the experiment, the controller opened the cold-air bypass valve by 25% in 13 s, using a maximum rate of 0.7% in a single time-step (80 ms), whereas in simulation, the valve was able to provide a 40% change in less than a couple of seconds. Such inconsistency was probably due to underdamped dynamic effects that were not completely captured in the input–output model between the valve and the process. For instance, even increasing the transport delay in the system model between the valve opening and the process did not improve the response. This simulation created an even faster opening in the valve to compensate a negligible change in the process reaching the saturation point more quickly.

Although the fuel cell was still electrochemically inactive, an automated control of the cold-air bypass valve created a larger coupling on the turbine speed compared to the quantification shown in the simulation of the model. The setpoint change in the cathode airflow was equal to 0.2 kg/s, and almost 1000 rpm of coupling occurred in the turbine operation even without electrochemical dynamics taken into account during the experimental
Such a coupling is primarily due to the action of the cold-air bypass valve that reduces the turbine inlet temperature bypassing cold airflow from compressor discharge to the turbine inlet.

6.3 Simultaneous Control of Actuators. Once the performance of each control loop was individually validated, the multi-variable control strategy with the communicative effect presented in Fig. 4 was simultaneously connected to the hardware system. Considering the test matrix of Table 1, all of three tests in the third experiment were experimentally performed.

6.3.1 Experiment 3—Test 1: Turbine Speed Set-Point Tracking Maneuver. Figure 10 presents a turbine speed setpoint tracking maneuver equal to 1.2\% of the full range of operation during a simultaneous control of actuators. This test was considered important to quantify the coupling effect on the cathode airflow when the turbine speed changes, and simultaneously, the cold-air bypass controls the cathode airflow.

Similarly to the single-input single-output case, the multi-variable control strategy presented in Fig. 10 shows different performance between the simulation and the experimental test. The simulated response was characterized by an overall overdamped behavior for both actuators, as properly defined in the design process. On the other hand, the experimental response presented an underdamped behavior in the operation of the turbine and a more pronounced overdamped behavior in the response of the cold-air bypass valve to reject the coupling on the cathode airflow compared to the simulation trend. As previously described, such inconsistency was due the mathematical model of the system that was used to design the response of the control strategy. In other words, the turbine speed model lacked underdamped dynamics that exists in the real power plant, and the model of the cold-air bypass valve does not reproduce hardware limitations in the actuator and in the sensor measurement.

However, in this case, the turbine speed overshoot is 190 rpm, and similarly to Fig. 7 represents a significant deviation considering only 500 rpm of setpoint change. Similarly to the single-input single-output case, the maximum rate of change of the electric load command was 1.5 kW in a single time-step (80 ms). As in the previous case, the electric load response produced an excessive overshooting in the turbine speed that was not properly quantified in the off-design tuning of the controller and in the modeling of the system. Such a turbine oscillation also caused an instantaneous fluctuation on the cathode airflow, which was not similarly predicted by the model. This coupling effect was not entirely rejected due to the slow response of the overdamped design used in Eq. (21), and due to hardware limitations existing in the actuation of the valve and in the sensor measurement. However, the simultaneous operation of the cold-air bypass valve avoids cathode airflow steady-state difference that existed in the single-input single-output case shown in Fig. 7 (experiment 1, test 1).

The maximum rate of change of this actuator was only equal to 0.14\% for a single time-step (80 ms). The fuel cell would be able...
to tolerate this oscillation because the coupling caused by the turbine speed on the cathode airflow was at the limit of 0.05 kg/s. However, a larger change in the speed would cause a higher coupling on the cathode airflow that could be detrimental to the fuel cell, especially if such an overdamped design is still used.

6.3.2 Experiment 3—Test 2: Fuel Valve Disturbance Rejection. Similar to the single-input single-output case of Fig. 8, Fig. 11 presents a fuel valve step perturbation that reproduces a waste heat variation due to an increase of a hypothetical fuel cell load change, but during a simultaneous control of actuators.

As well as for the single-input single-output case, the significant variation of thermal energy (200 kWth) that affected the turbine speed operation was properly handled by the electric load. The controller provided a change of 18.5 kW in about 4.08 s, and the maximum rate of change was 1 kW in a single time-step (80 ms). Similarly to the previous case, the quick response and the pronounced rejection capability on the turbine speed provided the most important benefit on the minimization of the coupling on the cathode airflow. Indeed, a deviation of just 300 rpm affected the turbine speed when the heat perturbation was applied, and the airflow was kept steady during such a transient.

Compared to Fig. 9, the cathode airflow did not present steady-state difference because it was controlled by the cold-air bypass valve, but presented a similar oscillation when the fuel flow perturbation was applied. Even in this case, the over-damped design limited the rejection of the coupling effect on the cathode airflow.

The maximum rate of change of the cold-air bypass valve was only equal to 0.1% for a single time-step (80 ms). The coupling effect of a hypothetical fuel cell load perturbation on the cathode airflow is just 0.03 kg/s, and the fuel cell would be able to tolerate this oscillation. If the fuel cell had been electrochemically active, the oscillation in the cathode airflow would have affected the turbine speed even more, due to the self-propagated effect existing in the hybrid configuration. However, the turbine speed controller responded pretty quickly, and this confirms that constant turbine speed operation, or an excellent rejection capability for turbine speed perturbations, minimized the coupling effect on the cathode airflow.

As well as for the single-input single-output case, a larger deviation in the turbine speed and a more consistent electric load change were shown in the experimental test compared to the simulation results. In this case, such inconsistency does not affect significantly the controller performance, but still presents the need for a better quantification of the dynamic response in the modeling of the system and during the design of the controller.

6.3.3 Experiment 3—Test 3: Cathode Airflow Set-Point Tracking Maneuver. Similarly to the single-input single-output case of Fig. 9, Fig. 12 presents the cathode airflow setpoint tracking variation, but during simultaneous control of actuators. The amplitude for the cathode airflow variation was kept equal to 0.2 kg/s.

Figure 12 shows that the simultaneous control of the electric load rejected 1000 rpm of coupling on the turbine speed during the automated control of the cold-air bypass valve to track the cathode airflow setpoint change presented in Fig. 9. Specifically, the control strategy was able to open the cold-air valve by 30%.
and reduced the electric load from 40.5 kW down to 33.5 kW to keep the turbine speed constant. The maximum rate of change for the cold-air bypass valve was equal to 0.6% in a single time-step (80 ms), and for the electric load command to maintain the turbine speed constant was 0.5 kW. However, a few spikes showed in dashed circles in Fig. 12 affected the turbine speed around nominal conditions. A robust design in future work could be preferable to avoid detrimental instabilities on the turbine shaft.

Similarly to the single-input single-output case, the multivariable control strategy presented the same limitation in the comparison between the simulation and experimental results. For instance, the simulated response of the cold-air bypass valve showed a more reactive behavior that provided an accurate tracking of the airflow set-point change. On the other hand, the physical actuator has a slower response that affected the airflow tracking due to an inaccuracy in the model of the valve.

7 Summary and Conclusions

This paper showed an experimental evaluation of a 2 × 2 multivariable control strategy applied to a gas turbine recuperated cycle designed for a fuel cell/gas turbine hybrid concept. State-space methodology was used to quantify the dynamic performance of the physical plant. In this scenario, the fuel cell was electrochemically inactive, but the control of the physical hybrid plant was still a challenge due to the large size of the compressor plenum volume, which represents the cathode and anode pressure drop in the hybrid configuration. In the experiments, the electric load was used to control the turbine in order to evaluate the relationship between the speed and the power generator, whereas the cathode airflow was controlled using the cold-air bypass valve, which showed a strong coupling on the turbine speed. The control strategy was evaluated under heat disturbance perturbation that represented a hypothetical fuel cell load variation, and turbine speed and cathode airflow setpoint tracking maneuver.

An overall inconsistency was shown between the designed performance in simulation environment and the response of the power plant due to hardware limitations in the sensor measurement, actuation of the valves, and underdamped dynamic effects that were difficult to be captured with the first-order dynamic model that was used to design the control strategy. For instance, during turbine speed setpoint tracking change, an overdamped designed produced an underdamped behavior, and a significant peak overshooting from the setpoint occurred. In general, it was shown that for hybrid systems, a turbine speed deviation for more than a few seconds generates strong coupling on the cathode airflow that can be detrimental for the fuel cell operation. Furthermore, the same set of gains that well perform for disturbance rejection can cause an excessive response for set-point tracking operation.

During heat perturbation, the shaft power required to maintain constant turbine speed operation was equal to 16 kW of change in the electric load when a hypothetical 10% (200 kWth) of fuel cell load turn down operation was reproduced. The coupling in the cathode airflow was minimized and kept below a 5% limit that avoids fuel cell damage. However, the cold-air bypass valve showed a slow response due to hardware limitations and inaccuracy in the underdamped dynamics of the first-order model used to characterize the cold-air bypass-cathode airflow response to tune the controller.

Finally, although the fuel cell was electrochemically inactive, a 0.2 kg/s of cathode airflow perturbation caused almost 1000 rpm...
of change in the turbine speed operation due to the effect of the cold-air bypass valve. During the simultaneous control of actuators, the multivariable control strategy was able to keep the turbine speed constant. This test presented the most significant difference in the response of the controller between simulation and experimental behaviors due to hardware limitations in the physical system. However, the overall comparison between simulation and experimental results of the multivariable control strategy presented in this paper shows that the coupling in hybrid power systems can be strong enough such that even linear transfer functions can be inadequate to describe fluid and turbomachinery behaviors occurring during transients.

8 Future Work

As mentioned, in this work, the fuel cell was electrochemical inactive to avoid critical damaging to the turbine equipment during the simultaneous operation of the actuators. However, future work will evaluate the multivariable control strategy during the electrochemical activity of the fuel cell. In this case, the coupling between cathode airflow and turbine speed can be very detrimental due to the self-propagated effect between fuel cell and turbine transient [25]. Gains optimization and the communication effect in the multivariable control strategy will definitely play a critical role to minimize the coupling interactions. Higher order mathematical models in the discrete time domain will be directly included in the state-space representation to identify a larger range of frequencies in the system, and to more precisely quantify the time delay as time steps of delay [35]. Optimal control tuning, robust and adaptable design of novel control strategies will be also compared to this baseline control problem [36].

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Nomenclature

- $A$ = states matrix in the state-space formulation
- $B$ = inputs matrix in the state-space formulation
- $C$ = output matrix in the state-space formulation
- $g$ = transfer function in the Laplace domain
- $G$ = matrix transfer function
$k$ = transfer function gain

$p$ = pole location in the frequency domain

$s$ = Laplace variable

$u$ = input vector

$u_0$ = input vector augmentation

$u_d$ = disturbance input vector

$u_e$ = electric load

$u_v$ = cold-air bypass valve

$u_w$ = fuel valve gas turbine combustor

$u_q$ = hot-air bypass valve

$x$ = state vector

$x_o$ = state vector augmentation

$x_1$ = turbine speed error

$x_3$ = cathode airflow error

$x_5$ = turbine state feedback

$x_6$ = cathode airflow feedback

$y$ = output vector

$y_1$ = turbine speed error

$y_2$ = cathode airflow error

$\Delta u = $ input variation

$\Delta y = $ output variation

$\delta = $ time delay

$s*$ = Hadamard product operator, element by element multiplication

**Acronyms**

FOPDT = first order plus delay time transfer function

IMO = internal model control

RGA($G$) = relative gain array matrix of $G$

RGA$_{number}$ = relative gain array number

**References**


