INVERSION OF EDDY-CURRENT DATA AND THE RECONSTRUCTION OF FLAWS USING MULTIFREQUENCIES*

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INTRODUCTION

We describe a model, together with the results of numerical experiments, that uses multifrequencies to acquire and invert eddy-current data for reconstructing flaws in tube walls. The model that we describe here uses sixty frequencies, from 200 kHz to 16 mHz (though more or fewer frequencies can be used, spanning a greater or smaller spectrum), and allows the reconstruction of flaws on a grid whose cells measure 0.002" by 0.005". A single coil wound on a ferrite core is simulated for excitation and detection; thus the system is monostatic (the ferrite core is used to achieve satisfactory field concentration). The method of solution is based on minimizing the squared error between the measured data and the model data. The mathematical algorithm that is used for inversion is a constrained least-squares technique using a Levenberg-Marquardt parameter for smoothing. The numerical experiments indicate that the model performs satisfactorily in reconstructing simulated 'high' and 'low' contrast flaws in the presence of data uncertainty. The grid consists of a single column of twenty-five cells spanning the wall thickness of the tube.

One important aspect of this problem is how to collect data. In [1] and [2], we used a two-coil system. In this paper and in [3], we acquire data for inversion using a single stationary coil but vary the frequency of the exciting current and measure the resulting EMF induced into the coil. This approach has greater flexibility

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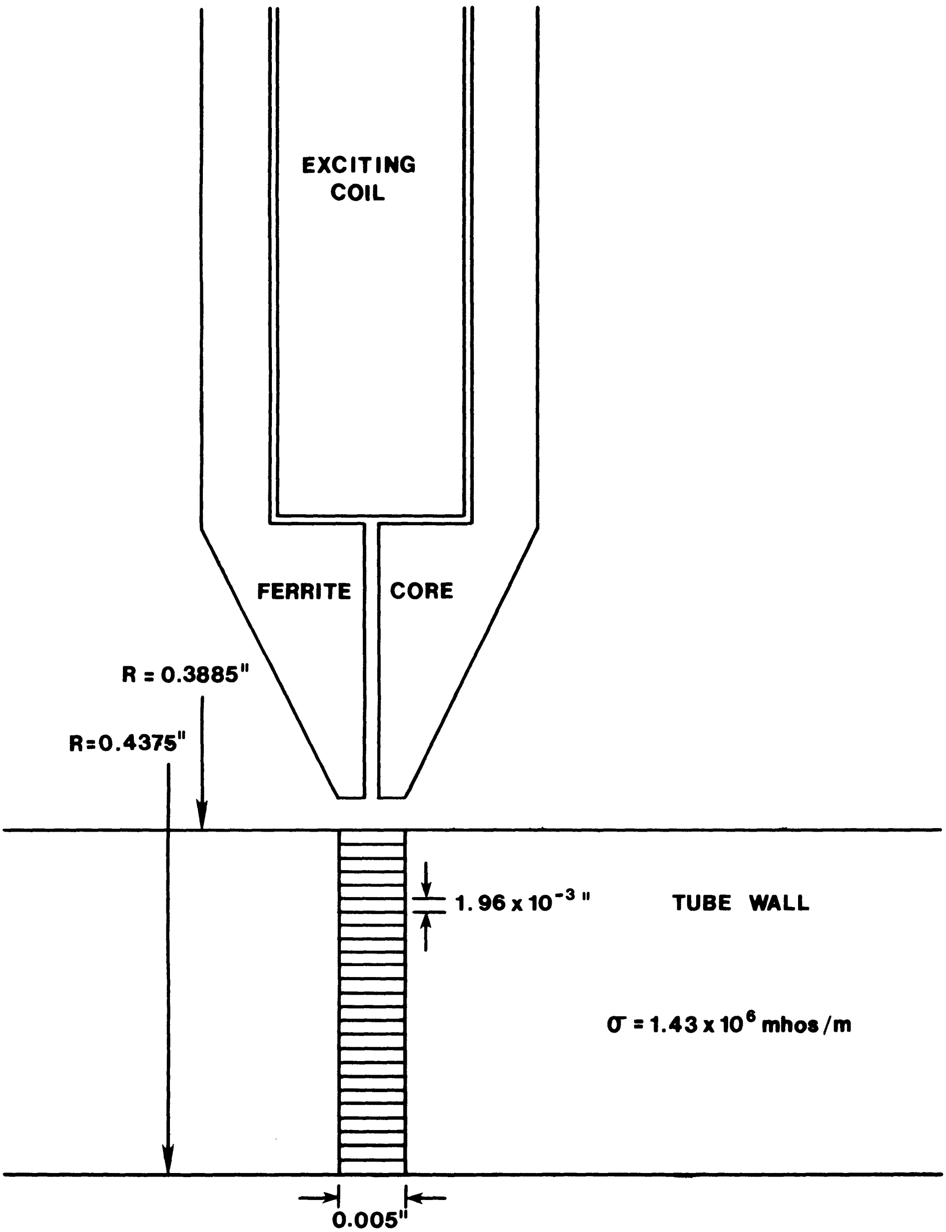


Fig. 1 Illustrating the physical system.

than the multicoil (or multiposition) method of [1] and [2], and may permit greater resolution and accuracy.

The coil we use in this model is shown in Figure 1. By using this idea, we can concentrate the exciting field so that only one column of the mesh is excited at a time.

The model integral equations are derived from basic electromagnetic theory and are fully described in [1]. These integral equations are discretized by means of the method of moments. The resulting vector-matrix version of the linearized equation is

$$T(\sigma E_0) = EMF \tag{1}$$

which in component form is

$$\sum_{j=1}^{N_c} (T_{kj}E_{0kj})\sigma_j = EMF_k \tag{2}$$

where

EMF_k = EMF induced in the probe coil at frequency k;
E_{0kj} = electric field at frequency k, incident on cell j, due to exciting coil (scattered field is small);
T_{kj} = EMF at frequency k induced in probe coil, due to current at frequency k flowing in cell j;
σ_j = conductivity of cell j (the unknown), $-1 \le \sigma_j \le 0$;
N_c = number of cells in grid.

The matrices generated by these inversion techniques are inherently ill-conditioned. Thus some form of smoothing is needed. In [3] we discuss the use of a Levenberg-Marquardt (L-M), or regularizing, parameter to help combat this problem. In addition, our approach converts a least-squares problem into a 'least-distance' quadratic programming problem (see [3] and [4]).

RESULTS

We performed a variety of numerical experiments with various combinations of frequencies. We discovered that the following combination of sixty (60) frequencies produced the best results. That combination was:

200 kHz - 500 kHz in 10 kHz steps (31)
600 kHz - 1900 kHz in 100 kHz steps (14)
2 mHz - 16 mHz in 1 mHz steps (15).

The condition number of the matrix produced was 1.05×10^4.

According to our model, if a flaw fills a cell entirely then the conductivity assigned to that cell has value -1, whereas if a

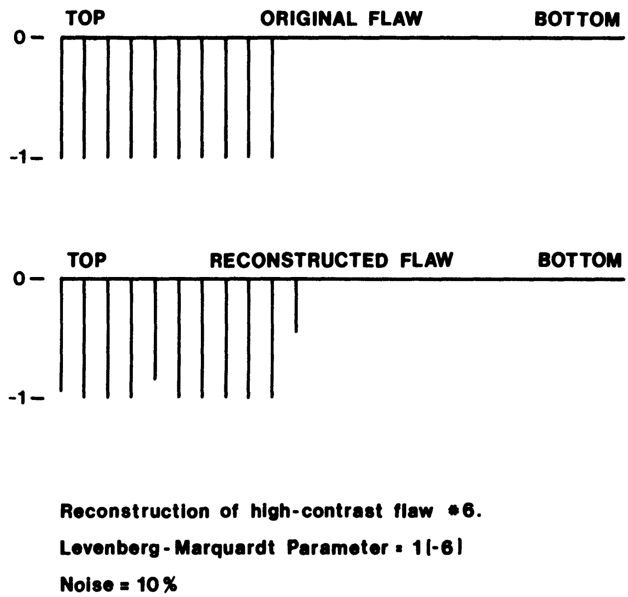


Fig. 2(a). Reconstruction of high-contrast flaw #6.
L-M parameter = 0. Noise = 10%.

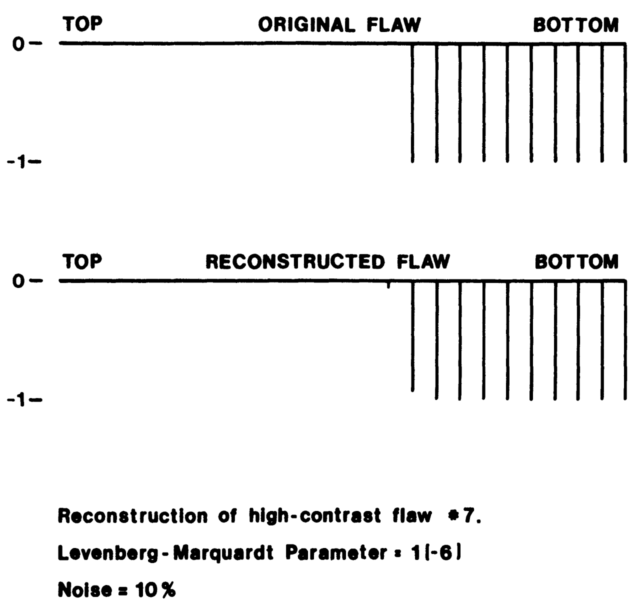


Fig. 2(b). Reconstruction of high-contrast flaw #7.
L-M parameter = 0. Noise = 10%.

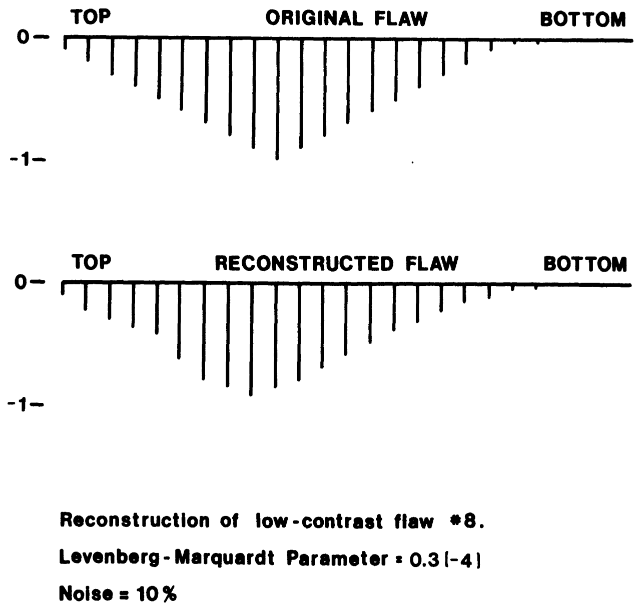


Fig. 2(c). Reconstruction of low-contrast flaw #8. L-M parameter = 0.3[-4]. Noise = 10%.

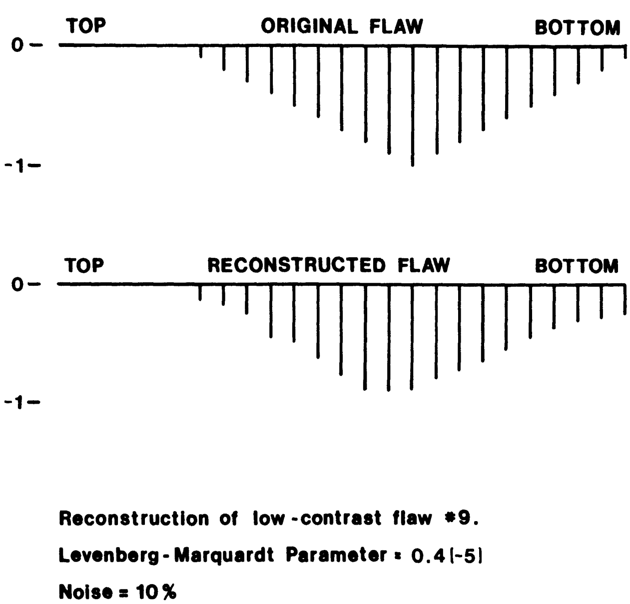
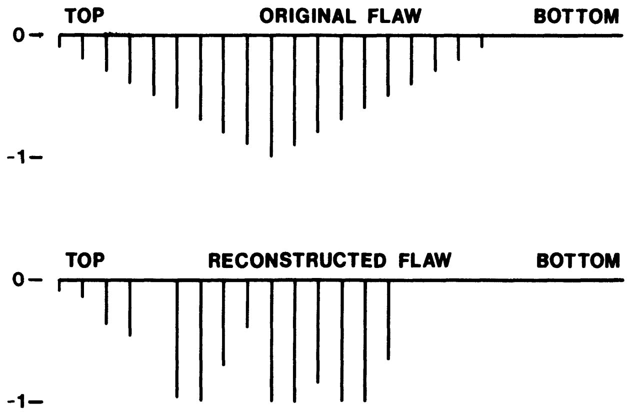


Fig. 2(d). Reconstruction of low-contrast flaw #9. L-M parameter = 0.4[-5]. Noise = 10%.



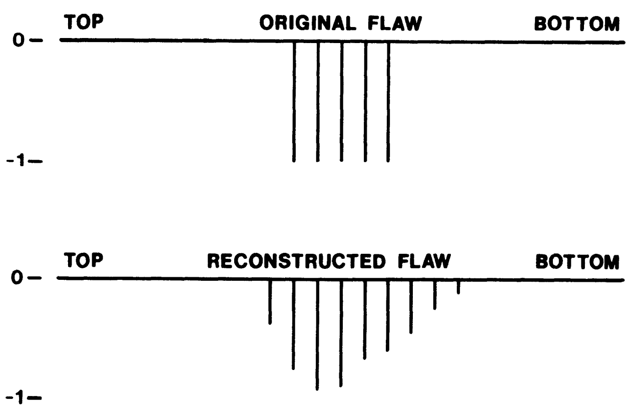
Reconstruction of low-contrast flaw #8.

Levenberg-Marquardt Parameter = 1[-6]

Noise = 10%

This figure illustrates the deleterious effects of using a Levenberg-Marquardt Parameter that is too small; the reconstruction is less stable.

Fig. 3(a). Reconstruction of low-contrast flaw #8. L-M parameter = 1.0[-6]. Noise = 10%.



Reconstruction of high-contrast flaw #3.

Levenberg-Marquardt Parameter = 1[-5]

Noise = 10%

This figure illustrates the deleterious effects of using a Levenberg-Marquardt Parameter that is too large; the peak value of the reconstruction is a little smaller, and the reconstruction is spread over more cells (i.e., the resolution is degraded.

Fig. 3(b). Reconstruction of high contrast flaw #9. L-M parameter = 1.0[-5]. Noise = 10%.

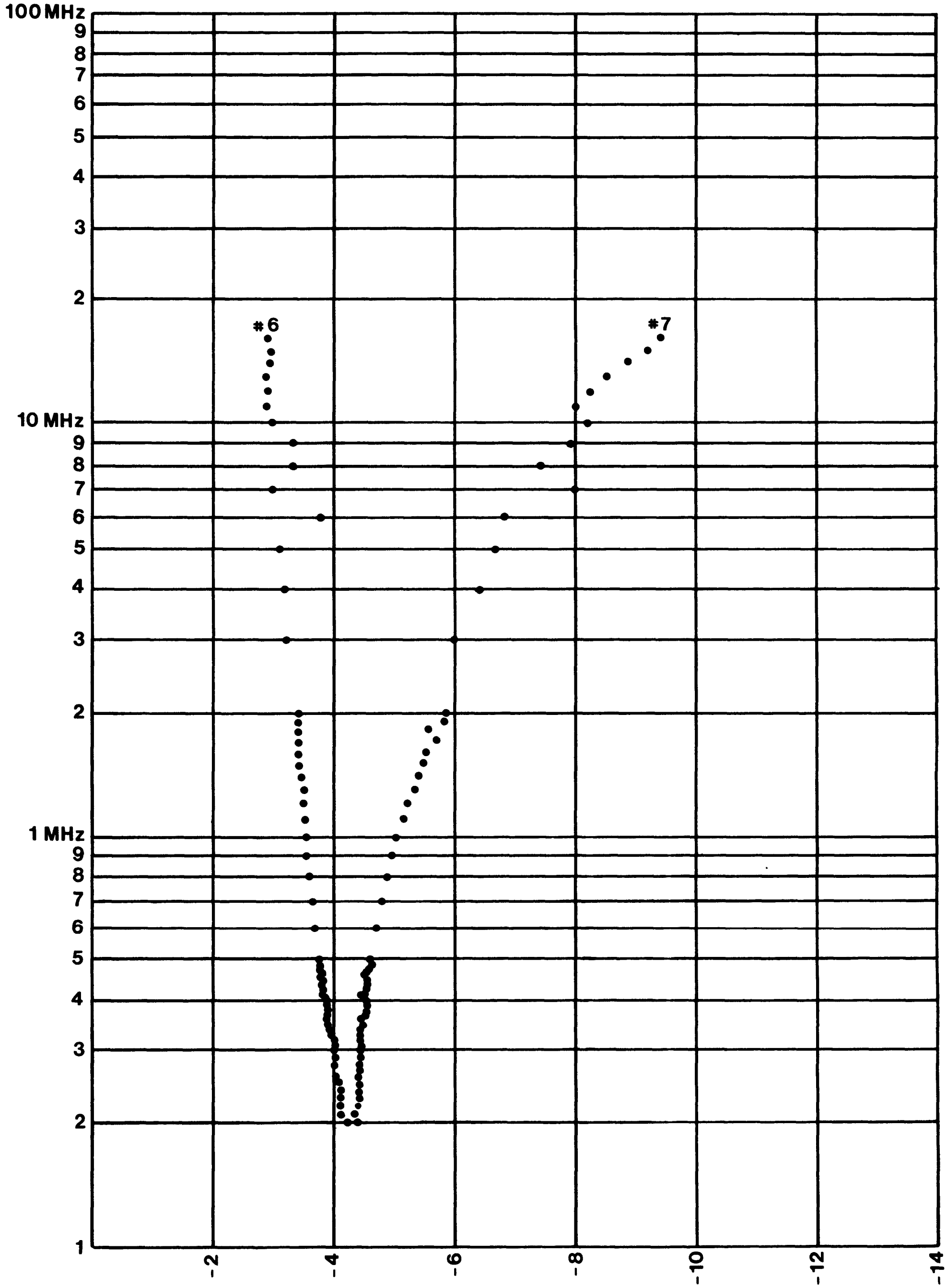


Fig. 4. Logarithm of the magnitude of the perturbed EMF vs. frequency, produced by high-contrast flaws #6 and #7.

cell is void of a flaw, then the conductivity of the cell has value 0. These are the limiting values of cell conductivity. If a flaw partially fills a cell, then the cell conductivity is intermediate to these two values. Flaws that partially fill a cell are called 'low-contrast' flaws, while those that either completely fill a cell or leave a cell empty are called 'high-contrast' flaws. A graphical representation of some of our numerical experiments on both high and low contrast flaws is presented in Figures 2 through 4.

In Figure 2, we show the results of experiments in which two high-contrast flaws are reconstructed with a L-M parameter of zero and two low-contrast flaws which are reconstructed with different L-M parameters. In Figure 3, we show a low-contrast flaw with a L-M parameter that is too small and a high-contrast flaw with a L-M parameter that is too large. Thus one conclusion we draw is that for high-contrast flaws, the L-M parameter should be small, possibly even zero, to maintain resolution. A second conclusion we draw is that for low-contrast flaws, the L-M parameter should be 'tuned' to the flaw, and the value of this parameter will change, depending on the location of the flaw, even if the flaws are otherwise identical.

As mentioned before, it is advantageous to know before-hand if a flaw is concentrated in one-half of the column mesh or the other. We can certainly tell if the flaw is located in the bottom-half of the mesh by looking at the high-frequency behavior of the EMF data. In Figure 4, we show the frequency responses of several flaws that are identical, but have different locations.

Based on our present work, [3], we feel that this current approach is superior to the approach of [1] and [2]. More analysis is needed to 'fine-tune' the algorithms, for example, in choosing an optimal Levenberg-Marquardt parameter. In addition, other algorithms, such as the algebraic reconstruction techniques and robust statistical estimators, need to be more fully investigated. However, our experimental results lead us to believe that the algorithms described in [3] give a correct technique for solving our problem.

REFERENCES

[1] H. A. Sabbagh and L. D. Sabbagh, 'Development of a System to Invert Eddy-Current Data and Reconstruct Flaws,' Final Report: Contract No. N60921-81-C-0302 with Naval Surface Weapons Center Code R34, White Oak Labs, Silver Springs, MD 20910, 18 June 1982.

[2] L. David Sabbagh and Harold A. Sabbagh, 'Development of a System to Invert Eddy-Current Data and Reconstruct Flaws,' presented at the Review of Progress in Quantitative Nondestructive Evaluation, held at the University of California at San Diego, August 1982.

[3] Harold A. Sabbagh and L. David Sabbagh, 'Inversion of Eddy-Current Data and Reconstruction of Flaws using Multi-frequencies,' Final Report: Contract No. N60921-82-C-0139 with Naval Surface Weapons Center Code R34, White Oak Labs, Silver Springs, MD 20910, 22 April 1983.

[4] Charles L. Lawson and Richard J. Hanson, Solving Least Squares Problems, Prentice-Hall, Englewood Cliffs, 1974.

DISCUSSION

J.H. Rose (Ames Laboratory): I'm very interested in this Levenberg-Marquardt parameter that you have been talking about. What's happened is you have a bunch of non-unique solutions, right, when you have the situation that you are in? If you didn't have this parameter, you'd have a whole class of possible solutions that would be valid within the noise.

Normally, there are two ways of fixing that. One is a mathematician's "just do something and I'll fix it" or you can fix in on *a priori* physical grounds. You can pick a method of picking a unique solution on some prior information physically. It would be very interesting to try and relate the choice of your parameters to some underlying physical prior information.

H.A. Sabbagh: The only prior physical information we have are the bounds, minus one to zero. Now, I showed you what happens if you base some flaw reconstructions only on that data. You get some oscillations; you remember the low-contrast flaw. So, we do still need some regularization even in the present step higher information.

I'm not going to answer your question directly, Jim, because I am not sure I have an answer that's any better than what I'm telling you now. Conversely, for that high contrast flaw, we found that we needed no regularization, that the bounds themselves were sufficient. I don't know exactly why that appears, but right now I'm willing to take it and run. But your point is quite well-taken and one of these days, I'll have a better answer for you.

S.G. Marinov (Dresser Industries): My question is about the conductivities. You show the absolutely certain piece of your specimen which you worked with; it certainly had some conductivity. Did you try to apply your model for different conductivities?

The second question: What happens if you move in the lower part of your range of the frequencies, are you still able to distinguish between the depth of defects as you showed?

H.A. Sabbagh: First question, Sam, if I understood, last year when we first started doing this inversion, we were running it on aluminum. Nobody makes steam tubes out of aluminum but I didn't know that at the time. (Laughter) All right. Let's be serious now. And that has a conductivity which is about 10 times greater than this. So the modeling and the theory will hold independent of the conductivity. You have to choose a reasonable frequency or frequency range. This frequency range obviously was chosen with this conductivity in mind. I suspect that if we went back to aluminum, you would probably not want the high frequencies as high as what I showed you there.

Now, maybe the second question is the frequency range. The frequency range depends on the particular material, the conductivity. If this were a ferromagnetic material then obviously, you don't have to worry about the permeability.

Yes, right. I think you really like to have a range of frequencies. You remember the last curves that I showed you, when the flaw was concentrated at one end or the other, the high frequencies really allowed you to discriminate. That's what gives you the orthogonality of your columns, and I really think that you need the high frequencies there to help discriminate flaw position at the top or the bottom. And this, I presume, is well known.

People use high frequencies to deliberately stay away from the lows so that you can concentrate at the top. This is sort of an interesting thing. It shows that by using all frequencies simultaneously, you can reconstruct a flaw given no prior knowledge of where the flaw is in the tube wall. But I think you do need a range of frequencies for that.