ADAPTATION OF ECONOMIC PRODUCTION LOGIC
TO FEED UTILIZATION BY LIVESTOCK

by

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INTRODUCTION

Farm livestock may be raised with a variety of feeds each of which differs in cost to the farmer. If the farmer is to achieve greatest possible profit, he must choose from among the various available feeds that combination which will produce a given output of livestock product with least cost. To accomplish this purpose, technical information regarding the relative nutritive values of the many feeds important to livestock are needed. The nutritive value of a feed need not be proportional to its cost; the lowest priced feed need not have the lowest nutritive value relative to other available feeds.

Relatively little is known about the effects on the quantity and quality of livestock product from substituting various levels of one feed for another in an animal's ration, yet without such information it is impossible to designate the rations which will minimize feed costs for a given output. The design of experiments to obtain more accurate information on the nutritive value of various feeds lies in the field of the physical scientist, the animal husbandryman, the nutritionist. However, the objectives of the economist and nutritionist are not always the same; the technical data necessary for economic analysis are often not readily available. The economist must then adapt such data as is available and also publicize the necessity for, and the importance of, more information on certain aspects of the technical conditions of production.

These technical conditions determine the scope and limitations of production; the output of a producing unit is dependent upon the kinds and quantities of the productive factors and the techniques of production. If there existed only one technique and fixed ratios of specific factors to produce a particular product, the economist would have little to do other than indicate the most profitable output or level of production. In most cases, however, the technical conditions of production permit some freedom of choice regarding the factors of production that can be used to achieve the same output. The different combinations of factors are likely to have different costs attached during any one time-period. A knowledge of the alternative technical conditions which will achieve the same output is, therefore, necessary to minimize costs for a given output.

Objectives of the Study

The main objective of this study is to show how production economics logic used in determining maximum efficiency in production may be adapted to the technical conditions of feed utilization by livestock. While some empirical evidence is presented, it is not the main purpose of this thesis to solve empirically the entire problem of choosing the most efficient rations for livestock. Instead, it is an attempt to adapt the models (theory) and tools of production economics to the specific problems of livestock feeding. As will be brought out later, much of the information needed to arrive at satisfactory solutions to specific livestock feeding problems is not available and it may take years of experimentation to furnish such data.

The logic presented in this thesis is, therefore, aimed to indicate: (1) just what information is needed to attain maximum economic efficiency in livestock feeding (2) the form the data should take to be most useful in solving production economics problems (3) the methods of adapting available data to the problem in lieu of more satisfactory information which will be obtained only by further experimentation (4) hypotheses and models for feed utilization studies yet to follow.

PHYSIOLOGICAL BASES FOR FEEDING STANDARDS

Animal Physiology

A brief review of some differences in the physiological and anatomical make-up of animals is helpful to a better understanding of livestock feed requirements. It also provides the basis for hypotheses about technical coefficients in substitution and hence the economic optimum ration. Definite differences between species exist as indicated by biological classification of animals as ruminant herbivorous, omnivorous and carnivorous animals and birds. Animals and birds have differences in mouth parts, teeth, beaks, etc. which are adapted to different feeding habits. In mammals the food passes directly from the mouth to the stomach while in birds the food is first transported down the esophagus to the crop. And, although the general pattern of the digestive systems is similar among the various species there are important physiological and anatomical differences. The bird has a gizzard for grinding hard foods. Ruminants have the rumen and the reticulum where the semi-masticated food is subjected to the actions of digestive juices and microbial action. This trip through the rumen, reticulum and omasum to the abomasum permits a high degree of breakdown of plant cellulose. In all species the food passes from the stomach to the small intestine where fats, proteins, sugars and starches but not celluloic substances are broken down and absorbed. The large intestine is of relatively little digestive importance in ruminant herbivora and carnivora, but in non-ruminant herbivora (horses) further digestion of plant material continues there. The ruminants are probably more efficient in the use of microbial agents since they can also utilize the starch and protein which the bacteria synthesize for themselves in small intestine while the non-ruminant herbivora haven't this advantage. Ruminants can make more efficient use of celluloic feeds and can eat relatively more of them. There are also differences between ruminants themselves and between non-ruminants in ability to utilize various feeds.

Maintenance Requirements

The ordinary bodily functions of an animal necessary to life involve a certain heat output by the animal and it must receive digestible food energy at least equal to this rate in order to live. For mammals this rate is approximated by[1]

$$B.M.\ (Cal.) = 39.5\ W^{0.7}$$

where B.M. is the basal metabolic rate of the animal in calories and W is the weight of the animal in pounds. An animal,

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[1]R. G. Linton and John T. Abrams. Animal nutrition and veterinary dietetics. 3rd ed. Edinburgh, W. Green and Son, Limited. 1950. p. 248.

therefore, requires a certain minimum maintenance ration which in practice is affected by the nature of exercise performed, environmental conditions, and the nature of the food. An animal may well vary 20 percent above or below the standard, depending upon the conditions and the animal.[1] However, if an animal were fed entirely on a starch ration it could not maintain its body weight even though the starch ration contained enough digestible food energy. Other nutrients such as protein, fats, vitamins and minerals must also form a part of the ration. Feeds are seldom pure protein, starch, etc. Instead feeds are composed of combinations of feed-nutrients that also vary in quality. One feed may, therefore, have a relatively low substitution rate for one feed and a high substitution rate for another.

Growth takes place at different rates at different times. The composition of the body also changes. These changes affect the substitution coefficients and hence the rations which are economic at different levels of growth. Usually the skeleton and vital organs develop first, then the muscles and finally the fat. With a given ration the development is in that order regardless of the plane of nutrition, although the rate may differ with the plane of nutrition. The value of energy needed from protein varies at different

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[1]Ibid., p. 250.

stages of animal growth, tending to be greatest in the earlier stages or with younger animals as well as with the physiological state of the animal. Proteins may also be broken down in the digestive process and used to some extent as work energy, although carbohydrates are the main source. On this basis, it is economical to substitute carbohydrates at greater rates for protein feeds with mature animals or animals needing energy for work.

Nutritive Value of Feeds

No two feeds are alike in nutritive value to an animal since they differ in amount and quality of protein, fat, carbohydrates, minerals and vitamins. Comparison of their nutritive values becomes difficult and especially so when it is remembered that the nutritive needs of livestock differ with species, age, sex, condition, and individuals.

It is possible to calculate the gross energy of a feed (heat liberated when one gram of a substance is completely oxidized), but animals are not 100 percent efficient in absorbing food from the alimentary canal. Furthermore, livestock have ability to break down certain feeds into two or more substances some of which may be utilized and others voided. A step further toward the actual energy value of the feed involves subtraction of that portion of energy value lost through faecal excretion. However, the digestible energy thus determined depends both on the feed and the animal. The true value of proteins cannot be indicated by their digestible energy, since almost all of the nitrogen of metabolized proteins is voided in the urea (uric acid in the case of birds). By taking account of the heat of combustion lost in this manner, metabolizable energy of the feed may be measured.[1]

That portion of the metabolizable energy used to carry on work, lactate, lay eggs, grow and fatten is called net energy. The species, size, age, sex, environment, plane of nutrition, combination of feeds and activity of the animals as well as the physiological state affect the net energy value of feed to animals. No one set of net energy values is entirely satisfactory for all species and types of livestock production. Hogs cannot be expected to utilize forages to the same extent as ruminants. The net energy value of forage must necessarily differ for these animals.

Similarly hogs that are fed high protein rations will utilize carbohydrate feeds more fully than hogs fed little or no protein. When the pig's intake of protein is a limiting factor in growth, he oxidizes or eliminates the extra feeds he may eat. When a hog is fed a protein which is lacking in certain amino acids, the hog's body attempts to

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[1] Ibid., p. 283.

synthesize the needed compounds from the protein which is present. Thus it is that the net energy value of feeds fed to a hog separately may not equal the net energy of the feeds fed in combination. The combination may have a higher net energy value than the two feeds fed separately. It is also possible that certain feeds fed in combination may release compounds into the digestive system of the animal that hinder digestion. Net energy values of feeds calculated on an individual basis are not additive. A logical conclusion is that feed utilization for each species, during each physiological state of importance such as fattening with each different production technique,must be studied separately for accurate information as to the relative value of feeds. Feeder cattle in dry-lot, milk cows and feeder sheep must each be fed various rations and the relative nutritive values of the feeds measured in combination.

Present feed value or feed utilization tables are various direct and indirect methods of arriving at estimates of one or another of the above energy values of feeds. The digestible nutrient systems usually approximate digestible energy to some degree under given conditions but are expressed in terms of carbohydrate equivalents instead of starch (carbohydrate) required to produce the same amount of fat as would 100 kilograms of the feed. The Swedish barley equivalent system arrives at similar estimates in terms of barley equivalents based upon the relative value (net energy) of feeds to lactating dairy cows. All of the feeding standards including Armsby's net energy values establish with a single index figure an estimate of the nutritive value of one feed relative to another subject to certain restrictions. In most instances it is assumed that a balanced ration is fed, i.e., all of the nutrients are present in the ration in proportion to the need for them in carrying on the body function in question. Within some range, then, the tables offer constant ratios at which feeds substitute for each other. Usually these tables are computed for ruminants and cattle specifically. The starch equivalent tables are based on animal gains when fattening; the Swedish barley equivalents are based on milk production. The users of the tables often ignore the above restrictions without realizing that different species, different planes of nutrition, different sexes and different physiological conditions mean different feed utilization values. When properly used the tables serve as a valuable guide to livestock feeding in lieu of more accurate information. When the economic feeding of farm animals is considered, several difficulties arise because both the nature and amounts of proteins, fats, carbohydrates, minerals and vitamins differ from feed to feed.

Thus, while present feeding standards are undoubtedly of great value, they have the following limitations:

(1) Feeds are assumed to substitute for one another at constant ratios although it is well known that replacing a small amount of carbohydrate with protein in a protein deficient ration will have greater effect on the animal's output (milk, fat, work or growth) than a similar substitution when the animal has a ration relatively high in protein. Such differences in the value of feeds can be revealed only through experiments with the particular kind of animals in question. The digestible nutrients and energy values as ordinarily calculated fail to indicate differences in feeds under these conditions.

(2) Substitution ratios apply to a limited portion of possible feed combinations. In using such tables one has to assume that the least cost rations are of the 'balanced' type since the table values were estimated from animals fed reasonably balanced rations.

(3) In most cases average nutritive values are given as if they applied to all classes of livestock even though it is known that ruminants such as cattle can utilize forage to a greater extent than can hogs.

(4) The same feed substitution rates are given for different weights and ages of animals. However, the authors usually recognize that their figures are extremely inaccurate unless protein levels are kept above minimums which differ with age and body function.

(5) The same feed substitution rates are advocated for animals whether fattening, lactating, working, growing or gestating. Such an assumption is not born out by nutrition experiments.[1]

[1]L. A. Maynard. Animal nutrition. 2nd ed. N.Y. McGraw-Hill Book Company Inc. 1947.

ECONOMIC THEORY RELEVANT TO FEED UTILIZATION BY LIVESTOCK

The brief review of animal nutrition in the preceeding sections indicates limitations in the use of production economics logic when applied to livestock feeding problems. This section presents a sketch of the general economics theory which will be adapted later to specific livestock feeding problems that arise from differences in animal physiology and differences in the livestock products produced.

General Production Economics Theory

Under given technical conditions of production, the output of the product from a given producing unit depends uniquely upon the variable productive factors used. The technical conditions may be flexible or inflexible, i.e., when flexible, various quantities of different factors may be combined to achieve the same output; when inflexible, the factors combine in fixed proportion. An example of the latter case exists when two molecules of hydrogen combine with one of oxygen in the presence of heat to form water. Such situations require little economic analysis; the only question is one of how much to produce, i.e., how much of the feed resource to use. Once this has been decided the quantities of the factors needed has been determined by the technical conditions of production. In the instance of flexible technical conditions, however, the big problem of what combination of factors to use also arises since there is freedom of choice as to the factors and quantities of factors used.

If a good Y is produced with varying amounts of the factors X_1, X_2, X_3,....., X_n the production function may be written as

$$Y = f(X_1, X_2, X_3, \ldots\ldots, X_n).$$

For the purpose of presenting the logic and theory of the production process, it is assumed that the factors of production are continuously divisible and that the production function is therefore a continuous function of the variables. Variable factors may, however, be used with certain fixed factors. The function may then be expressed as

$$Y = f(X_1, X_2, X_3, \ldots\ldots, X_r, X_{r+1}, \ldots\ldots, X_n),$$

where the amounts of factors X_{r+1},.....,X_n are fixed.

A graphic presentation such as appears in Figure 1 may be made when a two variable factor, single product function exists. X_1 and X_2 are the factors of production and Y is the product. Any point on the upper surface of the three dimensional figure represents the output of Y for some combination of X_1 and X_2. Different combinations of X_1 and X_2 may be used to achieve the same output. Line RS in Figure 2 represents all points at a given level of Y on the production surface. A combination of Oa of X_1 and Od of X_2 produce

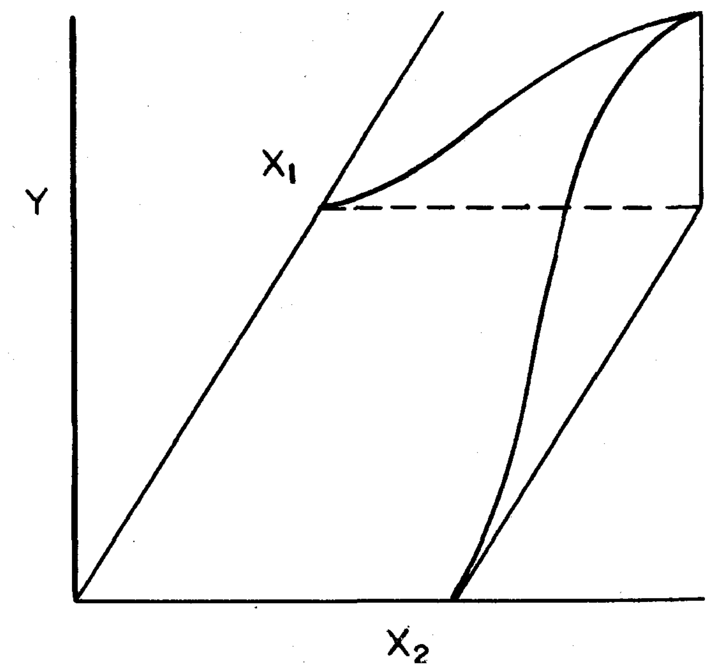

Fig. 1. Production function with two variable factors.

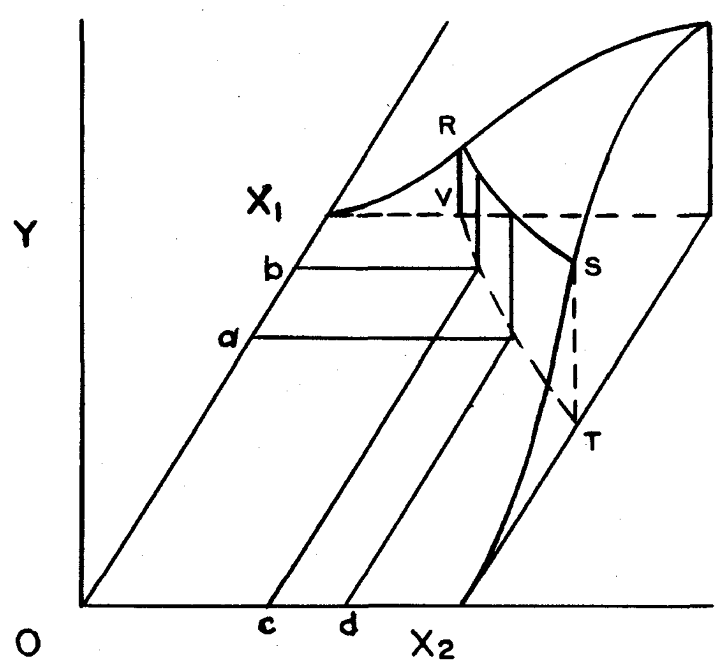

Fig. 2. Combinations of the factors producing a given output.

the same output as Ob of X_1 and Oc of X_2. A two dimensional contour map such as in Figure 3 may be used to illustrate this situation. In this diagram 10y, 20y and 30y are iso-product contours similar to RS in Figure 2. Various combinations of X_1 and X_2 such as ON and OR, OM and OS, OL and OT, and OH and OV respectively may be used to produce 10y of the product Y.

In Figure 3 there is a diminishing marginal rate of substitution between the factors, i.e., less and less of X_2 is needed to replace a unit of X_1 in the constant output combinations as relatively more of X_1 is used. While there are concepts other than that of a diminishing rate of substitution, it is one of the most useful and one which logically corresponds to fact. Another alternative, the combination of factors in fixed proportions, which was mentioned earlier is illustrated in Figure 4. Adding more of one factor will not increase output unless more of the other is also added. On the other hand, it is possible for factors to be perfectly substitutible. In feeding animals, white and yellow corn approach this condition very closely if the animals already have a source of necessary vitamins. Figure 5 illustrates perfect substitutibility by use of straight line iso-product lines. The proportion of X_2 that must be added in order to maintain the same output as a unit of X_1 is given up remains the same.

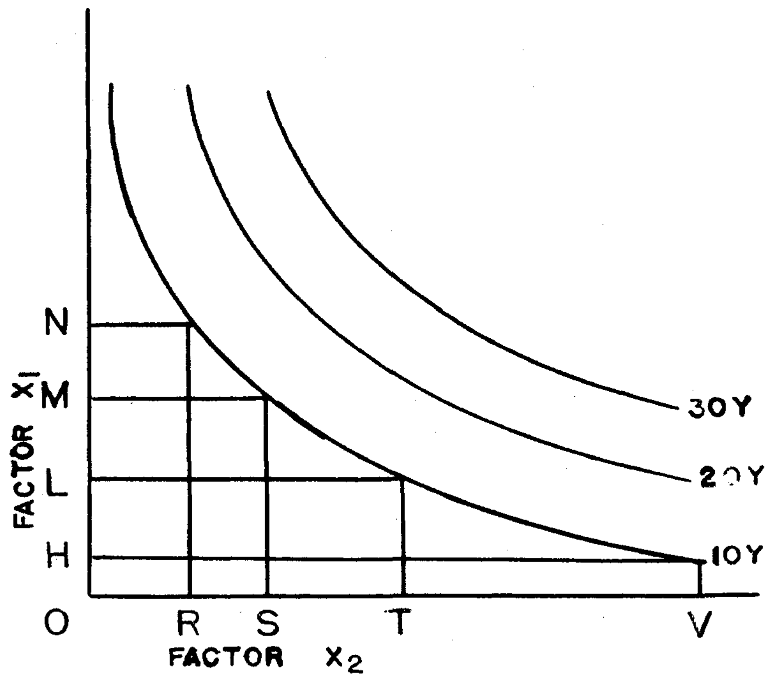

Fig. 3. Iso-product map.

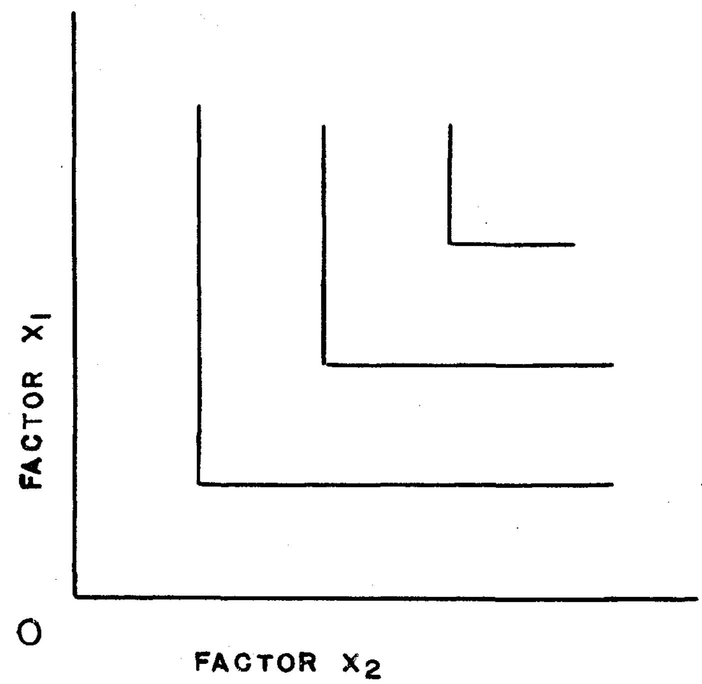

Fig. 4. Inflexible factor combinations.

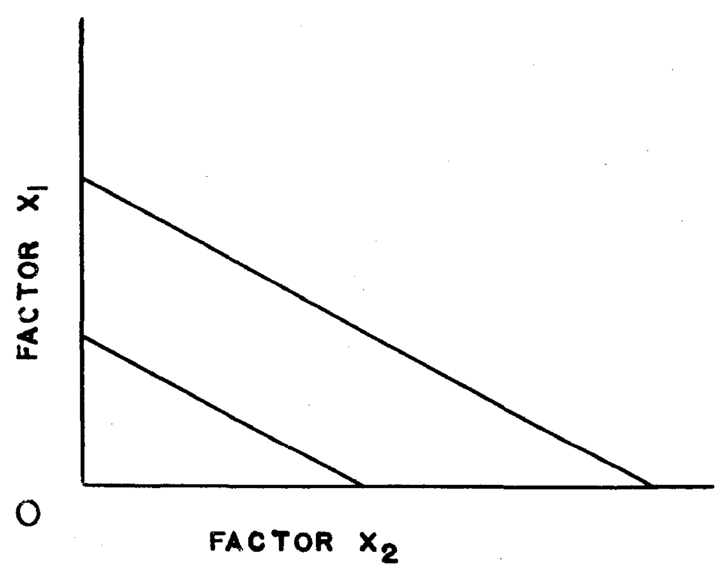

Fig. 5. Perfect substitution between factors.

In geometrical terms the slope of the straight line iso-product curve is the ratio at which one feed may be substituted for another and still maintain the same output. In Figure 5 output 1 may be produced with either OC of X_1, OA of X_2 or any combination of the two factors indicated by points on the output contour 1 or AC. The rate of substitution of X_1 for X_2 remains the same over the straight line segment. The ratio OC/OA is both the marginal rate of substitution and the average rate of substitution in the case of a straight line. The slope of the curved iso-product curve also represents the marginal rate of substitution geometrically, but the slope and hence the substitution rate is different at each point on the curve. The slope at these points may be approximated by taking the small change in one feed with respect to another feed, output unchanged, e.g., Δcorn/Δhay. It is often easier and more accurate to use calculus to find the $\lim_{\Delta h \to 0} \frac{\Delta \text{corn}}{\Delta \text{hay}}$ or $\frac{d \text{ corn}}{d \text{ hay}}$. Geometrically this is the slope of the tangent to the curve at a point. In this case the marginal rate of substitution need not be the same as the average rate of substitution. If the rate of substitution of one feed for another is diminishing (less of one feed is necessary to substitute for another at a constant output as more of the second is added), the iso-product contour must be convex to the origin.

Usually some limits to substitution exist, i.e., the product cannot be produced with all of one or the other of the factors. The rate of substitution becomes positive or infinite at some point depending upon whether the added units of the factor in question finally produce no effects without the addition of the second factor or whether the additional units of the factor actually detract from production by hindering the production process either through cluttering up the plant or requiring extra processing and handling from which no product results.

Within the limits of substitutibility, output may be expanded in several ways. Some factors of production may be held constant and output increased by adding another. Output also may be increased by adding factors in a haphazard manner. Of particular interest is the method of adding factors in constant (fixed) proportion. In livestock feeding this corresponds to feeding a ration made up of feeds in fixed proportion. Figure 6 illustrates the increase in output as more of particular rations are fed. If feed A and feed B are combined in the proportion of 3 parts of B to 4 of A, a straight line through the origin such that the vertical distance increases three units for every four units of increase horizontally is a geometric illustration of the constant proportion ration. The farther from the origin at which a point on this line is selected, the greater will be the amount of the combined quantities of the two feeds in question. The

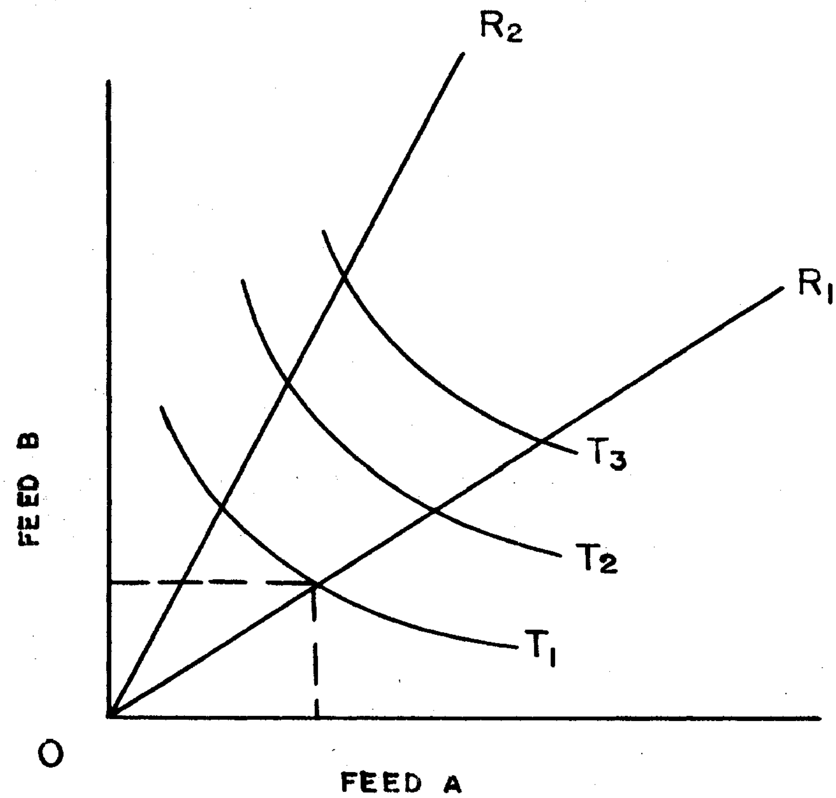

Fig. 6. Ration lines indicating inputs of feeds in fixed proportions.

iso-product contours, T_1, T_2 and T_3, are similar to the contour RS in Figure 2 projected to the base of the volume as shown by curve TV. As previously indicated, the points on such contours represent the combinations of factors which will produce the same output. The farther any such contour is from the origin, the higher is the iso-product contour which it represents on the surface of the production function. Thus, any number of such contours may be drawn which are analogous to lines on a contour map or to iso-therms on a weather map.

The ration lines which are straight lines because of the fixed proportions of the feeds cross iso-product contours of increasing degree as they extend outward from the origin. Feeds need not substitute for each other at the same ratios along ration lines. It is highly unlikely that they do so, since animals' feed requirements tend to change with time (age). In other words, the slopes of the tangents to the iso-product contours (marginal rates of substitution) at the points of intersection need not be equal.

After the technical conditions of production have been determined, factor prices may be introduced into the model to indicate the least cost combinations. The lines A_1B_1, A_2B_2, and A_3B_3 in Figure 7 represent the total amounts of the factors X_1 and X_2 that may be bought with given sums of money. For example, if OA_1 is the amount of factor X_1 which

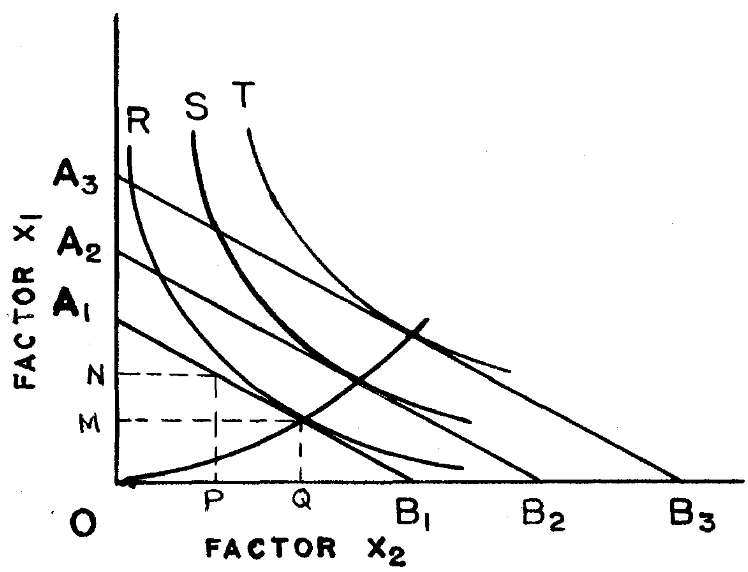

Fig. 7. Least cost combinations.

may be bought for \$100; OB_1 is the amount of factor X_2 which may be bought for the same sum of money. One hundred dollars will buy more of X_2 than of X_1. In fact, factor X_1 is more expensive by the ratio of OB_1/OA_1. This ratio, which is equal to the ratio of the factor prices, is the same as the slope of the iso-cost line A_1B_1.[1]

The iso-cost lines cut across many product contours and are tangent to the highest one they touch. Thus, each point of tangency represents the largest amount of product which can be produced with a given sum of money represented by the iso-cost lines A_1B_1, etc. Thus, while ON of X_1 plus OP of X_2 costs the same as OM of X_1 plus OQ of X_2 (Fig. 7), the first combination produces some output less than that represented by contour R. The second combination produces exactly R. No greater output can be produced with any combination of factors represented by the iso-cost line A_1B_1.

At the point of tangency the slope of contour R is just equal to the slope of A_1B_1. In economic terms, the marginal rate of substitution of one factor for another in production is represented by the slope of R and is just equal to the inverse ratio of the factor prices represented by the slope of A_1B_1. When this condition holds, the least cost combina-

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[1]In a strict sense it is the negative reciprocal of the ratio of the prices which is equal to the slope of the iso-cost line. The terms "price ratio" and "ratio of the prices" refers to this negative inverse when used throught this thesis.

tion of factors has been attained for the given output.

Another way of looking at the situation is as follows: let P_1 and P_2 be the prices of factors X_1 and X_2 respectively.

$$\text{If } \frac{dX_1}{dX_2} > \frac{P_2}{P_1}, \quad \text{then} \quad dX_1\,P_1 > dX_2\,P_2$$

where dX_1 and dX_2 are small increments of the factors X_1 and X_2. When an increment of X_1 (output constant) costs more than the corresponding increment of X_2, it is profitable to substitute X_2 for X_1 in the production process.

$$\text{If } \frac{dX_1}{dX_2} < \frac{P_2}{P_1}, \quad \text{then } P_1\,dX_1 < dX_2\,P_2.$$

The reverse situation holds. By substituting X_1 for X_2 the same output may be attained at less cost.

$$\text{If } \frac{dX_1}{dX_2} = \frac{P_2}{P_1}, \quad \text{then} \quad dX_1\,P_1 = dX_2\,P_2.$$

The cost of a small increment of X_1 is the same as that of a similar quantity of X_2 (output constant). There is, therefore, no incentive to substitute one for the other. When this occurs the iso-cost line is tangent to the highest iso-product contour attainable with the given resources. At each point of tangency the total cost per unit of output will be a minimum. Any other point on a given outlay contour would give less output for the same outlay. Any other point on a given iso-product contour would cost more for the same output.

From a line drawn through the points of tangency, a series of least cost combinations of factors for different outputs can be read off. Such a line is termed an expansion path and indicates the most efficient use of resources to obtain various outputs. The rational firm that wishes to maximize profits will therefore produce with one of the combinations of factors indicated on the expansion path.

In agriculture the expansion path crosses all iso-product contours at points of equal marginal rates of substitution (slopes). No one farmer sells enough of any product to perceptibly affect market prices. The price ratios are the same regardless of the level of output he may choose. By definition the expansion path passes through the points where the slopes of the iso-cost lines (price ratios) are tangent to (equal to) the slopes of the iso-product contours. Thus, the expansion path need not correspond to a ration line, since the marginal rate of substitution of one feed for another may vary along a ration line. Animals utilize various feeds with different efficiency at different stages of growth.

Discovery of the least cost combinations such as shown in Figure 7 do not, of course, designate the level of output from the producing unit which will return the greatest profit.

However, from the points on the expansion path representing the lowest cost to produce any given output, a total cost curve for the producing unit may be plotted. Figure 8 illustrates such a cost curve when some level of fixed costs is also assumed, i.e., the variable factors of production are combined with certain fixed factors as mentioned at the beginning of this section. Since maximization of profits means the same as maximization of the difference between total costs and total revenue, total revenue is introduced into the model. When as in agriculture relatively free competition exists, the total revenue may be assumed to be represented by a straight line as shown in Figure 8. Geometrically the difference between total revenue and total costs is greatest where a tangent to total cost is parallel to total revenue as at output OM. This point can be shown to be the same as the output where marginal cost is equal to marginal revenue or price under free competition.[1]

While the production relationships shown have been of a simple type amenable to two dimensional graphic illustration, the logic may be extended to multi-factor, multi-pro-

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[1]R. G. D. Allen. Mathematical analysis for economists. London, Macmillan and Co., Limited. 1949. p. 197-198.

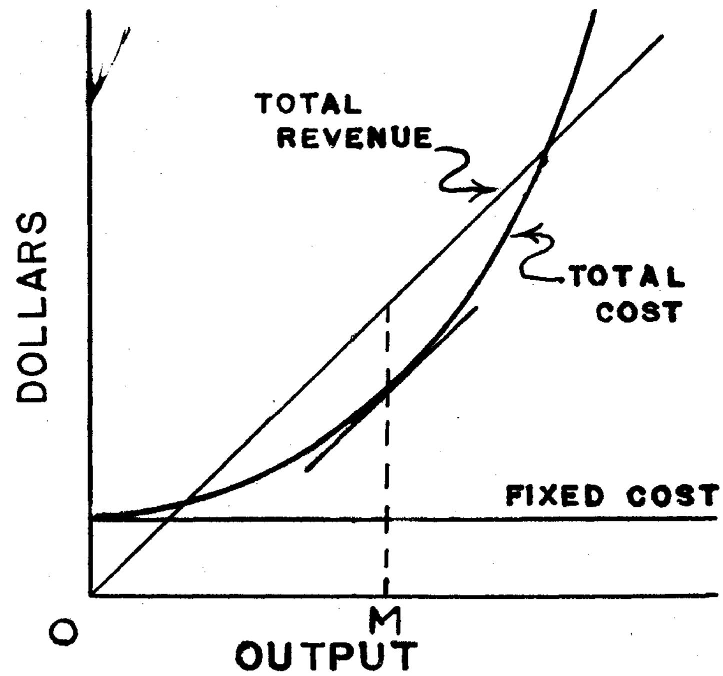

Fig. 8. Maximization of profit.

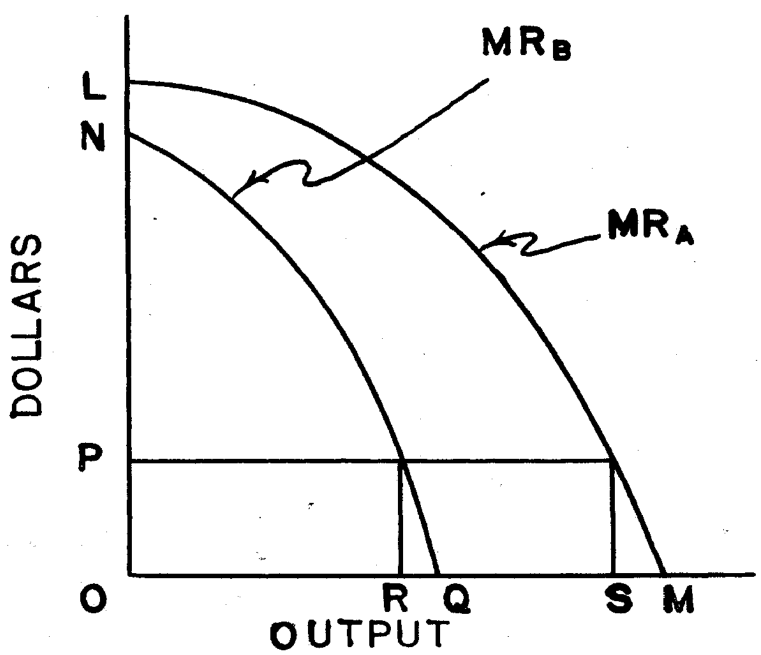

Fig. 9. Equation of marginal returns.

duct cases in the manner of Hicks and Reder[1]: In order to maximize profits, the marginal rate of substitution between any two factors must be equal to their price ratio, the marginal rate of transformation between any factor and product must be equal to their price ratio; the marginal rate of substitution between products must be equal to their price ratio. Second order conditions and "total conditions" of the firm as Hicks terms them are also presented by the authors.

Sometimes an individual farmer is prevented from operating at the long run optimum output level because of limited capital or other resources. Such a farmer is restricted to a specified area of the production surface, but otherwise follows the above logic, although he may think of it more in line with the model shown in Figure 9. LM represents the marginal returns from product A when all resources are variable; NQ represents the returns from product B. If capital or other limited resource used to produce both products is utilized in a manner to equate the marginal returns from both products A and B, OP represents the level of return and OM and OQ represent the outputs of products A and B respect-

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[1] J. R. Hicks. Value and capital. 2nd ed. Oxford, Clarendon Press. 1948. ch. VI; and M. W. Reder. Studies in the theory of welfare economics. N.Y., Columbia University Press. 1949. ch. I.

ively. It would not pay the farmer to push output of product B past OR, since to do so would force him to forego the higher return from product A. Since he hasn't enough capital to push all products to output levels where they no longer return a profit, he will equate the returns from the products as indicated at some level short of this point.

Application of Economics Theory to Livestock Production

In this section the various models such as the production function with its iso-product contours and isoclines will be presented as they appear when adapted to various classes of livestock. Steers, dairy cattle and other livestock do not have exactly the same production functions. Dairy cattle are mature animals producing a product, while feeder steers are usually immature animals and the product is a part of the steer's growth. In the first instance the maintenance ration per day remains nearly constant regardless of output level, but in the second the maintenance ration becomes a larger proportion of the feed fed as output is increased. However, we first discuss the adaptation of the production function concept to livestock in general. The latter parts of this section deal with differences peculiar to individual classes of livestock.

Physical production relationships

Production function. From a feed utilization view-point, feeder stock, dairy stock and poultry are technical units of production through which various feeds are transformed into products over a period of time. Peculiarities of the various kinds of producing units cause this production function to differ from the simple case illustrated in Figure 1 where two factors of production were shown. For comparison purposes a simplified hypothetical production function for slaughter steers is presented in Figure 10. Gains are represented by the height of the volume. Forages are combined on one axis and concentrates on the other. Other factors are assumed fixed or constant, and the problem of heterogeneity and indivisibility of factors is ignored. When a steer is fed to capacity on forage, OB is consumed to produce EB pounds of gain in the slaughter steer.

Because of the animal's limited stomach capacity for feed, the greater nutritional value of the concentrates for a given bulk results in higher gains as more grain is substituted for forage. The output CF when the steer is fed to capacity on a high grain ration is greater than output EB. Another peculiarity of the production function when applied to animals is caused by the animal's maintenance requirements. A certain amount of feed is eaten which does not

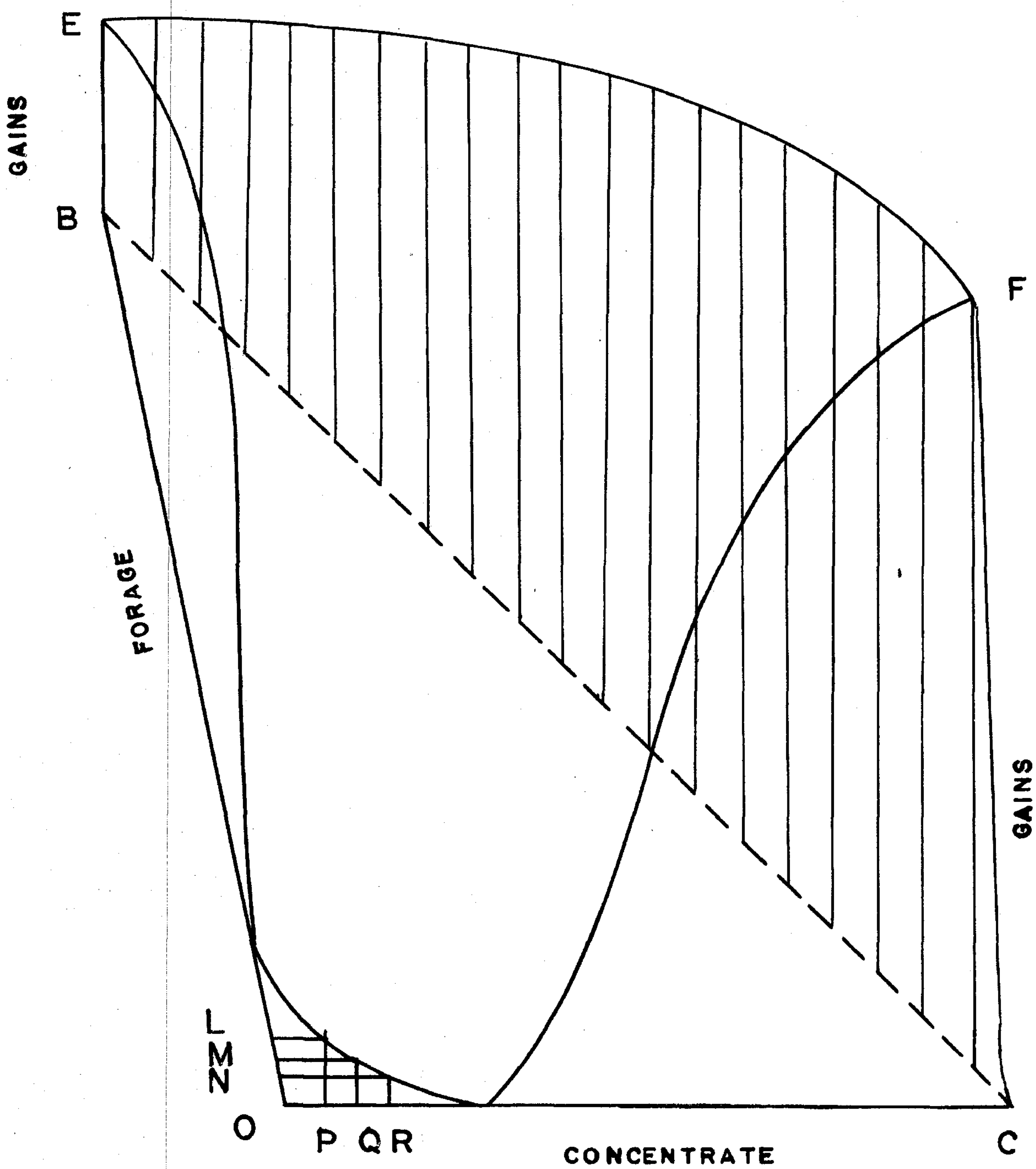

Fig. 10. Hypothetical production function for a slaughter steer.

result in gains, milk, eggs, etc., i.e., produces zero product. ON of forage and OR of concentrate, OM of forage and OQ of concentrate, and OL of forage and OP of concentrate are a few of the maintenance rations that could be fed before products are produced. Anything less than a maintenance ration would result in deterioration of the producing unit (steer, cow, pig). It may also be necessary to have some forage in the diet of a ruminant in order to maintain efficient digestive processes. This possibility is illustrated in Figure 10 by the concentrate side of the function not rising vertically from the axis to the surface. Thus, as a little forage is added to the ration, gains increase rapidly until that point where healthy digestive processes can be maintained.

Limited feeding. The function shown in Figure 10 was based on an assumption of production over a given time period. Not all combinations of feed fed permitted the animal to eat its fill. It was also assumed that the animal was fed each combination over the time period in a manner to achieve the most efficient gains. Since the steer's maintenance requirements depend largely upon time, it is possible to feed an animal a ration barely above the maintenance level and have it consume as much feed as when the same ration is fed over a shorter period of time to achieve greater gains. Thus, if time were disregarded, the same combinations of feeds could be responsible for two points on the production surface. There would not be a unique solution to the problem of what would be the steer's gains fed a particular ration or combination of feeds. In order to avoid indeterminancy and have a more general picture of production, time may be included as a factor of production. Then the output forthcoming with different feed combinations may be predicted for various time periods both for limited and full feeding. This procedure would be useful, since the time used in production is usually related to the time of marketing and, hence, differences in product prices and prices of feeder steers. This problem is discussed further in a later section.

Full feeding. Under full feeding only those gains represented as points along the production surface edge, EF, (Fig. 10) are achieved. If the steer ate only forage and all of the forage it desired, OB would be consumed. As concentrate is substituted for forage, i.e., a unit of grain is introduced into the ration but the animal permitted to fill up on hay afterward, a movement toward C takes place along the steer's feed capacity line BC. The shape and position of line BC depends both upon the animal's physical capacity for feed and its appetite. The animal may choose to eat more of certain combinations and kinds of feeds. When line BC is straight as in Figure 10, the steer's capacity for feed is of the same nature as that of a feed bin. The feeds substitute for each other in a constant ratio as far as quantity consumed is concerned. That the capacity line is of the constant ratio type generally is doubtful, since animals show a preference for some feeds and combinations of feeds and eat relatively more of them.

The different feed combinations along the steer's feed capacity line result in different gains. EF need not be a straight line, i.e., the proportion of grain in the ration (animals eating to capacity) necessary to obtain a unit increase in output need not stay the same for all points on the capacity line. The fiber content of the forage may be useful to digestion when present at certain levels. Similarly, vitamin content of the various feed elements vary. Some combinations will result in greater gains than others.

When full feeding capacity lines from a number of such production functions, each for a different time period, are examined, a new possibility arises. Time may be excluded, and a production function assuming full feeding may be derived. It is possible under full feeding to use different combinations of feeds to achieve the same output, although the time taken for an animal to eat them may vary. Under such conditions, full feeding may be looked upon as a separate technique of production. A production function similar to Figure 10 exists.[1] The points on the surface of this function correspond to points on the capacity lines of various production functions each for a given time period.

Functions based on full feeding as a technique are useful where the gains achieved at different times are of equal value, e.g., when the government fixes prices and time of marketing is not important except as it influences costs incurred in producing over different lengths of time. Even then, the costs of labor and other factors associated with time may be already available and considered as fixed costs. As such they need not enter into short run analysis of choosing optimum feed combinations to maximize profits. There is also some evidence that full feeding is one of the most practical techniques of feeding fattening animals. Many farmers follow this method and research has devoted a large part of its resources to it. More investigation is needed in this area, however.

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[1]L. J. Bodensteiner. Marginal rate of substitution of grain and forage in production of beef. Unpublished M.S. Thesis. Ames, Iowa, Iowa State College Library. 1952; and E. O. Heady and R. O. Olson. Marginal rates of substitution and uncertainty in the utilization of feed resources with particular emphasis on forage crops. Iowa State College Jour. of Sci. 26: 49-70. 1951.

Generalized function. So far only the production function of a single animal has been considered. A herd or lot of animals may also be looked upon as a technical unit of production. The number of steers or other animals then becomes a variable factor of production.[1] The effects of crowding in given lot and building space would be taken into account. Theoretically, a function of this sort exists and would be useful if derived empirically. The costs of research where size of herds, lots, time and feeds all are varied would be great. Information of nearly as great value may be obtained for practical purposes from the production function of individual steers raised in lots of two or more. Although the production functions might be derived on the basis of individual steers in lots, several steers per lot would permit empirical derivation of production functions more nearly like the conditions under which steers are fed on farms. Steers' appetites are affected by companionship.[2] For purposes of handling economic problems of how big a lot or herd of steers to feed, the information from the production function of an individual steer may be multiplied.

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[1]E. O. Heady. Economics of agricultural production and resource use. N.Y. Prentice Hall. 1952. p. 157.

[2]T. B. Keith and others. The optimum ratio of concentrate to alfalfa hay for fattening steers. Idaho Agr. Exp. Bul. 290. 1952.

Problems of crowding and labor would have to be given separate consideration.

Time also is an important factor of production in the more generalized concepts. Time may be substituted for feed in the cases where limited feeding is practiced.[1] This is a condition peculiar to animals where maintenance rations must be considered. Also, in full feeding, time is of importance in making economic decisions where different costs and product prices are associated with different lengths of production periods. Later sections deal with this problem.

Average daily gains. An alternative way of introducing time rather than to enter it directly as a factor of production is to consider the average daily gains as output rather than total gains. Time then becomes a fixed factor; i.e., is fixed at one day. Feeding different rations for specified time periods results in different rates of average daily gain. (Average milk production per day, eggs per day, etc. are comparable.) Often when limited feeding is practiced, different amounts of various rations may be fed to achieve the same average daily gains. Figure 11 illustrates these relationships. OA represents the minimum amount of forage needed

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[1]J. F. Lasley and others. Full vs. limited feeding of growing-fattening pigs. Mo. Agr. Exp. Sta. Progress Report 17: 12-15. 1952.

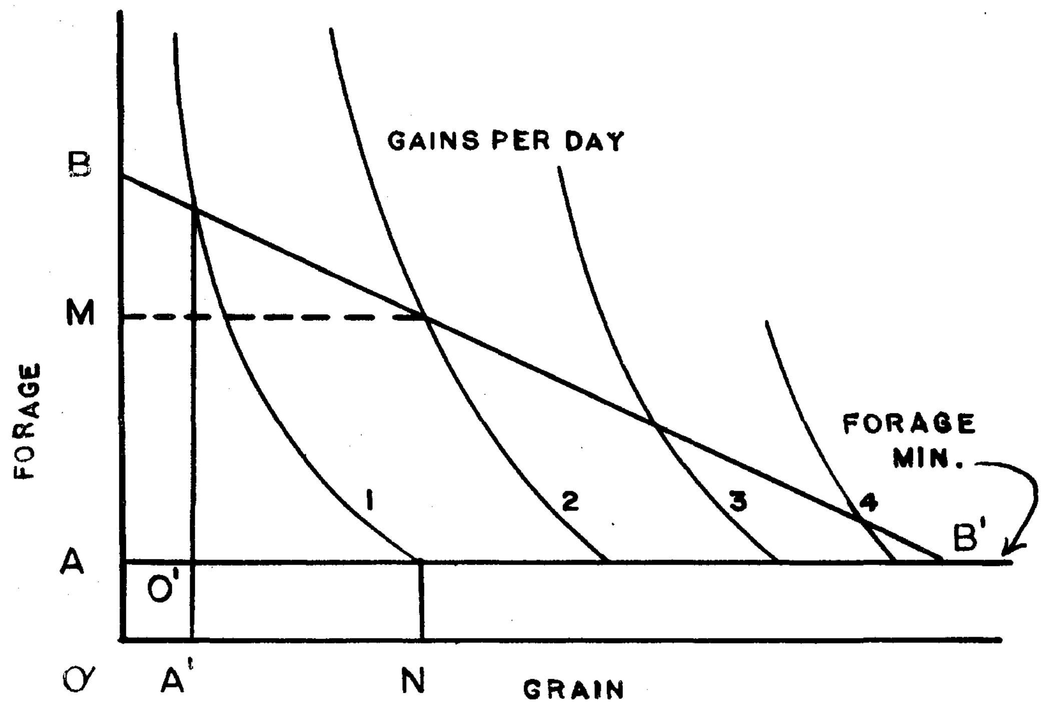

Fig. 11. Average daily gains.

in the diet of a steer on the average per day; OA' represents the minimum amount of concentrates if good-choice beef is to be produced. Output contour 1 represents the production level which starts where a minimum of grain and the necessary forage are fed to produce a given quantity of beef. Gains per day on this contour are lower than on contour 2. OM indicates the upper limit of forage which may be fed to the steer and still obtain an output of choice beef represented by contour 2. Where greater output of choice beef per day than indicated by contour 2 is desired, it is necessary to feed relatively more concentrates. Maximum forage in the ration then would be reduced along line BB'. At levels 1 and 2 there are a number of rations which achieve the same output. As the gains per day are increased and more and more concentrates are fed until the maximum gain per day specifies a point of minimum forage, OA, which can be combined with the concentrates to maintain a healthy animal. Point B' will depend upon the different concentrate mixtures which are combined with a particular forage or vice versa. Where a more general function is considered with more than two feeds, B' may not be a point. However, the combinations of feeds (rations) that could achieve highest possible daily gains would always be fewer than those necessary for lower gains.

When an animal is full fed combinations of two feeds, e.g., corn and soybean oilmeal in the case of hogs, average daily gains would not form iso-product contours. No two combinations of two feeds are likely to produce the same average daily gain if the animal eats to capacity. However, when three or more feeds are fed, there are several possible combinations of the several feeds which will produce the same output. If all but two are fixed the situation remains the same as with two feeds.

Different products from the same producing unit. In the previous section it was noted that limitations exist regarding rations that may be fed to attain a given quality of products (steer, milk, etc.). Such quality differences are usually called grades in the case of fattening animals.[1] Milk is graded on fat content and eggs on the basis of size, shape and shell structure. (Egg qualities affected more by handling than feeds fed are disregarded here.) In Figure 11 that portion of the product contour map representing good to choice steers is enclosed in triangle O'BB'. Similar diagrams could be used to differentiate other animal products on a production surface.[2]

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[1]Grading systems are not generally used for hogs at present in the United States. From the viewpoint of the farmer, slaughter hogs are a product of a single grade because no price differential for quality is made in the market. A few minor exceptions occur.

[2]Heady, op. cit., p. 157.

Although a definite line is drawn between the grades, caution must be exercised in applying such an interpretation to expected results from the various combinations of feed fed to different animals. Feeder steers of a specified grade have individual differences, and the slaughter steers into which they are transformed are not perfectly homogeneous with respect to grade. Individual differences between animals result in differences in their responses to different kinds and quantities of feeds and to techniques of feeding. It is possible, therefore to start with the same quality of feeders (good for example) but arrive at the market stage with some utility, some good and some choice fat steers as well as considerable variation within each grade. The ration fed undoubtedly affects the proportions of these various products to a large extent. That is, the choice of the ration fed to a specified grade of feeder steers might be expected to alter the expected or most probable proportion of the finished cattle grades that would be produced. Diagramatically, when fed a high concentrate ration, a lot of feeder steers (e.g., grade of good) might be expected to produce a proportion of good and of choice steers indicated by line OL in Figure 12. With a relatively low grain ration OM, the proportion might tend toward more good and less choice steers. Other ration lines might be introduced. Several different rations could have the same line determining the

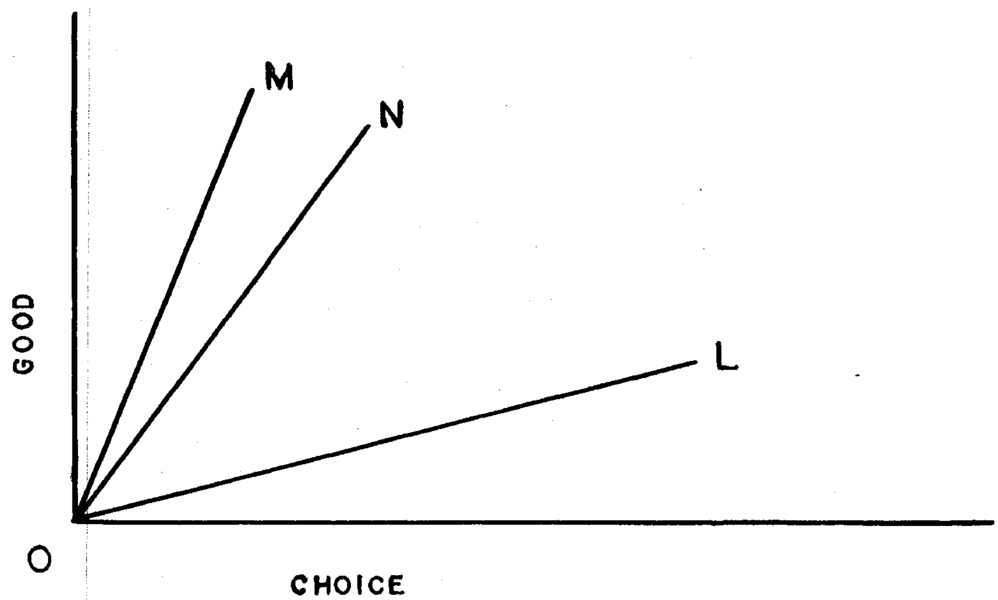

Fig. 12. Different products with different rations.

most likely proportion of good and choice steers that could be produced.

Economic implications

Little empirical investigation has been carried on in a manner to give complete quantitative measures of the physical production relationships discussed in the preceeding sections. In this section economic implications of these physical relationships are presented (1) to offer a theoretical solution for the problem of using feeds efficiently to maximize profits in livestock production, (2) to indicate the type and kind of physical relationships that are important to economic problems of feed utilization by livestock and (3) to encourage more work in empirical measurement of the production relationships outlined.

These economic problems of maximizing profit from livestock feeding operations fall into three categories. First, the ration which will produce a given output with least cost or maximize output from a given outlay is desired. Second, a level of output must be selected. Third, when several products may be produced a selection of a product or a product combination must be made.

Least cost ration. Feed prices must be included in our models in order to determine the least cost ration. In Figure 13 which represents average daily gain contours for ani-

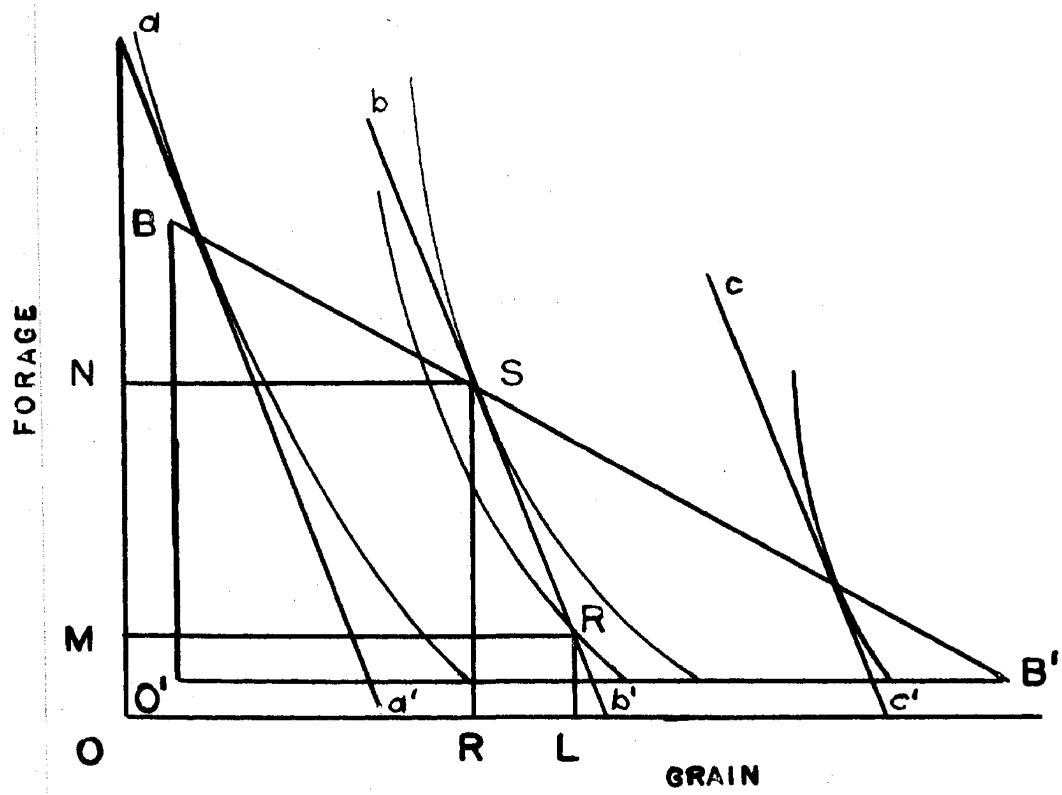

Fig. 13. The maximum forage ration a least cost ration.

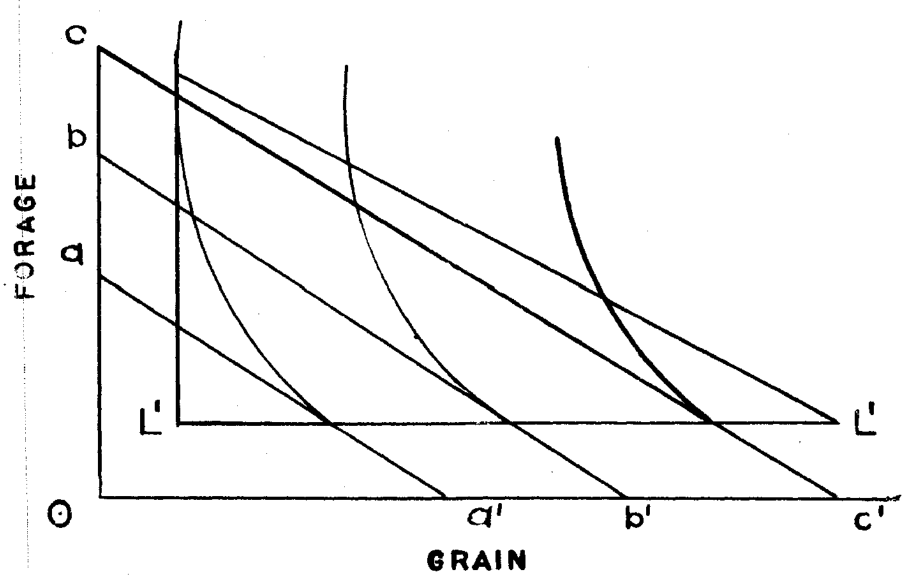

Fig. 14. The maximum grain ration a least cost ration.

mals such as steers and pigs, the price ratio between grain and forage is indicated by the slope of lines aa', bb' and cc'. When grain and forage are purchased at these price ratios it will be most profitable to produce on the maximum forage expansion path BB'. Other ration combinations will not achieve the greatest possible output for a given cost or will not produce good-choice beef. For example, feed combination OM of forage and OL of grain may be had for the outlay represented iso-cost line bb' and produces 1.9 pounds of gain per day. The maximum forage ration, ON of forage and OR of grain, which costs the same produces two pounds per day.

On the other hand, if price ratios of the nature of Figure 14 exist, it will be most profitable to expand production along the maximum concentrate expansion path LL'. Where the price ratios lie between these extremes[1], some combination within triangle LCL' would represent the least cost combination for a given output of good-choice steer. The expansion path may actually curve about within the triangle and even outside of it. In the latter instance a different product is indicated as being most profitable at certain outputs with given price ratios.

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[1]The condition referred to is that which occurs when the iso-cost lines are tangent to the iso-gain curves within triangle LCL' rather than to points outside of the triangle or its edges.

<u>Different substitution rates at different output levels</u>. Growing animals need more proteins while older animals can utilize proportionately more carbohydrates.[1] Feeds often substitute at different rates at different levels of output (stages of maturity in many cases). Since animals make more efficient use of different feeds at different stages of maturity, the expansion path need not correspond to a scale line (ration line).[2] It may, therefore, be profitable and efficient to feed different rations at different levels of output. In Figure 15, R_1 is the ration which will achieve output 1 with least cost. To produce the output 2, ration R_2 is least costly. There is a possibility of feeding ration R_1 until output 1 is reached and then a different ration to reach output 2.

Although R_2 is the least cost ration to achieve output 2 when the animal is fed R_2 all of the time, it may not be the least cost ration to feed over the interval from output 1 to output 2. The substitution rate indicated by slope of contour 2 is an over-all substitution rate used in producing output 2 by given techniques of feeding. That is, output

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[1]L. L. Madsen. Factors affecting nutrition, feed utilization, and health of farm animals. U.S.D.A. Yearbook of Agriculture, 1939: 431-449. 1939. p. 438.

[2]Heady, op. cit., p. 192.

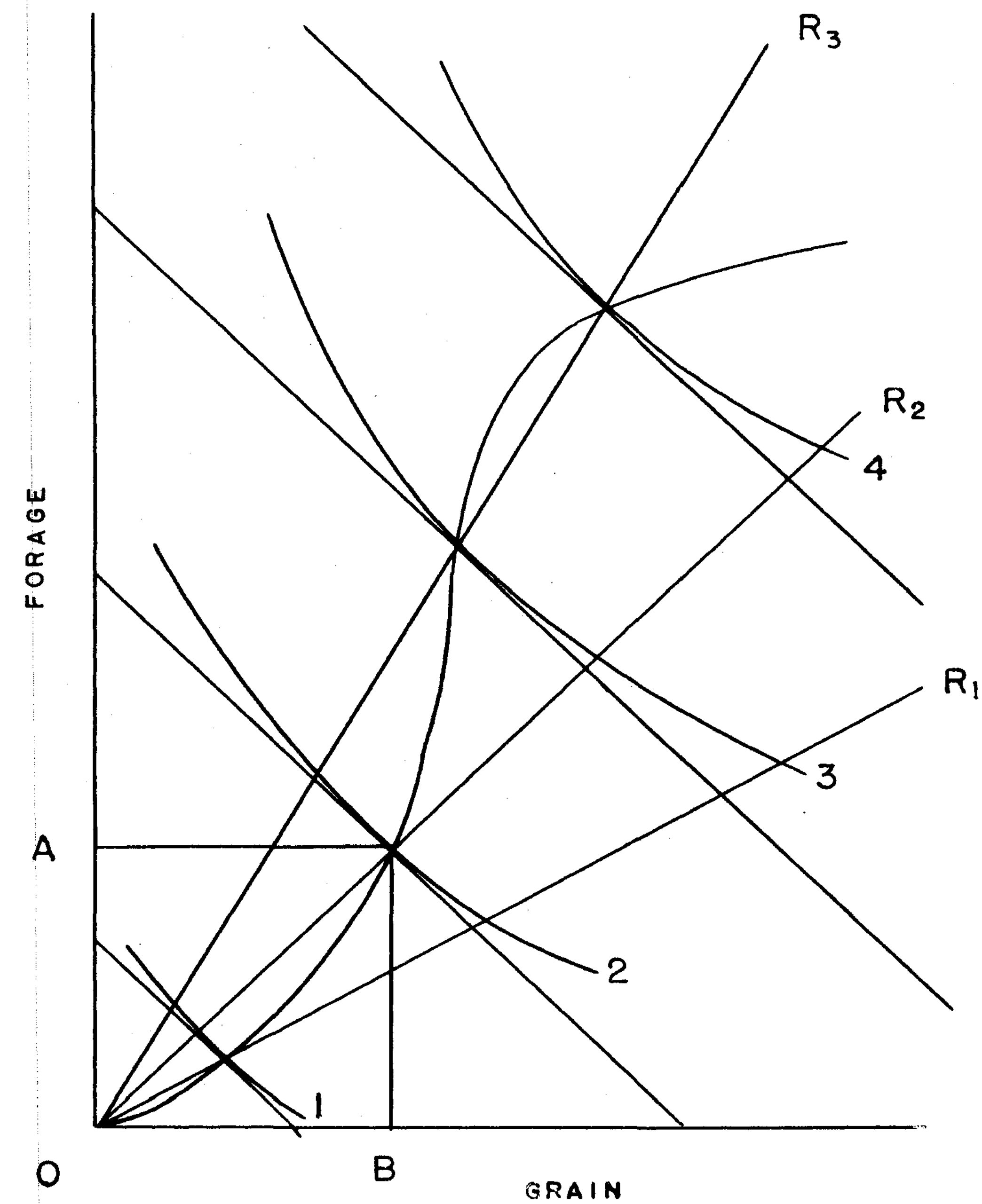

Fig. 15. Expansion path that differs from ration lines.

contour 2 represents various quantities of grain and forage that produce the same output when fed in a specified manner. The method which will result in greatest gains from given feeds is often assumed. However, an empirically derived function refers only to the technique used. For example, points on the production surface might represent gains when the ration fed any one animal is unchanged over the entire production period. In this instance, grain and forage needed to achieve output 2 could be counted upon as being OA and OB respectively only when R_2 is fed over the entire period of production. When an animal is fed first one ration and then another, it may use less or more of some or all of the feeds to achieve output 2.

A production function derived from observations on animals fed constant proportion combinations of feed would be one of the most useful in determining the least cost ration. The same proportional combination of feed could be considered as composing separate rations when full fed and when fed at various degrees of limited feeding. From such a function it is possible to estimate the results of first limited feeding and then full feeding; first feeding high protein rations, then low protein rations; first forage, then grain and vice versa. One ration can be traced part way up the production surface and then another followed. However, certain difficulties of interpretation occur. An iso-product curve as ordinarily diagramed in Figure 15 does not illustrate the rate at which an animal can substitute feeds over the last feeding interval. It gives the average substitution conditions over the entire production interval from the beginning of the production process (zero output) up to the output represented by that contour.

This point may be illustrated in simple terms if straight line substitution contours are assumed, i.e., feeds are perfectly substitutable at fixed ratios. In Figure 16, 100 pounds of grain or 100 pounds of forage are assumed to produce 100 pounds of gain starting with a 400-pound feeder. The feeds substitute for each other in a ratio of 1:1. (While these figures are unrealistic the principle demonstrated is the same as if more realism were introduced at the cost of using more complicated arithmetic.) Assume also that during the second section of development, the animal is able to achieve another 100 pounds of gain but using 100 pounds of grain or 200 of forage with one pound of grain substitutable for two of forage. Figure 17 illustrates an output contour, when the initial technical unit is taken as weighing 500 pounds (400 pounds plus 100 pounds of previous gain). Figure 18 illustrates the substitution relationship when these data are combined into a single production function. Note that the substitution rate of the 600-pound contour (200 pounds of gain) is 2:3 representing the average

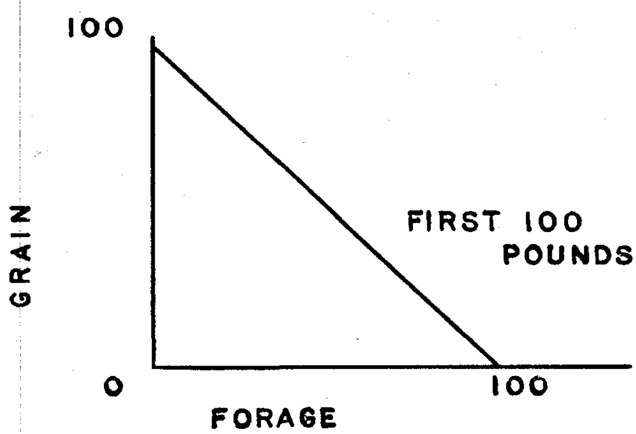

Fig. 16. The first 100 pounds of gain.

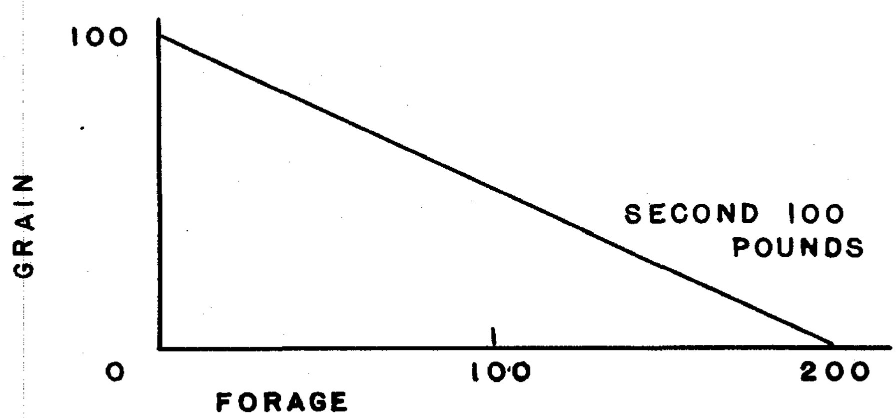

Fig. 17. The second 100 pounds of gain.

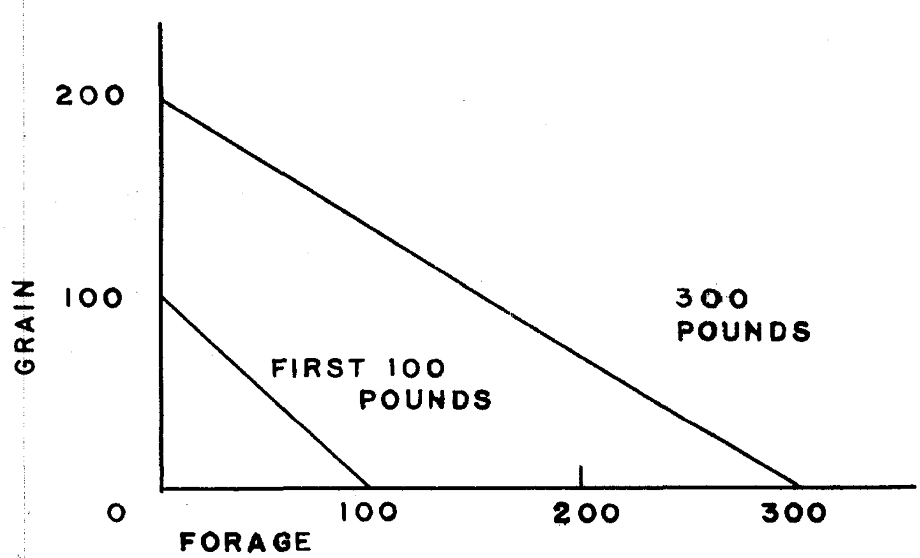

Fig. 18. The first and second 100 pounds of gain with the same origin.

rate at which the animal substitutes grain for forage over the entire production process of achieving output of 200 pounds of gain. Thus, the 1:1 ratio of the first contour would be satisfactory to determine the ratio at which the animal substitutes grain for forage over the first production interval, but the slope of the second contour does not indicate the substitution rate for the second interval.

Indication of changing least-cost rations from interval to interval with diminishing rates of substitution appears in Figure 15, but the exact least-cost ration to feed from one weight to another is not readily apparent. Separate production functions for each interval like those in Figures 16 and 17 would, of course, give this information. A later section deals with empirical derivation. The economic implications when choosing a least-cost ration are apparent. That ration would be chosen for each interval which equated the marginal rate of substitution of the feeds to the price ratios in that interval. This procedure may not be possible when feeds substitute in a linear fashion. Usually one or the other feed is most profitable except for the special case when the substitution rate happens to equal the price ratio. Then any combination of the two feeds is equally satisfactory.

<u>Choice of products from the same producing unit</u>. Rations affect the product quality as well as the rates of gain. Whether a steer finishes good or choice depends largely on the ration fed. This consideration is introduced into the model of a production surface in Figure 19. The division between good and choice may be considered an average when numbers of feeders are fed. Also different livestock judges and buyers tend to vary some in their placement of steers in this area. When the division is fairly definite, the choice of products may be made in the following manner. The iso-product contours (initial weight of the feeder plus gains) are transformed into iso-value curves but with a break or jog in each. For example in Figure 20, the 900-pound contour (800 pounds of feeder plus 100 pounds of gain) is valued at $20 per hundredweight for choice and $18 for good. The 900 pounds of choice steer is worth $180 and the 900 pounds of good steer is worth $162. To achieve $180 worth of grade good, 1000 pounds of steer must be produced. Thus, the 180-dollar iso-value curve follows the 900-pound contour through the choice area of the production map and the 1000-pound contour through the good area.

Price ratios of the feeds may be used as indicators of whether the 1000 pounds of good or the 900 pounds of choice can be produced with least cost. A price ratio of 2:1 of forage to grain indicated by the slope of iso-cost line BD

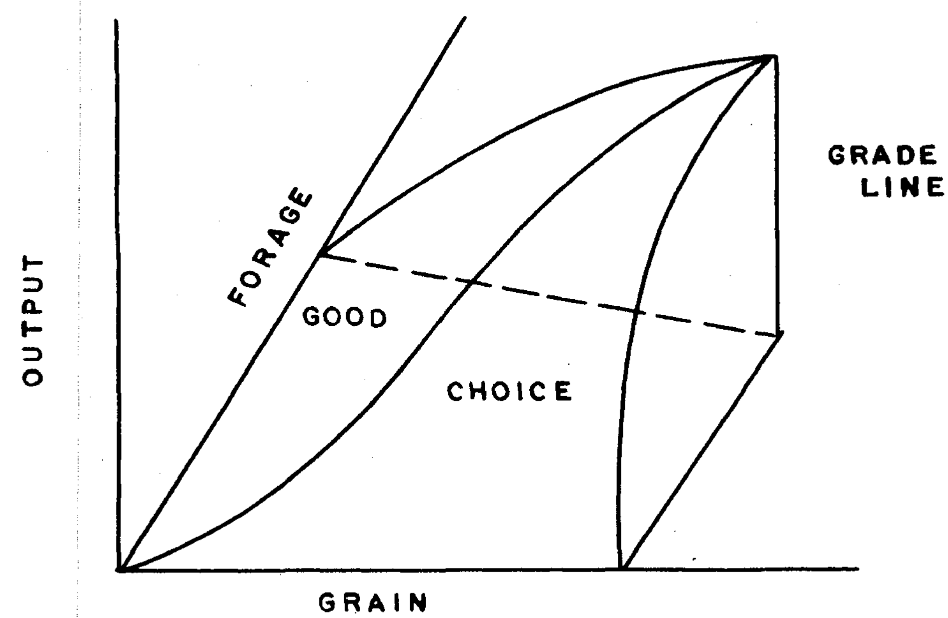

Fig. 19. Production surface showing grades.

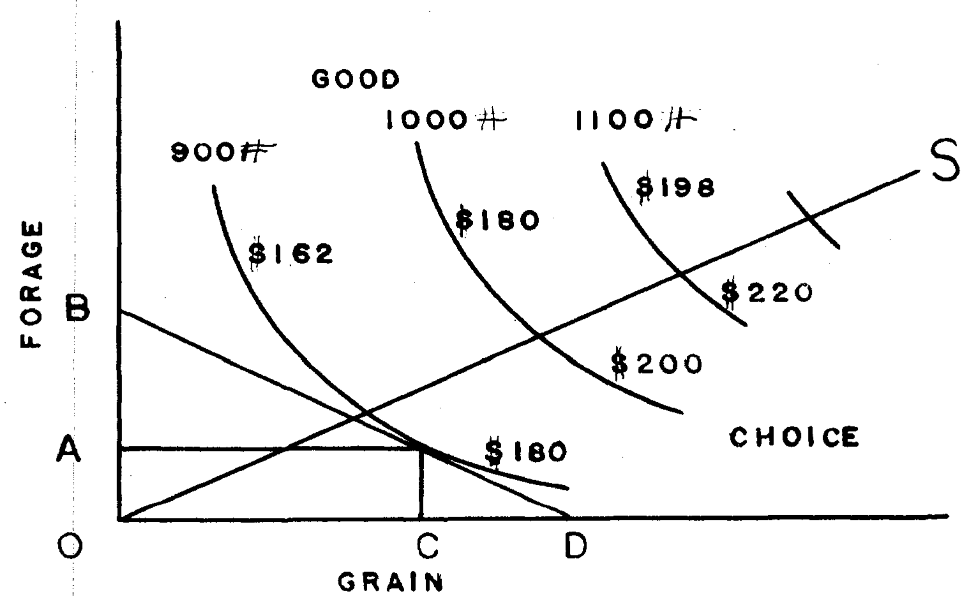

Fig. 20. Iso-value map.

permits $180 worth of choice steer to be produced with OA of grain and OC of forage. Any other combination of the feeds that can be purchased for an equal outlay (indicated by any other point on line BD) would produce less than $180 worth of steer. Other price ratios could favor another grade.

If different grades of feeder steer were used, the analysis would not hold. Unless it is assumed that the steer or steers in question are of the same initial value, the $180 worth of steer might represent less profit than the $162 worth of steer of the same weight. The analysis applies in the case where the cattle feeder has a choice of producing slaughter cattle of the same grade as his feeder cattle, a lower grade than his feeder cattle, or a grade higher than his feeder cattle.[1] Maintaining the same grade or attempting to raise the grade is probably most important. A cattle feeder would seldom lower the grade deliberately if it were possible to buy feeders of a lower grade at less cost to start with.

The possibility of using an average daily output model can be investigated as a solution to the choice of product problem. In Figure 21, average daily gain contours for a slaughter steer are transformed into the corresponding value

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[1]A. L. Anderson. Introductory animal husbandry. N.Y. The Macmillan Co. 1943. p. 114.

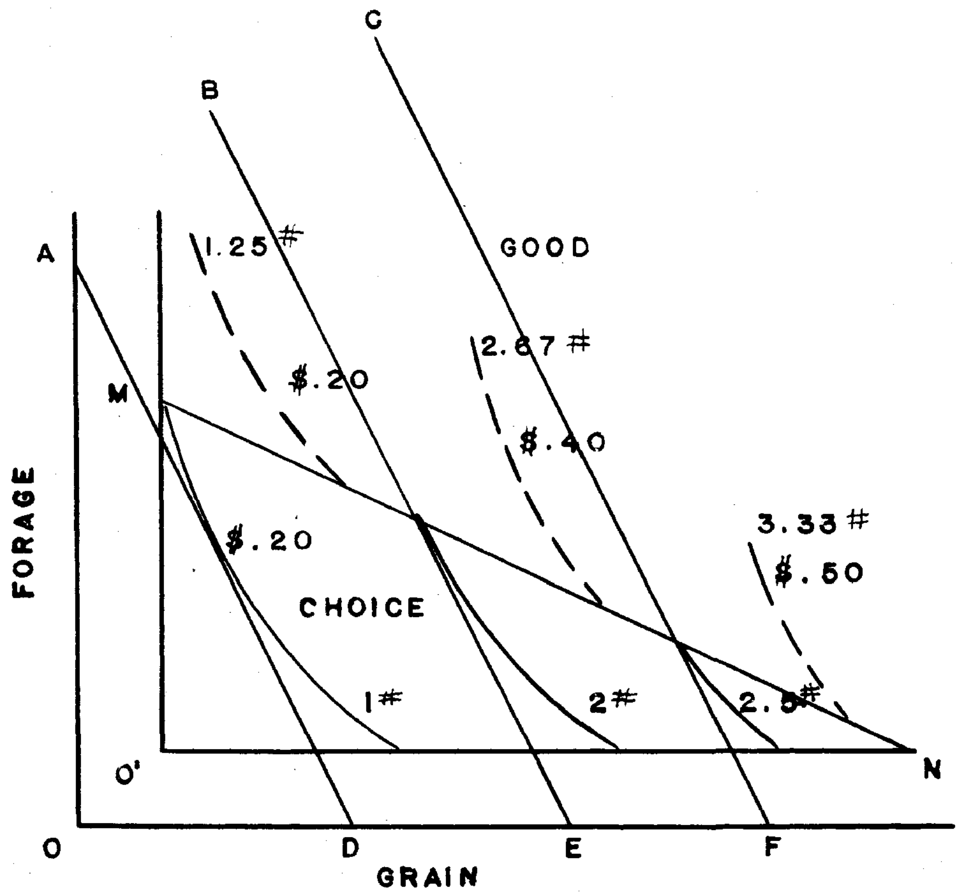

Fig. 21. Grades introduced into the average daily output model.

gains per day. The price used for grade of choice was $20 per hundredweight and for good the price was $15. The introduction of price ratios indicates the least cost combinations to achieve given values of product per day (points of tangency of iso-cost lines AD, BE and CF to the iso-value curves or the lowest cost point at which contact can be made with a given value curve). Unfortunately, this analysis is true only for the actual gains in the case of steers. Their initial weights are disregarded and the effects of margins correspondingly absent. Since a portion of the return from transforming an 800-pound feeder into a 1000-pound slaughter steer results from the difference in the price of the feeder and the price of the slaughter steer, the methods used to choose the most profitable product must take into account this characteristic of the producing unit being sold as a part of the product. On the other hand, milk, eggs and other products from producing units not marketed with the product present no difficulties to the use of this model.

In both of the above models, a definite division between grades was assumed. While this is true of an individual steer, the dividing line OS in Figure 20 is only the average expected when a large number of feeders are considered. As discussed in a previous section, some range of variability in the reaction of a given grade of feeders to various feed combinations exists. If such information were available, the type of transformation curve in Figure 22 could be constructed.[1] Various proportions of good and choice grades result from feeding a lot of steers different feed combinations. When all of these cost the same, the points lie on transformation curve RS. The point of tangency between iso-revenue line LM and the transformation curve indicates the highest revenue possible from the given feed outlay. The expected proportion of good and choice animals would amount to OA of good and OB of choice.

Even though the least cost combination necessary to achieve any given output and the most profitable product to produce at that output is determined, the level of output that will maximize income has not been determined. In fact, the product chosen and the ration may vary when the level of output is changed. It is possible that if a low level of output is chosen, a lower quality steer and a high forage ration might be most profitable. When a high level of output is desirable, a high concentrate ration and a higher quality steer might be most profitable.

Margins and marketing dates. If different lots of feeder steers were put on feed at the same time but fed different rations, it is possible for the same quantity and quality of beef to be produced from each lot. However, the

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[1]Heady, op. cit., p. 260.

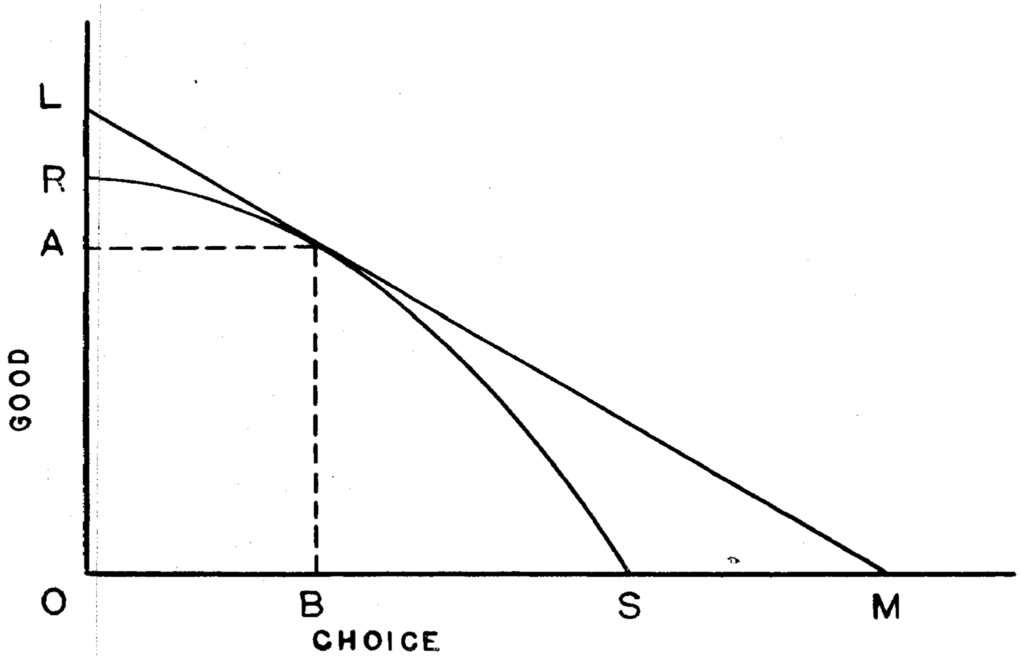

Fig. 22. Feed transformation curve for slaughter steers.

product from different lots might be forthcoming on the same or different dates. Prices for the given quality of beef usually differ on different dates. Hence, the steers finished on different days may be considered as different products. The cattle feeder is confronted with this problem when he decides to purchase feeder cattle. Should he try to buy the cattle at the time of lowest market prices and then hope to market them at a profit? He may, of course, vary the finishing date to some extent by the manner in which he feeds the steers. In some instances, it might be most profitable to buy at somewhat higher than the market low in order to sell at a price which would offer the greatest margin. The cost of feeding over the production interval also has to be considered, however.

There are at least two ways in which the problem may be approached. (1) The purchase date may be chosen first. Various sale dates may be considered with different lengths of feeding periods. (2) The date of sale may be chosen first and different possible purchase dates and the corresponding lengths of feeding periods considered. Farmers have various reasons for choosing a purchase date and then deciding upon the most profitable sale date. Cattle feeding may be a supplementary enterprise on the farm, i.e., a method of utilizing labor and equipment during an otherwise slack season. A farmer may obtain a bargain in feeder cattle that he doesn't wish to pass up. More knowledge of the purchase prices and supply of feeders than of slaughter cattle may be available;in which case the farmer may feel that he is reducing the risk of the enterprise to some extent by buying at what he considers the periods of lowest feeder prices. On the other hand,some of the reasons for choosing a sale date and then varying the production period by the date of purchase are evident. The feeding operations may have to be terminated before cropping operations or other work starts. Higher prices may be expected in some months than in others and the farmer may wish to take advantage of the market highs in selling. The cattle feeder may run the cattle he plans to feed on pasture for a time. In view of small differences in costs of keeping animals in that stage of production for varying lengths of time, the cattle feeder may feel that a lower purchase price may well offset any added expense of a longer pasture period necessary to reach the date he expects to market his animals. In such a case he may set his least cost fattening ration ignoring time to some extent. That is, if the production period falls during a slack season,the ration chosen may not be the one that achieves the desired quality in the least time. The full feeding production function would be useful in this situation, although some indication of the time required to reach the desired finish would be needed in order to choose the starting date of feeding.

An example of the first type of decision appears in a thesis by Bodensteiner[1] where the effects of margins that occurred in the past on profits from feeder operations are illustrated. Bodensteiner concludes that the weight gains of the steer may not be as important as the change in grade (quality) which occurs. Therefore, the least cost ration in terms of gain is not as important as the least cost ration in terms of quality change. There is, of course, no reason to ignore the concept of least cost rations just because margins often play a larger part in determing total profits than the profits on weight gains. In times of narrow margins ,the difference between profit and loss may be a matter of careful feeding. The point is that the objective of feeding is not one of maximizing weight gains from a given quantity of feed in this instance but rather the gains in weight when the desired grade is achieved. More information on the quality changes of the product over the production surface are needed as discussed in previous models. However, some information from experiments designed to produce a given grade of cattle are available. If the range in product quality is narrow, a production function may be derived from such ex-

[1]Bodensteiner, op. cit., p. 52-65.

periments. A least cost ration that will achieve a particular grade can be found. (Bodensteiner's ration was not shown to be the least costly possible.)

In order to study the relationship between the least cost ration and margins, a number of 2-year-old steer feeding experiments that produced good to choice cattle starting with good to choice feeders were chosen. An exponential production function was assumed and a regression of the gains on feeds fed computed. The result was as follows:

$$Y = 4.703X_1^{.3286}X_2^{.2350}X_3^{.0333}$$

where Y represents pounds of gain and X_1, X_2, X_3 represent pounds of corn, alfalfa, and linseed oilmeal respectively. Within the ranges of the original data the rations fed achieved different amounts of good to choice beef. The technique of full feeding alone was considered. It is recognized that with experiments not designed primarily for grading purposes the range in quality may be wider than desired. Some range in quality occurs within any grade designation. Further, there is some tendency to relax market classifications in times of beef shortages and vice versa. Thus, the classification of good-choice is open to some criticism. Nevertheless, the data are among the best available and their limitations not unduly serious if recognized.

A least cost ration based on average prices of feed from 1933-51 was found. It was assumed that the cattle feeder could have always used the previous years feed supplies. The ratios of the prices set equal to the marginal rates of substitution between feeds indicate the least cost ration under ordinary circumstances. However, the 1.3:1 ratio of alfalfa to corn was outside the range of the data used. Within the range of the data, a 1:1 ration of alfalfa to corn was the highest that achieved good to choice slaughter steers. It was, therefore, assumed that a 1:1 ration of alfalfa to corn was the extreme forage ration possible to maintain good-choice grades. The actual ration considered as the least cost ration of raising good-choice steers was, therefore, 1:1 of alfalfa to corn and 20:1 of corn to linseed oilmeal.[1]

The derivation of a single least cost ration for the entire production surface is a characteristic of the exponential type function used. While this would be unrealistic with lighter cattle and hogs, it may not be unreasonable with the shorter feeding periods of the heavy type steers under consideration. If longer feeding periods were to be considered, interval functions of a nature discussed previously would be needed. Then several rations might be fed, one over each interval, in order to achieve any given output with least cost.

With the two-year old steers, however, the one function

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[1]Appendix A

was used as an indicator of a least cost ration. Since this ration would cause different gains over different lengths of time, the effects of feeding time on gain were needed. All experiments were for full feeding; hence, time taken to eat various quantities of feed was a function of the feeds fed. The following equation resulted when the regression of the logarithms of the feeding times was run on the logarithms of quantities of the various feeds eaten:

$$T = 0.13945X_1^{.486}X_2^{.425}X_3^{.025}$$

where T represents time in days and X_1, X_2, X_3 represent pounds of corn, alfalfa, and linseed oilmeal respectively.

If the function is accepted as an accurate indicator of feeds fed over time, the period of feeding may be set at 30, 60, etc. days and the amounts of any ration eaten calculated. By solving simultaneously with the gain function, both gains and the amounts of the least cost ration that would have been eaten were found for various time intervals from 30 to 150 days.[1] These figures and the values of the feeds fed based on the 1933-51 average prices were entered in Table I.

[1] Appendix A

Table I. Amounts of feeds fed, their values and the corresponding weight gains.

Days on feed	Corn[a] lbs.	Corn[a] $	Alfalfa[b] lbs.	Alfalfa[b] $	Linseed oilmeal[c] lbs.	Linseed oilmeal[c] $	Total cost $	Gain in weight lbs.	Average daily gain lbs.
30	338	5.37	338	2.96	17	.51	8.84	119.1	3.97
60	708	11.25	708	6.21	35	1.05	18.50	185.2	3.09
90	1090	17.32	1090	9.55	54	1.62	28.49	239.6	2.74
120	1483	23.56	1483	13.00	74	2.21	38.78	288.0	2.40
150	1882	29.90	1882	16.50	94	2.81	49.21	332.1	2.21

a .89 per bushel; \$0.01589 per pound.
b \$1753 per ton; \$0.008765 per pound.
c \$2.99 per hundred weight; \$0.0299 per pound.

The profits from these gains when the steers were sold in different months appear in Table II. Since the profits received per steer also depend on margins as well as on profits from weight gains, the average of past margins when cattle were bought in various months and sold 30, 60, 90, 120, and 150 days hence were tabulated (Table III). The profits from gains (Table II) were added to the profits from margins (Table IV) to find total profit per steer (Table V).

As was expected, the major portion of the profits was derived from margins resulting from changes in grade. In fact, where a sale date is chosen and the animals fed different lengths of time but all gains valued at one price, more profit to feed gains occurred during the first 60 days than during the longer feeding periods (Table II).

The point of maximum profit will, of course, vary with the price ratios. Total costs form a relatively straight line when plotted against length of feeding period (Fig. 23). Two-year old steers tend to eat about the same amount per month once they are on full feed. The feed cost curve (Fig. 23) remains the same for all marketing periods when the feeds were purchased from last year's supplies. On the other hand, returns per 100 pounds of feed fed decline with the length of the feeding period or as in Figure 24, the total physical product increases at a slightly diminishing rate over time. The returns-to-weight gain curves have the gen-

Table II. Average profit from weight gains from different feeding period lengths.

Days fed	Month of sale											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
30	11.37	10.76	11.01	11.08	11.13	11.43	12.18	12.62	12.94	13.47	13.59	13.37
60	12.93	11.98	12.37	12.48	12.56	13.02	14.19	14.87	15.37	16.19	16.22	16.04
90	12.17	10.95	11.45	11.59	11.69	12.29	13.80	14.69	17.73	16.39	16.43	16.19
120	10.09	8.62	9.23	9.40	9.52	10.24	12.05	13.12	13.90	15.16	15.22	14.93
150	7.15	5.45	6.15	6.35	6.48	7.31	9.41	10.63	11.53	12.99	13.06	12.73

Table III. Average margin per hundredweight for 1000-1200-pound steers of good to choice grade at Chicago. 1933-1951.[a]

Month of Purchase	Month of Sale											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.		3.40	3.61	3.67	3.71	3.96						
Feb.			3.23	3.29	3.33	3.58	4.21					
Mar.				2.80	2.84	3.09	3.72	4.09				
Apr.					2.60	2.85	3.48	3.85	4.12			
May						2.40	3.03	3.40	3.57	4.11		
June							3.24	3.61	3.78	4.32	4.34	
July								4.08	4.35	4.79	4.81	4.71
Aug.	2.95								4.27	4.71	4.73	4.63
Sept.	2.87	2.36								4.63	4.65	4.55
Oct.	3.10	2.59	2.80								4.88	4.78
Nov.	3.34	2.83	3.04	3.10								5.02
Dec.	3.29	2.78	2.99	3.05	3.09							

[a]U. S. Production and Marketing Administration. Livestock market news statistics and related data. U. S. Dept. Agr. Statistical Bul. Series. 1934-1952.

Table IV. Average profit from margin for 950-pound, 2-year-old steers at Chicago. 1933-1951.

Month of Purchase	Month of Sale Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.		32.30	34.30	34.86	35.24	37.62						
Feb.			30.68	31.26	31.64	34.01	40.00					
Mar.				26.60	26.98	29.36	35.34	38.86				
Apr.					24.70	27.08	33.06	36.58	39.04			
May						22.80	28.78	32.30	33.92	39.04		
June							30.78	34.30	35.91	41.04	41.23	
July								38.76	41.32	45.50	45.70	44.74
Aug.	28.02								40.56	44.74	44.94	43.98
Sept.	27.26	22.42								43.98	44.18	43.22
Oct.	29.45	24.60	26.60								46.36	45.41
Nov.	31.73	36.38	28.88	29.45								47.69
Dec.	31.26	26.41	28.40	28.98	29.36							

Table V. Average profit from gains plus margin per steer for 950-pound 2-year-olds at Chicago. 1933-1951.

Month of Purchase	Month of sale											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.		43.06	46.67	46.45	44.76	44.93						
Feb.			41.69	43.74	43.33	44.25	49.41					
Mar.				37.68	39.54	41.65	47.39	49.49				
Apr.					35.83	40.10	46.86	49.70	50.67			
May						34.23	42.97	46.99	47.82	52.03		
June							42.96	49.17	53.64	56.20	54.29	
July								51.38	56.69	61.89	60.92	60.42
Aug.	35.17								53.50	60.93	61.37	58.91
Sept.	37.35	27.87								57.45	60.40	59.41
Oct.	41.62	33.22	32.75								59.95	61.45
Nov.	44.66	47.33	38.11	35.80								61.06
Dec.	42.63	38.39	39.85	38.38	35.84							

eral shape of the physical product curve. In Figure 23 the physical product curve of Figure 24 has been multiplied by various constants (values per hundred-weight of steer). The lower the expected value, the less the feeding time which will achieve maximum profit from weight gains.

Thus, there is a tendency for weight gains to be less profitable during a time when they could be used to offset low margins. It is possible for losses to be suffered on the weight gains at the most profitable length of feeding period. Margins can make up the difference. Similarly, returns to weight gains may be profitable when margins result in loss to the enterprise as a whole. With heavy steers a few cents difference in margin makes a great difference in profits because of the large initial weight of the steer. For example, a 2-cent negative margin with a 950-pound feeder would have wiped out the highest of the average profits from weight gains tabulated in Table I. The use of weight gains to offset unfavorable margins is thus limited. On the other hand, with younger steers and longer feeding periods, returns for gains make up a larger portion of the profits or losses. Some possibility of minimizing losses by using returns to gains to offset losses from margins when price expectations change during the feeding period exists.

In any case, there is some most profitable feeding period from the standpoint of weight gains of a particular grade.

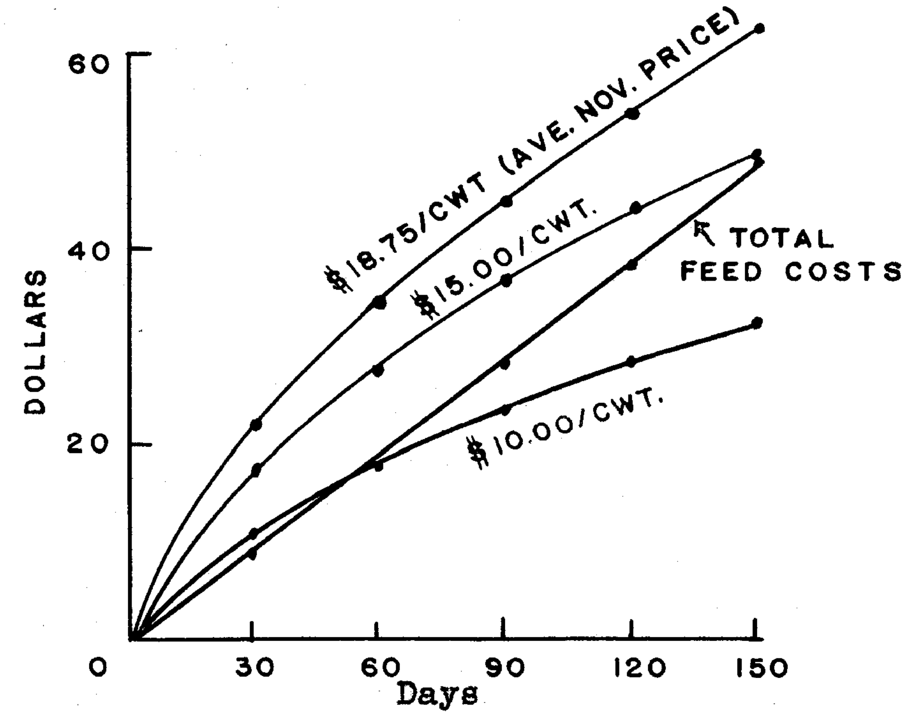

Fig. 23. Costs and returns curves for weight gains.

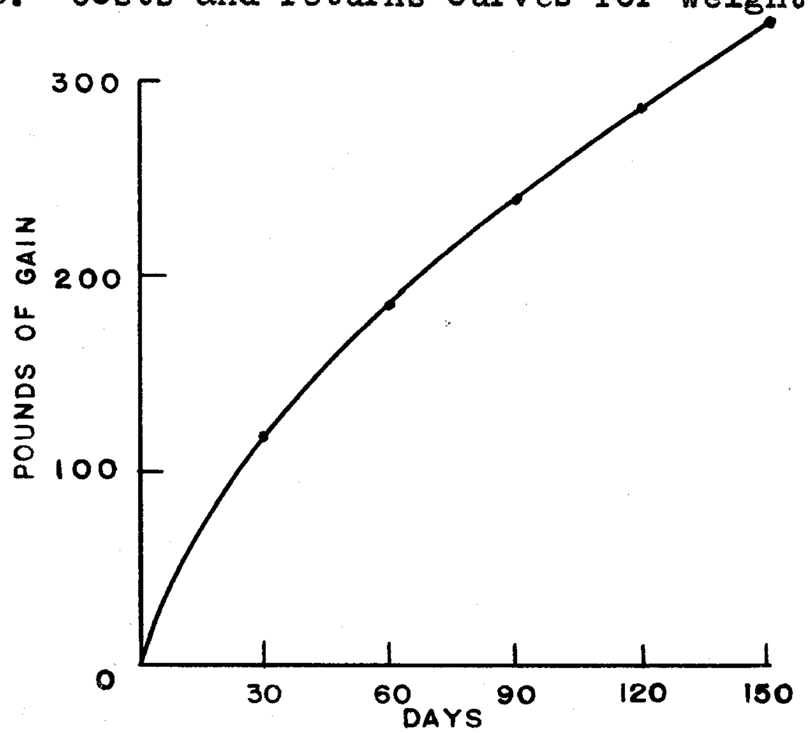

Fig. 24. Weight gains with different lengths of feeding periods.

After this point, profits from gains decline or losses become greater. It follows that the man who purchased his animals and is examining his expectations of future margins to determine the length of production period must have somewhat higher margins after the point of maximum profit to weight gains is achieved. Otherwise the decline in profits from this factor would result in lowered total profits. The declining profits over time when gains from different time periods are valued at the same monthly sale price in Table I illustrate this situation. However, prices expected several months hence may be higher than the intervening months; in which case the declining physical product is offset by the price increases. This has happened on the average in the past. Reading from left to right in Table VI, there is relatively less decline in the value of weight gains when each period's gain is valued at the price in the ending month. As indicated in the lower section, though, prices may also be lower in the future which would accelerate the diminishing nature of the profits. The feeder who selects his sale date and varies his purchase date is, of course, faced with the same problem. In the past the most profitable sale date has occurred on the average in the late fall and early winter for heavy cattle (Table V). Prices have been higher and margins over possible purchase prices (average) have been greater at that time (Table III). The figures in the tables

Table VI. Average profit from weight gains.

Month of Purchase	Month of Sale											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.		10.76	12.37	11.59	9.52	7.31						
Feb.			11.01	12.48	11.69	10.24	9.41					
Mar.				11.08	12.56	12.29	12.05	10.63				
Apr.					11.13	13.02	13.80	13.12	11.53			
May						11.43	14.19	14.69	13.90	12.99		
June							12.18	14.87	17.73	15.16	13.06	
July								12.62	15.37	16.39	15.22	12.73
Aug.	7.15								12.94	16.19	16.43	14.93
Sept.	10.09	5.45								13.47	16.22	16.19
Oct.	12.17	8.62	6.15								13.59	16.04
Nov.	12.95	10.95	9.23	6.35								13.37
Dec.	11.37	11.98	11.45	9.40	6.48							

are not meant to be set forth as future expectations other than as an indication of a past pattern which may recur. Even then the figures must be considered only as averages and qualified in the light of conditions existing in any specific period.

ALTERNATIVE METHODS OF DERIVING THE PRODUCTION FUNCTION

The models outlined in previous sections indicate a need for systematic empirical estimation of all points on the production surface. The physical relationships of feed utilization by livestock have not been completely derived in this fashion. Feed substitution tables such as Morrison's[1] are valid only within a limited range of feed variation around a "balanced ration". The many experiments which establish a difference in treatment effects, however useful for other purposes, do little to help with the over-all production picture. At best single points or lines across the production surface are estimated.

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[1] F. B. Morrison. Feeds and feeding. 21st ed. Ithaca, N. Y. The Morrison Pub. Co. 1948.

Single Producing Unit Approach

Various alternative means of deriving the production relationships exist. A single producing unit (steer for example) could be fed various combinations of feeds and its gains observed. In practical feeding, however, it is often impossible to vary the rations and quantities of feeds fed and observe the output from a given animal in a meaningful manner. For example, a steer cannot be fed one ration to reach 200 pounds of gain and then returned to its former status in order to test another ration. That particular producing unit is destroyed when it is slaughtered. Some variation in this manner can be observed with milk cows. Even then limitations occur. A cow's production ability changes over the lactation period and may be affected to some extent by previous rations. Also, when the substitution relationship between feeds is desired, maintenance of a given output while one feed is substituted for another is practically impossible. Hence, the information gained in feeding experiments with one animal must be assumed to apply reasonably well to other similar animals or little can be gained from experimenting. Fortunately, animals of a particular class, breed, and age are fairly homogeneous.

Multiple Producing Unit Approach

When the objective is one of estimating the structural relationships of production for a particular type of livestock unit, useful information may be gained by placing a number of such animals each on a different ration. A regression may then be run to estimate the structural parameters of production. Small differences between ration effects often confounded with chance variation are of interest and are estimated from such a production surface.

The experiment could be of replicated design. Sums of squares of deviations between replications could be accounted for. The object would be to explain as much of the deviation from regression as possible. While this object is worthy of consideration, any experiment of the nature proposed requires large numbers of animals. If more lots and more animals are available, it is questionable whether the information gained by replicating would be worth as much as the greater quantity of structural estimates that would occur if more animals, each animal eating rations different from any other in the experiment, were added.

If one had unlimited resources, animals might be fed many different rations and the production surface built from the results. Replications might then be advisable to account for effects of uncontrolled variation in physical conditions to which the steers were subjected. Many variable besides feed affect production and must be carefully controlled or entered into the function as measurable variables. When physical conditions are well controlled, estimates of feed utilization coefficients would be useful even though an estimate of variation due to the differences in physical conditions between lots were not accounted for.

Feeding rations of fixed proportions

One of the most convenient methods of obtaining production data is to hold the proportions of various feeds constant over the entire feeding period. In Figure 25, seven such rations are indicated. Only one or two animals need to be used per ration and the accumulated data taken as the results of feeding. For example, the animals could be weighed every ten days and the feeds fed totalled, for the weights achieved. Dots on the ration lines indicate possible positions or combinations of feeds on successive weighing dates. In this manner many observations could be gained with a minimum number of animals on experiment.

Since animals fed together may tend to eat more than when fed separately, more than one animal per lot is desirable. Under farm conditions animals are seldom fed out alone. Of course, other methods that result in the animal's acting as they normally would in a group feed lot would be a satisfactory substitute for more than one animal per lot. Dairy cat-

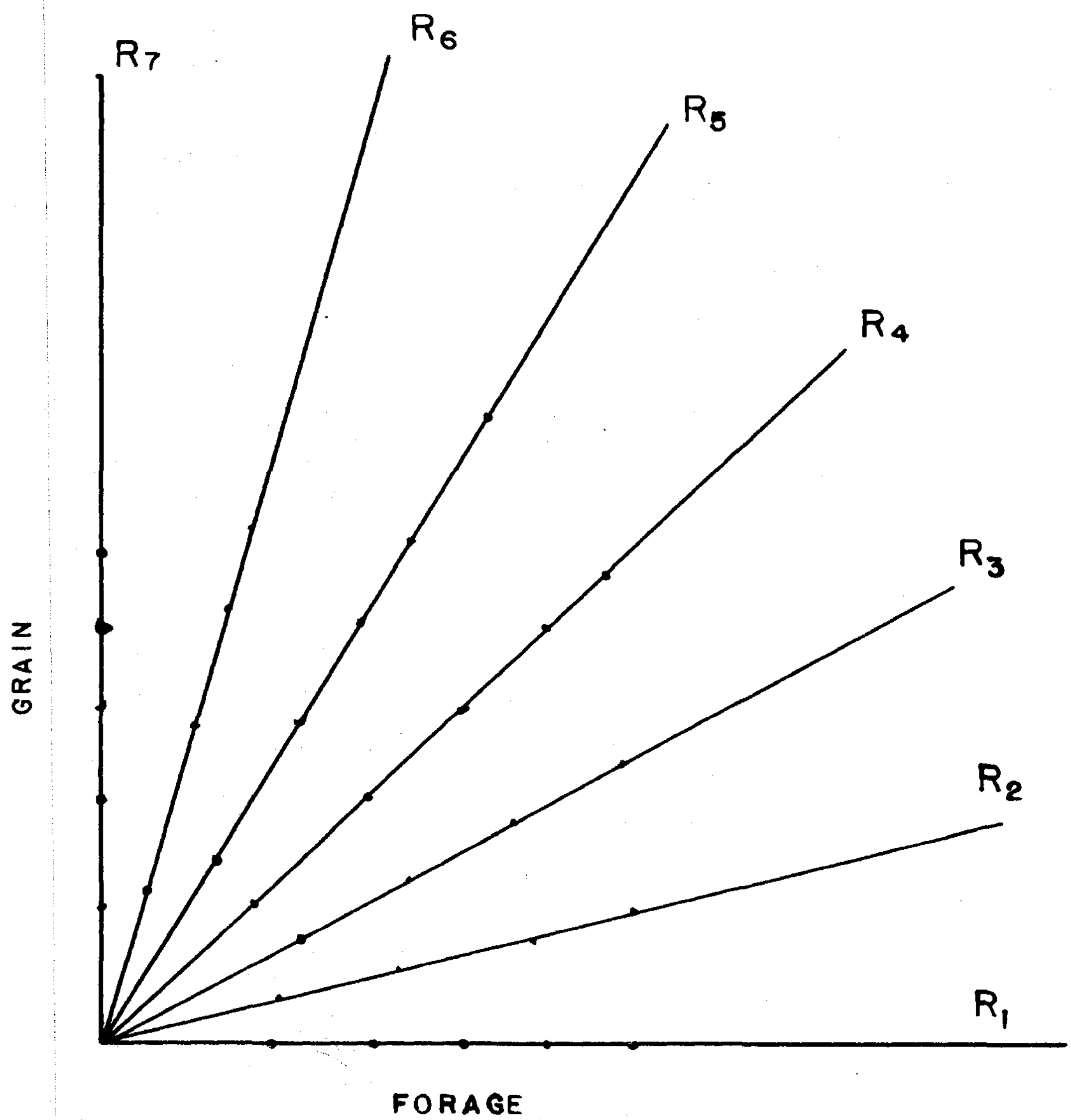

Fig. 25. Ration lines indicating the ratio of one feed to another.

tle, for example, present no problem since they can normally be fed different rations even while handled as a herd.

Even with limited resources the controlled experiment approach may be used. The production relationships for a few feeds and a number of rations may be observed first. Ten or so rations (combinations of feeds in fixed proportions) could be used requiring only ten animal units or twenty if two per lot are fed. Ten lines over the production surface (two independent variable model viewpoint) could be estimated. From these the production surface could be constructed. Other combinations and feeds could be added as more resources became available.

Possible Mathematical Models

Various mathematical functions may be used to approximate physical feeding relationships. The cases of production discussed previously place certain restrictions on the production surface. In general, as less of feed B is used, more of feed A will have to be fed to maintain the same output. As more of feed A is substituted for feed B, increasingly larger amounts of A must be added to compensate for the reduction in B. The iso-product contours must, therefore, form a system of downward sloping curves convex to the origin as in Figure 26.[1] Ordinarily some range or limits exist in

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[1]Allen, op. cit., p. 286.

tion of the animal has the capacity and desire to consume more hay. Of course, high producing dairy cattle would be unable to maintain high production on an all hay ration. The situation illustrated would apply only to lower producers whose stomach capacity relative to production was sufficiently large enough to permit the output in question to be approached with an all hay ration. Otherwise, the condition discussed in the preceeding paragraph would tend to occur.

In some models the iso-product contours may extend from axis to axis or from one axis to some point in the plane defined by the feed axes (Fig. 29). This condition is often realistic. For example, hay may be replaced by corn in the diet of a pig and output held constant, but some point is finally reached after all hay has been replaced where the concentrated nutrients of the corn not only replace all hay, but further addition results in increased gains. It is impossible to stay on the same iso-product contour after all hay has been replaced by corn, i.e., the iso-product contour reaches the corn axis.

Two functions that meet production requirements reasonably well are the quadratic and an exponential function often called the Cobb-Douglas production function.[1] Both may be

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[1]C. W. Cobb and P. H. Douglas. A theory of production. Am. Econ. Rev. 18: 135-165. 1928. p. 139.

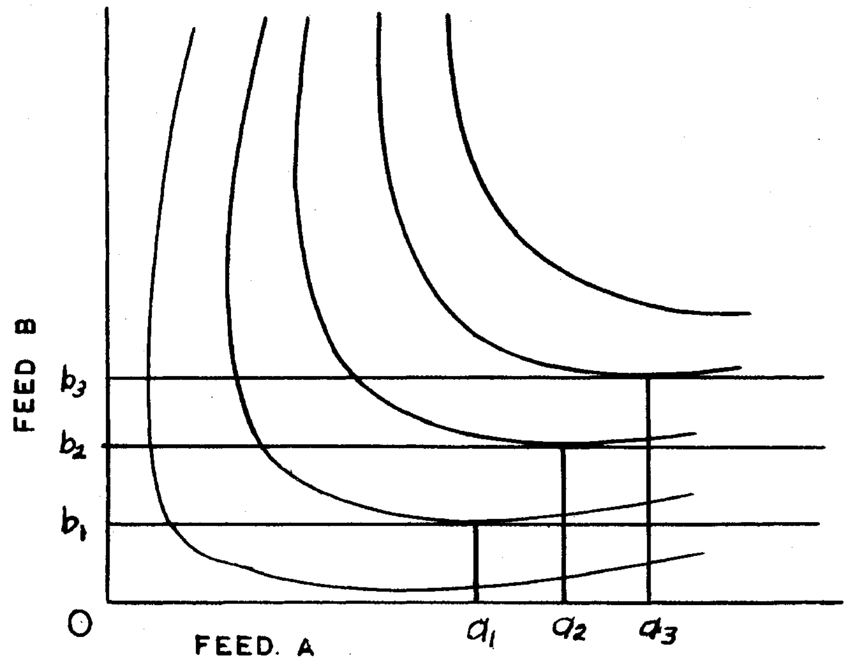

Fig. 26. An ordinary type of iso-product contour map.

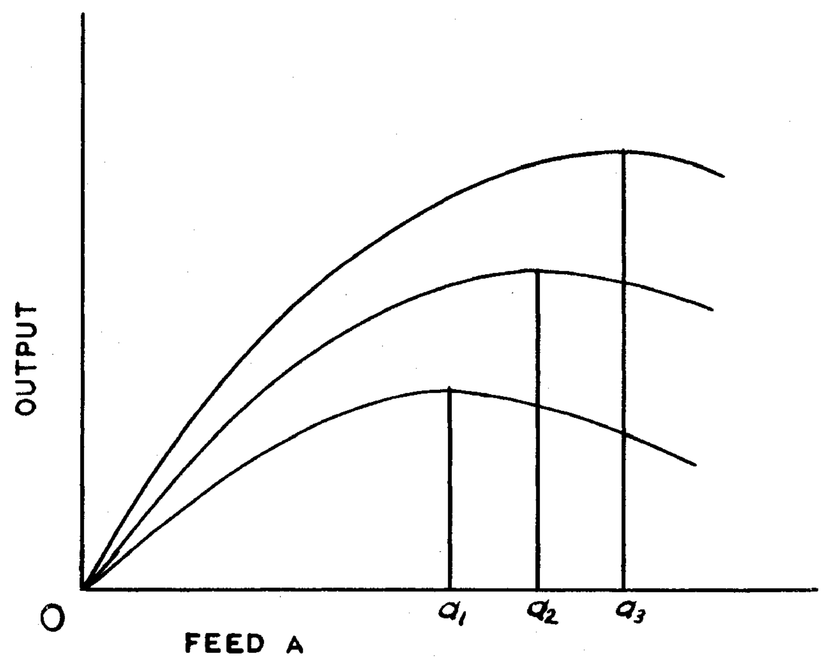

Fig. 27. Input-output curves corresponding to the ordinary contour map.

which the above relationships apply. Outside of this range or at the extreme edges of this range several possibilities exist. Both feeds may have to be increased to maintain output. For example, as more and more of feed A is fed with feed B held constant at b_3, successively higher contours are reached until a_3 is being fed (Figs. 26, 27). After this point gains decline if more of A is fed. Such a situation occurs with feeds like peas fed to hogs. When the proportion of the ration fed is extremely high in peas, digestive disturbances may result causing loss of weight rather than gain.

In other models the iso-product contours may become asymptotic to the feed axes (Fig. 28). A point may be reached under heavy feeding of one feed where the animal makes less and less efficient use of that feed relative to another. If the animal can be induced to eat more without "going off feed", smaller and smaller amounts of the second feed are needed to compensate for a unit amount of the first. Finally, very small quantities approaching zero of the second will substitute for very large quantities of the first. Perhaps feeding hay to ruminants approaches this situation. The animals finally reach a point in the consumption of hay where a very small amount of grain is needed to offset a pound of hay when maintaining the same output (milk, for example). However, some small amount of corn may be taken from the ra-

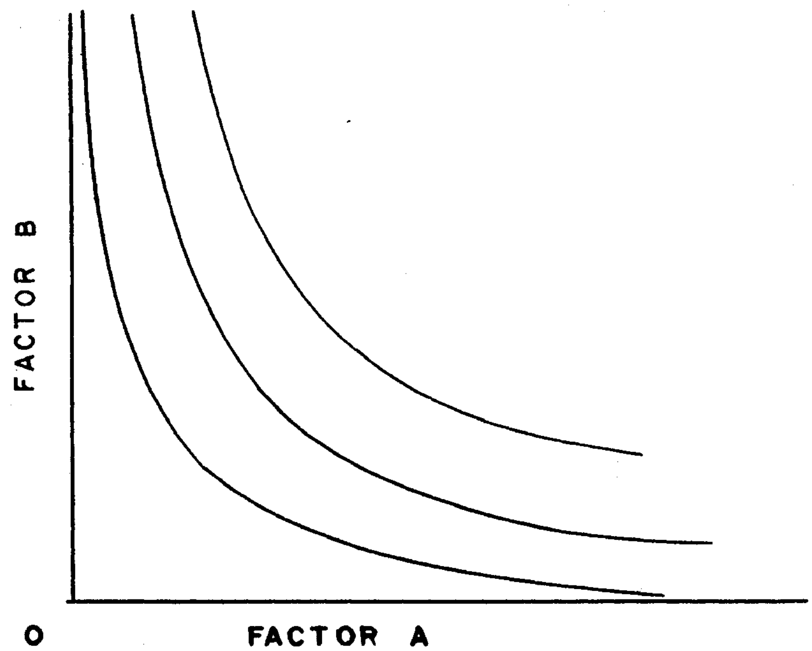

Fig. 28. Iso-product contours asymptotic to the axes.

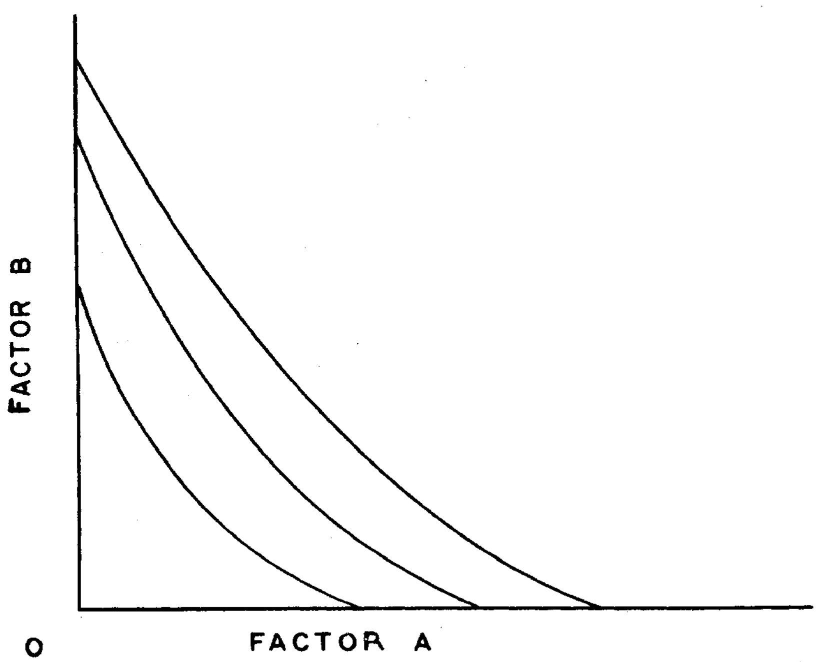

Fig. 29. Iso-product contours that intersect the axes.

fitted to the data with least-squares regression methods. The Cobb-Douglas function which is linear in the logarithms permits the use of more variable factors with relatively fewer computations. This function permits the phenomenon of diminishing returns required by nutritional theory and also permits substitution of feeds at diminishing rates. The regression coefficients conveniently give the elasticities of production, but constant elasticities are assumed for the entire range of data. The relative change in a variable is accompanied by the same relative change in output, other variables held constant at any given level of input. This characteristic does not correspond to fact except as an average over the entire production function. Nutritionists have demonstrated that animals at different stages of maturity utilize feeds with different relative efficiencies.[1] Hence, the substitution rates of one feed for another along a ration line are different at different levels of output, although the Cobb-Douglas function represents them as the same.[2] On the other hand, the regression equation is reasonably easy to compute for several independent variables. Only one degree of freedom is lost for each variable as compared to functions such as the quadratic where the squared

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[1]Morrison, op. cit., pp. 140-141.

[2]Earl O. Heady. Use and estimation of input-output relationships or productivity coefficients. Jour. Farm. Econ. Proceedings 34: 775-786. 1952.

and cross product terms each take another degree of freedom. When more than two feeds are considered at one time, squared and cross product terms necessary to show inter-relation effects of feeds become numerous, in the case of the quadratic. It is much less expensive and less tedious to run regressions for a Cobb-Douglas of a comparable number of variables.

R. G. D. Allen[1] points out that the equation $x = 2Hab - Aa^2 - Bb^2$, where A, B, and H are positive constants such that $H^2 > AB$, will produce iso-product curves of the type shown in Figure 26. It is often found empirically, however, that a better fit of the data in animal feeding is obtained if the more general form of quadratic function is used,[2] i.e.,

$$F(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F.$$

Output is a function of two factors x and y. The capital letters are constants representing coefficients of production. The iso-product contours of this function are apt to cross the axes in the manner of Figure 29 which is not theoretically correct in every instance. Usually the range of the data in which the researcher is interested is somewhere between the extremes of feeding all of one feed or the other. The central portion of the iso-product contours estimated with a quadratic function would then be satisfactory. No

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[1]Allen, op. cit., p. 286.

[2]Heady, Use and estimation of input-output relationships or productivity coefficients, p. 780.

assumptions of constant elasticity or constant rates of change in curvature are made with such quadratic functions. Also, ration lines do not necessarily cross iso-product contours at points of equal marginal rates of substitution as is characteristic of Cobb-Douglas functions. It is therefore possible to obtain indication of a changing least cost ration as output is increased. This situation is an advantage over the Cobb-Douglas function which indicates the ration that approximates least cost on the average. No reason for changing rations at different output levels is indicated by the Cobb-Douglas function. Young animals are relatively more efficient users of protein than of corn and older animals relatively more efficient users of corn, but the Cobb-Douglas function pictures them the same unless separate regressions are run to cover different intervals of output. Quadratic equations are therefore more flexible and realistic from this viewpoint.

Quadratic equations often indicate diminishing total returns to a feed. That is, as excessive amounts of a particular feed are fed, the animal looses weight. This condition is realistic if there is a possibility of the animal "going off feed". In many cases, though, the animal merely approaches some maximum gain. Exponential type functions are more appropriate in such cases, although the quadratic will be satisfactory theoretically if the point of diminishing total returns to a feed (point a_3b_3 Fig. 26) is reached beyond the practical range of the data.

Another production function has received considerable attention in fertilizer analysis. This production function,

$$Y = M - AR^x,$$

is usually termed the Spillman production function after its originator, W. J. Spillman.[1] Y is the yield or output obtained with x units of input. M is a limit approached by Y as x increases indefinitely. A is a theoretically maximum increase in output. R is the ratio of successive decreasing increments of Y corresponding to successive unit additions of x. The function may be extended to multivariable cases by using the following form:

$$Y = A(1-R^{x_1})\ (1-R^{x_2}).$$

In this form the equation corresponds to the Mitscherlich production function.[2] These functions permit diminishing returns but not diminishing total returns. The input-output curve becomes asymtotic to a maximum M (Fig. 30). The increments of output added with each successive input of feed or fertilizer form a decreasing geometric series. Each such increment of output is a fixed percentage of the preceeding increment.

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[1]W. J. Spillman. Use of the exponential yield curve in fertilizer experiments. U.S.D.A. Tech. Bul. 348. 1933.

[2]Ibid., p. 54.

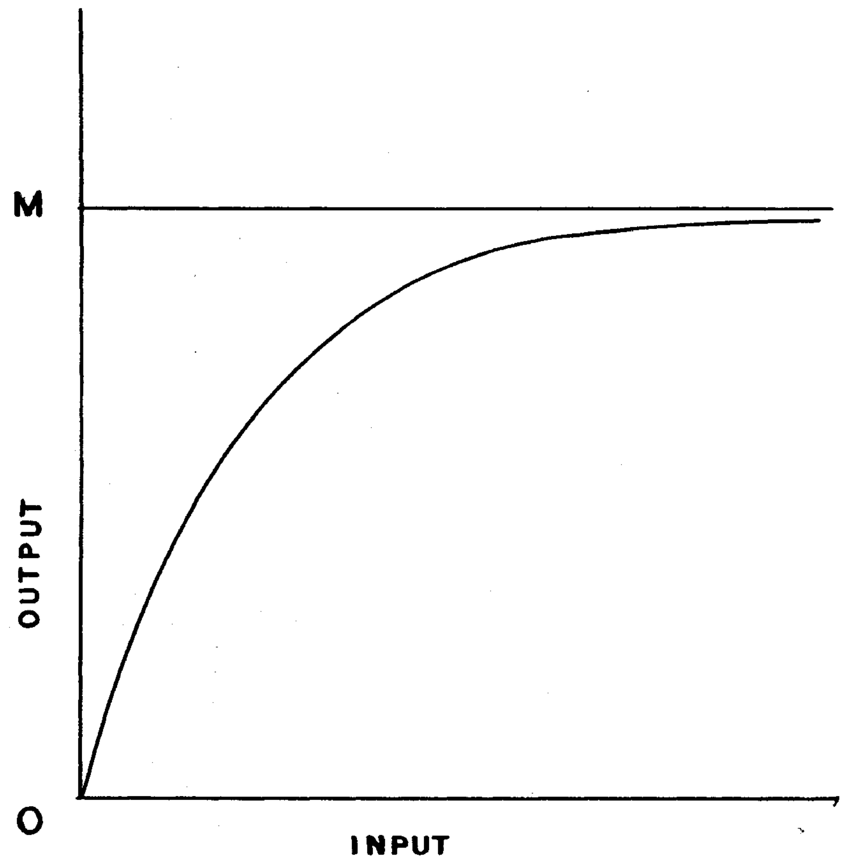

Fig. 30. Approach to a maximum output by the Spillman-type production function.

The use of this function involves varying the inputs of one factor while all others are present in sufficient amounts to produce approximately 90 percent of maximum. The individual estimates of R obtained in this manner are used to derive a function of the several factors when all are variable. A peculiarity of this equation requires that the inputs of the variable factors be applied in equal increments. It is ,therefore,difficult to apply in animal experiments. The feeding of 20-pound increments of corn can be done in many ways. It can be fed in units per day, in fixed proportion to other feeds, or even alternately with other feeds. This is somewhat different from applying increments of fertilizer to plots of soil which can also absorb large amounts of other fertilizers. Animals have stomach capacities which limit the amounts of feed that can be eaten at any one time. The level of input of one feed is,therefore,limited by the amounts of other feeds eaten by animals. These functions are better adapted to fertilizer experiments in this respect.

The Spillman and Mitscherlich type functions are also limited in usefulness by the procedures needed to fit empirical data. Tedious trial and error processes are often required except in some special cases.[1] Another disadvantage is that tests of significance are not yet readily available.

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[1]Spillman, Ibid., p. 7-9.

While only two factor cases were illustrated, the principles may be generalized into multivariable cases. If all but two of the feeds are fixed at given levels, the relationships as illustrated exist between the variable feeds.

Some of these functions fit the data at hand better than others which brings up a problem. Should that function be used which best fits the data? Since the data are a sample which may or may not be entirely representative of actual production relationships, fitting the data may mean deviating from the true relationships. Unless the inferences to be made are hypothesized before the sample is taken, nothing can be added upon discovery of other patterns within the sample. All single samples may be expected to contain some spurious results.[1] A better policy is, therefore, to choose a function that fulfills the theoretical demands. One would not be in a position to reject it for another that gave a better fit, but did not meet the theoretical requirements as well. This does not preclude the possibility of fitting various curves as a basis for future hypotheses.

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[1] A. M. Mood. Introduction to the theory of statistics. N. Y. McGraw-Hill Book Co., Inc. 1950. p. 317.

USE OF DATA FROM EXPERIMENTS DESIGNED FOR OTHER PURPOSES

Available Data

While studies are under way at the present time to obtain data which can be used to estimate the production functions of hogs, dairy cattle and beef cattle, such experiments are still in the idea stage for other animals. In the meantime useful information may be extracted from previous experiments with different original objectives. The data from many of these experiments may be brought together to form a production surface. Limitations and disadvantages that attach to such data are: (1) Conditions of the experiments are not the same, especially between years. Feed quality varies from year to year. Hence, such data will have greater variation than would data from experiments designed for estimation of production coefficients. (2) Wide ranges in the proportions of the feeds fed are not available. (3) Quantity and kinds of minerals fed vary. If minerals are used as a variable, this is an advantage. However, the number of feeds in the rations are so great that they much be aggregated in order to make computation of the functions mechanically practical with limited resources. (4) Many combinations of a few feeds are desirable and not always available in experiments. Instead a few using linseed oilmeal, a few using various soybean oilmeal products, a few using tankage and a few using other protein concentrates are to be found. A similar situation exists with hay. In order to accumulate enough observations of hay protein and carbohydrate feeds over a wide range of feeding levels, feeds within classes must be transformed to their equivalent in one of the feeds. This transformation procedure constitutes a serious limitation. *A priori* knowledge as to the substitution relationships between members of a class of feeds over the entire ration range must be assumed. For want of better estimates, linear relationships of Morrison's may be used. However, substitution is not entirely a matter of digestible nutrients even within classes of feeds. The effects of palatibility may appear within feed classes as well as between feed classes. On the other hand, data from past experiments are available. Until new experiments are performed, these old experiments are the best source of feed utilization information. Where a sufficiently wide range of rations (factor combinations) are available and care is taken to obtain a homogeneous set of experiments with regard to physical conditions, useful information may be obtained.

In order to obtain a useful range of rations, information from a number of experiments must be drawn upon. In some instances only a few of the lots of an experiment are useful. Check lots fed ordinary rations where the experiment was one of determining effects of drugs or various vitamin and/or minerals could be used. In an overall sense minerals are of interest too. However, given the present knowledge of mineral requirements, information on feeds other than minerals is more useful economically. Minerals may be provided in sufficient supply at a nominal cost whereas other feeds are relatively higher priced and make up a larger portion of the diet. A practical method of reducing the number of variables that must be handled is to choose only such of these past experiments as are known to have permitted the animal access to a plentiful supply of minerals. Minerals are not then a limiting factor. Later, more refined work may build up those areas of the production surface in which various substitution rates between minerals are obtained and the substitution rates of various feeds determined when minerals are a limiting factor.

Most feeding experiments do not record the performance of individual livestock units. Instead the average performance or group performance of a number of animals in a lot are obtained.[1] These averages are satisfactory for establishing points on a production surface. While more efficient results would be obtained if each of the animals were used to estimate a different point, less variation from the true surface is expected of these averages than from a num-

[1]Lots of 20 to 30 animals are not uncommon although 8 to 10 is the most frequent number.

ber of single animal recordings equal to the number of averages.

Difficulties Involved in the Use of Data From Experiments Not Designed for Production Function Derivations

In order to obtain enough data from experiments with a specified class of animals such as hogs, it is often necessary to use experiments conducted a number of years ago. If one goes back as much as twenty or thirty years, there is a possibility that animal types have changed to some extent. This problem is of relatively minor importance. Of greater importance is the differences in methods of feeding encountered. In a number of experiments in hog feeding, the animals were (1) permitted free choice, (2) limited in one feed and permitted to eat all they chose of others, or (3) fed one ration for a time and then another during the latter stages of fattening. The ration path of the pigs appears in Figure 31. In the case of (1), one hog might eat ration OA for a day while another hog was eating ration OB. Similarly, the next day or period of time the first hog might eat ration CD while the second hog was eating ration EF. In the case of (3), the hogs could stay on one ration such as OG for a half or so of the feeding period and then change to a ration such as HI. In the case (1) the hogs individually would wander up the production surface according to their own inclination,

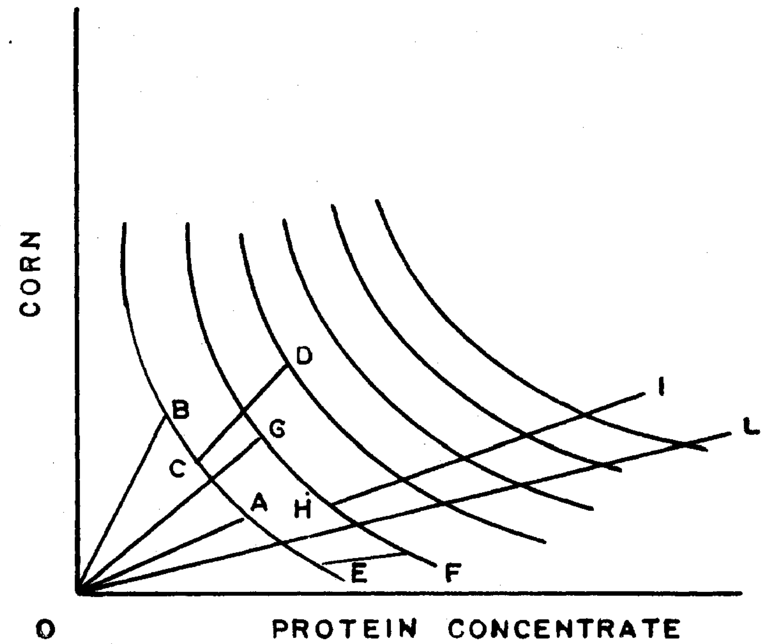

Fig. 31. Various ration paths resulting from different feeding techniques.

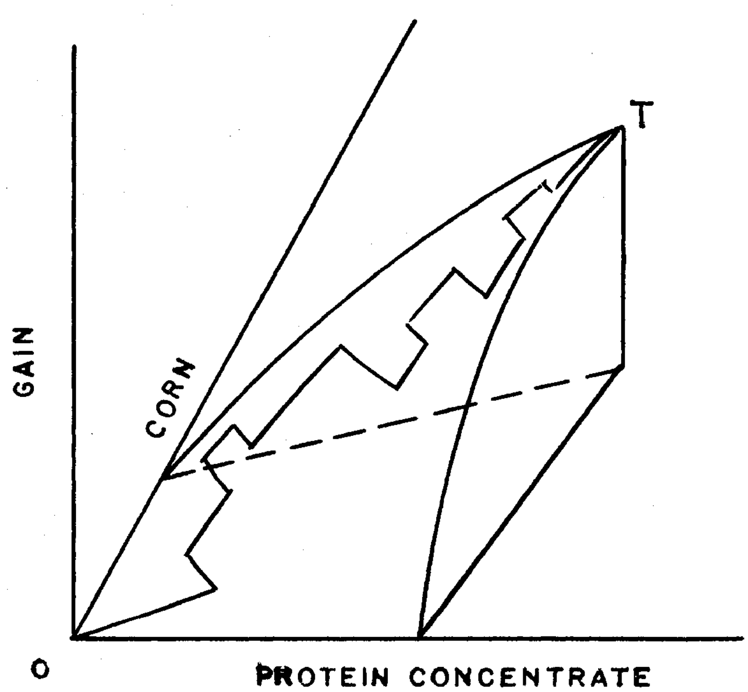

Fig. 32. A possible ration path when the animal has free choice of feeds.

e.g., along a path such as OT (Fig. 32). In case (2), they would follow much the same procedure except the amount of one or more of the feeds eaten per day is set at some maximum amount. Under case (3), all hogs would have to follow the same ration path, e.g., OL, (Fig. 31).

When data from one or more of the above situations is used to derive a production function, some difficulty of interpretation is encountered if the effects of feeding a ration of feeds in fixed proportions is desired. When the data may be used to estimate a production function based on specified feeds and fixed rations instead of the system used in the experiments, an important assumption must be made. It must be assumed that pigs or other livestock can change rations from day to day, at least when they do it voluntarily, and not affect gains differently than if each were kept on a specified ration over the time. Nutritionally there is some basis for such an assumption. It is likely[1] that pigs are unable to store protein. Yesterday's high intake of protein should have little effect on growth a day or so later.

The logic used is that one pig is used to measure the effects of a specified ration. Next day a different pig eats that ration while the first pig furnishes an observation on

[1]Maynard, op. cit., pp. 106-107.

the effects of a different ration. Since the pigs are not weighed each day in practice an average of the rations eaten during the time between weighings is used. Some loss of information occurs, but the results are probably still close enough to the instance of fixed rations to be useful in practical feeding.

The fact that the pigs seemed to eat a wide variety of rations indicates that either pigs have wide ranges in taste and/or (2) the pigs were fed different feeds which varied in palatibility from lot to lot and experiment to experiment. Even though the pigs may have different tastes, they may eat to capacity of a fixed-proportion ration when there is no choice. Pigs probably eat to capacity regardless of the ration as long as it is not actually distasteful. Thus, it is likely that estimates of a production surface intended to show the amounts which a pig will eat of various fixed-proportion rations to achieve specified gains can be derived from observing pigs fed free choice.

Different Substitution Rates at Different Output Levels

Since young growing stock need more protein than mature animals, the relative nutritive value of feeds changes as the animals approach maturity for market hogs and other livestock when both growth and fattening processes are being carried on together. When output (quantity of live pork for example) is a function of the various feeds and age (maturity), complications in the use of the Cobb-Douglas type of function appear if it is used to estimate production surfaces for livestock. The Cobb-Douglas function with its assumption of constant elasticity over the entire production surface does not take into account the fact that the nutritive value of protein relative to carbohydrate feeds is higher in the young animals than in the older animals. To compensate for this descrepancy and still make use of the Cobb-Douglas function with its ease of calculation, the feeding period may be divided into intervals and a function derived for each interval. It is less serious to assume a constant elasticity for an interval than for the entire production surface. In fact it may be useful to look upon the production relationships in this manner. Livestock feeders usually do not find it practical to change rations more than a few times during the feeding period. Hence, the average substitution rates over these intervals would be information they could use in choosing rations.

Fitting curves to the interval data is accomplished in the same manner as over-all production function estimation. However, the division of the data must be on the basis of the feed inputs. In Figure 33, if OA is the best least squares fit to the observations of Y for each X, limiting the Y observations to an interval OY_1 can cause the curve fitted

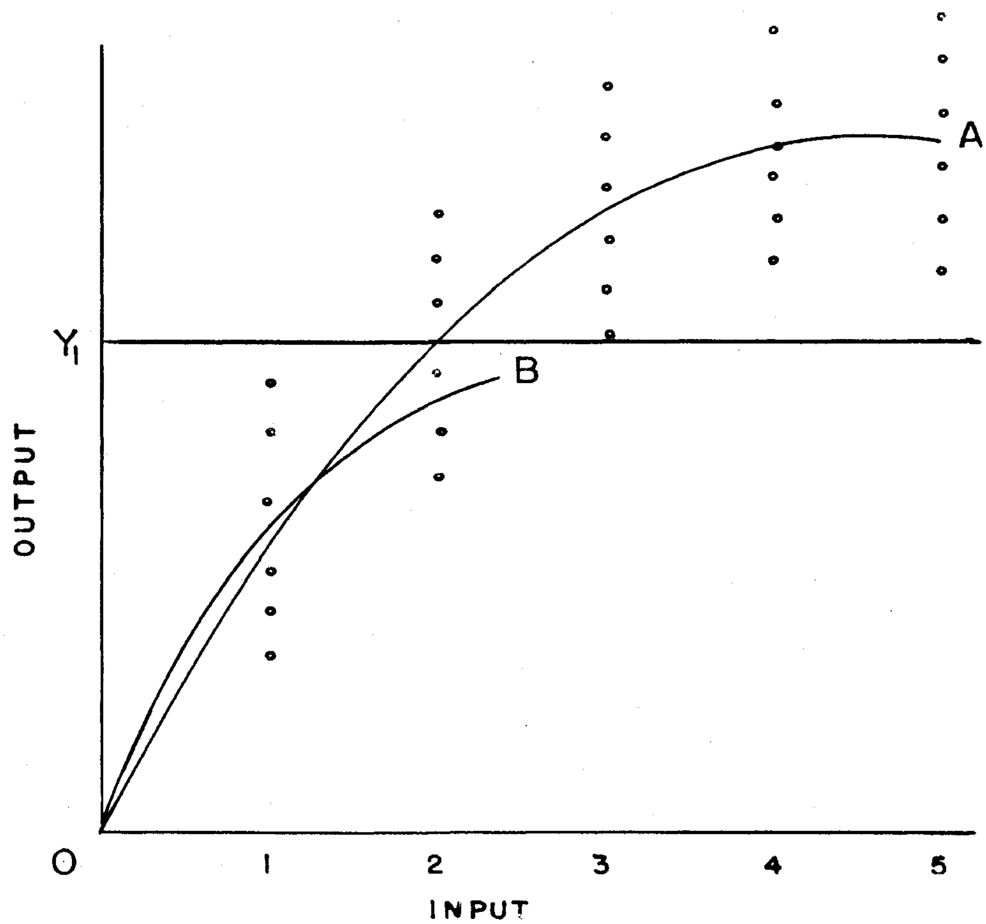

Fig. 33. Effect of setting intervals by output division.

to these observations to differ from the actual interval production curve. OB is an example of what might occur if the intervals were set directly on the basis of output. The estimate of the production surface obtained is distorted. It remains then to choose an interval based on the independent variables. In order to include the outputs in the interval desired, the data may be graphed. An interval of the independent variables which corresponds to that portion of the production surface which is desired may be selected. For example, in Figure 34 observations from yearling steer feeding experiments conducted in drylots are graphed.[1]

OR_A represents the highest alfalfa ration fed while OR_C represents the highest corn ration. Observations of gains do not always fall on the ration lines. This condition is peculiar to experiments where the animals are not fed rations in which the feeds are combined in fixed proportions. The cattle in these experiments were not allowed free choice, however. Instead the rations were changed at the discretion of the feeder. In this case line MR is drawn to include output observations less than 200 pounds. In deriving the interval function, all observations of gains from combinations of feed falling within the triangle OMR are included in the estimate of the product contour. A second function

[1]Only enough data are plotted to indicate the method.

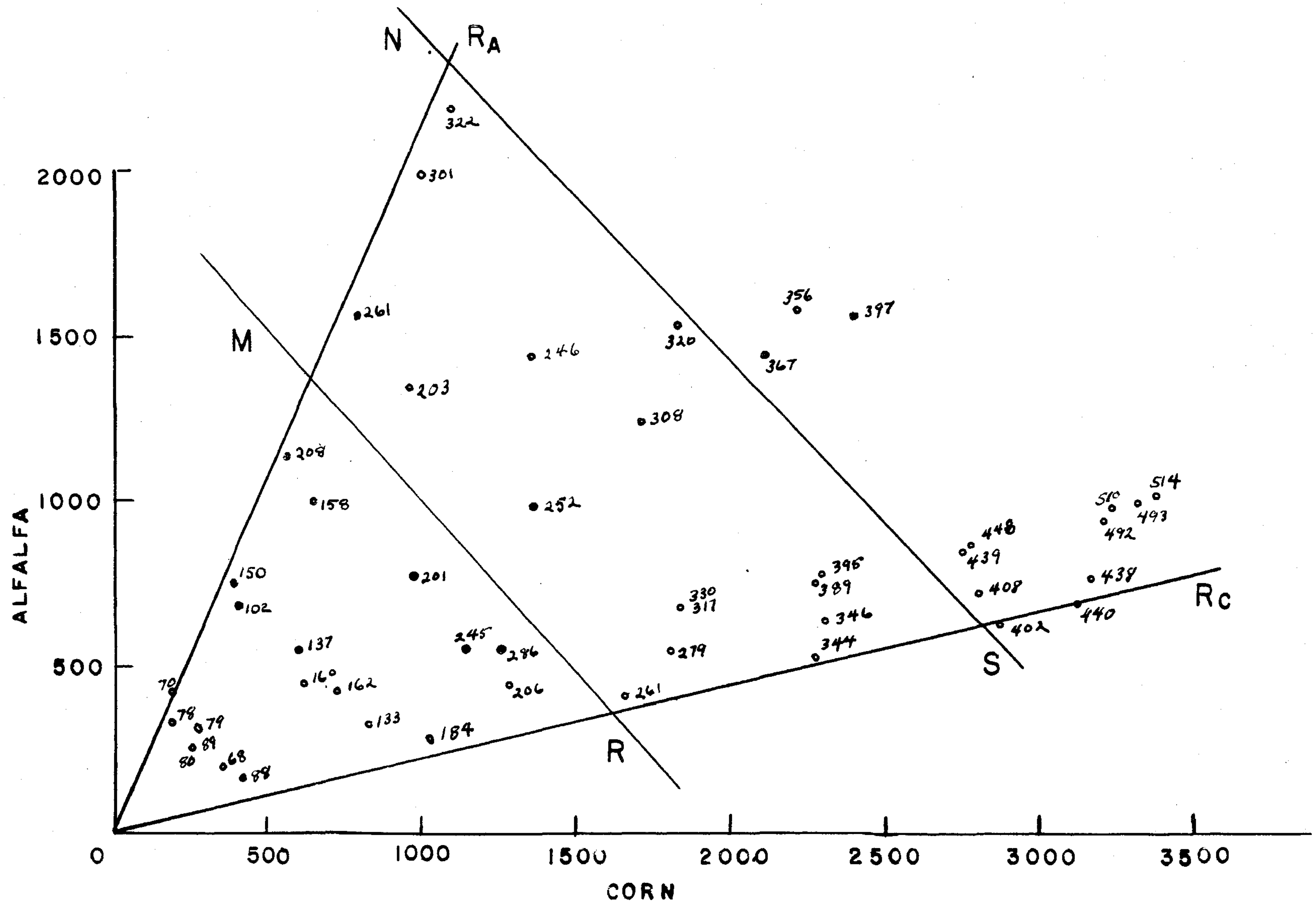

Fig. 34. Scatter diagram of weight gains in pounds relative to pounds of feeds consumed.

may be fitted to the remaining observations. Enough observations must be available in each area to give reasonable estimates.[1] A slight over-lap of the intervals can be used to increase the number of observations. If estimates of more than two intervals were desired, another line such as NS could be added bounding the second interval on the upperside. The observations in the three areas OMR, MNSR, and everything beyond NS could then be used to derive production functions for each interval respectively. Since the observations within each interval would determine the elasticity of production for that area, a different elasticity could occur for each of the three intervals of the production surface even though the function used estimates only an average one in each case.

When more than two feeds are fed, a simple equation may be used to separate the rations for each interval. Let k_1 be the slope of line MR in Figure 34. Then as any other feed such as linseed oilmeal which was fed would also yield a similar line when plotted against corn. Let k_2 be the slope of that line. Then

$$M = C + k_1A + k_2L$$

where C is the quantity of corn in the ration, A is the quantity of linseed oilmeal. M is merely chosen large enough to

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[1]George B. Snedecor. Statistical methods. 4th ed. Ames, Iowa. The Iowa State College Press. 1948. pp. 359-360.

include the rations that achieve the maximum output desired in the interval. It may be estimated from some of the borderline cases along MR. All of the rations that can be fitted into the formula without exceeding M are accepted for the lower interval calculations. Those that result in a figure larger than M,when entered into the formula,would be used in the calculation of a second interval function.

While interval production functions can indicate a possible difference in substitution rates of feeds in different production intervals, this may be done in two ways, each having a different meaning. On the one hand, the amounts of feed fed during the interval of gains in question may be used to derive the interval function. That is, where the pigs (or other stock) are started at 35 to 45 pounds (weaning weights or beginning feeder weights), all input combinations which are capable of producing 100 pounds of gain could be selected. The second interval for a 200-pound contour would require calculations of just that feed eaten after the pigs reached the weight of 100 pounds of gain.

In practice, weight observations are not taken at exactly the time a pig reaches 100 pounds. It is hard to separate feed eaten before and after the pig gains 100 pounds. Observations of each ration at a point approximating the beginning of the desired production interval and those approximating the end of the interval may be used. There will be some variation in the starting weights of various animals but this is not objectionable since an average over the interval is the only objective. This method is useful with data where rations have not been held in fixed proportions as well as those where they have.

The second method of deriving an interval function uses the same ways of defining the beginning and ending of the interval. It does not, however, entail excluding feed fed during previous intervals. Instead the regression of gains in the interval desired is run directly on the feeds fed to attain total production indicated by this interval. For example, the 150-300 pound gain interval of steers would call for observations of gains from quantities of feed fed that achieve gains in this area. The resulting production function gives marginal rates of substitution which are based on feeding rations over the entire interval from 0 to 300 pounds although the gains in the area of 0 - 150 pounds are not used as observations. It is not possible to derive the least cost ration for the 150-300 pound interval directly from this function and prices. An iso-product contour derived from this function when mapped in the manner of the contours in Figure 15, indicates the average rate of substitution over the interval from 0 to the contour, but takes into account the change in average rate of substitution from interval to interval. An expansion path could be drawn after the manner of Figure 15.

The results of this procedure, when using the observations on yearling steers, are as follows. The first function covering an interval up to 200 pounds of gain was

$$Y = .5331X_1 + .0868X_2 + .22668X_3 - .2136$$

and the second function covering an interval which included 150 to 300 pounds of gain was

$$Y = .5653X_1 + .0602X_2 + .1932X_3 - .0451.$$

The X's and Y are logarithms of the feeds and gain respectively. X_1 represents corn; X_2, linseed oilmeal; X_3, alfalfa hay. Original measurements were in pounds.

Since the equations are in logarithmic form, the coefficients are elasticities. An increment of corn is slightly more effective in the second stage than in the first. On the other hand, the relative effectiveness of adding alfalfa or protein has decreased as might be expected. During the final finishing process, the more carbohydrate concentrate that can be fed, the greater the gain. In the beginning period, some growth is still taking place and the animals need the protein and hay. Also, animals are not yet adapted to a high concentrate ration at the beginning of a feeding period and may require more forage.

The product contour for the second interval alone may be derived indirectly from the results of these two functions. (A contour derived from the first interval would not be any different under either method.) An estimate of feed used to achieve the two outputs using the same ration is calculated from the above functions. The difference is an estimate of feed fed to achieve the gains made during the second interval alone. As many such points as desired can be calculated and graphed. When more than two feeds were fed, all but two must be held at fixed levels. The iso-gain contours in Figure 35 for the second feeding interval was derived from the yearling steer production function.[1]

The problem of deriving marginal rates of substitution from this function is difficult. For practical purposes arithmetic estimates such as the average substitution rates between ration points as shown in Table VII may suffice. These do not, of course, apply to points on the ration lines but to the intervals between.

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[1]Appendix B

Table VII. Marginal rates of substitution of alfalfa for corn.

Alfalfa	Corn	△ Alfalfa	△ Corn	△ Corn / △ Alfalfa
147.1	1181.0			
278.7	1022.1	131.6	-158.9	-1.21
319.2	990.4	41.1	- 31.7	- .77
367.4	958.4	48.2	- 32.0	- .66
427.2	925.0	59.8	- 33.4	- .56
499.0	891.8	71.8	- 33.2	- .46
572.2	864.4	73.2	- 27.4	- .37
686.5	824.7	114.3	- 39.7	- .35
815.3	790.9	128.8	- 33.8	- .26
1407.1	690.0	591.8	-100.9	- .17

Iso-Product Contours for Yearling Steers

One hundred fifty-pound iso-gain contours were derived for both the first 150 pounds and for the second 150 pounds gained by yearling steers. Protein was fixed at two levels (100 and 140 pounds) in each case. As expected, the 150-pound iso-product contour with a fixed level of linseed oilmeal of 140 pounds lies below that of the contour with 100 pounds of linseed oilmeal (Fig. 35). That is, less corn and hay are needed to maintain a constant output if more protein is added to the ration.

The vertical distance between iso-product contours A and B and between C and D is greatest at the left ends of the curves, i.e., when alfalfa makes up a relatively large

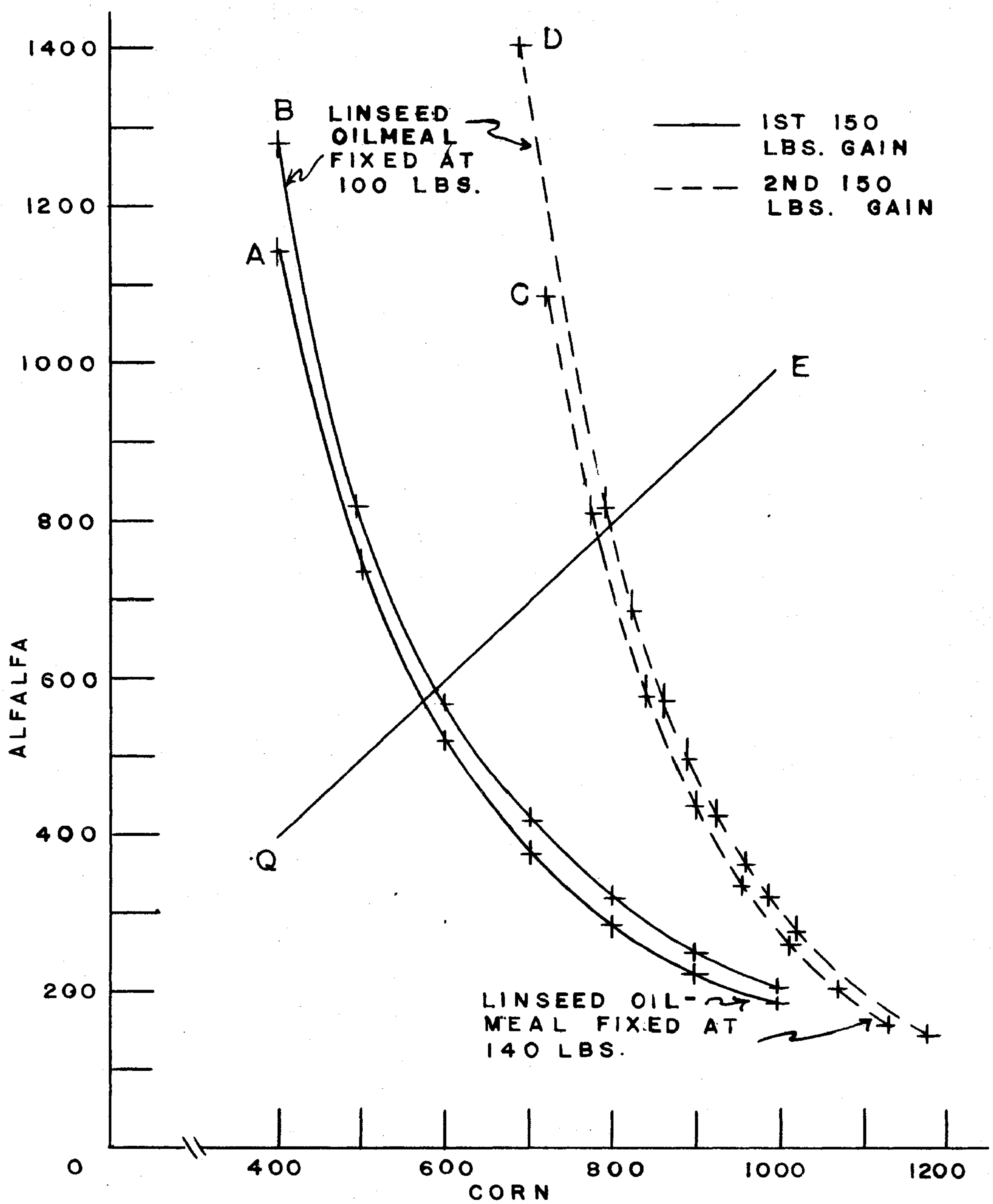

Fig. 35. Iso-gain contours for yearling steers.

proportion of the ration. Additional protein substituted for alfalfa (amount of corn constant) has greater effect in reducing the amount of alfalfa needed to maintain constant output when the amount of alfalfa relative to protein is greatest.

This situation agrees with the theoretical notion that in order to maintain a constant output as more and more of one factor (alfalfa) is added to a ration relative to another (protein), the amount of the second (protein) displaced by a unit of the first (alfalfa) becomes less. In other words, there is a diminishing marginal rate of substitution of alfalfa for linseed oilmeal or vice versa. Similarly, the horizontal distance between iso-product contours A and B and between C and D is greatest at the lower ends of the curves. As more and more corn relative to protein is included in the ration, the amount of protein displaced by a unit of corn, when maintaining constant output, becomes less. The marginal rate of substitution of corn for proetin is a diminishing rate or vice versa.

A diminishing rate of substitution is also presented between corn and hay. When the level of alfalfa (contour A) is 1000 pounds, it requires approximately 50 pounds of corn to replace 200 of alfalfa. However, in order to maintain the same output when the alfalfa level is 400 pounds, addition of 50 pounds of corn replaces only about 50 pounds of alfalfa. (Protein was fixed at 140 pounds throughout the illustration.) It is recognized that a constant output is not actually maintained with feeder animals. With animals it is, however, realistic to consider the various substitutions as possibilities in achieving the given outputs.

Similar diagrams could be drawn with one or the other of the feeds fixed at given quantities while the product contours for two other feeds were examined. This method is a simple way of attacking the problem of multiple feed inputs. In some instances, simultaneous equations in which all feeds are permitted to vary may be resorted to for more exact relationships. However, where the simple relationship between two feeds at a time is sufficient, fixing the rest of the feeds at various levels is appropriate.

The difference in marginal rates of substitution between feeds during different parts of the production period is illustrated by the difference in the slope of A compared to C and B compared to A. The contours A and C were derived on the basis of 150-pound gains by yearling steers weighing an average of 671 pounds. C and D represent additional feed required for gains of an additional 150 pounds by the same steers. With the protein level the same for contours A and C, the slope of the two curves at points on the same ration line is different, i.e., rates of substitution are different. For example, a casual inspection of the points where the curves intersect ration line QE shows curve C to be steeper than A. Thus, when alfalfa and corn are fed in equal parts, the average rate of substitution of alfalfa for corn ($\Delta C/\Delta A$) is less in the latter stages of production or fattening. A relatively greater amount of alfalfa is required to replace a unit of corn. With the same relative prices for alfalfa and corn, it would pay to feed more corn in the latter stages of feeding than in the earlier stages. Since the animals are engaged mainly in adding fat during the latter part of the feeding period, these results are logical. The concentrated carbohydrate feed, corn, can be much more easily converted to fat than can hay. During the earlier part of the feeding period when the animal was growing as well as fattening, corn could not be utilized as efficiently relative to hay.

It may also be noted that the contours for the second 150 pounds of gain lie to the right and above the first 150 pounds of gain. This condition implies that a pound of gain during the finishing process requires relatively more feed. Fat, the product of the second stage of feeding, contains considerably more food energy (calories) per pound than does muscle, which makes up a larger portion of the product during the first part of the feeding period. Also, as greater quantities of feed are consumed by the animal, there is a biological limit to the weight which the animal can reach.

The effect of using information on price ratios and marginal rates of substitution of one feed for another are tabulated in Tables VIII and IX. The least cost ration of corn and alfalfa that will achieve 150 pounds of gain are underlined in the column headed "Total cost of C & A." for two levels of protein. In the first period this is a combination of 600 pounds of corn and 569 pounds of alfalfa with 100 pounds of protein and 600 pounds of corn and 510 pounds of alfalfa with 140 pounds of protein. At these points the marginal rate of substitution of corn for alfalfa is approximately equal to the inverse price ratio of corn and alfalfa. Theoretically, the marginal rate of substitution is equal to the inverse price ratio when the least cost combination is obtained. In practice, it is often convenient to tabulate the data at descrete intervals and obtain an approximation to equality. The ration containing only 100 pounds of protein is less costly at the stated prices than the ration containing 140 pounds.

Only two possible levels of protein supplement are shown. It is possible that some other level is even more profitable. It is difficult to illustrate the most profitable point by graphic methods when three feeds are allowed to vary. Simultaneous equations could be used. However, it is simpler to make up tables or diagrams in which two feeds vary and the rest are held constant, first at one level and

Table VIII. Cost of producing the first 150 pounds of gain on yearling steers.

Linseed oilmeal constant at 100 pounds								
$\frac{dC}{dA}$[a]	$\frac{PA}{PC}$[b]	Corn	Alfalfa	$\frac{C}{A}$[c]	Cost of corn	Cost of alfalfa	Total[d] cost C&A	Total[e] cost C,A&P
-2.44	.552	1000	205.0	4.9	15.890	1.797	17.887	20.877
-1.78	.552	900	253.0	3.6	14.301	2.530	16.831	19.821
-1.25	.552	800	320.3	2.5	12.712	2.807	15.519	18.509
- .84	.552	700	418.2	1.7	11.123	3.666	14.789	17.779
- .53	.552	600	568.7	1.0	9.534	4.985	14.519	17.509*
- .30	.552	500	819.2	.6	7.945	7.180	15.125	18.115
- .16	.552	400	1278.8	.3	6.356	11.209	17.565	20.555

Linseed oilmeal constant at 140 pounds								
$\frac{dC}{dA}$[a]	$\frac{PA}{PC}$[b]	Corn	Alfalfa	$\frac{C}{A}$[c]	Cost of corn	Cost of alfalfa	Total[d] cost C&A	Total[e] cost C,A&P
-2.62	.552	1000	184.7	5.4	15.890	1.619	17.509	21.695
-1.98	.552	900	226.8	4.0	14.301	1.988	16.289	20.475
-1.39	.552	800	287.1	2.8	12.712	2.516	15.228	19.414
- .93	.552	700	374.8	1.9	11.123	3.285	14.408	18.594
- .59	.552	600	510.0	1.2	9.534	4.470	14.004	18.190
- .34	.552	500	734.0	..7	7.945	6.434	14.379	18.565
- .17	.552	400	1146.0	.3	6.356	10.045	16.401	20.587

[a]Marginal rate of substitution of corn for alfalfa.

[b]Price ratio of alfalfa to corn. The average of prices for 1933-51 was used. See B. French and W. Chryst. Prices affecting Iowa farmers. Ames, Iowa. April, 1950. (Mimeo.)

[c]Ratio of corn to alfalfa in the ration fed.

[d]Total cost of corn and alfalfa fed.

[e]Total cost of corn and alfalfa plus protein.

Table IX. Cost of producing the second 150 pounds of gain on yearling steers.

Linseed oilmeal constant at 100 pounds								
$\frac{dC}{dA}$ [a]	$\frac{pA}{PC}$ [b]	Corn	Alfalfa	$\frac{C}{A}$ [c]	Cost of corn	Cost of alfalfa	Total cost C&A [d]	Total cost C,A&P [e]
-1.21	.552	1181.0	147.1	8.0	18.766	1.289	20.055	23.045
- .77	.552	1022.1	278.7	3.7	16.241	2.443	18.684	21.674
- .66	.552	990.4	319.2	3.1	15.737	2.789	18.535	21.525
- .56	.552	958.4	367.4	2.6	14.229	3.220	18.449	21.439
- .46	.552	925.0	427.2	2.2	14.698	3.744	18.442	21.432*
- .37	.552	891.8	499.0	1.8	14.171	4.374	18.545	21.535
- .35	.552	865.5	572.2	1.5	13.735	5.015	18.750	21.740
- .26	.552	824.7	686.5	1.2	13.104	6.017	19.121	22.111
- .17	.552	790.9	815.3	1.0	12.567	7.146	19.713	22.703
	.552	690.6	1407.1	.5	10.974	12.333	23.307	26.297

Linseed oilmeal constant at 140 pounds								
$\frac{dC}{dA}$ [a]	$\frac{pA}{PC}$ [b]	Corn	Alfalfa	$\frac{C}{A}$ [c]	Cost of corn	Cost of alfalfa	Total cost C&A [d]	Total cost C,A&P [e]
-1.36	.552	1129.7	162.5	7.0	17.951	1.424	19.375	23.461
-1.04	.552	1073.9	203.6	5.3	17.064	1.784	18.848	23.034
- .78	.552	1016.5	259.0	3.9	16.152	2.270	18.422	22.608
- .58	.552	957.9	334.2	2.9	15.221	2.929	18.150	22.336
- .41	.552	878.2	437.8	2.0	14.272	3.837	18.109	21.946
- .30	.552	840.0	579.1	1.4	13.348	5.076	18.424	22.610
- .19	.552	772.0	808.0	1.0	12.267	7.082	19.349	23.535
- .15	.552	718.7	1085.6	.7	11.420	9.515	20.935	25.121
- .07	.552	658.3	1538.8	.4	10.460	13.488	23.948	28.134
	.552	597.4	2438.5	.2	9.493	21.373	30.866	35.052

[a] Marginal rate of substitution of corn for alfalfa.

[b] Price ratio of alfalfa to corn (1933-51 average).

[c] Ratio of corn to alfalfa in the ration fed.

[d] Total cost of corn and alfalfa fed.

[e] Total cost of corn and alfalfa plus protein.

then at another. Instead of just two 150-pound iso-product contours for a feeding period as shown, many more would be included. This is realistic in cases where one feed or another is considered to be of fairly fixed amount by the feeder. Farm raised feeds are sometimes looked upon in this fashion when the farmer considers livestock enterprises as a means of marketing primary products.

Tests of Significance

So far, methods have been indicated that estimate the various coefficients of the production function. Various functions were presented from which product contours could be obtained at fixed levels of gain. Empirical examples illustrated that estimated marginal rates of substitution along such contours weren't the same for all points on ration lines. The estimates of substitution rates were, however, derived from sample data. Therefore, the regression coefficients for different production intervals are subject to variability. It is logical to test the differences between regression coefficients from the different intervals of production to determine if the differences are great enough to be significant. If the difference is great enough to occur by chance less than some acceptable level of probability such as 5 percent, the hypothesis that the coefficients are the same would be rejected. This result would substantiate the conclusion that least cost rations vary for different parts of the feeding period, i.e., substitution rates vary with output level.

A straight forward test of the differences between regression coefficients is possible if the observations on output are independent with normally distributed errors. This condition in the strictest sense requires that each animal or lot of animals be used only once to estimate a point on the production surface. An element of randomness would also be necessary in the selection of animals. An example of such a test was made using the two-year-old steer experimental data from the Iowa State College Animal Husbandry Department's records previously discussed in connection with margins. Two regressions were run. The first was for observations of gains on feed inputs needed to estimate the first 175 pounds of gain. The second regression was for a second 175 pounds of gain or from 175 to 300 pounds of gain. The methods of slicing the production surface into these two intervals were discussed previously. The areas estimated by the two regressions were overlapped slightly in order to be sure of including all feed combinations that could achieve 175 pounds of gain. The main objective of this procedure was to obtain estimates of the average substitution rates for the respective areas. The dividing line for feed combinations included in the first interval was set slightly above that estimated as necessary to achieve 175 pounds of gain. Similarly, in dividing the function to obtain an estimate of the second 175 pounds of gain the dividing line was set just below the amounts of the feed combinations that could achieve 175 pounds of gain. Some correlation between the two regression estimates was introduced by this method but was assumed negligible. Each lot average gain was used only once to indicate an output resulting from a specific combination of feeds.

The interval production functions derived were as follows:

$$Y_1 = .2909 \log X_1 + .1992 \log X_2 + .0427 \log X_3 + .7971$$

$$Y_2 = .3432 \log X_1 + .3068 \log X_2 + .0248 \log X_3 + .2495$$

where X_1, X_2 and X_3 represent pounds of corn, alfalfa and linseed oilmeal respectively. Y_1 and Y_2 are pounds of gain in the first and second interval.

In order to make a test of the differences between regression coefficients with pooled variance, the individual variances should be homogeneous. An F test was used to test the hypothesis that there was no difference between variances.

$$F = \frac{s_1^2}{s_2^2} = \frac{.003278}{.003122} = 1.05$$

where s_1^2 and s_2^2 are the sample variances from regression[1] of the first and second intervals respectively. F was not significant (10 percent level), so the null hypothesis was not rejected.

Differences between corresponding b values (regression coefficients) of the interval functions were then tested with the t test,

$$t_j = \frac{b_j^{(i)} - b_j^{(i)}}{s_{y.123}\left(C_{jj}^{(i)} + C_{jj}^{(i)}\right)^{\frac{1}{2}}}$$

where i is the equation number, i.e., 1 stands for the first interval and 2 for the second. The subscript j refers to the particular b's for which the difference is being tested. Thus, b_1 is the regression coefficient for corn, b_2 for hay and b_3 for protein supplement. Sums of squares of the deviations from regression were pooled in calculating $s_{y.123}$, the standard error. The C_{ii} are Gauss multipliers.[2]

The t's with 86 degrees of freedom were as follows:

$t_1 = 1.34$

$t_2 = 1.31$

$t_3 = .37$

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[1]Snedecor, op. cit., 368.

[2]Snedecor, op. cit., pp. 364-373.

Since all t's were non-significant, the null hypotheses of no differences between pairs of b's was not rejected. It is likely that the assumption of average substitution rates for the entire feeding period in the discussion of margins is realistic with two-year-old steers. Substitution rates of one feed for another are the same at all points on a ration line or nearly so.

The above analysis requires experiments with large numbers of animals. An animal is used for only one observation on the production surface. Feeding large numbers of animals is expensive. When the main objective is one of obtaining estimates of the production coefficients alone, the system of making numerous observations on each animal is satisfactory.

Tests of significance between regression coefficients obtained in such a manner are complicated, however. Since the observations are made on the same animal's gains, they are correlated. A series of regression lines, one for each animal, is obtained in theory. The gain observations from which each is estimated are correlated although the group of lines are independent and lead to a good estimate of the true feeding relationship. The estimate of a standard error term for the individual interval regression equations becomes involved because of this correlation. Besides this difficulty, there is correlation between the two production interval groups of observations since the same animals are carried over into the second period. Animals that eat heavily in the first and make relatively large gains may have a tendency to taper off during later stages of fattening more than do the lighter eaters in the first period.

A test of difference between regression coefficients for the yearling steer function is affected by such correlations. Tests can be made but the cost of labor and time involved would be relatively large. However, evidence from other sources is available to substantiate the desirability of obtaining estimates of substitution rates for different intervals of production rather than one over-all average.[1]

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[1]Abrams, op. cit., p. 336; and Morrison, op. cit. p. 355.; and Maynard, op. cit., p. 335.

SUMMARY AND CONCLUSIONS

No two feeds are alike in nutritive value to animals since they differ in amount and quality of protein, fat, carbohydrates, minerals and vitamins. Comparison of their nutritive values is difficult because the nutritive needs of livestock differ with species, age, sex, condition, and individuals. While present feeding standards are of great value, they have the following limitations: (1) Feeds are assumed to substitute for each other in constant ratios. (2) Substitution ratios apply to a limited portion of possible feed combinations. (3) In many cases average nutritive values are given as if they applied to all classes of livestock. (4) The same substitution ratios are given for different weights and ages of animals. (5) The same substitution ratios are often offered for animals whether fattening, lactating, working, growing or gestating.

The use of these feeding standards to discover least cost rations or maximum output from a given set of resources is limited. Production economics theory calls for production functions from which the marginal rates of substitution between feeds in any combination and at any output level may be derived. At present such functions are not available for livestock feeding.

Various peculiarities appear in production functions when adapted to livestock. The animal's limited stomach capacity and its minimum feed requirement for maintenance are two modifying factors. The problem of getting the nutritive elements into a small package occurs when the greater intensity of production is desired. On the other hand, animals continue to eat whether or not they are producing. Thus, the same feed combinations fed over different lengths of time will produce different outputs. A true production surface can only be derived if limited feeding as well as full feeding is included. However, a production surface can be derived from full feeding experimental data if the lengths of production periods are disregarded. Such a function has value under certain economic conditions. Time may be directly incorporated into the functions as a variable or introduced into the output side of the picture as average daily gains, average daily milk output, or other average daily output.

Since weight gains or milk output of the same quantity need not be of the same quality product differences should account for deriving production functions. Little empirical investigation has been carried on in a manner to give complete quantitative measures of the physical production relationships in this fashion. If such functions were available, the least cost ration which would achieve a desired grade or quality of product could be found by equating the inverse ratio of the feed prices to the marginal rates of substitution of the feeds. If different marginal rates of substitution occurred at different output levels, the least cost ration would be achieved by changing the ration fed at the appropriate times.

The choice of the most profitable grade or quality of product to produce from a given livestock unit depends upon the price ratios of the products and feeds. An over-all production surface with grades outlined on it would be useful in this respect. The marginal rates of change along iso-value lines could be equated to the price ratios of the feeds to discover the optimum product.

An empirical example of the average least cost ration over a past period of 19 years and the margins that occurred for two-year-old steers was made. Margins were able to offset losses incurred in increasing the weight of the cattle and vice versa. Profits from weight gains are usually small in heavy cattle making them most important in times of narrow margins. Because of the tendency to diminish with length of feeding period, profits from weight gains are not very useful in offsetting unfavorable margins with heavy steers. With yearlings and lighter cattle, where profits from weight gains make up a larger proportion of total profits and where gains per 100 pounds of feed do not fall off as rapidly, weight gains are more important.

Various alternative mathematical models are available for deriving production functions. The advantages of most are offset by complicated methods of fitting them to the empirical production data. The Cobb-Douglas type of function is one of the simplest to use. It meets the requirements of production theory in so far as indicating diminishing returns and marginal rates of substitution between feeds. Unfortunately, it assumes constant elasticity over the production surface, i.e., feeds are assumed to substitute for each other at the same rate at any level of output if fed in the same proportions. This limitation may be eleiminated for practical purposes by dividing the production surface into intervals. Various methods of accomplishing this and some examples using empirical observations were presented.

Another equation, the quadratic, is realistic in many cases of production but difficult to use empirically. Many more computations are necessary in order to obtain estimates of its coefficients than are necessary with the Cobb-Douglas function of the same number of feed variables. The Spillman function has possibilities, but is better adapted to fertilizer experimentation than to livestock production. Also, ordinary regression techniques of estimating its coefficients cannot be used except in special situations.

Alternative means of obtaining the necessary data to estimate empirical production functions include direct experimentation and use of past experiments which were conducted for other purposes. For future experiments, the use of fixed proportion rations with limited feeding and full feeding are recommended. However, past experimental data on feeding are the best source of information available now. Such data has limitations. Nevertheless useful information can be derived from this source until other data is available.

As an example, empirical estimates of iso-product contours for yearling steers were derived for the first and second 150 pounds of gain. Feed prices were assumed and the economic implications of diminishing marginal rates of feed substitution were illustrated. Least cost rations differed for different parts of the feeding period because the substitution rates were not the same between feeds in the same ration for the entire period.

Since the estimates of the various coefficients of the production functions are based on sample observations, statistical tests of the difference between coefficients estimated for different parts of the feeding period can be made. An example with two-year-old steers presented no evidence of feed substitution rates changing with output level. Hence, the same least cost ration would to the entire production period. However, this is not true of less mature animals with longer production periods. Unfortunately, the least costly and easiest methods of obtaining data for good estimates of the coefficients of production complicate the tests of differences between the coefficients. For estimation of coefficients alone,a number of observations of each animal's gains with respect to feed eaten may be made. The correlation between observations introduced by this method makes necessary more involved statistical tests.

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APPENDIX A.

THE LEAST COST RATION

The least cost ration is found by equating the inverse price ratios of the feeds to the marginal rates of substitution. Thus, for two-year-old steers with a production function of

$$Y = .328589 \log X_1 + .23506 \log X_2 + .0333254 \log X_3 + .60983,$$

the least cost ration is found when

$$\frac{dX_1}{dX_2} = \frac{PX_2}{PX_1} \qquad \text{and} \qquad \frac{dX_1}{dX_3} = \frac{PX_3}{PX_1}.$$

X_1 represents pounds of corn; X_2, pounds of hay; X_3, pounds of linseed oilmeal; Y, pounds of gain.

The average prices of corn, linseed oilmeal and alfalfa for the period 1933-51 were \$.01589, \$.008765 and \$.0299 per pound respectively.[1] The marginal rates of substitution for feeds placed equal to the price ratios follow:

$$\frac{dX_1}{dX_2} = -\frac{.2351X_1}{.3286X_2} = -\frac{.008765}{.01589}$$

$$\frac{dX_1}{dX_3} = -\frac{.03332X_1}{.3286X_3} = -\frac{.0299}{.01589}.$$

Hence, $X_2 = 1.30103X_1$ and $X_3 = .053898X_1$

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[1]Derived from figures compiled by Chryst and French brought up to date. See B. French and W. Chryst. Prices affecting Iowa farmers. Ames, Iowa. April, 1950. (Mimeo.)

The values for X_2 and X_3 are substituted into the production function to find the outputs for various quantities of the least cost ration. Similarly, they may be substituted into the time function to find the length of time animals on full feed take to eat the various amounts of the ration. However, the ration of alfalfa equal to 1.3 times the amount of corn was assumed too high to achieve good-choice beef. A 1:1 ration was used instead because this was within the range of the original data whereas 1:1.3 was not. Also, the corn-protein ration was rounded to 20:1 for convenience in computation and because practical rations are usually kept easy to understand percentage terms.

The time and gain functions derived were as follows:

$$Y_T = .4863X_1 + .4252X_2 + .0253X_3 - .8586$$

$$Y_G = .3286X_1 + .2350X_2 + .03332X_3 + .6099$$

where Y_T is the logarithm of time in days and Y_G is the logarithm of gain in pounds. X_1, X_2 and X_3 are the logarithms of the pounds of corn, alfalfa and linseed oilmeal respectively. Substituting, the least cost ration results in

$$\log Y_T = .9368 \log X_1 - .891577$$

$$\log Y_G = .5969 \log X_1 + .05665.$$

Letting Y_T = 30, 60, 90, 120 and 150 days in succession and solving simultaneously obtained the results tabulated in Table I in the text.

APPENDIX B.

DERIVATION OF A 150-POUND ISO-PRODUCT CONTOUR FOR THE INTERVAL OF 150-300 POUNDS OF GAIN

The following production functions were derived from yearling steer feeding experiments as indicated in the text:

$$Y_2 = .5653X_1 + .0602X_2 + .1932X_3 - .0451$$

$$Y_1 = .5332X_1 + .0868X_2 + .2668X_3 - .2136$$

where Y_2 is gain from feed combinations achieving gains approximating 150-300 pounds of gain and Y_1 represents the corresponding gains in the 0-200 pound interval. The variables are expressed in logarithms.

An iso-product contour for the second interval with linseed oilmeal at a fixed level (100 pounds) was derived as follows: It was assumed that rations of fixed proportion were fed and that the protein supplement would be mixed with or combined in fixed proportions with the other feeds in feeding. Thus, with each change in corn and alfalfa, the ratio of protein supplement to corn changes. Amounts of linseed oilmeal were arbitrarily set at the figures indicated in Table I and the amounts of alfalfa and corn eaten during the second interval were computed (Table X).

Table X. Feed combinations that achieve 300 and 150 pounds of gain with yearling steers.

300 pounds gain			150 pounds gain		
Alfalfa	Corn	Linseed oilmeal	Alfalfa	Corn	Linseed oilmeal
294.2	2362	200	147.1	1181.	100
529.5	1942	190	250.8	919.9	90
600.1	1862	188	280.8	871.6	88
683.4	1783	186	316.0	824.6	86
786.0	1702	184	358.8	777.0	84
908.2	1623	182	409.2	731.2	82
1103.	1556	180	457.8	691.6	80
1222.	1468	178	535.5	643.3	78
1435.	1392	176	619.7	601.1	76

The equations used to compute the quantities of feeds fed during the different intervals to achieve 150 pounds of gain in each were obtained in general as follows:

$$Y_1 = a_1 X_{11}^{b_1} X_{12}^{c_1} X_{13}^{d_1} \qquad (1)$$

$$Y_2 = a_2 X_{21}^{b_2} X_{22}^{c_2} X_{23}^{d_2} \qquad (2)$$

where (1) and (2) are general equations corresponding to the production functions for yearling steers. Y_2 and Y_1 represent weight gains. The first subscript refers to the equation and the second to the feed factor. X_{11} represents pounds of corn fed in the first interval, i.e., to achieve 200 pounds of gain starting with the original feeder. X_{21} represents pounds of corn fed to achieve from 150 to 300 pounds of gain.

$$\frac{X_{12}}{X_{11}} = \frac{X_{22}}{X_{21}} = R \qquad (3)$$

$$\frac{X_{22}}{X_{23}} = \frac{X_{12}}{X_{13}} \qquad (4)$$

$$\frac{X_{21}}{X_{23}} = \frac{X_{11}}{X_{13}} \quad . \qquad (5)$$

The same fixed proportions of corn to protein and corn to hay are maintained in both equations.

Solve simultaneously for X_{22}, hay, fed for gains of 150 to 200 pounds. First, substituting from (3) for X_{11} and X_{12}

$$Y_2 = a_2 X_{22}^{b_2+c_2} R^{b_2} X_{23}^{d_2} \qquad (6)$$

$$Y_1 = a_1 X_{12}^{b_1+c_1} R^{b_1} X_{13}^{d_1} \quad . \qquad (7)$$

Using logarithmic form and eliminating R

$$K = b_1(b_2+c_2)\log X_{22} - b_2(b_1+c_1)\log X_{12} + b_1 d_2 \log X_{23} - b_2 d_1 \log X_{13}$$

where $K = b_1(\log Y_2 - \log a_2) - b_2(\log Y_1 - \log a_1)$.

From (4) $0 = \log X_{22} + \log X_{13} - \log X_{12} - \log X_{23}$

$$\log X_{22} = \frac{K - (b_1 d_2 + b_2 b_1 + b_2 c_1 \log) X_{23} - (-b_2 b_1 - c_1 b_2 - b_2 d_1)\log X_{13}}{b_1 c_2 - b_2 c_1} \quad .$$

Then substituting a series of linseed oilmeal quantities (X_{23}) within the range of the data for the second equation and making X_{13} 100 pounds less, quantities of hay were found as in Table I. Substituting these quantities back into equation (2) resulted in quantities of corn X_{21}. Substituting into the equation (4) and (5) results in values for X_{11} and X_{12}.

A check occurs when the feed quantities derived in this manner are substituted into the two original equations (1) and (2). The gains, Y_1 and Y_2 should then equal 150 and 300 pounds, respectively, if the feed quantities are correct.