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Nikolaus Karpinsky and Song Zhang

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Composite Method for Discontinuous 3-D Surface Measurement: Simulations

Nikolaus Karpinsky and Song Zhang

Department of Mechanical Engineering, Iowa State University, Ames, IA 50011 USA. Email: song@iastate.edu

Abstract. This paper presents a method for discontinuous surface measurement by using only four fringe images. The technique has the advantage of measurement speed over conventional methods with similar capabilities that rely on utilizing at least nine fringe images. Simulation results demonstrated that the technique can be used to effectively measure steep height variations in an object as well as discontinuous surfaces.

Keywords: Step height; Phase shifting; Real time; Fringe Analysis.
PACS: 43

INTRODUCTION

3-D shape measurement is extremely important to numerous disciplines, and over the years a number of techniques have been developed, such as stereo vision; structured light; and digital fringe projection with phase-shifting [1]. Due to increases in the computational power of a personal computer, real-time 3-D shape measurement is being realized, and is soon to become a requirement of such systems. [2]. In real-time 3-D shape measurement systems that use the digital fringe projection and phase-shifting technique, the number of fringe images used to reconstruct one 3-D shape is vital. In general, as more fringe images are used, the slower the overall measurement speed of the system is. Therefore, reducing the number of fringe images required to perform the measurement becomes a key issue to address.

For a real-time 3-D shape measurement system, a three-step phase-shifting method is typically used because it uses the minimum number of fringe images to uniquely solve for the phase pixel by pixel. However, because a three-step phase-shifting algorithm is essentially a single-wavelength method, it cannot be used to measure a discontinuous surface or a surface with steep height variations of more than π [3]. To measure arbitrary 3-D shapes with surface discontinuities, a multiple-wavelength phase-shifting technique [4] is required. In practice, at least three-wavelength phase-shifted fringe images are needed [5]. Assuming a three-step phase-shifting algorithm is applied for each wavelength, at least nine fringe images are required to perform one 3-D shape reconstruction. For a real-time 3-D shape measurement system, this is not desirable, as it will drastically reduce the measurement speed. Single-wavelength phase-shifting algorithms have the advantage of measurement speed, since only three phase images are needed; yet it is limited by a phase unwrapping stage. The phase unwrapping is essentially to detect the 2π discontinuities in the wrapped phase map and remove them by adding or subtracting multiples (integer number, k) of 2π [6]. Traditional solutions attempt to solve this problem though the use of phase unwrapping algorithms, yet the proposed technique addresses this problem with an alternative route, identifying the integer number k using only four fringe images, maintaining real-time speed.

A digital fringe projection system provides the solution to this problem, as it allows for the generation of arbitrary profile fringe patterns. The composite phase unwrapping algorithm proposed in this paper uses such a system, solving the problem of discontinuous surfaces using only four fringe images, assuming that the surface has uniform reflectivity. An additional stair fringe image augments the existing three phase-shifted fringe images, resulting in four fringe images being used. The stair image has the integer number k phase changes precisely aligned with the phase discontinuities, providing a unique grayscale encoded value for each pixel value. Therefore, from this stair image, the unique integer number k can be determined, and the phase can be unwrapped point by point. In this research, we simulate the algorithm through the use of a virtual fringe projection system called Holoimage [7]. Our simulation has successfully verified the success of the proposed algorithm. Compared with a traditional multiple-wavelength phase-shifting algorithms, this new algorithm only requires four instead of more than nine fringe images to reconstruct arbitrary 3-D shape. Therefore, it is better suited for real-time 3-D shape measurement applications.

The paper is organized as follows: we will first explain the principles of the proposed algorithm, show the experimental results by simulations, and finally summarize the results and address directions of future research.

PRINCIPLE

Phase-shifting methods are widely used in optical metrology because of their speed and accuracy [8]. For the real-time 3-D shape measurement system we developed, a three-step phase-shifting algorithm with a phase shift of $2\pi/3$ is used [9]. The intensity of three fringe images can be written as:

$$\begin{aligned} I_1(x,y) &= I'(x,y)+I''(x,y)\cos(\phi-2\pi/3), && (1)\\ I_2(x,y) &= I'(x,y)+I''(x,y)\cos(\phi), && (2)\\ I_3(x,y) &= I'(x,y)+I''(x,y)\cos(\phi+2\pi/3). && (3)\end{aligned}$$

Where $I'(x,y)$ is the average intensity, $I''(x,y)$ the intensity modulation, and $\phi(x,y)$ the phase to be solved for. Simultaneously solving Eq. (1)-(3), the phase can be obtained

$$\phi(x,y)=\tan^{-1}\left[\sqrt{3}(I_1-I_3)/(2I_2-I_1-I_3)\right]. \tag{4}$$

Equation (4) gives a phase value within the range of $[-\pi, +\pi)$ with modus of 2π. A traditional approach to remove the 2π discontinuity is to apply a spatial phase unwrapping algorithm [6]. Essentially, by analyzing the neighboring pixel phase values and assuming a smooth surface, a phase unwrapping algorithm can detect 2π phase jumps. These phase jumps are then removed by adding or subtracting multiples of 2π. That is, the relationship between the wrapped phase and the unwrapped one can be written as,

$$\Phi(x,y)=\phi(x,y)+k(x,y)\times 2\pi. \tag{5}$$

Here, $\Phi(x,y)$ denotes the unwrapped phase, and $k(x,y)$ is an integer number.

If a stair image is used that is perfectly aligned with the 2π phase discontinuities, the $k(x,y)$ can be determined from the stair images. Figure 1 illustrates the proposed algorithm. Assuming vertical fringe stripes, the resulting stair image can be generated as,

$$I_s^p(x,y)=floor[(x+P/2)/P]\times S. \tag{6}$$

Here P is the fringe pitch (number of pixels per fringe period). $floor()$ removes the decimals of floating point data keeping the integer part. S is the intensity level of each stair.

Because the object surface reflectivity might not be uniform, normalizing the structured images might be necessary to accurately determine the integer $k(x,y)$. The normalization procedure is straightforward because from Eq. (1)-(3), the maximum and minimum intensity for each pixel can be obtained,

$$\begin{aligned} I_{min}(x,y) &= I'(x,y)-I''(x,y), && (7)\\ I_{max}(x,y) &= I'(x,y)+I''(x,y). && (8)\end{aligned}$$

Where $I'(x,y)=(I_1+I_2+I_3)/3$, and $I''(x,y)=\sqrt{3(I_1-I_3)^2+(2I_2-I_1-I_3)^2}/3$.

Assume the captured stair image is $I_s(x,y)$, $k(x,y)$ can be determined from the stair image

$$k(x,y)=\frac{I_s(x,y)-I_{min}(x,y)}{I_{max}(x,y)-I_{min}(x,y)}\times\frac{R}{S} \tag{9}$$

Here R is the fringe intensity range generated by the computer. Once $k(x,y)$ is determined, the phase can be unwrapped point-by-point using Eq. (5).

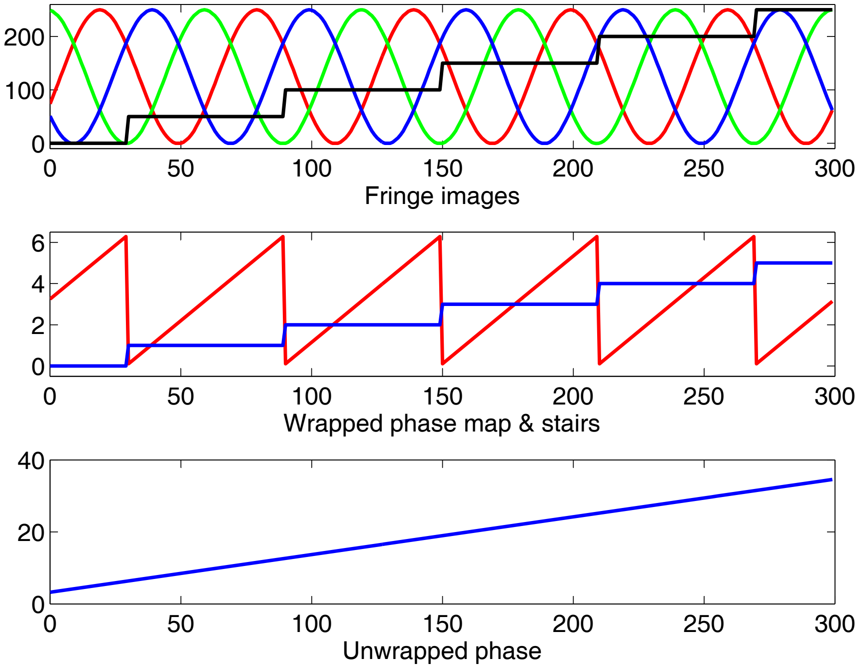


FIGURE 1. Composite phase shifting algorithm. Top image illustrates four fringe images used: three phase-shifted sinusoidal fringe images, and one stair image. Middle image illustrates the 2π discontinuities of the phase precisely aligned with the normalized stair image intensity changes. By applying the $k(x,y)$ obtained from the stair image to the wrapped phase map, the phase can be unwrapped point by point.

EXPERIMENTS

To verify the performance of this approach, we simulated the algorithm using a Holoimage system [7]. A Holoimage system simulates a digital fringe projection and phase-shifting system in an advanced digital graphic environment, so that all aspects can be controlled and measured. Thus, it is easy to implement any phase-shifting algorithm in this system and measure its effectiveness. Since the proposed approach reconstructs 3-D shapes on a point-by-point basis, it can be used to measure arbitrary 3-D shapes including surfaces with sharp changes or discontinuous surfaces. To verify this, we tested a step-height surface: a flat object with a squared table block standing on its top. Figure 2 shows the result. Figure 2(a)- 2(c) show the phase-shifted fringe images, and Figure 2(d) shows the stair image. From the phase-shifted fringe images, the wrapped phase map can be obtained, as shown in Fig. 2(e). From the stair image, integer $k(x,y)$ can be obtained for each pixel, as shown in Fig. 2(f). The wrapped phase is then unwrapped by referring to integer number $k(x,y)$. Figure 2(g) shows the unwrapped phase map. Figure 2(h) shows the phase map after removing the tilt of the unwrapped phase map.

The unwrapped phase map is then plotted in 3-D as shown in Fig. 3(a). One cross section of the phase map is shown in Fig. 3(b). It clearly shows that the there are sharp surface changes, but the shape can still be correctly recovered.

The previous experiment has demonstrated that the proposed technique can be used to recover 3-D geometry with steep height variations. To further demonstrate the new techniques capability, we apply the same technique to reconstruct multiple separate objects: four identical spheres. Figure 4(a) shows the fringe image, with red, green, and blue respectively representing the three phase-shifted fringe images, I_1, I_2, and I_3. Figure 4(b) shows the stair image. From these four images, the unwrapped phase map can be obtained. Figure 4(c) shows the unwrapped phase map after removing its tilt. To better visualize these spheres, a 3-D plot is shown in Fig. 5. This experiment clearly shows that this technique can indeed be used to measure a discontinuous surface.

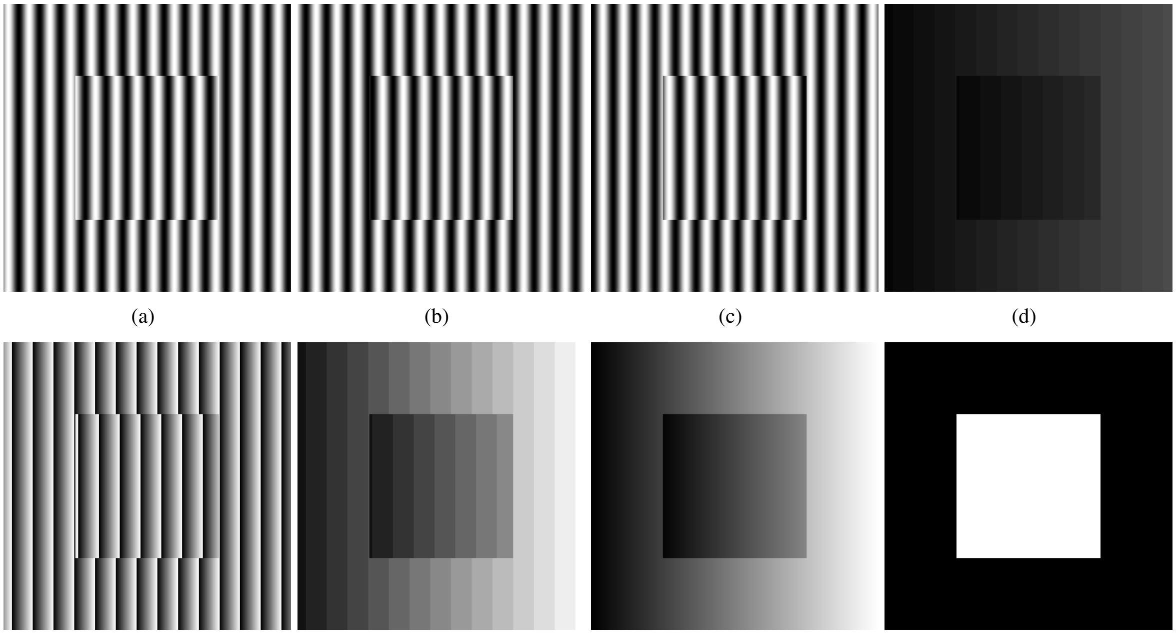
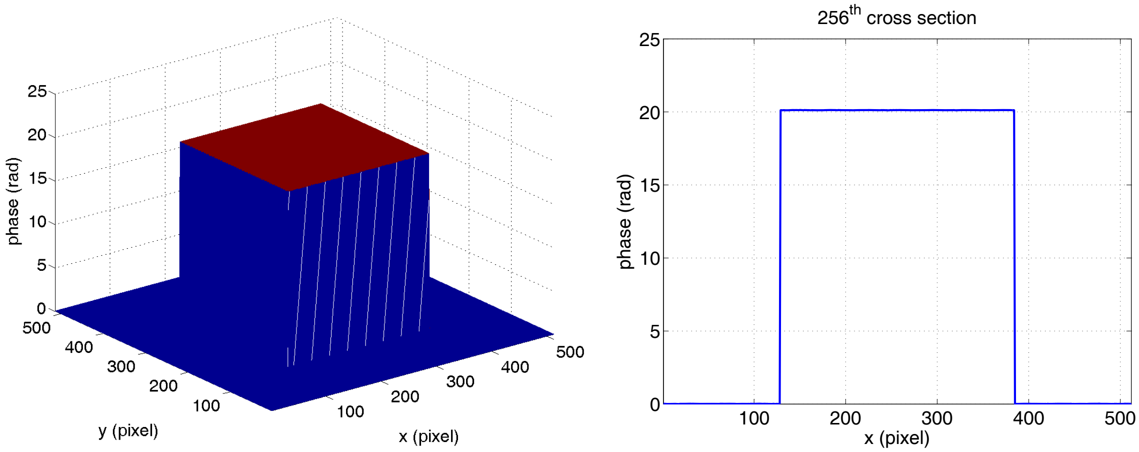

(a) (b) (c) (d)
(e) (f) (g) (h)

FIGURE 2. Experimental results of a step height surface. (a)-(c) Three phase-shifted fringe images; (d) Stair image; (e) Wrapped phase map; (f) $k(x,y)$ map to unwrap the phase; (g) Unwrapped phase map; (h) Phase map after removing its tilt shown in (g).



(a) (b)

FIGURE 3. (a) 3-D plot of the step height object; (b) Cross section of the step height object.

CONCLUSIONS AND FUTURE WORK

This paper has presented a technique to measure steep height variations and discontinuous surfaces by utilizing a forth fringe image. Simulations have been presented, and have verified the techniques success. Because this proposed technique only uses four fringe images to recover one 3-D shape, it is desirable for real-time applications. In the future, we will integrate this technique into our existing real-time 3-D measurement systems. If successful, it would be highly useful for rapid 3-D shape measurement.

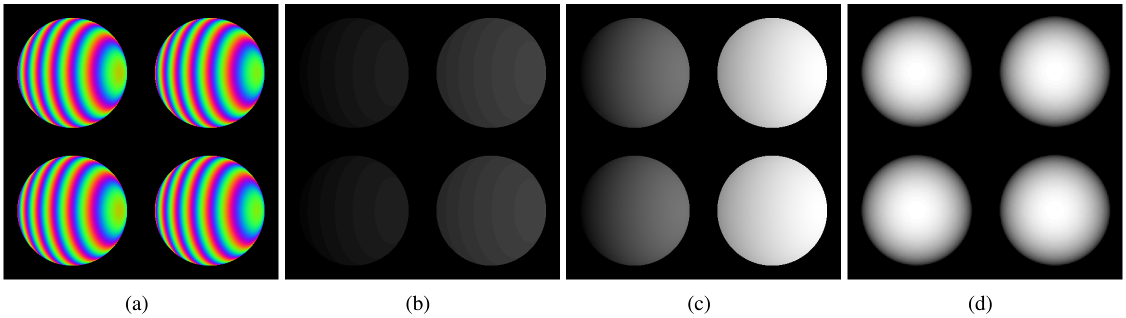


FIGURE 4. Experimental results of four separate objects. (a) Fringe images: R, G, B are encoded as I_1, I_2, and I_3; (b) Stair image; (c) Unwrapped phase map; (d) Unwrapped phase map after removing its tilt.

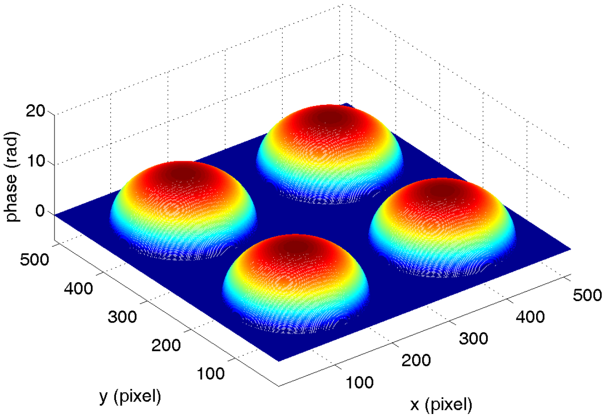


FIGURE 5. 3-D plot of the four spheres measured at the same time.

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