OPTIMAL STOCHASTIC ADVERTISING STRATEGIES

FOR THE U.S. BEEF INDUSTRY

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ABSTRACT

An important decision variable in the promotional strategy for the beef sector is the optimal level of advertising expenditures over time. Optimal stochastic and deterministic advertising expenditures are derived for the U.S. beef industry for the period 1966 through 1980. They are compared with historical levels and gains realized by optimal advertising strategies are measured. Finally, the optimal advertising expenditures in the future are forecasted.

I. INTRODUCTION

Advertising has a cumulative effect on sales which depreciates over time. Also, advertisement has been shown to be an effective method of increasing demand. Because of this effectiveness the optimal level of advertising expenditures over time can be important information to an industry.

Beef producers have endured more than their share of problems in recent years. A ban on DES in 1972, a national consumers boycott and the government's price freeze in 1973, the beginning of energy price increases in 1974 and more recently, a drought in 1980 causing early sale of cattle, high interest rates and negative publicity have all combined to cause bad times for U.S. beef farmers. Increased promotional expenditures have been suggested as one method to mitigate the problems of beef farmers. These promotional efforts are largely financed by beef farmers based on a per head sale charge or checkoff. The producer controlled Beef Councils of America coordinates national promotional strategy. An important decision variable in the promotional strategy for the beef industry is the optimal level of advertising expenditures over time.

Currently, the beef industry through the Beef Industry Council of the National Livestock and Meat Board is increasing its advertising expenditures. Determining the optimal level of spending on advertising this year and future years is a problem facing the beef industry. Using an inefficient trial and error process, i.e., advertising expenditures are increased when sales are poor and they are decreased when sales are good, is not advisable especially when dealing with millions of hard earned beef farmer dollars. Thus, knowledge of the optimal advertising expenditure target can provide the beef industry with important information.

The economics of advertising has an enormous amount of literature although the majority was published in the 1960s and 1970s. According to Boynton and Schwendiman, there are five theoretical approaches: (1) demand theory; (2) theory of the firm; (3) welfare theory; (4) information theory, and (5) industrial organization (Boynton and Schwendiman, p. 3). The second approach is employed in this study.

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Dorfman and Steiner made a pathbreaking study on optimal advertising. They showed that profit is maximized when the marginal value product of advertising equals the price elasticity of demand if the advertising budget is currently positive, and the marginal value product of advertising should be less than or equal to the price elasticity of demand if the advertising budget is currently zero (Dorfman-Steiner Theorem). Nerlove and Arrow derived an optimal advertising policy under dynamic conditions. It was shown that the optimal stationary solution implies a constant ratio of advertising to sales[1] (Nerlove-Arrow Theorem). Jacquemin generalized both Dorfman-Steiner and Nerlove-Arrow Theorems using optimal control theory.

The first empirical application of optimal advertising in agricultural products was done by Hochman, et al. They estimated the optimal advertising level in the Florida citrus industry and concluded that some redistribution of advertising budgets from the winter quarters to the summer would yield gains.

This study has four major purposes. First, it derives optimal stochastic advertising expenditures over time in the U.S. beef industry. Second, these expenditures are compared with historical levels, and gains derived by the optimal advertising policy are estimated. Third, a competitive market model and a price-controlled model are compared. Finally, the optimal advertising expenditures in the future are forecasted.

II. DYNAMIC OPTIMIZATION MODEL

Commodity advertising has a carryover effect as is well-known, i.e., advertising influences the future sales of a product as well as those of the present (Gould and Lee). In this paper control theory[2] is applied to derive the optimal level of advertising expenditures.[3]

[1] $\frac{A^*}{pq} = \frac{\beta}{\eta(\alpha+\delta)}$ where A^* is the optimal stock of goodwill which summarizes the effect of past advertising outlays, p is the price charged, q is quantities sold, β is the elasticity of demand with respect to goodwill, η is the elasticity of demand with respect to price, α is a fixed rate of interest, and δ is the constant proportional rate of depreciation of goodwill.

[2] See Dorfman, Intriligator, and Kirk.

[3] The model is mainly based on Jacquemin's work. However, it is simplified to get empirical results in this study.

Let $\pi(t)$ be the profit at time t:

$$\pi(t) = (P(t) - C(t))\,q(t) - S(t)$$

where P(t) is the price at time t, C(t) is the unit cost at time t, q(t) is the quantity sold by the industry at time t and S(t) is advertising expenditures at time t. Let K(t) be the stock of goodwill which summarizes the effects of current and past advertising outlays. Then q(t) is written as a function of P(t), S(t), K(t), and time t:

$$q(t) = q(P(t), S(t), K(t), t).$$

It is assumed to be of the form:

$$(1)\quad q_p < 0,\ q_s > 0,\ q_k > 0$$
$$q_{ss} < 0,\ q_{kk} < 0,\ q_{ks} = 0.$$

q_p indicates a downward sloping demand, q_s and q_{ss} exhibit that q is an increasing function of s with a decreasing rate, and q_k and q_{kk} indicates that q is an increasing function of k with a decreasing rate. It is also assumed that goodwill depreciates over time with the constant proportional rate, δ, so that

$$(2)\quad \dot{K}(t) = S(t) - \delta K(t).$$

The industry wishes to maximize the present value of the future stream of profits with respect to P and S given equation (2). Thus, the problem is formulated as follows:

$$(A)\quad \underset{S}{\text{Max}}\ J = \int_0^\infty \pi(P, S, K, t)e^{-rt}dt \quad \text{Subject to } \dot{K} = S - \delta K.$$

The problem (A) can be solved by the Maximum Principle. Let $\lambda(t)$ be a co-state variable or a dynamic shadow price which may be interpreted as the value, at time zero, of the marginal unit of the level of goodwill at time t. The Hamiltonian function can be written as follows:

$$(3)\quad H(P, S, K, \lambda, t) = ((P(t) - C(t))q(P, S, K, t) - S)e^{-rt} + \lambda(S - \delta K).$$

The necessary conditions for maximization are as follows:

(a) $\frac{\partial H}{\partial S} = ((P - C)\,q_s - 1)e^{-rt} + \lambda = 0$

so that: (4) $1 = (P - C)q_s + \hat{\lambda}$

where $\hat{\lambda} = e^{rt}\lambda$.

(b) $\dot{\lambda} = -\frac{\partial H}{\partial K} = -((P - C)q_K)e^{-rt} + \lambda\delta$

so that: (5) $\hat{\lambda}\,(r + \delta) = (P - C)q_K + \dot{\hat{\lambda}}$.

(c) $\dot{K} = \frac{\partial H}{\partial \lambda} = S - \delta K$

so that: (6) $\dot{K} = S - \delta K$.

The left-hand side of equation (4) is the present value of the last dollar of investment in advertising. The first term of the right-hand side of (4) is the current effect of increased advertising on profits and the second term is the long-run effect of the current advertising outlay on future profits. The sum of the two terms implies the present value of total net contribution over time of S to J. Thus, the optimal rate of advertising expenditures is that rate at which the present value of the last dollar of investment in advertising equals the present value of the total net contribution of current advertising outlays to profits. The left-hand side of equation (5) is the marginal opportunity cost of investment in K. On the right-hand side of (5), the term $(P - C)q_K$ implies the marginal revenue from increased goodwill and the term $\dot{\hat{\lambda}}$ implies the capital gain. Therefore, along the optimal path, the marginal opportunity cost of investment in goodwill equals the sum of the marginal revenue from increased goodwill and the capital gain. Given the initial level of goodwill, K_o, the optimal path of the advertising expenditure, $S^*(t)$, can be calculated from equations (4) through (6).

III. APPLICATION OF THE MODEL AND RESULTS

The dynamic advertising model described in the previous section is applied to the U.S. beef industry for the years 1966 to 1980. As portrayed in the introduction section, the U.S. beef industry is trying to improve the demand for its product by increasing advertising expenditures.

Prior to applying the dynamic control model to the beef industry, two equations are estimated; one is the net profit function (profit margin equation) which is used in a criterion function of the dynamic control model. The other is an equation of motion which explains the intertemporal relationship between the level of total sales (state variable) and advertising expenditures (control variable).

Using regression techniques, the net profit function is estimated with 1966-1980 data (Cooperative Extension Service, Hovland et al., National Livestock and Meat Board, and U.S.D.A.) as:

$$(7)\quad \pi(K, S, P, SPR, t)$$
$$= 771{,}548.2483 + \underset{(0.00000122)^4}{0.00000371K^2} - \underset{(0.8622)}{2.5277\ K{*}P}$$
$$- \underset{(0.8236)}{2.2524\ K{*}t} - \underset{(9.0448)}{47.7951\ S^2} + \underset{(8.8871)}{29.0175\ S{*}SPR}$$
$$- \underset{(1{,}696.8377)}{8{,}482.3945\ S{*}t} + \underset{(154{,}153.8743)}{425{,}823.0755\ P^2} + \underset{(281{,}028.3145)}{821{,}786.8122\ P{*}t}$$
$$+ \underset{(187{,}624.9951)}{459{,}430.8721\ t^2}$$

$R^2 = 0.9105$ MSE $= 9.3223 \times 10^{11}$.

π is the net profit of beef producers in thousand dollars;[5] K is total sales in thousand dollars; S is advertising expenditures of the beef industry in thousand dollars; P is the price of beef in dollars per CWT; SPR is the advertising expenditures of the pork industry, and t is time. The independent variables of equation (7) explain 91 percent of the variance in the net profit of beef producers. The coefficients are all significant at the 5 percent probability level. The equation of motion is also estimated with 1966-1980 data. (National Livestock and Meat Board, and U.S.D.A.) and has the following form:

$$(8)\quad K_{t+1} = 4{,}131{,}161 + 3{,}752{,}951\ S_t + 0.636075\ K_t + U_t$$

$R^2 = 0.8220$ S = 26,354.23 DW = 2.0189

where K_{t+1} is the total sales of the beef industry at time t+1 in thousand dollars, S_t is the advertising level of the beef industry at time t in thousand dollars, and U_t is a disturbance term with standard deviation S. R^2 is the coefficient of determination and DW is the Durbin-Watson D statistic. Equation (8) indicates that about

[4] Figures in parentheses are standard errors.

[5] The net profit function, $\pi(t)$ is calculated using the following formula: $\pi(t) = (P(t) - C(t))\,q(t) - S(t)$ where P(t) is the price at time t, C(t) is the unit cost, except the advertising cost, at time t, q(t) is the quantity sold, and S(t) is the advertising cost at time t.

two-thirds of the previous year's goodwill is carried over to the next period.

The estimated coefficients from these two equations are plugged into the optimal control model and it is solved by Chow's optimal control formulation (Chow, 1974, pp. 149-174). The model is solved for the years 1966-1980. The optimal control problem is decomposed into two parts: a deterministic control solution and a stochastic control solution. The former does not take into account the random disturbance in the equation of motion. The latter incorporates the stochastic factor in the equation of motion.

Table 1 presents the comparison of the optimal stochastic solution (combined with the optimal deterministic solution) to historical trends. The optimal stochastic solution has a larger quantity sold with a higher level of advertising expenditures and results in a greater net profit. The gains derived from the optimal advertising strategies are 466.9 million dollars with no discount and 378.8 million dollars with a 5 percent discount rate. The gains are 2.2 and 2.7 percent of the actual total net profit under no discount and a 5 percent discount rate, respectively.[6]

The annual levels of optimal advertising expenditures and actual expenditures are exhibited in Figure 1. The optimal deterministic solution indicates the steady state growth. On the other hand, the optimal stochastic solution shows the fluctuation over time with high advertising costs in the good years and preceding years. By good years we are referring to years with high sale prices and low production costs. In 1979 and 1980, the actual costs spent for advertising beef are relatively low and both optimal solutions (stochastic and deterministic) recommend higher levels of advertising. Both optimal solutions indicate that more than two million dollars should be used in 1980.

The stochastic solution and deterministic solution are compared in Table 2. In the stochastic solution, beef farmers sell less beef and spend less on advertising than in the deterministic solution. Also, higher net profits are earned for the stochastic solution since beef farmers spend less on advertisements and sell less product in bad years and spend more and sell more in good years.

Through collective marketing practices or some other method, it might be possible for beef farmers to gain control over the price of their product. Although this scenario is not considered to be a realistic possibility in the near future, it was considered as an interesting comparison to the competitive market solution. Table 3 reports results of the price controlled solution and the competitive market solution. The price-controlled

Table 1. A comparison of the optimal stochastic solution to total actual advertising expenditures and total net returns 1966-1980 to the U.S. beef industry

	Quantity sold million CWT	Advertising expenditures \$ million	Net Profit \$ million	Net present value of profit[a] \$ million
Optimal stochastic (OS)	6,039.4	12.8	21,496.6	14,620.8
Actual (A)	5,846.0	11.6	21,029.7	14,242.0
Gains due to optimal strategies (OS-A)	-	-	466.9	378.8

[a]A 5 percent discount rate.

Figure 1. Optimal stochastic, deterministic, and actual advertising expenditures

[6]The study of optimal advertising expenditures in the Florida Citrus Industry shows 4.7 and 4.6 percent increases in net gains under no discount and a 7 percent discount, respectively, although it involves only the optimal deterministic control (Hochman et al., pp. 704-5).

solution indicates less sales and advertising expenditures with higher prices and higher net profits. The price-controlled solution is similar to the monopoly solution in that it provides gains to producers, but consumers will be worse off since higher prices are charged by producers under the price-controlled system.

The optimal levels of advertising expenditures are forecasted for the years 1981 to 1985 (Table 4). Results indicate that advertising expenditures should increase at an 8 percent average annual rate.[7] Because of uncertainty in the future, there are no drastic changes in expenditures and they remain within a 2.4-3.3 million dollar level.

Finally, an analysis of the sensitivity of our model to varying levels of discount rates is conducted. The experiment results are reported in Table 5. As the discount rate is increased, advertising expenditures also increase and more beef is sold in the beginning years while less is sold in the later years. This is explained by the fact that the objective function is the sum of the time-discounted net profit stream and it has more weight in the beginning than in later years.

IV. CONCLUSIONS

Comparing the optimal advertising strategy to the actual advertising expenditures reveals an opportunity for beef producers to increase economic gains by 467 million dollars in actual value and by 379 million dollars in net value. The higher advertising expenditures under the optimal strategy yield the larger net profits. However, as the comparison of the stochastic to the deterministic results exhibits, the best strategy for advertising is not to spend more money for advertising in general but to adjust expenditure levels according to the expected fluctuations of price, cost, and sales. A recommended strategy is to increase expenditures in the years of expected high marginal revenue from advertisements.

Lack of monthly or seasonal data prohibited our analysis of the seasonal allocation of advertising expenditures. It might be expected that advertising expenditures would be allocated unevenly across seasons since the demand for beef has a seasonal fluctuation. Also, we were not able to obtain satisfactory advertising expenditure data for the poultry industry. Poultry has become an important substitute for beef and thus the behavior of the poultry industry might have a significant impact on the beef industry.

Use of the optimal control approach showed some definite advantages. It allows the linkage of all years and determines the optimal strategies over time in a systematic way. Particularly, the stochastic control technique takes into account stochastic influences due to uncertainty of the effectiveness of advertisements, and price and demand fluctuations. Application of this technique has great potential to significantly increase the net income of beef producers by improving the allocation of their advertising expenditures.

Table 2. A comparison of the optimal stochastic to optimal deterministic solution 1966-1980

	Quantity sold million CWT	Advertising expenditures $ million	Net profit $ million	Net present value of profit[a] $ million
Stochastic	6,039.4	12.8	21,496.6	14,620.8
Deterministic	6,149.3	15.3	21,400.5	14,576.3

[a]A 5 percent discount rate.

Table 3. A competitive model versus price-controlled solution 1966-1978

	Quantity sold million CWT	Average price level $/CWT	Advertising expenditure $ million	Net profit $ million
Competitive	5,301.9	36.18	8.97	19,352.6
Price-controlled	5,070.9	36.79	7.99	21,694.4

Table 4. Optimal advertising expenditures forecasted

	Advertising expenditures $ million
1981	2.45
1982	2.62
1983	2.86
1984	3.11
1985	3.27

Table 5. Optimal stochastic solutions with different discount levels

	Quantity sold million CWT	Advertising expenditure $ million	Net profit $ million	Net present value of profit $ million
5 percent	6,039.4	12.77	21,496.6	14,620.8
10 percent	6,035.5	12.71	21,484.0	10,486.4
No discount	6,043.5	12.83	22,050.6	22,050.6

[7]Our forecasted advertising expenditures are considerably lower than the planned beef advertising budget. Our forecasted expenditures are based on an optimally planned systematic strategy while currently the beef industry has a somewhat erratic advertising strategy.

REFERENCES

Boynton, R. D. and L. Schwendiman. "Theoretical Approaches to the Economics of Advertising." The paper presented at NC-117 Symposium on Advertising and the Food System, Airlie, Virginia, Nov. 6-7, 1980.

Chow, G. C. Analysis and Control of Dynamid Economic Systems. New York, John Wiley & Sons, 1975.

Cooperative Extension Service. Estimated Returns from Cattle Feeding in Iowa Under Two Alternative Feeding Programs. M-1195 and Addendum to M-1195. Iowa State University, 1978.

Dorfman, R. "An Economic Interpretation of Optimal Control Theory." American Economic Review 59 (1969): 817-831.

Dorfman, R. and P. O. Steiner. "Optimal Advertising and Optimal Quality." American Economic Review 44 (December 1954):826-836.

Gould, J. P. "Diffusion Processes and Optimal Advertising Policy." In Microeconomic Foundations of Employment and Inflation Theory, eds. E. S. Phelps, et al., New York: W. W. Norton and Co., 1970.

Hochman, E., U. Regev, and W. W. Ward. "Optimal Advertising Signals in the Florida Citrus Industry: A Research Application." American Journal of Agricultural Economics 56 (1974):697-705.

Hovland, Craig, Gene Futrell, and J. M. Skadberg. Return and Risk of Iowa Cattle Feeding Under Two Alternative Programs, 1967-1980. Economic Report Series # 13. Department of Economics, Iowa State University, 1980.

Intriligator, M. D. Mathematical Optimization and Economic Theory. Englewood Cliffs, Prentice Hall, 1971.

Jacquemin, A. P. "Optimal Control and Advertising Policy." Metroeconomica 25 (1973):200-207.

Kirk, D. E. Optimal Control Theory: An Introduction. Englewood Cliffs, Prentice Hall, 1970.

Lee, K. C. "Application of Control Theory to Economic Analysis of Agriculture" (mimeo). The Center for Agricultural and Rural Development, Iowa State University, 1981.

National Livestock and Meat Board. Annual Report 1962 to 1980.

Nerlove, M. and K. Arrow. "Optimal Advertising Policy Under Dynamic Conditions." Economica 29 (May 1962): 129-142.

U.S. Department of Agriculture. Agricultural Statistics 1962 to 1980.