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Surrogate-based Performance Evaluation Strategy for High Performance Control Systems under Uncertainties

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ABSTRACT

High-performance control systems (HPCSs) are sophisticated vibration mitigation strategies that include active, semi-active and hybrid systems. They generally outperform passive supplemental damping systems by relying on a feedback mechanism enabling adaptability, resulting in better control reachability over a wide excitation frequency bandwidth. HPCSs are therefore ideal for multi-hazard mitigation. However, the performance of these systems is highly dependent on the controller design, which requires appropriate tuning based on assumptions or on knowledge of dynamic parameters, long-term performance of sensors, excitations, etc. The quantification of performance based on possible uncertainties on such assumptions and/or knowledge could be a powerful tool in financially or technically justifying the use of an HPCS, or simply to benchmark the long-term performance of a given control algorithm. This paper investigates a methodology to assess the performance of control algorithms under various sources of uncertainties. To reduce the computational demand of the uncertainty quantification process, surrogate models are employed to map the nonlinear relationship between structural response and controller configurations. Long-term performance is quantified using life-cycle cost analysis. The investigation is conducted on a 39-story building, located in Boston (MA), and equipped with a set of semi-active friction devices. Results demonstrate that the proposed framework can be used to assess the performance of a given control algorithm considering various sources of uncertainties.

Keywords: Kriging, semi-active control, control uncertainties, surrogate model, life-cycle cost, vibration mitigation

1. INTRODUCTION

Structural resilience can be improved by incorporating auxiliary motion control devices in the structural system. High-performance control systems (HPCSs), including active, semi-active and hybrid systems,\textsuperscript{1–4} are sophisticated damping strategies that can be deployed to enhance structural performance against wind, seismic, and blast loads.\textsuperscript{5–8} They employ feedback mechanisms and adaptive laws designed to improve vibration mitigation performance with respect to traditional passive dampers.\textsuperscript{9,10} However, uncertainties in the closed-loop configuration including those on estimated dynamic parameters, sensors noise, and assumed external load intensity, can reduce the performance of HPCSs.\textsuperscript{11,12} In particular, the controller design is usually dependent on assumption or on knowledge of dynamic parameters, long-term performance of sensors, types of excitations, etc. For example, the authors had studied the multi-hazard mitigation performance of mis-tuned controllers caused by limited sensor feedback\textsuperscript{13} and uncertain dynamic parameters.\textsuperscript{14} Results showed that the mis-tuned controller design can lead to a 60\% variation of multi-hazard mitigation performance. The quantification of structural performance based on possible uncertainties on such assumptions and/or knowledge could be a powerful tool in financially or

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technically justifying the use of an HPCS, or simply to benchmark the long-term performance of a given control algorithm.

The performance of HPCSs under uncertainties has been investigated in some studies. In previous work, the authors evaluated the performance of an HPCS under uncertainties in the closed-loop configuration (i.e., sensor failure and mechanical degradation) and external loads using traditional uncertainty analysis techniques, including deterministic- and stochastic-based methods. Results showed that uncertainties in the HPCS affect the life-cycle cost (LCC) of the structure and that traditional analysis techniques are computationally demanding. Such demand increases exponentially with the number of uncertain variables and damping devices considered. Micheli et al. proposed the use of surrogate models (metamodels) to reduce the computational time of the uncertainty quantification process of HPCSs. The authors employed a Kriging surrogate and an adaptive wavelet neural network to estimate the average and variance in structural response under uncertainties in HPCS configuration and wind load. Results demonstrated that the metamodels were able to predict these statistical values accurately with a significantly reduced computational time.

This paper extends previous work by presenting a surrogate-based evaluation strategy to assess the performance of HPCSs under various sources of uncertainties, focusing on the controller performance under uncertain knowledge of dynamic parameters. In the proposed procedure, a Kriging metamodel is used as a replacement of the numerical simulation model of the structure to reduce the computational demand of the uncertainty quantification process. Specifically, Kriging is employed to map the nonlinear relationship between uncertainties (input of the surrogate) and structural response (output), and to predict the output for a large number of new inputs. The predicted outputs are then utilized to perform a probabilistic LCC assessment, evaluating lifetime economic gains arising from better vibration mitigation. The life-cycle performance of a wind-sensitive 39-story building equipped with a set of semi-active friction devices is investigated, where uncertainties in wind load and dynamic properties are examined. The performance of two control algorithms, namely a linear quadratic regulator (LQR) and a sliding mode control (SMC), is evaluated and compared in terms of LCC.

The remainder of the paper is organized as follows. Sec. 2 describes the surrogate-based performance evaluation strategy. Sec. 3 introduces the case study building and the simulation models. Sec. 4 presents the Kriging and the LCC results. Sec. 5 concludes the paper.

2. SURROGATE-BASED PERFORMANCE EVALUATION METHOD

In the surrogate-based evaluation strategy, the performance of an HPCS under various sources of uncertainties is assessed using a Kriging surrogate model. The metamodel is employed to reconstruct the nonlinear relationship between uncertainties and structural response. The uncertainties in the structure (e.g., dynamic properties), external load (e.g., wind intensity) and controller configuration (e.g., tuning parameters) are taken as inputs of the Kriging surrogate, while the peak acceleration experienced by the building (i.e., performance target), $a_{\text{peak}}$, is its output. The life-cycle cost of the structure is selected as performance metric, allowing to financially quantifying the long-term mitigation performance of the HPCS. The surrogate-based performance evaluation strategy consists of the following steps:

1. Identify the uncertain variables $x$ in the system and their range of variability;
2. Create a Kriging metamodel with $x$ as input and $y = a_{\text{peak}}$ as output based on $n$ training observations;
3. Verify the accuracy of the Kriging metamodel with $n_t$ testing observations;
4. Employ Kriging to predict $a_{\text{peak}}$ under a large number $n_v$ of new uncertainties scenarios $x_{\text{new}}$;
5. Evaluate the life-cycle cost of the structure based on the $n_v$ predicted $a_{\text{peak}}$ values;
6. Fit the $n_v$ life-cycle cost values with a probability distribution.

Note that the Kriging surrogate is employed as a replacement of the numerical simulation model of the structure equipped with HPCS to reduce the computational demand of the uncertainty quantification process. The Kriging algorithm assumes that the observed response of a system $y(x)$ derives from a stochastic process $Y(x)$ of mean $\mu(x)$ and deviation from the mean $Z(x)$.
\[ Y(x) = \mu(x) + Z(x) \quad (1) \]

where \( Z(x) \) is assumed to follow a Gaussian process with zero mean and covariance matrix \( D \):

\[ D = \sigma^2 \Psi \quad (2) \]

where \( \sigma^2 \) is the variance of the process, and \( \Psi \in \mathbb{S}^{n \times n} \) is the correlation matrix. For \( n \) observations, \( \Psi \) is expressed as:

\[ \Psi = \begin{pmatrix} \psi[Y(x^{(1)}), Y(x^{(1)})] & \cdots & \psi[Y(x^{(1)}), Y(x^{(n)})] \\ \vdots & \ddots & \vdots \\ \psi[Y(x^{(n)}), Y(x^{(1)})] & \cdots & \psi[Y(x^{(n)}), Y(x^{(n)})] \end{pmatrix} \quad (3) \]

In Eq. 3, the Gaussian correlation function \( \psi \) is written:

\[ \psi[Y(x^{(p)}), Y(x^{(q)})] = \exp \left( -\sum_{j=1}^{k} \theta_j |x_j^{(p)} - x_j^{(q)}|^2 \right) \quad (4) \]

where \( \psi[Y(x^{(p)}), Y(x^{(q)})] \), \( p, q = 1, \cdots, n \) are observations of the stochastic process, \( k \) is the number of input variables, and \( \theta_j, j = 1, \ldots, k \) are the hyper-parameters of the correlation function. The hyper-parameters of the Kriging function can be estimated employing the maximum likelihood method,\(^\text{20}\) where the likelihood function can be written as a function of the observations:

\[ \mathcal{L} = \frac{1}{(2\pi\sigma^2)^{n/2} |\Psi|^{1/2}} \exp \left[ -\frac{(y - 1 \mu)^T \Psi^{-1} (y - 1 \mu)}{2\sigma^2} \right] \quad (5) \]

It can be shown that the maximum likelihood estimates of the mean and standard deviation are:\(^\text{20}\)

\[ \hat{\sigma}^2 = \frac{1}{n} (y - 1 \mu)^T \Psi^{-1} (y - 1 \mu) \quad (6) \]

\[ \hat{\mu} = \frac{1^T \Psi^{-1} y}{1^T \Psi^{-1} 1} \quad (7) \]

where the hat denotes an estimate, and \( 1 \in \mathbb{R}^{n \times 1} \) is a vector of ones. Note that \( \Psi \), and therefore \( \hat{\mu} \) and \( \hat{\sigma}^2 \), depends on the unknown hyper-parameters \( \theta_j \), which can be evaluated using an optimization algorithm, such as a genetic algorithm, to maximize Eq. 5 after substituting Eqs. 6 and 7 into Eq. 5. When two observations are very close to each other, the nearest symmetric positive matrix\(^\text{21}\) is used for matrix \( \Psi \) to prevent the correlation matrix in Eq. 3 from becoming poorly conditioned. In this paper, the observations \( x \) and \( y \) required to train and test the Kriging surrogate are created by numerically simulating the structure equipped with the HPCS under \( n \) different uncertainties scenarios and recording \( a_{\text{peak}} \) for each scenario. It follows that the input of the surrogate is a matrix \( x \in \mathbb{R}^{n \times k} \) and its output a data vector \( y \in \mathbb{R}^{n \times 1} \).

3. CASE STUDY

This section presents the case study building investigated in this paper, along with simulation techniques, cost models, and control strategies considered.
3.1 Building Description

The case study building is a wind-sensitive 39-story office tower located in Boston (MA).\textsuperscript{10,22} The inter-story heights are 7.4 m at the ground and roof levels, and 3.9 m at the other floors, for a total height of 163 m. The lateral resisting system is a steel-moment resisting frame. The structure is equipped with 15 sets of two passive viscous dampers installed at every other floors, starting from the 5\textsuperscript{th} floor up to the 33\textsuperscript{th} floor, to mitigate wind-induced accelerations.\textsuperscript{23,24} This structure was selected because it represents an opportunity to benchmark the mitigation performance of an HPCS against an existing passive damping system.

The building is numerically simulated in its weak direction using a spring-dashpot-mass model and the state-space formulation. The equation of motion for the 39-story building is:

\[
M\ddot{u} + C\dot{u} + Ku = E_w W - E_f F
\]  

(8)

where \( u \in \mathbb{R}^{39 \times 1} \), \( \dot{u} \in \mathbb{R}^{39 \times 1} \), and \( \ddot{u} \in \mathbb{R}^{39 \times 1} \) are the displacement, velocity and acceleration vectors, respectively, the dot denotes a time derivative, \( W \in \mathbb{R}^{39 \times 1} \) is the wind load vector, \( F \in \mathbb{R}^{15 \times 1} \) is the control input vector, \( M \in \mathbb{R}^{39 \times 39} \), \( C \in \mathbb{R}^{39 \times 39} \), and \( K \in \mathbb{R}^{39 \times 39} \) are the mass, damping, and stiffness matrices, respectively, \( E_w \in \mathbb{R}^{39 \times 39} \) and \( E_f \in \mathbb{R}^{10 \times 15} \) are the wind load and the control location matrices, respectively.

Eq. (8) can be represented in the state-space as:\textsuperscript{25}

\[
\dot{U} = AU + B_w W - B_f F
\]  

(9)

where \( U = [ u \ u \dot{u} ]^T \in \mathbb{R}^{78 \times 1} \) is the state vector, with:

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{78 \times 78} \quad B_f = \begin{bmatrix} 0 \\ M^{-1}E_f \end{bmatrix}_{78 \times 15} \quad B_w = \begin{bmatrix} 0 \\ -E_w \end{bmatrix}_{78 \times 39}
\]

The solution of Eq. 9 follows the discrete form of the Duhamel integral:\textsuperscript{25}

\[
U(t+1) = e^{A\Delta t} U(t) + A^{-1}(e^{A\Delta t} - I)[B_w W(t) - B_f F(t)]
\]  

(10)

where \( \Delta t \) is the simulation time interval and \( I \in \mathbb{R}^{39 \times 39} \) is the identity matrix. The inter-story drift is expressed as \( \delta_1 = u_1 \) at the first floor and \( \delta_j = u_j - u_{j-1} \) at the other floors. The dynamic properties (\( M, K, \) and \( C \)) of the 39-story buildings can be found in Cao et al.\textsuperscript{10}

3.2 Wind Load Simulation

The wind load vector \( W \) in Eq. 9 is taken as:

\[
W_j = \rho C_D A_j (V_{m,j} + V_{i,j})
\]  

(11)

where \( W_j \) is the dynamic component of the along-wind forces acting at the \( j \)-th floor in its weak direction, \( \rho \) is the air density (1.25 kg/m\(^3\)), \( C_D \) is the drag coefficient (1.5), \( A_j \) is the projected area of the building normal to the wind flow, \( V_{m,j} \) is the mean wind speed, and \( V_{i,j} \) is the fluctuating wind velocity generated by the wind turbulence. The value of \( V_{m,j} \) can be found applying the logarithmic law:\textsuperscript{26}

\[
V_{m,j} = V_{m,10} \frac{\ln(z/z_0)}{\ln(10/z_0)}
\]  

(12)

where \( V_{m,10} \) is the mean wind speed at a reference height \( z = 10 \) m above the ground, and \( z_0 \) is the terrain roughness (0.03 m). Synthetic wind speed time histories are generated as function of \( V_{m,j} \) using the spectral approach outlined in Deodatis\textsuperscript{27} and reported in Micheli et al.\textsuperscript{28} Fig. 1 reports two examples of wind load time histories acting on the 36\textsuperscript{th} floor of the building.
3.3 Motion Control Devices

The damping device employed in this study is a Banded Rotary Friction Device (BRFD), presented in Downey et al. The BRFD is a semi-active, variable friction device, based on a double wrap band brake system. The dynamic behavior of the BRFD is characterized by the following 3-stages dynamic model:

**Stage 1** is a typical friction dynamics and the damping force $F_d$ is modeled using the LuGre friction model:

\[
F_d = \sigma_0 \zeta + \sigma_1 \dot{\zeta} + \sigma_2 \dot{\eta}
\]

\[
\dot{\zeta} = \dot{\eta} - \sigma_0 \frac{|\dot{\eta}|}{g(\dot{\eta})} \zeta
\]

\[
g(\dot{\eta}) = F_c + (F_s - F_c)e^{-\frac{(\frac{\dot{\eta}}{\eta_s})^2}{\rho_2}}
\]

where $\sigma_0$ represents the aggregate bristle stiffness, $\sigma_1$ the microdamping, $\sigma_2$ the viscous friction, $\zeta$ an evolutionary variable, $\dot{\eta}$ the device velocity, and $g(\dot{\eta})$ is a function that describes the Stribeck effect in which $\eta_s$ is a constant modeling the Strubeck velocity, $F_s$ the static friction force, and $F_c$ the kinetic friction force. The value of $F_c$ is taken as the maximum friction force $F_{\text{max}}$ (e.g., nominal capacity of the device), and $F_s = C_s F_{\text{max}}$, with $C_s$ being the static friction coefficient.

**Stages 2 and 3** are modeled as two linear stiffness regions that represent the backlash effect in the BRFD. The damping force $F_d$ is modeled as linear stiffness elements $k_2$ and $k_3$ during displacements $d_2$ and $d_3$ in stages 2 and 3, respectively.

**Transition regions** are modeled with the following smoothing function $\Omega$

\[
\Omega(\eta) = \frac{1}{1 + e^{-\frac{\rho_1(\eta - \eta_0)}{\rho_2}}}
\]

where $\eta_0$ is the reference displacement when transitioning to a new stage, and $\rho_1$ and $\rho_2$ are constants. The damping force $F_d$ within the transition from stage $i$ to stage $j$ is written as:

\[
F_d = [1 - \Omega(\eta)]F_{d,i} + \Omega(\eta)F_{d,j}
\]

Table 1 reports the values of the constants used in the 3-stages dynamic model for the BRFD simulations. The location of the HPCS devices along the building height follows the passive viscous damping configuration. It consists of 15 sets of two BRFD devices installed at every other floors, starting from the 5th floor up to the 36th floor of the building: (a) $V_{m,10} = 18$ m/s; (b) $V_{m,10} = 23$ m/s.
33\textsuperscript{th} floor. The nominal capacity for each set of BRFDs, $F_{max}$, is taken as 1,350 kN for the dampers below the 26\textsuperscript{th} floor, and 900 kN for the devices above the 26\textsuperscript{th} floor, following the motion-based design provided by the authors in previous studies to satisfy the serviceability-based requirement of the maximum acceleration ($a_{peak} \leq 25 \text{ mg}$) under frequent wind hazards.\cite{1,28}

### Table 1. BRFD parameters used in the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>7005</td>
<td>kN$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0017</td>
<td>N$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.0017</td>
<td>N$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\eta}_s$</td>
<td>0.002</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$C_s$</td>
<td>1.065</td>
<td>$-$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.03</td>
<td>kN·mm$^{-1}$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>2.3</td>
<td>kN·mm$^{-1}$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>3.05</td>
<td>mm</td>
</tr>
<tr>
<td>$d_3$</td>
<td>1.52</td>
<td>mm</td>
</tr>
<tr>
<td>$\rho_1$</td>
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<td>mm</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.01</td>
<td>mm</td>
</tr>
</tbody>
</table>

The force exerted by a generic passive viscous damper is simulated using:

$$F_v = c_v \dot{u}$$  \hspace{1cm} (18)

where $c_v$ denotes the damping coefficient and $\dot{u}$ is the relative velocity. The damping coefficients of the viscous devices are taken as 52,550 kN·s/m for the dampers below the 26\textsuperscript{th} floor, and 35,000 kN·s/m for the devices above the 26\textsuperscript{th} floor.\cite{1,24}

### 3.4 Control Strategies

Control strategies under consideration include a linear quadratic regulator (LQR) and a sliding mode controller (SMC). For the LQR algorithm, objective function $J$ is taken as:

$$J = \frac{1}{2} \int (U^TQU + F^TRF)dt$$  \hspace{1cm} (19)

with:

$$Q = \begin{bmatrix} q_d I & 0 \\ 0 & q_v I \end{bmatrix}_{2N_f \times 2N_f}$$  \hspace{1cm} (20)

where $Q$ is the regulatory weight matrix with positive definite diagonal elements $q_d$, $q_v$ and $R = q_r I_{N_f \times N_f}$ a weight matrix with positive constant $q_r$. The LQR parameters are set as $q_d = 19.5$, $q_v = 78$ and $q_r = 0.29$.

For the SMC, the sliding surface $S \in \mathbb{R}^{39 \times 1}$ is taken as

$$S = \Lambda (U - U_d)$$  \hspace{1cm} (21)

where $U_d$ is the desired state ($U_d \equiv 0$ for civil structure) and $\Lambda = [\Lambda I I] \in \mathbb{R}^{39 \times 78}$ is a user-defined weight matrix that includes strictly positive constants $\Lambda$ and identity matrix $I \in \mathbb{R}^{39 \times 39}$.

The required control force from the SMC takes form:\cite{10}

$$F_{d,req} = -\Gamma \left( [\Lambda B_f]^T [\Lambda B_f] \right)^{-1} [\Lambda B_f]^T S$$  \hspace{1cm} (22)
where $\Gamma$ is a positive pre-defined control parameter. The numerical values of the SMC controller parameters used in the simulations are $\Gamma = 110$ and $\lambda = 0.1$.

For both the LQR and the SMC algorithms, the actuation force $F_{d,act}$ is assumed to be linear as a function of the actual voltage $\nu_{act}$:

$$F_{d,act} = F_{d,0}\nu_{act}$$  \quad (23)

$$\nu_{act} = -\nu_{delay}(\nu_{act} - \nu_{req})$$  \quad (24)

where $F_{d,0}$ is a voltage scaling constant, $\nu_{req}$ is the required voltage computed from the required control force, and $\nu_{delay}$ is a positive constant taken as 200 sec$^{-1}$. The required voltage $\nu_{req}$ is computed employing a bang-bang control rule, where the voltage is set to maximum if the generic required force $F_{d,req}$ is higher than the BRFD capacity and set to zero if the signs of $F_{d,req}$ and $\delta_{max}$ are equal:

$$\nu_{req} = \begin{cases} 
\nu_{max} & \text{if } |F_{d,req}| > F_{d,max} \\
0 & \text{if } \text{sign}(\delta_{max}) = \text{sign}(F_{d,req}) \\
\frac{|F_{d,req}|}{F_{d,0}} & \text{otherwise}
\end{cases}$$  \quad (25)

where $\nu_{max}$ is the maximum allowable voltage, taken as 12 V.

### 3.5 Uncertainties

Two sources of uncertainties are considered. The first is the variability of the wind load over the lifetime of the structure, modeled with a variation of the mean wind speed $V_{m,10}$ (Eq. 12) between 5 and 28 m/s. This range of wind speeds is based on meteorological data collected for the area of Boston. The second source of uncertainties is related to knowledge of dynamic parameters. Specifically, the mass of each floor of the building is assumed to vary between $\pm 10\%$ of its initially assumed value.

The space-filling Latin hypercube method is employed to generate $x$ by sampling $N$ different values of $V_{m,10}$ and floor masses from uniform distributions normalized in $[0, 1]$. The $N$ samples are then divided in $n$ training and $n_t = 0.25\%$ $n$ testing datasets, utilized to create the Kriging surrogate model. Observations $y$ are generated by propagating the uncertainties into the system through Eq. 12 (wind load) and Eqs. 9 to 10 (structure). The performance of the HPCS is investigated in two cases: case 1, where only the wind speed is considered as uncertainty and $x \in \mathbb{R}^{N \times 1}$, and case 2, where both wind speed and floor masses are taken as uncertainties and $x \in \mathbb{R}^{N \times 40}$ (39 floor masses plus the mean wind speed). These two cases are studied for each control strategy, except for the passive viscous dampers, where only uncertainties in the wind speed are considered.

### 3.6 Life-Cycle Cost Analysis Model

The LCC of a structure equipped with an HPCS is given by:

$$\text{LCC} = C_0 + C_I + C_M + C_F$$  \quad (26)

where $C_0$ is the initial construction cost of the structure, $C_I$ is the initial cost of the HPCS, $C_M$ is the maintenance cost of the devices, and $C_F$ is the annual failure cost. The cost $C_I$ is given by the sum of the mechanical devices $C_D$ cost, the installation cost, and costs of sensors and electronics. For a generic viscous damper, $C_D$ (USD) is estimated as:

$$C_D = 0.77F_{max}^{1.207} + 2806$$  \quad (27)

The cost of the BRFD is taken as 70% of the cost of a viscous damper with an equivalent $F_{max}$. The 30% discount factor stems from the lower fabrication cost due to the relative mechanical simplicity of the BRFD. The cost of a sensor is assumed as 2,900 USD, including a lumped cost for the data acquisition systems. It
follows that the cost $C_I$ for the HPCS is 200,647 USD, including 39 sensors (one sensor per floor) and 15 damping devices. Similarly, the passive viscous strategy results in a $C_I$ being equal to 124,138 USD. The maintenance cost of the HPCS, $C_M$, is taken as 59,983 USD (adapted from $^{32}$) and considers a regular system check and an annual hardware check for 50 years (lifetime of the structure). It is assumed that no maintenance is required for the passive dampers, yielding $C_M = 0.33$. The cost $C_F$ quantifies the economic losses occurring when the structure does not meet the prescribed performance objective, and it is expressed as: $^{28}$

$$C_F = \sum_{i=1}^{n_I} C_{\text{fail}} (1 + r)^{-i\tau}$$

where $\tau$ is the cost analysis time interval (1 year), $n_I$ is the lifetime of the structure (50 years), $r$ is the expected rate of return (3%), and $C_{\text{fail}}$ is the annual cost due to failure in meeting the expected performance:

$$C_{\text{fail}} = \sum_{j=1}^{n_h} \sum_{k=1}^{n_{DS}} P_{h,j} P_{DS,k} C_{DS,k}$$

where $P_{h,j}$ is the annual probability of occurrence of the $j$-th wind hazard event, $n_h$ is the number of hazard events considered, $P_{DS,k}$ and $C_{DS,k}$ are respectively the probability occurrence and the repair costs associated with the $k$-th damage state ($DS$), and $n_{DS}$ is the total number of damage states considered. The probability $P_{h,j}$ are derived from the site-specific wind hazard curve$^{28}$ illustrated in Fig. 2(a), while $DS$ and $P_{DS,k}$ are estimated combining the structural response of the building, expressed in terms of $a_{\text{peak}}$ (computed with the Kriging surrogate) with the three fragility curves reported in Fig. 2(b). These fragility curves are associated with different levels of $a_{\text{peak}}$ and represent $n_{DS} = 3$ damage states of increasing severity. The damage states costs, $C_{DS,k}$, are related to the indirect financial losses caused by wind-induced motion sickness and discomfort of the building occupants. Details on the procedure adopted to estimate $C_{DS,k}$ are reported in Ref.$^{28,34}$

![Wind hazard curve related to the area of Boston: Weibull distribution with scale parameter 14.9 and shape parameter 6.4](image-a)

![Fragility curves representing the effects of motion sickness discomfort in the building occupants. Mean parameters of the curves: $DS_1 = 10$, $DS_2 = 20$, $DS_3 = 35$, and standard deviation of 0.12 for all curves. Corresponding costs (USMD): $C_{DS_1} = 0.10$, $C_{DS_2} = 0.20$, and $C_{DS_3} = 0.35$.](image-b)

**Figure 2.**

4. RESULTS AND DISCUSSION

This section reports the results of the evaluation procedure applied to the case study building. First, the acceleration mitigation capability of the control strategies is discussed. Then, the accuracy of the Kriging surrogate model at predicting the peak acceleration is described. Finally, the LCC results are presented as a function of the control strategy.
4.1 Accelerations Mitigation Performance

Fig. 3 plots a typical acceleration time history recorded at the 36\textsuperscript{th} floor of the building under a frequent wind hazard event (design wind intensity) for the structure equipped with BRFDs controlled by an LQR (Fig. 3(a)) and SMC (Fig. 3(b)) algorithm. The LQR yields slightly lower accelerations than the SMC.

![Figure 3](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

Figure 3. Acceleration time series at the 36\textsuperscript{th} floor of the building under wind load $V_{m,10} = 18$ m/s and an (a) LQR; and (b) SMC control algorithm.

Fig. 4 reports the maximum acceleration profile for the structure equipped with BRFDs controlled under LQR (labeled as “LQR no uncertainties” in the figure), using passive viscous dampers, and the building without damping devices (uncontrolled) under two different wind speeds. The figure shows that the BRDF controlled with the LQR provides a better mitigation performance than the passive strategy under both wind speeds. Furthermore, the figure illustrates the maximum acceleration profile for the LQR under two typical scenarios which differ for the values assumed for the floor masses (scenarios 1 and 2), demonstrating that the performance of the BRFD controlled with LQR is affected by uncertainties in the structure’s mass, in particular at the top floors of the building. Also, one can notice that, in comparison with the case with “no uncertainties” in floor masses, the performance reduction highly depends on the scenario and wind speed considered. Fig. 5 is a similar plot for the BRFD controlled with the SMC. One can also notice under this case that uncertainties in floor masses influence the structural response.

![Figure 4](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

Figure 4. Acceleration profiles for BRFD controlled with an LQR algorithm under two different wind speeds: (a) $V_{m,10} = 18$ m/s; (b) $V_{m,10} = 23$ m/s. Scenarios 1 and 2 have different floor mass values.
Figure 5. Acceleration profiles for BRFD controlled with an SMC algorithm under two different wind speeds: (a) $V_{m,10} = 18$ m/s ; (b) $V_{m,10} = 23$ m/s. Scenarios 1 and 2 have different floor mass values.

4.2 Kriging Surrogate Accuracy

The Kriging surrogate accuracy is quantified with two error metrics, namely the root mean square error (RMSE) and the normalized maximum absolute error (NMAE). The RMSE quantifies the accuracy of the metamodel globally and it is given by:

$$RMSE = \sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} (y_i - \hat{y}_i)^2}$$  \hspace{1cm} (30)

where $\hat{y}_i$ denotes the maximum structural response estimated by the Kriging surrogate, and $y_i$ the real maximum structural response derived from numerical simulations. The NMAE is related to the local accuracy of the metamodel and it is defined as:

$$NMAE = \frac{\max |(y_i - \hat{y}_i)|}{n_t \sigma_y}$$  \hspace{1cm} (31)

where $\sigma_y$ denotes the standard deviation of the testing data set. Both the RMSE and NMAE are estimated on the testing data set.

Table 2 reports RMSE and NMAE values for each control strategy and uncertainty case, along with the corresponding number of variables, training, and testing data set sizes. Results in Table 2 show that LQR - case 2 and SMC - case 2 lead to the largest RMSE and NMAE values. This could be attributed to the larger number of variables ($k = 40$) involved in the metamodeling process. Also, LQR - case 1 and viscous strategy yield similar values of RMSE and NMAE, while SMC - case 1 presents slightly higher errors, although $k = 1$. This might be attributed to the nonlinear behavior of the SMC controller. In terms of number of training observations, in case 1 both LQR and SMC require a relatively low number of observations to obtain an acceptable performance (RMSE $\leq 5\%$), while in case 2, $n = 800$ samples are necessary.

4.3 Life-Cycle Cost Results

The LCC results of the LQR controller are reported in Fig. 6 and Table 3. Fig. 6(a) plots the probability distribution of the failure cost $C_{\text{fail}}$, while Fig. 6(b) reports the probability distribution of the LCC under the two uncertainty cases. Table 3 lists the mean and standard deviation of the LCC. Results demonstrate that the standard deviation of $C_{\text{fail}}$ and LCC increases significantly when uncertainties in the floor masses are considered (case 2) in comparison with the case where only uncertainties in the wind load are evaluated (case 1). Results
Table 2. Kriging accuracy results.

<table>
<thead>
<tr>
<th>case</th>
<th>k</th>
<th>n</th>
<th>n_t</th>
<th>RMSE (%)</th>
<th>NMAE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LQR - case 1</td>
<td>1</td>
<td>100</td>
<td>25</td>
<td>2.80</td>
<td>0.90</td>
</tr>
<tr>
<td>LQR - case 2</td>
<td>40</td>
<td>800</td>
<td>200</td>
<td>4.80</td>
<td>1.80</td>
</tr>
<tr>
<td>SMC - case 1</td>
<td>1</td>
<td>100</td>
<td>25</td>
<td>4.40</td>
<td>1.65</td>
</tr>
<tr>
<td>SMC - case 2</td>
<td>40</td>
<td>800</td>
<td>200</td>
<td>5.00</td>
<td>1.70</td>
</tr>
<tr>
<td>Viscous</td>
<td>1</td>
<td>100</td>
<td>25</td>
<td>2.36</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Table 3. Life-cycle cost statistics.

<table>
<thead>
<tr>
<th>case</th>
<th>mean LCC (USMD)</th>
<th>std LCC (USMD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LQR - case 1</td>
<td>4.20</td>
<td>1.60</td>
</tr>
<tr>
<td>LQR - case 2</td>
<td>4.36</td>
<td>1.97</td>
</tr>
<tr>
<td>SMC - case 1</td>
<td>4.46</td>
<td>1.53</td>
</tr>
<tr>
<td>SMC - case 2</td>
<td>4.55</td>
<td>1.87</td>
</tr>
<tr>
<td>Viscous</td>
<td>5.00</td>
<td>2.35</td>
</tr>
</tbody>
</table>

in Table 3 confirm that the standard deviation of LCC increases of 23.10 % from case 1 to case 2. Also, one can notice that the mean LCC is slightly affected by the uncertainties in the system, as expected. Also, Fig. 6 reports the probability distribution curves of C_fail and LCC for the passive viscous case. A cross-comparison between the semi-active and passive strategies shows that the BRFD controlled with the LQR provides a lower mean and standard deviation of the LCC, with and without considering uncertainties in the floor masses.

![Figure 6. LQR results: (a) cost of failure probability distribution; (b) LCC probability distribution.](image)

Table 3. Life-cycle cost statistics.

<table>
<thead>
<tr>
<th>case</th>
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<tbody>
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</tr>
<tr>
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<tr>
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<td>1.53</td>
</tr>
<tr>
<td>SMC - case 2</td>
<td>4.55</td>
<td>1.87</td>
</tr>
<tr>
<td>Viscous</td>
<td>5.00</td>
<td>2.35</td>
</tr>
</tbody>
</table>

Fig. 7 reports the C_fail and LCC probability distributions curves for the SMC controller. Results demonstrate that also in this case the standard deviation of C_fail and LCC increases when uncertainties in the floor masses are considered. Specifically, from Table 3 one can notice that in case 2 the LCC standard deviation increases of 18.20 % in comparison with case 1. Similarily than in the LQR case, the mean LCC attains similar values in cases 1 and 2. In addition, results in Fig. 7 demonstrate that the SMC yield a lower mean and standard deviation LCC than the passive viscous case. A comparison between the statistics of the LQR and SMC controllers shows that under case 2 the SMC leads to a slightly higher mean and standard deviation of the LCC than the LQR.
5. CONCLUSION

In this paper, a surrogate-based performance evaluation strategy for high-performance control systems (HPCSs) was investigated. The Kriging surrogate model (metamodel) was employed to assess the effects of uncertainties in the knowledge of dynamic parameters on the HPCS performance. The metamodel was used to map uncertainties in the dynamic properties of the structure and external load intensity (inputs to the surrogate) to the structural response (output), expressed in terms of peak acceleration. The prediction capability of the Kriging surrogate was exploited to predict the structural response in correspondence of a large number of new inputs. The predicted outputs were then utilized to estimate the life-cycle cost of the structure equipped with HPCS, where the long-term savings in peak acceleration mitigation were related to motion sickness and building occupants’ discomfort reduction. The surrogate-based performance evaluation strategy was applied to a 39-story case study building, located in Boston (MA), and equipped with a set of semi-active friction devices for wind-induced vibrations mitigation. The life-cycle performance of two controllers, namely a linear quadratic regulator (LQR) and a sliding mode control (SMC), was evaluated in two uncertainties cases. In case 1 only uncertainties in the wind load were considered, while in case 2 uncertainties in wind load and dynamic properties were examined.

Results demonstrated that the semi-active friction devices controlled with LQR and SMC provided a better acceleration mitigation performance than traditional passive damping devices. However, their performance was affected by uncertainties in floor-mass and wind speed. In terms of metamodel accuracy, results showed that under case 2, both the LQR and SMC required a larger number of training observations \( n = 800 \) to obtain an acceptable accuracy (root mean square error \( \leq 5\% \)) in comparison to case 1, where only 100 samples were necessary. This was attributed to the larger number of uncertain variables under case 2. The LCC analysis results demonstrated that, for both the LQR and SMC, the standard deviation of the LCC increased significantly when uncertainties in the floor masses were considered (case 2) in comparison with the case where only uncertainties in the wind load were examined (case 1). A comparison between LQR and SMC controllers showed that the LQR presented a slightly better life-cycle performance than the SMC, yielding a lower mean and standard deviation of the costs. In addition, the LCC of LQR and SMC were compared with that of an equivalent passive strategy, demonstrating that the semi-active control systems were able to provide a lower LCC than the passive viscous case although the uncertainties in the dynamic properties of the structure. Future work will entail the integration of a larger number of uncertainties in the performance evaluation strategy.

REFERENCES


