A DISCUSSION OF THE INVERSE PROBLEM IN ELECTROMAGNETIC NDT

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INTRODUCTION

The principal components of a nondestructive testing (NDT) system are shown in Figure 1. The specimen to be tested is energized by a transmitting transducer. The response of the energy-specimen interaction is picked up by a receiving transducer. This signal is then processed suitably and analyzed for defect characterization. The most critical step here is the inverse problem which involves the characterization of the specimen parameters given an NDT probe response signal. This paper is mainly concerned with the solution of the inverse problem.

Ideally, one wishes to develop an analytical model that gives a closed form functional representation of the probe signal in terms of the test specimen parameters. In electromagnetic NDE, however, the material nonlinearities and awkward testing geometries render the problem analyticaly intractable. This has resulted in the increasing popularity of numerical methods such as the finite element model. However, numerical methods do not give a closed form solution, and hence are not directly useful for solving the inverse problem.

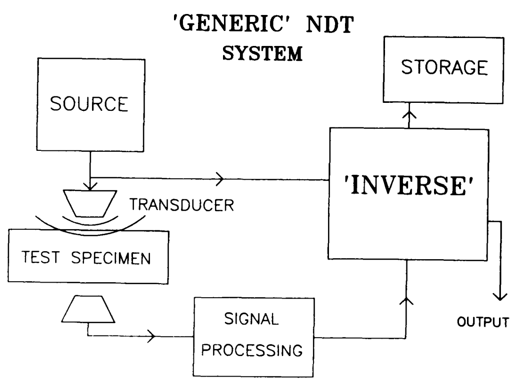

Fig. 1: The components of an NDT system.

In the following sections, a typical electromagnetic NDT problem is first formulated using finite elements and Green's function methods. The two methods are compared with regard to their advantages and limitations. A procedure, using the Green's function, for solving the inverse problem in eddy current NDT is then described and compared to an Artificial Intelligence (AI) approach that uses the finite element model to solve the general inverse problem.

FINITE ELEMENT FORMULATION

The major aspects of finite element modeling for electromagnetic NDT problems are described in a number of references [1,2]. The region of interest is subdivided into a finite number of triangular elements connected to each other at a discrete set of nodal points. The variation of the continuous field quantity is approximated by a polynomial in such a way that the approximated function is continuous across the inter-element boundaries. The nodal point values are determined by minimizing an energy related functional derived from the governing equation, which yields a set of linear equations in the unknown nodal point values. Since the unknown value at each node is expressed in terms of the values at the adjoining nodes the resulting matrix is sparse, banded, symmetric and diagonally dominant. These attributes make the numerical computation robust and stable. The finite element formulation for the magnetostatic NDT problem is discussed by Lord and Hwang [3], and the formulation for the eddy current NDT problem is described by Lord and Palanisamy [4].

INTEGRAL FORMULATION USING GREEN'S FUNCTIONS

The general governing equation in an electromagnetic NDT problem, under linear and isotropic conditions, can be written as

$$(\nabla^2+k^2)f(\underline{x}) = g(\underline{x}) \tag{1}$$

subject to boundary conditions. In (1) $g(\underline{x})$ is the source in terms of current or charge distribution, $f(\underline{x})$ represents the field (E/H) or potential (scalar/vector) distribution in the region of interest and k^2 is a real, imaginary or complex number derived from the material properties and operating frequencies. When k^2 is pure real, the equation reduces to the Helmholtz wave equation and in the case of the eddy current problem k^2 is a pure imaginary number.

A powerful analytical tool for studying boundary value problems is provided by Green's function and Green's theorem [5,6]. Basically the Green's function is a two-point function which is the kernel of the integral equations solution of the given differential equation. It can also be considered as the impulse response of the system, satisfying the equation

$$(\nabla^2+k^2)G(\underline{x},\underline{x}') = \delta(\underline{x}-\underline{x}') \tag{2}$$

where the primed coordinates correspond to the source and the unprimed coordinates, to the observation point. Considering an infinite medium with boundaries at infinity and applying Green's theorem, the solution for (1) is obtained as

$$f(\underline{x}) = \int_V g(\underline{x}')G(\underline{x},\underline{x}')dv' \tag{3}$$

the superposition volume integral that can be evaluated analytically or more often, numerically. The integral equation formulation using Green's function, for the magnetostatic and eddy current field problems can be found in the publications of Simkin and Trowbridge [7,8] and McWhirter et al [9].

ADVANTAGES AND LIMITATIONS OF THE TWO METHODS

In principle, the Green's function method has several advantages over the finite element method of solution for the forward problem in electromagnetic NDT.

1. The Green's functions are independent of the shape of the source or source distribution.
2. The integral equation reduces to much smaller systems of equations and affords considerable reduction of data required to run a problem.
3. Representation of boundary conditions is more accurate than in the finite element formulation.
4. Finally, the integral equation formulation provides a means to solve the inverse problem. Writing (3) in discrete-space as the matrix equation

   $$\{f\} = [G]\{g\} \tag{4}$$

   one can solve for the unknown source {g} by computing

   $$\{g\} = [G]^{-1}\{f\} \tag{5}$$

   provided [G] is well-conditioned and invertible.

However, the Green's function method also suffers from some severe limitations which make the method difficult to apply in practice.

1. The derivation of the Green's function is extremely difficult except in the case of very simple geometries such as an infinite medium or axisymmetric geometries that can be treated in one dimension. In realistic NDT situations the geometry is often complicated by randomly shaped defects and interface boundaries making analytical computation of the Green's functions impossible.
2. Implicit in the formulation of the superposition volume integral is the requirement of linearity. This immediately precludes the use of the method in magnetostatic NDT used for testing ferromagnetic materials. In the active leakage field technique, the ferromagnetic test specimen is first magnetized by passing a current through it. The magnetic flux lines are set up as shown in Figure 2. The presence of a defect results in a small leakage field that can be detected by any flux sensitive probe. The governing equation for the active leakage field phenomenon is given by

   $$\bar{\nabla}\times\left(\frac{1}{\mu}\bar{\nabla}\times\bar{A}\right) = \bar{J} \tag{6}$$

   where μ is the permeability and $\bar{A}$ is the magnetic vector potential. The equation (6) together with the magnetization characteristics in Figure 3 describe a nonlinear system which cannot be solved using a single superposition integral.

Fig. 2: The Active leakage field phenomenon.

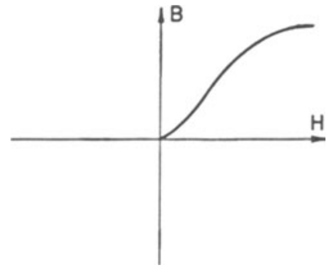

Fig. 3: A typical magnetization curve.

3. Finally the matrix [G] in (4) is full and severely ill-conditioned and the inversion required in (5) to evaluate the unknown source is a formidable problem.

These limitations have resulted in the increased popularity of the finite element method which is not limited by complex defect geometries, nonlinear material characteristics or numerical instabilities. The major limitation of the numerical model is the lack of a parametric expression that can be used for solving the inverse problem.

In the following sections two methods for solving the inverse problem in electromagnetic NDT are presented. The integral equation approach using Green's functions is first described and compared with an AI approach using the finite element model.

INTEGRAL EQUATION INVERSION FOR EDDY CURRENT DATA

In the basic eddy current test illustrated in Figure 4, the probe coil excited by an alternating current sets up a time varying magnetic field. When the probe is brought close to a conducting medium, in accordance with the Maxwell-Faraday law, the time varying H field induces an emf in the test medium resulting in an eddy current distribution, such that the secondary flux associated with the eddy currents opposes the primary flux of the probe. The presence of a flaw in the medium perturbs and redistributes the eddy currents, altering the net flux linkages and thereby the impedance of the coil. These impedance changes, measured using an AC bridge, constitute the eddy current test data. In the case of nonferromagnetic test specimens the governing equation is a linear equation

$$(\nabla^2+k^2)\overline{E}(\underline{x}) = \overline{J}_s \tag{7}$$

where

$$k^2 = j\omega\mu\sigma , \tag{8}$$

μ is the permeability, σ the conductivity and ω is the frequency of excitation. Using Green's theorem we can obtain the integral equation

$$\overline{E}(\underline{x}) = \int_v G(\underline{x},\underline{x}')\overline{J}_s(\underline{x}')dv' \tag{9}$$

Since the change in probe impedance due to a flaw can also be defined as

$$\Delta z_{coil} = \frac{\text{Emf induced by the perturbed secondary flux}}{\text{Excitation current}}$$

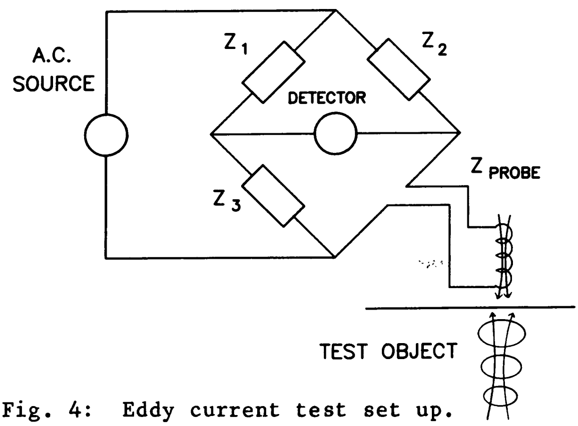

Fig. 4: Eddy current test set up.

the flaw can be modeled by an equivalent current source $J_f(\underline{x})$ which produces a perturbation field at the coil which in turn is responsible for the eddy current signal. Given the perturbation field $E_p(\underline{x})$ and the Green's function for the system, the integral equation

$$\bar{E}_p(\underline{x}) = \int_{flaw} G(\underline{x},\underline{x}')\bar{J}_f(\underline{x}')dv' \tag{10}$$

can be used to solve for the equivalent current source $J_f(\underline{x})$. Considering Maxwell's equations in the test material with and without a defect the equivalent current source can be deduced as [10]

$$\bar{J}_f(\underline{x}) = \{\sigma_f(\underline{x}) - \sigma_0(\underline{x})\}\bar{E}_f(\underline{x}) \tag{11}$$

where σ_f is the spatial distribution of the conductivity in the presence of the flaw and σ_0 is the conductivity distribution of the defect free material. Hence the solution for J_f also gives the conductivity profile or defect profile. The implementation of this method consists of the following steps.

1. Compute the Green's function for the given geometry.

2. Calculate the unperturbed field given by

$$\bar{E}_0(\underline{x}) = \int_{coil} G(\underline{x},\underline{x}')\bar{J}_0(\underline{x}')dv' \tag{12}$$

3. Calculate the equivalent current source for the flaw by solving

$$\bar{E}_f(\underline{x}) = \int_{flaw} G(\underline{x},\underline{x}')(\sigma_f-\sigma_0)\bar{E}_f(\underline{x}')dv'+E_0(\underline{x}) \tag{13}$$

4. Compute the perturbed field at the coil according to

$$(\bar{E}_f-\bar{E}_0)(coil) = \int_{flaw} G(\underline{x},\underline{x}')(\sigma_f-\sigma_0)E_f(\underline{x}')dv' \tag{14}$$

5. Compute the emf induced in the coil by

$$Emf_{model} = 2\pi n \int_{coil} (\bar{E}_f-\bar{E}_0)dv \tag{15}$$

6. If $|Emf_{model}-Emf_{expt}| < \varepsilon$, σ_f is the desired flaw profile.

   Else update σ_f and go to 3.

Apart from the issue of deriving a closed form Green's function, in step 3 is a nonlinear integration in the unknowns $\underline{E}_f$, σ_f and the flaw volume. Steps 3 to 6 have to be executed iteratively using specialized numerical techniques for inverting the ill-conditioned matrices.

AI APPROACH FOR SOLVING THE GENERAL INVERSE PROBLEM IN ELECTROMAGNETIC NDT

One of the commonly used problem-solving techniques in Artificial Intelligence is the method of trial-and-error or heuristic search [11] where one searches for the optimal solution out of a set of allowable solutions. The general search procedure can be explained with the help of Figure 5.

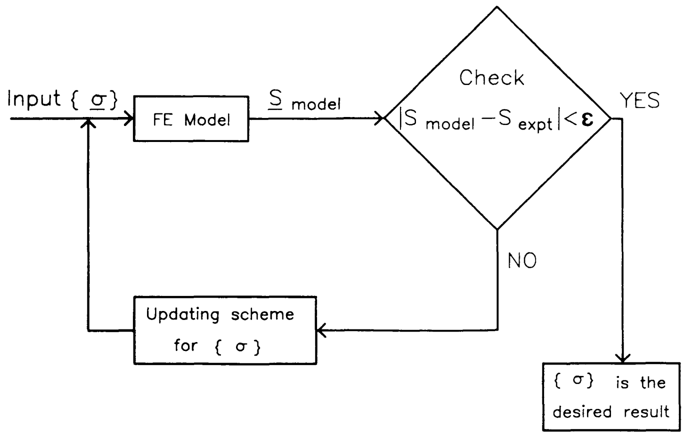

Fig. 5: Block diagram for the inversion.

The problem can be defined as a state-space search, where the state-space contains all possible configurations of the system. Given an initial state, the conductivity distribution vector $\{\underline{\sigma}\}$ and a termination criteria which requires the FE model output to be close to the experimental signal, a procedure for searching a "tree" of logical possibilities is designed using the feedback loop in Figure 5. The operation consists of a generation phase and an evaluation phase. In the generation phase, based on the rules of the updating scheme, the successors to the start node are generated. In the evaluation phase, these new successor vectors $\{\underline{\sigma}\}$ are evaluated by obtaining the FE model response to the input $\{\underline{\sigma}\}$ and comparing it with the experimental signal. If the goal node is not found, the tree expansion continues. Depending on the nature of the updating scheme or the node expansion scheme the search will be exhaustive or minimal. With the test material discretized into cells and the input vector $\{\underline{\sigma}\}$ being the conductivity values of each cell, the search space for $\{\underline{\sigma}\}$ can be exceedingly large. One way to reduce the search effort is by introducing domain-specific heuristics which will limit the number of successors of a node and minimize the search effort. Such an algorithm was implemented for magnetostatic active leakage field data. Two geometrical constraints were imposed viz.,

1. The defect boundary can be represented by a sequence of edges that correspond to the states of a discrete-valued Markov process.

2. The defect (crack) grows only narrower with depth.

The transition probabilities associated with the underlying Markov process together with the second constraint can be represented by matrices that govern the tree search procedure. The (FEM) defect signal in Figure 6a corresponding to the defect (rectangular slot) in Figure 6b was then input to the algorithm.

Fig. 6a: Leakage field signal for a defect in Figure 6b.

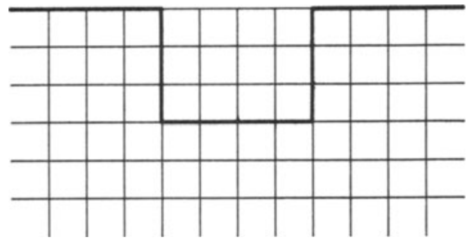

Fig 6b: Discretization of bar with rectangular defect.

The surface extent in terms of the right and left edge of the slot was first established by a simple edge-detection algorithm. The tree searching procedure was then executed to determine the depth profile of the defect. Figure 7 shows the tree expansion using the constraints 1 and 2. The desired defect profile was obtained after 20 incorrect alternatives.

The algorithm was then made more intelligent by requiring that at each stage of expansion the node with the smallest cost function be expanded first. The search was then minimized to 9 nodes as shown in Figure 8. Additional information in the signal can also be embodied into the system to further reduce the search.

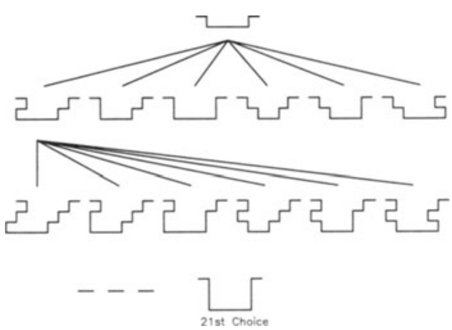

Fig. 7: Tree expansion of the exhaustive search.

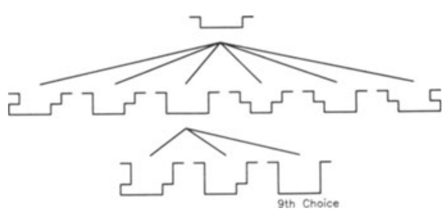

Fig. 8: Tree expansion of the ordered search.

CONCLUSIONS

An AI approach to solving the inverse problem has several benefits over an integral equation formulation. Since the method essentially employs the finite element model, it is not limited by complex geometries or non-linearities of material properties. Secondly, the matrices associated with the FE model being sparse and well-behaved, this method is numerically stable. Finally, the approach introduced here can be applied to a variety of problems such as eddy current NDT or ultrasonic NDT by switching to the appropriate FE model. In conclusion, the Green's function approach is a very powerful tool for linear problems with simple geometries. For more realistic NDT geometries involving non-linearities the AI approach appears to have significant potential.

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