Strategic underproduction and ownership limit policy in cap-and-trade

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Abstract

Cap-and-trade regulations enable tacit coordination and strategic underproduction of a regulators targeted cap. Strategic underproduction increases industry profits but lowers consumer surplus and creates cost inefficiency. These outcomes are prevented by limiting the quantity of production/emissions permits owned by individual firms. Ownership limits that are too stringent or too lax however can reduce welfare below levels that obtain without ownership limits. We demonstrate these results for the U.S. west coast groundfish fishery. Fishing permit ownership limits currently range between 2%-5% of total available permits. Welfare maximizing limits range between 10% - 50%. We predict a 25% reduction in welfare, due to foregone consumer surplus, under limits that are too lax. Results find that overly stringent ownership limits create scale inefficiency in groundfish production. Our results provide new directions for preventing market power inefficiency under cap-and-trade regulations.

JEL Classification: L13, Q2

Keywords: Quantity-based regulation, market power, strategic underproduction, ownership limits.

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1 Introduction

Quantity-based regulation, or cap-and-trade, is used in fisheries in the United States\(^1\), and throughout the world (Newell et al., 2005) to cap aggregate fishing mortality at levels that are deemed sustainable. The resource manager issues tradeable fishing permits that in aggregate sum to the desired total harvest during a given calendar period. A concern among managers and stakeholders who adopt these regulations is concentration of permit ownership by a small number of resource users (Anderson, 1991, 2008; Hatcher, 2012; Newell et al., 2005). Permit ownership concentration has been linked to undesirable outcomes including unfair distribution of the economic benefits among resource users and potential for market power and its attendant inefficiency (Hahn, 1984; Anderson and Holliday, 2007; Hatcher, 2012; Helgesen, 2022; Malueg and Yates, 2009).

Concerns over excessive concentration of individual fishing quota (IFQ) permits in fisheries has effected the introduction of ownership limit regulation. The U.S. Magnuson-Stevens Fishery Conservation and Management Act (MSFCMA) § 303A (5) states that all quota-based regulations in U.S. fisheries shall “(D) ensure that limited access privilege [individual fishing permit/quota] holders do not acquire an excessive share of the total limited access privileges in the program by (i) establishing a maximum share, expressed as a percentage of the total limited access privileges, that a limited access privilege holder is permitted to hold, acquire, or use; and (ii) establishing any other limitations or measures necessary to prevent an inequitable concentration of limited access privileges; and (E) authorize limited access privileges to harvest fish to be held, acquired, used by, or issued under the system to persons who substantially participate in the fishery, including in a specific sector of such fishery, as specified by the Council. This paper evaluates the design and efficacy of these policies in redressing market power inefficiency in industries that adopt quantity-based regulations.

A first contribution of this paper is to expose a neglected channel of market power inefficiency in cap-and-trade (CAT) industries. We hereafter term this channel, *strategic underproduction*. We

\(^1\)See [https://www.fisheries.noaa.gov/national/laws-and-policies/catch-shares](https://www.fisheries.noaa.gov/national/laws-and-policies/catch-shares).
present a two-stage game of production under an individual fishing quota regulation. In stage one, firms trade individual fishing quota (IFQ) permits until mutually beneficial exchanges are complete, i.e., permit trade is Pareto efficient in our model. In a second stage, firms engage in non-cooperative, permit-constrained Cournot competition. We characterize model primitives, i.e., numbers of firms, firm-specific production costs, the extent of \textit{ex ante} cost heterogeneity, the size of the aggregate production cap, and the demand for the industries product that support a strategic underproduction Nash equilibrium of our two-stage game.

Strategic underproduction equilibria arise through tacit coordination of individual firm production capacities.\(^2\) A strategic underproduction equilibria requires one or more firms acquire permits in excess of their own production, and leave these permits unproduced during the Cournot production stage subgame. Idling IFQ permits raises the downstream market price for the industry output. Total and individual firm revenues increase as does industry profit. Consumer surplus and cost efficiency of production both decline. The fully efficient outcome, which in our model aligns with the social welfare maximizing production plan, requires production of the entire cap with per-firm production satisfying a well-known equi-marginal cost condition \cite{Montgomery1972}. An under-production equilibrium allocates production across firms such that the incentive to idle IFQ permits is maintained. The equilibrium allocation of production does not minimize total cost. The two channels of inefficiency that arise in equilibrium, underproduction and cost inefficiency, can substantively reduce total welfare.

We calculate lost consumer surplus and cost efficiency across a range of model primitives for which strategic underproduction equilibria obtain. A surprising result is that the total production by CAT-regulated firms can decline when the regulated cap exceeds a threshold value.\(^3\) Welfare loss

\(^2\)Eső et al. \cite{Esőetal2010} presents a model of a vertically coordinated industry in which upstream firms supply a fixed quantity of an essential factor input to downstream producers. Symmetric downstream producers engage in the essential input-constrained Cournot competition. The equilibrium, as our model, is characterized by a single capacity unconstrained producer and a subset of capacity-constrained producers. Our paper is the first to evaluate strategic underproduction under CAT regulations.

\(^3\)Eső et al. \cite{Esőetal2010} derive this same result in the context of a vertically coordinated and capacity-constrained industry. They show that industry production may decline when available capacity increases.
tends to be largest when firms are heterogeneous in own costs, when own firm costs increase sharply with own firm production and, not surprisingly, when the demand for the final product is inelastic.

A second contribution of our paper is guidance for designing welfare-maximizing ownership limit policies under CAT. Ownership limits, when correctly set, can prevent strategic underproduction. They can also, under specific conditions that we identify, restore full efficiency in production. The design of the optimal ownership limits vary in complex ways across a range of model fundamentals. Limits that are set too stringently or too lax can reduce efficiency/welfare below levels that obtain when no ownership limits are imposed.

We show that total welfare under a CAT regulation is non-monotonic in the stringency of the permit ownership limit. As the ownership limit is reduced it first binds for the largest quota holder in the industry, which is also the most cost efficient firm in the industry. As the ownership limits becomes too stringent and crosses a threshold that we define below, the lowest cost producer defers its permit idling role to some other firm(s). The role switching reallocates production to higher cost firms and can amplify the cost inefficiency in strategic underproduction equilibria. We derive industry conditions under which total welfare drops discretely as the ownership limit policy cross the overly stringent threshold. The policy guidance that we provide is currently absent from the literature.

We demonstrate the challenges of designing an effective ownership limit policy in the context of the U.S. west coast groundfish quota management program. We calibrate our model to the U.S. west coast groundfish fishery, which currently imposes strict limits on the quantity of permits owned by individuals. We simulate equilibrium outcomes and welfare under competing scenarios for firm conduct, and under welfare-maximizing ownership limits. The results find that a strategic underproduction equilibria would result in total welfare losses in the range of 16% - 25%.

Standard measures to detect market power such as industry concentration and Lerner indices, which are used regularly by the U.S. Department of Justice to detect non-competitive markets, do not detect tacit collusion and/or strategic underproduction in CAT-regulated markets. A byproduct
of our results is guidance for regulators who are concerned with strategic underproduction.

The next section reviews related literature and background to support our assumption of efficiency in permit trading in real world CAT. Section 3 presents the model. Section 4 characterizes the underproduction equilibrium of the two-stage permit trade and production game. Section 5 presents results on the design of a welfare-maximizing ownership limit policy. Section 6 presents an application of our model to the west-coast groundfish fishery. Section 7 summarizes and discusses extensions. Proofs and supporting materials are collected in an appendix.

2 Related literature and background

One strength of CAT regulation is their ability to implement an aggregate pollution emissions cap or an aggregate production target at minimum cost in settings where individual firm costs may be unknown to the regulator. CAT permits must however be traded in frictionless, competitive markets (Montgomery, 1972) to achieve cost efficiency. Transactions costs and market power in permit trading can prevent efficiency-improving permit exchange, which raises the cost of producing the cap that is set by the regulator.

The workhorse model of market power in permit trading is attributed to Hahn (1984). The model assumes CAT permits are traded in a one-shot, single-price market. The model assumes that a large price-making firm will be, depending on an initial allocation of permits, either a net seller or a net buyer of permits. If a net seller - a monopolist - the firm curtails its permit sales to a set of price taking fringe firms, to prop up the permit trading price and increase its own permit sales revenues. If the firm is a net buyer of permits - a monopsonist - it curtails its permit purchases to keep the permit trading price low and reduce its own expenditures on the additional permits that is acquires.

4See Stavins (1995); Singh and Weninger (2017) for studies on transactions costs in CAT.
5There are exceptions. Malueg and Yates (2009) assume permit trades are facilitated by a Walrasian auctioneer. Firms submit price-quantity schedules and an auctioneer solves for the price that equates aggregate demand with fixed permit supply. In Dickson and MacKenzie (2018) firms submit lump sum bids if they are permit buyers, or willing sale quantities if they are permit sellers. An equilibrium price is determined as the total money bid by all firms, divided by the total permits supplied. This mechanisms allows all traders to influence the price and avoids limitations of trade predicted under the dominant firm/competitive fringe model.
As with trading transactions costs, monopoly/monopsony power prevents what would otherwise be total-cost-reducing permit trades. The result is higher total cost of producing the regulated cap.

The implications of monopoly/monopsony power in permit trading is examined under varying market structures (Westskog, 1996; Okumura, 2016), in settings with incomplete information (e.g., Malueg and Yates (2009)), in the context of a tradeable fishing quotas (Anderson, 1991, 2008; Hatcher, 2012; Helgesen, 2022), and when the dominant firm attempts to raise rivals costs through its permit trading activity (Misiolek and Elder, 1989; Sartzetakis, 1997; von der Fehr, 1993; Fershtman and de Zeeuw, 1995).

Our model turns off the transactions costs and market-power distortions in permit trading altogether. There are several reasons for this. First, the implications of trading frictions in CAT are well-understood. Second, and perhaps due to the insights from the above cited literature, real world cap-and-trade regulations often include programs designed specifically to reduce permit trading frictions and/or explicitly promote centralized and/or bilateral permit exchange.\(^6\)

Our paper is not the first to consider conditions that support underproduction of a regulated cap. Anderson (2008) uses a single dominant firm and competitive fringe model to derive the share of fishing permits at which the dominant firm will choose to curtail production in order to raise its own profit. The model abstracts from permit trading element of the CAT regulation.\(^7\)

Under-production of a cap in a CAT regulated industry means that a portion of the total available production permits are not produced. Under producing the cap raises the price of the industry output. It however also raises the valuation of the unused production permits for all producers in

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\(^6\) Bilateral private permit exchange dominates many quantity-based regulations that currently exist. Permit trading in the European Union Emissions Trading System, for example, occurs primarily as over the counter exchanges, or trades conducted outside of formal exchanges, directly between buyer and seller. Over the counter trades accounted for 16% to 80% of all permit exchange during 2005-13 (Wallner et al., 2014). Permit trading in the U.S. Sulphur dioxide CAT program can be similarly characterized, with the bulk of the exchange occurring bilaterally. In IFQ-regulated fisheries, for example, harvest rights are delineated as shares of an annual allowable catch, which is set by management year to year depending on bioeconomic conditions. Trades of perpetual ownership shares and associated annual catch quantities are frequent, often conducted bilaterally at non-publicly disclosed terms/prices (Newell et al., 2005). We review these programs in detail in appendix 8.1.

\(^7\) Anderson (2008) derives IFQ permit ownership limits for which the models’ dominant firm produces the entire total annual allowable catch in a fishery. We calculate welfare-maximizing ownership limits for the U.S. west coast groundfish fishery. Ownership limits derived under the two starkly different models are not comparable.
the industry. Idling permits thus forfeits the sales revenue that would otherwise obtain by selling idled permits to some other firm(s). Pareto-efficient permit exchange must be a maintained feature of equilibrium production under CAT (see also Eső et al. (2010)).

Eső et al. (2010) present a model of vertically coordinated production. An upstream sector supplies a fixed quantity of capacity to a downstream sector. Firms in the downstream sector, which are assumed to be \textit{ex ante} identical, subsequently engages in capacity-constrained Cournot competition.\(^8\) Eső et al. (2010) derive conditions under which the downstream sector includes a single capacity-unconstrained firm and multiple capacity-constrained firms.

Our model of CAT regulation closely follows Eső et al. (2010). We extend Eső et al. (2010) to include \textit{ex ante} heterogeneous firms. We focus our analysis on economic performance under a CAT regulation and the role of ownership limit policies in production and efficiency.

3 Model

We consider a CAT-regulated industry during a single regulatory cycle. We adopt the convention of using lower case to denote the firm-level and upper case to denote industry aggregate components of our model. The model is developed in the context of an aggregate production quota regulation. This setting illustrates the features of CAT as a production capacity commitment mechanism, directly and simply. The model is easily extended to the case of a pollution emissions CAT regulation.

The industry produces \(M\) products each of which is regulated with a product-specific aggregate cap. There are \(I\) firms in the industry, indexed \(i = 1, \ldots, I\). The aggregate cap is denoted \(\bar{Q}\). Permit quantities held by individual firms are denoted \(q_i\) for firm \(i = 1, \ldots, I\). Both \(\bar{Q}\) and \(q_i\) are vectors of dimension \(M\). The consumer demand for the product, the production technology, and the regulation are exogenously given.

Firms trade permits and make production decisions to maximize their own profit. Firm-level and aggregate production vectors are denoted \(h\) and \(H\), respectively. The CAT limits \(h_i \leq q_i\) for

\(^8\)The allocation of fixed capacity in the first stage is assumed to be Pareto efficient.
each firm and thus $H \leq \bar{Q}$ for the industry.

We use $c_i(h)$ to denote the firm level production cost for firm $i$. We assume $c_i(h)$ is strictly increasing and convex in $h$. A convenient parametric form of costs is introduced in the next section.

Firms sell a homogeneous product at common price (vector) $p(H)$. We assume $p_m(\cdot)$ is non-increasing and differentiable in $H_m$. Assumptions for cross-price effects follow below.

We use $q = (q_1, q_2, \ldots, q_I)$ to denote a permit allocation. Regulations require $\sum_{i=1}^I q_{im} = \bar{Q}_m$ for all $m = 1, \ldots, M$, where $\bar{Q}_m$ is the maximum aggregate quantity of product $m$ that can be legally supplied to the market.

Production profit for firm $i$ is give as,

$$\pi_i = p(H)h_i - c_i(h_i) \text{ subject to } h_i \leq q_i \forall i. \tag{1}$$

### 3.1 Efficiency

There are two channels by which inefficiency can arise: reduced consumer surplus and production cost inefficiency. Unless explicitly stated we assume that the production of $\bar{Q}$ is socially optimal in the sense that producing $H = \bar{Q}$ cost efficiently maximizes total social welfare. Where hereafter refer to this outcome and the first best. We do not consider the problem of choosing $\bar{Q}$ to maximize social welfare in this paper.

When $H < \bar{Q}$ consumer surplus is lower than under the first best. Outcomes with $H < \bar{Q}$ are deemed aggregate output inefficient. We use $\Phi^S \leq 1$, calculated as the ratio of realized to first best consumer surplus, as our measure of aggregate output inefficiency.

Cost inefficiency occurs when the industry output $H$ is produced at higher than minimum feasible cost. Under our cost assumptions, there exists a vector of production quantities, such that

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9The model can be modified to consider a polluting industry. Replace the cost function in (1) with multiple output costs $c_i(h, b)$ where $b$ is the quantity of pollution by-production given goods production, $h$ (see Murty et al. (2012)). The CAT regulatory constraint becomes, $b_i \leq q_i$ for firm $i$, where $q_i$ is firm $i$’s emissions permit holding and $\bar{Q}$ is the aggregate emissions cap. If pollution by-production is proportional to $h$, our results below obtain directly under a re-scaling of the units of pollution. If pollution by-production and goods production are nonlinearly related, our results hold only qualitatively.
\[ \sum_i h_i = H, \text{ and for which total cost across all firms is minimized. Denote this minimum cost as } C^*(H). \text{ We define aggregate cost inefficiency for a production plan } h'_i \text{ for } i = 1, \ldots, I \text{ and } \sum_i h'_i = H, \text{ as } \Psi^C = \sum_i c_i(h'_i)/C'(H'). \text{ A value of } \Psi^C \text{ greater than unity indicates aggregate cost inefficiency.} \]

Finally, we use \( \Psi^W \) to summarize overall efficiency. We calculate \( \Psi^W \) as the ratio of realized producer plus consumer surplus to the maximum feasible producer plus consumer surplus given \( \bar{Q} \). Values of \( \Psi^W < 1 \) indicate foregone economic welfare.

We next define our two-stage CAT production game.

### 3.2 Stage 1: Permit trading

We model the first stage permit trading as a Nash bargaining game. Each firm \( i \) enters the trading subgame with permit endowment \( q^0_i \in [0, \bar{Q}] \) where \( \sum_i q^0_i = \bar{Q} \). Denote the endowment vector, \( q^0 = (q^0_1, \ldots, q^0_I) \).

It is natural to define a disagreement outcome of Nash bargaining as the stage two profits earned in the event that bargaining breaks down. For firm \( i \) the break down profit is, \( \pi_i(h^c(q^0)) \). Let \( q^E \) denote the equilibrium outcome of the permit trading subgame. We have,

\[
q^E = \arg\max_{q \in Q} \prod_{i=1}^I \left[ \pi_i(h^c(q^E)) - \pi_i(h^c(q^0)) \right], \tag{2}
\]

where \( Q \) denotes the closed and bounded set of feasible permit allocations.

### 3.3 Stage 2: Permit-constrained competition

At stage 2, firms simultaneously choose production quantities \( h_i \leq q^E_i \) to maximize own firm profit. Define the unconstrained best production response for firm \( i \) as,

\[
h^u_i(H_{-i}) = \arg\max_{h \geq 0} \{ p(H_{-i} + h)h - c_i(h) \}.
\]
Adding the permit constraint $h_i \leq q_i^E$ obtains the firm $i$ best response,

$$h_i^c(H_{-i}|q_i^E) = \min \left\{ h_i^E(H_{-i}), q_i^E \right\}.$$ 

The Cournot-Nash equilibrium of stage 2 requires all firms play permit-constrained best responses. Define $H_{-i}^c(q_{-i}) = \sum_{j \neq i} h_j^c(q_j)$. Stage two profit for firm $i$ is,

$$\pi_i(h^c(q^E)) = p \left( H_{-i}^c(q_{-i}) + h_i^c(q_i^E) \right) h_i^c(q_i^E) - c_i(h_i^c(q_i^E)),$$

(3)

where $h^c(q^E) = (h_1^c(q_1^E), h_2^c(q_2^E), \ldots, h_I^c(q_I^E))$. The expression in equation (3) completes a mapping from the equilibrium permit allocation, $q^E$, to production quantities, $h^c(q^E)$, and to individual firm and industry profit.

4 Results

The following proposition describes the equilibrium of the CAT production game:

**Proposition 1.** (i) An equilibrium of the stage 2 permit-constrained production subgame, $h^c(q)$, exists and is unique for all permit allocations, $q \in [0, \bar{Q}]$ with $\sum_i q_i = \bar{Q}$.

(ii) An equilibrium of the stage 1 permit trading subgame is a permit allocation $q^E$ that maximizes cumulative industry profit, $\sum_{i=1}^I \pi_i(h^c(q^E))$.

**Proof:** See appendix 8.2.

We summarize several implications of proposition 1.

First note that the details of stage 1 bargaining and in particular the trades and money transfers between trading partners are not explicitly defined in our model. What is important is that trading is voluntary and therefore no firm is made worse off by engaging in permit exchange. Voluntary trades can only increase firms’ total payoffs of the full game. This is insured by the requirement that the Nash bargaining solution be Pareto efficient.
A second property of the equilibrium is that $q^E$ does not depend on $q^0$. In this sense, the equilibrium outcome departs from previous literature where the initial permit endowment determines the trading outcome and overall efficiency. In our model, $q^0$ affects only the distribution of rent that is generated under the CAT regulation.

A third observation is that industry structure/conditions exist for which $q^E$ is not unique, although the maximum industry profit is. It is possible that multiple firms acquire permits in stage 1 that will not be produced in stage 2. In this case there are infinite re-distributions of unused permits across these hoarding firms which satisfy equilibrium conditions in 1.

We next develop intuition for a strategic underproduction equilibrium using a simple example. A three firm, one product example

Consider an industry with $I = 3$ firms that produce a single product ($M = 1$). The inverse demand is $p(H) = 10 - H$. Firms have identical costs which we assume are given as $c_i(h) = \frac{1}{2}h^2$ for firm $i$. For purposes of illustration we first derive the permit unconstrained Cournot equilibrium. The best output response for representative firm $i$ is $h^u_i(H - i) = 10 - H - i \frac{3}{2}$, where following standard notation $H - i$ denotes the total production by firm $i$’s rivals. The equilibrium is easily calculated as $(h_1, h_2, h_3) = (2, 2, 2)$. Total output is $H = 6$ and the price is $P(H) = \$4$. Per-firm profit is $\$6$ and total industry profit is $\$18$.

Now introduce a CAT regulation. We set $\bar{Q} = 7.5$ and assume permits are initially allocated equally to each firm, $q^0_i = 2.5$. It is noting that in a regulatory scenario where no permit trading is allowed, and thus each firm’s permit endowment is fixed at $q^0_i = 2.5$, the permit-constrained best response is $h^*_i(H - i) = \min\{h^u_i(H - i), q^0_i\} = \min\{10 - H - i \frac{3}{2}, 2.5\}$ for firm $i$. In the no-trade case, permit constraints do not bind and therefore the equilibrium outcome is as above. Aggregated production is $H = 6 < \bar{Q}$, and is aggregate output inefficient with $\Phi^S \approx 0.89$. With identical cost firms, and equal shares of aggregate production, the cost of producing $H = 6$ attain the minimum.

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10 Discontinuities in the equilibrium outcomes that we derive make analytical solutions difficult. The results that we report in this section and in sections 5.2 and 5 are derived analytically where possible and numerically otherwise.
value, i.e., $\Phi^C = 1$.

Suppose trading of permits is allowed. To solve for the equilibrium outcome we search for a per-firm permit allocation that yields the highest stage two industry profit and therefore the highest per-unit permit value. With $I = 3$ calculating $q^E$ requires that we examine four scenarios for stage two production: (i) permit constraints are slack for all firms; (ii) two firm’s permit constraints are slack while a third firm’s constraint binds; (iii) one firm’s permit constraint is slack and two firm’s constraints bind, and (iv) all firm’s permit constraints bind.

We have already solved for the outcome under scenario (i) above. Scenario (iv) does not satisfy individual rationality; at least one firm wants to reduce production below its permit allocation quantity.

Consider case (ii). We let firm 1 play the role of the permit-constrained producer. By assumption, firms 2 and 3 are permit unconstrained and play best responses, $h^u_i(H_{-i}) = \frac{10 - H_{-i}}{3}$. We can maximize industry profit under the assumption that $h_1 = q_1$. The solution is $q_1 = \frac{10}{13}$ which obtains $h_2 = h_3 = \frac{30}{13}$. Firm 1’s profit is $3.25$. Firm 2 and 3’s profit is $7.99$, and industry total profit is $19.23$. Aggregate production is at $H = 5.38$, which is roughly 72% of $\bar{Q}$. The outcome is aggregate output inefficient, with $\Phi^S = 0.84$. Note that production is not equal across the three identical cost firms. The outcome is therefore cost inefficient with $\Phi^C = 1.16$, i.e., realized cost is 16% above the minimum cost when $H$ is produced efficiently.

Consider case (iii). Assume firm 2 and 3’s permit constraints bind. Firms 1 will play the role of the permit hoarding firm. Let $Q_B = Q - q_1$ denote the total permits held by firms 2 and 3. Firms 2 and 3 are identical and therefore will produce $h_j = \frac{Q_B}{2}$ units each. The Nash equilibrium is $(h_1, h_2, h_3) = (2.59, 1.11, 1.11)$ which obtains total production $H = 4.81$. Stage two profits are $10.08$ for firm 1 and $5.14$ for firm’s 2 and 3. Thus, total industry profit is $20.37$. Again, $H$ is less than $\bar{Q}$ and is aggregate output inefficient; $\Phi^S = 0.78$. The outcome is also cost inefficient, with total industry cost 37% higher than under full efficiency ($\Phi^C = 1.37$).

Pareto efficiency in permit trading suggests that case (iii) is the equilibrium of the full game.
We make the following observations.

In this simple example, firm 1 played the role of the permit hoarding firm, with $q^E_1 = 5.28$ and production $h_1 = 2.59$. The firms are identical, and thus two additional equilibria exist with firm 2 or firm 3 playing the role of permit-hoarder.

The equilibrium permit allocation, $q^E = (5.28, 1.11, 1.11)$ obtains voluntarily through mutually beneficial trade. Firms entered the trading subgame with $q^0_i = 2.5$, $\sum_i q^0_i = \bar{Q} = 7.5$. When no permits trades were allowed, firm profit is $6$ and industry profit is $18$. Industry profit increases to $20.37$ with trading, generating a trade surplus of $2.37$. This surplus is shared among trading firms under mutually beneficial exchange. Firm 1’s production profit increases by $4.08$ with trading. Firm 2 and 3’s production profits decline from $6$ to $5.14$. The cumulative loss is $1.74$, which is less than firm 1’s gain.

Note that industry profit would increase further if firms could collude in the production subgame. Collusion involve each firm agreeing to produce an equal share of the industry-profit maximizing output quantity. This quantity is $H = 4.29$. Total industry profit under collusion is $21.43$. While collusion is illegal under antitrust regulation, permit trading is encouraged in most CAT programs (see appendix 8.1).

**Heterogeneous firm costs**

This section extends the model to include cost heterogeneity across firms. We assume that costs for firm $i$ are given as,

$$c_i(h) = c(h|\theta_i) = \theta_i h + \frac{1}{2} \eta h^2,$$

where $\theta_i$ is a firm-specific inverse productivity parameter and $\eta \geq 0$ is a marginal cost slope parameter that is common to all firms. We assume the inverse product demand follows $p(H) = \alpha - \beta H$, where $\alpha > \theta_i > 0$ for all $i$, and $\beta > 0$. We continue with the $M = 1$ case and thus all parameters are scalars. To simplify the presentation, much of what follows features duopoly competition.

Figure 1 shows permit holdings and production for two identical cost firms. Model parameters
Figure 1: Efficient equilibrium, symmetric firms \((M = 1, I = 2)\): Model parameters are: \(\alpha = 10; \beta = 1; \theta_1 = 3.5; \theta_2 = 3.5; \eta = 1; \bar{Q}_{low} = 1.5; \bar{Q}_{high} = 3.25.\)

used to generate the figure are reported in the figure caption.

Production and permit quantities are shown, respectively, on the horizontal axis for firm 1 and on the vertical axis for firm 2. Unconstrained best responses are indicated as downward sloping linear (heavy dashed) lines, \(h_i^u(h_{-i})\) for \(i = 1, 2\). Fine-dashed elliptical curves trace aggregate industry profit isolines. Point \(h^c\) is the profit maximizing production vector which would be produced if firms could collude. Profit isolines increasingly distant from \(h^c\) indicated lower industry profit.

Figure 1 shows two aggregate production caps. The solid line \(\bar{Q}_{low}\) identifies feasible per-firm permit allocations when the cap is small. The solid line \(\bar{Q}_{high}\) shows feasible permit allocations under a larger cap. Both caps intersect positive profit isolines which indicates existence of a profitable production plan. In other words, the first best outcome involves cost efficient production at \(H = \bar{Q}\) for both the low and high cap cases.
Under identical convex costs, efficiency obtains when total production is shared equally. The set of cost efficient production plans thus follows a $45^\circ$ ray from the origin through point $h^c$ and $\tilde{h}$. To reduce clutter this ray is not shown in the figure.

The intersection of unconstrained best responses occurs at $\tilde{h} = \tilde{q}$, where in the example shown, $\tilde{q}$ is feasible for cap, $\bar{Q}_{high}$. Note that the location and slope of the $h^u_i$ curves do not depend on the CAT regulation. We have chosen $\bar{Q}_{high} = \tilde{h}_1 + \tilde{h}_2$ to demonstrate key features of the equilibrium outcome.

Two equilibria are shown in figure 1. The equilibrium under the low cap scenario is at $(q', h')$ and under the higher cap scenario is at $(q'', h'')$. An equilibrium symmetric to $(q'', h'')$ but with firms switching positions exists but is not shown to reduce clutter. We next explain why these outcomes are equilibrium of the two-stage game conditional on their respective aggregate quotas.

Consider first the low cap equilibrium at $\bar{Q}_{low}$. Note that all permissible per-firm permit allocations along $\bar{Q}_{low}$ lie below the firms’ unconstrained best responses. At a particularly small cap, marginal profits are strictly and perhaps substantively positive. In this case, firms have incentive to increase production. Equilibrium in stage two has both firms producing their entire permit allocation. The permit allocation at $q'$ maximizes stage two industry profit as required.

The low-cap equilibrium does not include strategic underproduction. Notice that the permit constraint set $\bar{Q}_{low}$ intersects industry profit isoclines that increase with additional quantity produced. The equilibrium is both aggregate output and cost efficient.

Next consider the larger cap, $\bar{Q}_{high}$ with equilibrium $(q'', h'')$. Before we explain why this equilibrium obtains, it is instructive to ask why the outcome at $\tilde{q}, \tilde{h}$ is not an equilibrium of the two-stage game?

At the permit-production pair $(\tilde{q}, \tilde{h})$ both firms produce their unconstrained best response, $h^u_i$ for $i = 1, 2$. However notice that in contrast to the low-cap scenario, production at $\tilde{h}$ occurs in a region of production space in which industry profit is declining in $H$. Industry and individual firm profit can be increased if firms can reduce total production. Through permit trading the strategic
underproduction equilibrium \( q'' \), \( h'' \) obtains.

To see why this outcome is the equilibrium, contrast the two feasible permit allocations \( \tilde{q} \) and \( q'' \). Allocation \( q'' \) includes larger permit holdings for firm 1 than for firm 2. If firm 1 produces at \( h''_1 \), firm 2 maximizes its stage two profit by producing \( h_2 = q_2 \). We see that \( h''_1 \) is firm 1’s best response to firm 2’s production choice. \( h'' \) is therefore an equilibrium of the stage two subgame. The tangency between the feasible permit allocation set and the industry profit isocline at \( q'' \) confirms that \( q'' \) achieves higher industry profit, as required in the trading subgame.

The inefficiency of the strategic underproduction equilibria in figure 1 is clear: \( H < \bar{Q}_{\text{high}} \), and therefore \( \Psi_S < 1 \). Also, \( h'' \) lies below the 45° line in figure 1 with the permit hoarding firm producing more than the permit-constrained firm. Total costs of production therefore exceed the minimum required cost and we have \( \Psi_C > 1 \).

Under our cost assumptions, lowering production reduces own-marginal-costs. As additional permits are traded away from permit-constrained firm(s) the own marginal profits, and thus the permit valuation by these firms, increases. This property of the CAT regulation limits the extent of permit hoarding that can occur in equilibrium. In figure 1 for example, permit accumulation by firm 1 beyond \( q''_1 \) does not occur. At \( h'_{2} = q'_{2} \) firm 2’s marginal profit and firm 1’s stage two profit increase from additional permit hoarding are equal. We next investigate further how relative cost efficiency across firms determines hoarding and efficiency of strategic underproduction equilibria.

Figure 2 presents a two heterogeneous-cost firm example. Firm 1 is the low cost producer. The figure shows unconstrained best responses, as a heavy dashed line \( h^u_i(h_j) \) for firm \( i \neq j \), and industry profit isoclines, as short-dashed ellipses. As above, industry profits are highest at the full collusion production vector \( h^c \); profit isoclines nearing \( h^c \) trace higher profit. Profit isoclines are centered around production plans with \( h_1 > h_2 \) due to firm 1’s lower costs. Below we use figure 2 to characterize equilibria when a limit on permit ownership is imposed. For now, we assume that all permit allocations along the solid and dotted segments of \( \bar{Q} \) are permitted under the regulation. Figure 2 also shows combinations of cost efficient production along the lightly dashed line labelled.
Figure 2: Ownership cap regulations \((M = 1, I = 2)\): Model parameters are: \(\alpha = 10; \beta = 1; \theta_1 = 3; \theta_2 = 4; \eta = 1; Q = 4.\)

as \(\Psi^C = 1.\)

The equilibrium permit allocation is at \(q',\) with equilibrium production at \(h'.\) Firm 2’s production is permit constrained at \(h'_2 = q'_2.\) Firm 1 holds excess permits and thus produces its unconstrained best response, at \(h'_1(q'_2) = h'_1.\)

The equilibrium exhibits strategic underproduction. Total production is less than \(Q\) and therefore \(\Psi^S < 1.\) The quantity produced by firm 1 lies to the right of the \(\Psi^C = 1\) set and therefore total production is cost inefficient, with \(\Psi^C > 1.\)

Observe that under the equilibrium \((q', h'),\) the high cost firm is permit-constrained while the low cost firm produces its unconstrained best response. More generally, in cost-heterogeneous settings a strategic underproduction equilibria will involve permit hoarding by the low cost firm(s) in the industry. To understand why this is the case, observe that strategic underproduction outcome is
achieved through a commitment by firms to curtail own and thus aggregate production. Curtailment occurs either via the binding constraint \( h_i(q^E) = q^E_i \) for firm \( i \), or through an unconstrained best response \( h_i(q^E) = h^u_i(h_{-i}(q^E)) \). Recall further that strategic underproduction arises because it raises industry profit above the profit achieved when firms produce the entire cap, \( \bar{Q} \).

Suppose industry seeks to curtail aggregate production to a level \( H < \bar{Q} \). Total industry revenue will be \( p(H)H \). The challenge is to identify a permit allocation \( q^E \) that induces equilibrium production \( h(q^E) \) at low cost. Under this production plan some firms will be permit constrained, while the rest will be permit-unconstrained in stage two. Denote the set of permit constrained producers as \( B \subseteq I \), and the set of permit unconstrained producers as, \( U \subseteq I \). We make the following observations.

First, marginal costs will be equal for all permit constrained firms,

\[
\theta_i + \eta h_i(q^E) = \theta_j + \eta h_j(q^E) \quad \forall i, j \in B, i \neq j. 
\]  

This requirement is easily understood by recognizing that \( q^E \) disciplines the per-firm and total industry production such that industry profit is maximized. Aggregate production of the set of permit constrained producers is \( \sum_{i \in B} h_i = \sum_{i \in B} q^E_i \). The cost incurred by these firms is minimized only if condition 5 holds.

A second observation is that the marginal cost of a firm \( i \in U \) will be less than the common marginal cost of firm’s \( i \in B \). It is easy to see that if this is not the case, industry could redistribute \( H \) away from a permit unconstrained firm to the set of permit constrained firms. This would lower the total cost of producing \( H \) and increase industry profit.

We therefore conclude that the subset of permit-unconstrained producers \( i \in U \) will be comprised of lowest cost producers. The marginal costs for these firms, evaluated at equilibrium production quantities will satisfy,

\[
\theta_i + \eta h_i(q^E) \geq \theta_j + \eta h_j(q^E) \quad \forall i \in B, j \in U. 
\]
Finally, an implication of the above relationship between firm’s marginal costs implies $h_i(q^E) \leq h_j(q^E)$ for $\forall i \in B, j \in U$, i.e., permit hoarding firms will produce larger shares of aggregate industry production.

The next section explores the possibility of limiting the share of permits and corresponding production for a single firm in the industry, for the purposes of improving efficiency under a CAT regulation.

## 5 Permit ownership limits

We now impose an upper limit on permit accumulation by a single firm during the trading phase. This constraint takes the form $q^E_i \leq s\bar{Q}$ where $q^E_i$ is as defined above and $s$ denotes the limit as a share of total available permits.

An equilibrium with $s = 0.75$ is shown in figure 2. Observe that the set of permissible permit allocations is now a solid segment of the set $\sum_i q_i = \bar{Q}$. Note also that $q'$ in figure 2 violates this regulation.

The equilibrium outcome with $s = 0.75$ occurs at $(q'', h'')$ in figure 2. Observe first that stage two profit is lower at $q'', h''$ than at $q', h'$. The ownership limit binds and thus impedes industry’s ability to discipline aggregate production. The limit binds for firm 1 requiring more permits be allocated to firm 2 relative to the case with $s = 1$.

Firm 2 is permit-constrained with production at $h''_2 = q''_2$. Firm 1’s production is given as $h''_1(q''_2) = h''_1$. Total industry production at $h''$ is higher than at $h'$ and thus $\Psi^S$ is increased (relative to the $s = 1$ equilibrium). We see further that under $s = 0.75$ the equilibrium production outcome coincidentally lies in the cost efficient set with $\Psi^C = 1$.

These results confirm that an appropriately chosen value of $s$ can increase efficiency. Ownership limits cannot induce the fully efficient outcome generally. For example, total production under the CAT regulation will not exceed $\tilde{h} = \tilde{h}_1 + \tilde{h}_2 < \bar{Q}$ (figure 2). We see further that production at $\tilde{h}$ is not an element of the $\Psi^C = 1$ set.
In general, imposing \( s < 1 \) cannot resolve inefficiency that derives from two sources; underproduction of \( \bar{Q} \) and cost allocative inefficiency. This highlights a tradeoff for the regulator between increasing consumer surplus versus maintaining or increasing cost efficiency.

A poorly designed ownership limit policy can actually reduce efficiency relative to the \( s = 1 \) case. To illustrate suppose the regulator imposes the ownership limit, \( s = 0.68 \). The equilibrium outcome of the two stage game occurs at \((\hat{q}, \hat{h})\) in figure 2.\(^{11}\) Under \( s = 0.68 \), firm 2 holds the larger share of permits and becomes the permit-hoarding firm. Two features of the equilibrium at \((\hat{q}, \hat{h})\) should be noted. First, aggregate production at \((\hat{q}, \hat{h})\) and \( h'' \) are roughly equal. Thus \( \Psi^S \) is roughly equal for \( s = 0.75 \) and \( s = 0.68 \). Second, at production vector \( \hat{h} \), the high cost firm’s share of total production is roughly half of the industry total. We see further that industry profit is lower at outcome \((\hat{q}, \hat{h})\) than at \((q'', h'')\). The two equilibria generate lower industry profit due to higher total costs. This confirms that total efficiency can decline under a poorly designed permit ownership limit policy. The next section identifies industry conditions under which the problem may be quantitatively significant.

5.1 Policy design

Under what industry conditions will ownership limits achieve first best efficiency? To address this question, let \( s^* \) denote the ownership limit that maximizes \( \Psi^W \) under given industry conditions and \( \bar{Q} \).\(^{12}\) We characterize \( s^* \) across two dimensions of our model parameter space, \( \bar{Q} \) and \( \Delta\theta \), calculated as the difference in cost efficiency between the two lowest costs firms in the industry. In other words, we rank cost efficiency from smallest to largest (\( \theta_1 \leq \theta_2 \leq \ldots \leq \theta_I \)) and calculate \( \Delta\theta = \theta_2 - \theta_1 \).

Figure 3 summarizes results for an optimally chosen \( s^* \) over parameter space \( \bar{Q}, \Delta\theta \) with \( \bar{Q} \) on

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\(^{11}\)A further reduction in \( s \) reduces the length of the solid segment of \( \bar{Q} \) in figure 2. The change is not illustrated.

\(^{12}\)The ownership limit \( s^* \) solves the following problem:

\[
\max_{1/2 \leq s \leq 1} \left\{ \int_0^{H(s)} p(x)dx - \sum_i c_i(h_i(s)) \right\},
\]

where \( h_i(s) \) is firm \( i \) output given \( s \), and \( H(s) = \sum_i h_i(s) \).
the vertical axis and $\Delta \theta$ on the horizontal axes. Results for a two-firm duopoly are shown. We calculate $\Delta \theta$ holding $\theta_1$ fixed. Thus $\theta_2$ increases from left to right along the horizontal axis in figure 3. Remaining parameters (reported in the caption of the table) are chosen such that costs exhibit moderate diseconomies of scale, with elastic demand at smaller values of $\bar{Q}$ and inelastic demand at larger values.

The parameter space in figure 3 is separated into five regions, labelled $A$ through $E$. Region $A$ delineates combinations of $\bar{Q}$ and cost differences for which the entire cap is produced cost efficiently with $s^* = 1$. The equilibrium outcome is fully efficient ($\Psi^W = 1$) in region $A$. As noted earlier, when $\bar{Q}$ is sufficiently small, marginal production profits are everywhere positive. Industry profit is maximized at $H = \bar{Q}$ under equi-marginal cost production.

At cost differences below $\Delta_{\theta}',$ the upper boundary of region $A$ delineates aggregate production when firms are indifferent between producing the strategic underproduction equilibrium output and the first best output. Production is largest at $\tilde{H}$ where there are no cost difference across firms, $\Delta_{\theta} = 0.$ As cost differences increase, moving left to right, the unconstrained equilibrium production declines under higher firm 2 costs. Values of $\bar{Q}$ that are produced efficiently under $s^* = 1$ therefore also decline.

The monopoly outcome will emerge in equilibrium, under $s^* = 1$, when cost differences are
sufficiently large. In figure 3, cost differences exceeding $\Delta'_{\theta}$, result in a monopoly outcome with lowest cost firm 1 holding all permits. In this case firm 1 and aggregate production is at $H^M$. The implication is that values of $\bar{Q}$ exceeding $H^M$ will not be produced without limits on permit ownership.

Region $B$ of figure 3 is characterized by smaller production caps and relatively small cost differences. Over this parameter space, first best efficiency is possible under an optimally chosen $s^* \in (0, 1)$. For region $B$, equilibrium outcomes without ownership limits involve strategic underproduction. The optimal $s^*$ redistributes permits from the low cost to the higher cost firm, which increases aggregate production. The upper boundary of region $B$, shown as a dashed line segment, is the largest value of $\bar{Q}$ for which production under $s^*$ is cost efficient. As noted, permit ownership limits in general cannot simultaneously address two sources of inefficiency.

In region $C$, permit ownership limits can raise aggregate production above quantities that obtain with $s = 1$. However, while $\Psi^S$ can be increased, simultaneously attainment of cost efficiency is not possible. In region $C$, the optimal $s^*$ must trade off increases in consumer surplus with cost efficiency. The optimal $s^*$ cannot induce the first best outcome.

Region $D$ and $E$ are characterized by a relatively large $\bar{Q}$ and large cost differences. Full efficiency is not possible; in region $D$, we have $s^* < 1$ and $\Psi^W < 1$. In region $E$, cost differences are large. The optimal ownership limit occurs at $s^* = 1$. For parameters in region $D$, setting $s < 1$ increases aggregate production above the no-ownership-limit quantity. This increases consumer surplus. However, in region $E$ cost differences are significant. Tightening $s$ below unity redistributes permits and production to the high cost firm. The losses due to increased production costs can be larger than gains in consumer surplus in which case $\Psi^W$ attains a maximum less than unity at $s^* = 1$.

Figure 4 plots $\Psi^W$ against $s$ under varying conditions. Efficiency is plotted on the vertical axis, against values of $s$ which increase from 0.5 to 1. The permit ownership limit thus becomes less stringent moving from left to right. Results for the case with $M = 1$ and $I = 2$ are shown.
Figure 4: Efficiency and permit ownership limits. Panels show the value of $\Psi^W$ as the restriction on permit ownership varies from $s = 0.5$ to $s = 1$ (no restriction). The figure assumes $I = 2$ based on model parameters: $\alpha = 10$, $\beta = 1$, and $\eta = 1$ in all panels; in panel (a), $\theta_1 = \theta_2 = 0$, $Q = 7$; in panel (b), $\theta_1 = 0$, $\theta_2 = 1$, $Q = 6$; in panel (c) assumes $\theta_1 = 0$, $\theta_2 = 2$, $Q = 3$. 
Figure 4a considers an industry with cost-homogeneous firms and a large quota, where we define large as the case where $\bar{Q}$ exceeds the aggregate production that would arise under Cournot unconstrained competition. The conditions support a strategic underproduction equilibrium.\footnote{Note that when firms have identical costs, any firm may play the role of permit-hoarder.}

Referring to figure 4a, we see that values of $s > s'$ do not bind. Therefore, the equilibrium outcome involves one firm holding excess permits and producing its unconstrained best response production quantity in stage two. Aggregate production is less than $\bar{Q}$ and $\Psi^S < 1$. Values of $s < s'$ bind. As $s$ is tightened below $s'$, the permits that would otherwise remain idle are redistributed across firms in the industry. This increases total production in stage two. The rise in $\Psi^W$ operates through both efficiency channels: aggregate production increases with accompanying increase in consumer surplus and the quantities produced by the two firms equilibrate which, with cost-homogeneous firms, increases cost efficiency.

Figure 4a shows that $\Psi^W$ attains its maximum value of roughly 0.9 at $s = s''$. Permit ownership limits cannot induce production beyond individual firms’ unconstrained best responses. With a large cap, values of $s \leq s''$ improve cost efficiency but cannot raise aggregate production entirely to $\bar{Q}$.

Figure 4b considers an industry with cost-heterogeneous firms and a relatively large cap. These conditions support a strategic underproduction equilibrium. Following our convention, firm 1 is the lower cost and firm 2 is the high cost producer.

At values of $s > s'$, the permit ownership constraint is slack. The equilibrium satisfies $q_1 > q_2$, $h_1 < q_1$, and $h_2 = q_2$. Aggregate production is less than $\bar{Q}$. As $s$ is tightened below $s'$ the ownership limit binds for low cost firm 1. Lowering $s$ below $s'$ redistributes permits to firm 2, which raises aggregate production and consumer surplus. The effect of binding $s$ on cost efficiency is less clear.

Under the parameters assumed in figure 4b, cost efficiency requires $h_1 > h_2$, which is the case when $s$ does not bind. Tightening $s$ below $s'$ causes production quantities to become more equal. This increases (reduces) cost efficiency as $s$ redistributes production responsibilities toward (away) from the cost efficient set $\{(h_1, h_2) | \Psi^C = 1\}$. 

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Figure 4b shows that $\Psi^W$ falls discretely as $s$ is reduced marginally below $s''$. For $s < s''$, the equilibrium outcome involves a role reversal for the two firms: firm 2 becomes the holder of excess and unproduced permits and firm 1 becomes the permit-constrained producer. The discrete drop in $\Psi^W$ occurs due to the redistribution of production away from the low cost firm 1 and toward high cost firm 2.

As $s$ is reduced further below $s''$, the ownership limit binds for firm 2. This redistributes permits back to the lower cost firm 1. Aggregate production and cost efficiency both increase, causing $\Phi^W$ to increase.

Efficiency gains are exhausted at $s = s'''$ for reasons stated above. Thus, $\Psi^S < 1$ under the optimal $s^*$.

Figure 4c assumes an industry with cost-heterogeneous firms and a relatively small cap, i.e., the unconstrained aggregate production is less than $\bar{Q}$. Under these conditions, firms have incentive to produce the entire cap. The equilibrium satisfies $\Psi^S = \Psi^C = 1$ at $s^* = 1$. As shown, $\Psi^W$ declines as $s$ falls below $s'$. For values of $s < s'$, the ownership limit regulation distributes additional production to higher cost firms. If $s$ is set sufficiently small, aggregate production can fall below $\bar{Q}$ with further reduction in $\Psi^W$. This latter outcome arises when the high cost firm cannot profitably produce its residual permits, i.e., $h_2 < q_2 = (1 - s)\bar{Q}$.

To this point we have held the number of participating firms fixed. The model provides further insights for equilibrium industry structure. We next define demand and cost conditions under which strategic underproduction takes an extreme form with a single permit holder and the monopoly production outcome.

Suppose $I \geq 2$. We can rank firms based on their relative cost efficiency, $\theta_1 \leq \theta_2 \leq ... \leq \theta_I$. A monopoly equilibrium will emerge, i.e., $q_1^E = \bar{Q}$, if:

$$\frac{\theta_2 - \theta_1}{\alpha - \theta_2} > \frac{\eta}{2\beta},$$

(7)

The numerator of the left-hand size of equation (7) is a measure of the difference in cost efficiency
among the two top-ranked firms. The left-hand denominator is a measure of the profitability of the industry. Note that the firm that is ranked second in terms of cost efficiency, values permits highest among the set of firms who participate in the trading subgame. Firm 2 effectively sets the shadow price that firm 1 must pay to acquire permits and implicitly ensure production by rivals firms is zero.

The numerator on the right hand side of (7) measures of rate at which marginal costs increase. \( \beta \), is the slope of product demand. The right hand side of (7) will be large when diseconomies of scale in production are not severe (when \( \eta \) is small) and when demand is inelastic. These conditions are consistent with natural monopoly. If the right hand side of (7) is large, the cost difference across firms must also be large to support the monopoly outcome, and visa versa.

5.2 Multiple products

Two channels of production interdependence are possible in our model. Products \( m \) and \( m' \) can be substitutes or complements in consumption, in which case the price of product \( m \) will vary with \( H_{m'} \) and \( H_m \). A second channel is jointness in production, or cost complementarities in production. We next characterize key features of strategic underproduction equilibria when interdependencies of each type are present.

We use superscripts to distinguish separate products. A multiple-product CAT regulation requires \( h_{im} \leq q_i^m \) and \( \sum_i q_i^m = Q^m \) for firms \( i = 1, \ldots, I \) and products \( m = 1, \ldots, M \), where \( Q^m \) denotes the aggregate cap for product \( m \). Assuming vector conformability, the price vector with \( M \) products becomes \( P(H) = \alpha - BH \), where \( P(H) \) and \( H \) are \( M \times 1 \) vectors and \( B \) is an \( M \times M \) coefficient matrix. A multiproduct cost function can be written, \( c(h|\theta_i) = h\theta_i + 0.5h\Lambda h \), where \( h \) is an \( M \times 1 \) vector, \( \theta_i = (\theta_{i1}, \theta_{i2}, \ldots, \theta_{iM}) \) is a vector of firm and product-specific inverse productivities, and \( \Lambda \) is a positive \( M \times M \) semi-definite coefficient matrix.

We continue with the simplest \( M = 2 \) and \( I = 2 \) case. To develop intuition for the results under interdependent production we first define a Cournot duopoly equilibrium without a CAT regulation. We then contrast this outcome to one that emerges when one of the two products, only, is regulated.
under a CAT. The equilibrium outcome and efficiency under a two product CAT, i.e., a product specific CAT regulation, is considered thereafter.

Interdependence across products implies that changes in the production of product 1 cause a shift in product 2 best responses. To manage clutter, figure 5 describes these shifts and equilibrium results qualitatively.

Figures 5a and 5b show permit and production quantities across products 1 and 2, respectively. The example in the figure assumes firms have identical costs. The dashed 45° line identifies per-firm quantities that satisfy cost efficiency in production, i.e., $\Psi^C = 1$.

With $M = 2$, firm $i$’s profit maximizing choice for product 1, $h_{i,1}^u(h_j)$ in figure 5a depends on firm $i$’s and firm $j$’s production of both products. If products are complements in consumption or if the technology exhibits economies of scope, the unconstrained best response for product $m$ shifts north and east as the quantity of product $m'$ increases. If products are substitutes in consumption or exhibit cost diseconomies when produced jointly, the unconstrained best response for product $m$ shifts south and west when the quantity of product $m'$ is increased.

Suppose, initially, that no CAT regulation exists. Equilibrium production occurs at the simul-
taneous intersection of unconstrained best responses. Production is at $a = h^{u,1}$ in figure 5a and at $b = h^{u,2}$ in figure 5b. The outcome is symmetric across products and firms. It is therefore cost efficient. We do not take a stand on the extent of aggregate output efficiency.

Next suppose that a CAT regulation is introduced in the market for product 1 only (figure 5a). For this example, we set the aggregate cap equal to the unconstrained aggregate production quantity; $ar{Q}^1 = h^{u,1} + h^{u,2}$ which is shown at point $a$ in figure 5a. The set of feasible permit allocations is the line segment $\bar{Q}^1$ in the figure. When firms are allowed to trade permits, the equilibrium allocation and production outcome occurs at $(\hat{q}^1, \hat{h}^1)$. The corresponding equilibrium outcome in the market for product 2 is discussed shortly.

The equilibrium at $(\hat{q}^1, \hat{h}^1)$ exhibits strategic underproduction, with $\hat{h}_1 + \hat{h}_2 < \bar{Q}^1$. We see that firm 1 hoards permits and produces the larger share of production. The outcome is therefore aggregate output and cost inefficient. Note that since the two firms are identical, a second symmetric equilibrium exists with the two firms exchanging roles.

How will a CAT regulation for product 1 affect the equilibrium? If there is demand complementarity, the reduced aggregate output in market 1 (due to strategic underproduction) lowers the price of product 2. If the technology exhibits cost complementarity, the marginal cost of producing product 2 increases as firms reduce production of product 1. Unconstrained best response functions shift toward the origin in figure 5b resulting in equilibrium located south and west of point $b$. If products are substitutes in consumption and/or there are diseconomies of scope, strategic underproduction in the market for product 1 shifts unconstrained best responses north and east of point $b$ in figure 5b.

The welfare implications of a CAT regulation in market 1 depend on the source of interdependence across products. Suppose for example, that interdependence operates through the price channel only, i.e., the multiple product technology is non-joint. In this case, a CAT regulation in market 1 creates cost inefficiency only in market 1. Inefficiency in market 2 arises from the firm’s response to a change in the product 2 price. If the two products are complements (substitutes) in consumption, strategic underproduction of product 1 increases (decreases) the aggregate quantity,
and consumer surplus, in market 2.

Cost inefficiency caused by strategic underproduction in market 1 will spill over to market 2 under a technology that is joint in inputs. In figure 5a, firm 1 (2) increases (reduces) production of $h^1$ under the CAT regulation (relative the outcome at $a$). These adjustments affect the marginal costs of product 2, in opposing directions. If the technology exhibits economies of scope, the unconstrained best response for firm 1 (2) shifts north and east (south and west) relative to point $b$ in figure 5b. Shifts in unconstrained best responses are in the opposite direction if the technology exhibits dis-economies of scope. In general, firm-level adjustments to production of the second product diverge from the cost-efficient subset, along the $45^\circ$ dashed line in figure 5b.

We close this section with some additional observations:

Strategic underproduction in both markets can be expected when CAT regulations are present in both markets. The equilibrium outcome will involve permit hoarding and self-imposed capacity constraints across multiple products such that production stage total industry profit is maximized. Efficiency implications depend on the form and magnitude of price and production interdependency.

Under economies of scope, a firm that hoards product $m$ permits will also hoard product $m'$ permits. Under diseconomies of scope, each firm will specialize in hoarding of permits and increased production of a single product. In this way the industry achieves strategic underproduction at lowest cost.

Permit ownership limits obstruct strategic underproduction equilibria and can improve efficiency in multiple-product settings. Refer once again to figure 5. It is easy to see that a limit on permit ownership will increase production of product 1, which can raise welfare in the market for product 1 and, if the products are complementary, in the market for product 2. In the same vein, non-monotonicity of welfare in the stringency of an ownership limit will hold in a multiple product CAT regulation with ownership limits.

Finally, as with the single-product analysis above, permit ownership limits across multiple products can reduce total welfare particularly under technologies that exhibit economies of scale in pro-
duction. Welfare losses are expected when ownership limits prevent firms from exploiting available scale economies and when limits redistribute production to relatively high cost producers. A comprehensive summary of the welfare implications is an empirical matter and is reserved for future research.

6 CAT regulation and ownership limit policy in West Coast Groundfish

The U.S. West Coast groundfish fishery, hereafter the groundfish fishery, began an individual fishing quota (IFQ) regulation, known also as catch shares, in January 2011. IFQs grant their owner a right to harvest a specified quantity of fish during a calendar year. Program designers were concerned that tradability of harvest rights would result in IFQ ownership becoming concentrated into the hands of a small number of individuals/firms who would then be able to influence market outcomes in their favor. The IFQ regulation correspondingly adopted strict limits on the total IFQ shares that can be owned by a single entity.

This section has two goals. We review the structure, conduct and performance of the groundfish fishery using standard indicators of non-competitive behavior. We then consider the problem of designing an ownership limit policy that can prevent strategic underproduction and insure efficiency in groundfish production. We begin with a brief background of the groundfish fishery and its regulation. An empirical investigation of industry costs and policy simulations follow. Due to space limitations abbreviated results from our data and estimations are presented here. Further details and results are presented in appendix 8.3.

Industry background

The U.S. west coast groundfish fishery is managed by the Pacific Fisheries Management Council, a stakeholder body that formally advises the U.S. National Marine Fishery Service on management
of fisheries in federal waters 3 to 200 miles off the California, Oregon, and Washington coasts. The modern history of groundfish management began in 1976 with the passage of the Magnuson-Stevens Fisheries Conservation and Management Act (MSFCMA). Early management actions sought to maintain harvest and stock abundance at sustainable levels using controls on factors of production allocated to groundfish harvesting. Regulations during this controlled access regime included vessel entry limits, area and seasonal closures, gear restriction, bimonthly landings limits, and in 2003, an industry-funded permit/vessel buyback program that sought to remove fishing capital and improve the profitability of the capital that remained.\textsuperscript{14}

In January 2011, fishery managers distributed IFQ shares to eligible industry participants. Shares times the annual allowable catch (AC), which is chosen by management, determines the quantities of groundfish by species that can be legally harvested (a small amount of IFQ can be carried forward and used for subsequent year production). IFQs, i.e., production permits in the language of section 3, can be traded to any U.S. resident under restrictions on the total IFQs that can be controlled by a single entity.\textsuperscript{15}

The catch share ownership limits that were written into the original groundfish regulation were designed to address a broad set of management goals. The Pacific Fishery Management Council specifically designed the IFQ regulation, to “Create and implement a capacity rationalization plan that increases net economic benefits, creates individual economic stability, provides for full utilization of the trawl sector allocation, considers environmental impacts, and achieves individual accountability of catch and bycatch.” Fishery (2009). In accordance with guidelines provided by the MSFCMA, the IFQ regulation imposed limits on accumulation of IFQ by individual entities Seger et al. (2016). Ownership limits further sought to restrict the degree of industry consolidation and prevent market power.

Current ownership limits range between 2.5% to 20% of annual ACs depending on species.

\textsuperscript{14}See Warlick et al. (2018) for a complete history of regulations in the groundfish fishery.

\textsuperscript{15}The regulation also places limits on quantities of groundfish by species that can be harvested on a fishing vessel. Vessel limits are considered in Evans et al. (2019).
An additional 2.7% limit on all species’ IFQ that can be controlled by a single entity is imposed. Recently managers and stakeholders have questioned whether the ownership limits as currently set are meeting management goals.

Some additional institutional details of the groundfish fishery are relevant to our empirical investigations. The fishery produces numerous species of roundfish, flatfish, rockfish, sharks, and skates. Most fish species live on or near the ocean floor and are harvested primarily with bottom trawl gear. Some species, e.g., sablefish are harvested with fish traps as well as trawls. Pacific whiting is a large volume species that lives in the midwater column. A portion of the pacific whiting annual catch is produced by the same firms that produce groundfish.

To assist with monitoring and enforcement, the IFQ regulation requires landed catch be transferred to a buyer that holds a valid first receiver site license. Licenses are issued for every site and company that purchases IFQ groundfish and/or whiting. The number of licences are not restricted but attainment of the license involves time costs, e.g., operating the license comes with considerable reporting requirements. As we show below, the number of groundfish buyers has historically and continues to be relatively small.

We collect data on purchases of raw fish, production of final product, revenue, and expenditures at the individual first receiver, hereafter FR, level from the PacFIN and Economic Data Collection data bases. Data are available for 54 unique FR licensees that operated during our 2009-16 data period. While the number of unique FR’s appears large and therefore suggestive of a competitive market, the structure of groundfish processing is more concentrated. FR’s vary considerably in terms of size, which we measure as annual production or revenues, the mix of species produced, and in some cases the services provided to the industry. For example, some FR’s specialize in the transport of raw fish only. Some were active in the industry for short periods, and when active, accounted for small shares of total production. More germane is the much smaller number of parent companies that own and operate multiple FRs. Some parent companies are vertically integrated owning FR-licensed processing plants and fishing vessels.
It is worth noting that the harvest sector in the groundfish fishery is made up of a large number of typically single-owner and independent vessel operations. Evans et al. (2019) report fleet sizes ranging from 130 vessels in 2009-10 to 95 vessels in 2015-16. Our investigations of market power will therefore and focus on the processing sector, at the level of the parent company. Finally, it should be noted that while the amount of IFQ owned by parent companies, hereafter simply firms, is currently low, at approximately 33% for whiting and 13% for non-whiting, ownership limit policy continues to be debated, e.g., see Pacific Choice Seafood Co. v. Ross.16 Our policy design experiments below will therefore derive equilibrium outcomes under varying scenarios for IFQ ownership limits.

**Structure and conduct in the groundfish fishery**

**Under production of annual catch limits**

Figure 6 reports annual quota utilization rates, $H^m/Q^m$ from 2006-16. The period straddles the start date of the IFQ regulation in 2011, which we mark as a dashed vertical line in the figure. As is apparent, some but not all groundfish species AC’s have been underproduced since the introduction of the IFQ regulation.

Whiting production of the AC is high initially, from 2011-13, but considerably lower during 2014-16. It is notable that the decline occurs in years in which the whiting AC is set to the highest levels observed in the data period.

Production of sablefish and petrale sole AC’s has remained consistently high during 2006-16. Utilization of the thornyheads, Other groundfish species, and rockfish available quota is consistently low, with less than 50% of the AC produced in many years. Production of thornyhead species shows a notable decline immediately following the IFQ regulation start date.

Underproduction of ACs does not imply market power and inefficiency. It should be noted that groundfish AC’s are based on biological criteria under a ’maximize sustainable physical yield’

16Court of Appeals, 9'th Circuit, 2020. No. 18-15455.
management objective. If a species stock is believed to be in an overfished state, a lower AC will be chosen to rebuild the stock. Underproduction of an AC may simply signal that the marginal cost of producing the entire AC is above the product’s market price.

**Concentration**

Standard measures of industry concentration provide a first indication of potential non-competitive behavior and inefficiency. Market shares of individual firms is also relevant to strategic underproduction. IFQ hoarding will be incentive compatible, only if the hoarding firm’s profit attains attain its maximum at a production quantity that is less than its IFQ holding. The firm must perceive a an infra-marginal revenue loss from increasing own production that offsets the marginal production profit. Hoarding firms must be large in terms of own production.
Table 1: Concentration in the Groundfish Processing Sector, 2009-16. HHI - Herfindahl-Hirschman index; 4-F Conc. denotes four firm concentration. Concentration measures are based on revenue in $2016.

We measure concentration in the groundfish fishery for FR’s and parent companies that process and sell groundfish to downstream consumers (FRs that provide ancillary services are dropped). Table 1 reports the Hirfindahl-Hirschman index and four firm concentration ratio across data years. Results show a relatively stable industry structure. The four largest FR’s account for roughly 20% of revenues, with an exception in 2011 where their revenue share increased to 26%. The number of firms has remained stable at 10. The four largest firms control roughly 90% of industry revenue.

The U.S. Department of Justice considers markets with HHI indices exceeding 2,500 points to be highly concentrated. The firm level HHI in the groundfish industry well exceeds this threshold.

**Price cost margins**

We estimate a parametric multiple-product cost model at the level of individual FR under a random parameters specification that allows for cost heterogeneity across FRs (Greene, 2005). The data and estimation used in this analysis is described in appendix 8.3. We summarize the findings here with a caveat, that our data are proprietary, requiring some of the results to be reported in aggregated form to protect individual firm identity.

Revenue and production data are used to infer prices of processed groundfish. We calculate marginal production costs as the sum of marginal processing costs and marginal harvesting costs.\(^{17}\)

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\(^{17}\)Estimates of marginal harvesting costs are obtained from Evans et al. (2019).
Price data and cost estimates allow calculation of short run profit margins in the industry.

Results find that the price-marginal cost margin for whiting, rounded to the penny, is $0.00 per pound, i.e., the Lerner index is zero. Whiting is a high volume species that is sold primarily into international markets. The industry margin that we estimate suggest perfectly competitive pricing for whiting.

Estimates of sablefish and petrale sole margins are, respectively, $2.922/lb. and $2.555/lb. The Lerner index is 0.545 for sablefish and 0.649 for petrale sole. Margins for these species are large and in an unregulated industry could easily be interpreted as evidence of non-competitive pricing behavior. An important consideration is that positive marginal profits under a CAT regulation, when the production quota binds, may simply reflect equilibrium rent being generated under the aggregate production cap. The positive marginal profits observed for sablefish and petrale sole cannot not be construed as evidence of monopoly pricing behavior.

Price-marginal cost margins are $0.248/lb. (0.086) for thornyheads species; $1.425/lb. (0.490) for dover sole; and $0.173/lb. (0.078) for rockfish species. Margins for thornyheads and rockfish are small. The margin for dover sole is higher while concurrently its AC is under-utilized (see figure 6).

Summing up, evidence of monopoly pricing obtained from Lerner index analysis is mixed. Whiting margin estimates are at zero. Under utilization of whiting AC’s during 2014-16 therefore appear to be the result of low marginal profit for this species. Profit margins for sablefish and Petrale sole are large, in the range of $2.5 - $3/lb. These margins are consistent with quota utilization rates near 100% (figure 6). Profit margins for underutilized species, thornyheads, dover sole, and rockfish, are lower but for dover sole appear larger than is consistent with perfectly competitive pricing.

Cost heterogeneity

The analysis in section 4 finds that permit hoarding in strategic underproduction equilibria is facilitated by a low shadow price of quota. Shadow prices are lower when rival marginal costs are
high and when marginal costs increase sharply with quantities produced. We next examine cost differences across firms and (dis)economies of scale in production.

Figure 7: FR Marginal Costs of Production.

Figure 7 plots differences in estimated marginal costs across FR’s in our data. Differences are reported as percentages ranked from smallest, a value of zero, to largest.

For whiting production, we report cost differences for nine FRs that produced positive quantities of whiting during the data period.\(^{18}\)

Results for whiting find marginal costs across FR’s are within 0.5% of the lowest cost FR. Small differences are not surprising given the low profit margins and high production volume.

Estimates of cost differences in sablefish are larger with the marginal cost of the highest cost FR estimated at roughly 6% above the lowest cost FR. We find a larger than 2% difference between the lowest and second lowest cost FR.

\(^{18}\)The cost model is nonlinear in quantities produced. We evaluate marginal costs at the median production bundle in the data. Cost differences arise through FR-specific parameter estimates. To accommodate differences in whiting and groundfish specialists, whiting marginal costs are evaluated at representative production quantity of whiting-focused FR’s.
Differences in petrale sole marginal costs attain a maximum of roughly 12%. Cost differences among the five lowest cost producers is smaller, in the range of 4%. Marginal cost differences among FRs in the production of the other groundfish aggregate attain a maximum of approximately 7%.

In sum, results indicate, for most species, small differences in fish processing cost across FR’s. The larger cost differences estimated for sablefish may be more supportive of strategic underproduction. Products for which FR-level costs vary most are relatively under-utilized (figure 6).

**Scale economies**

The rate at which marginal costs rise with increased production informs the *costliness* of owning/accumulating IFQs. If marginal costs increase sharply with own firm production, equilibrium IFQ trading prices will increase sharply as IFQ becomes scarce. We measure the responsiveness of costs and marginal costs to variation in production by examining cost elasticities estimated from our FR cost analysis. The cost elasticities that we report hold capital fixed and are interpreted as short run measures.

FR cost elasticity estimates are not reported to preserve anonymity. Results indicate that all FR’s and even those that produce the largest shares of total production, between 10% - 25% of the total production depending on year, produce under increasing returns to scale. Recall that strategic underproduction requires *industry* marginal profits attain negative value for some \( H \leq \bar{Q} \). Our finding of increasing returns to scale in groundfish production suggests this condition is not likely present given current industry structure.

Observe that current limits on IFQ ownership are considerably below the production scale at which scale-efficiency is achieved. Production of the largest firm in our data is an order of magnitude larger than the currently imposed ownership limit, indicating a stark mismatch between IFQ ownership permitted by regulation and production at which scale economies are realized.

What is not clear is whether firms’ ability to exploit available scale economies is constrained
by explicit *ownership* of IFQs. Firms in the industry can legally process raw fish that is matched to IFQ owned by others, e.g., harvesters.\textsuperscript{19}

**Quota ownership limits: simulation and prescriptions**

We next consider the problem of setting IFQ ownership limits that prevent strategic underproduction and associated inefficiency in the groundfish fishery. For this purposes we conduct a counterfactual policy experiment in which IFQ ownership limits are relaxed, but not removed entirely. We develop a calibrated version of the model in section 3 to derive equilibrium outcomes and optimal ownership limits. The simulation assumes a 10 firm groundfish fishery with available external data plus our cost analysis results. The simulation model is calibrated to the groundfish fishery in 2016.

We first map our empirical estimation of FR production costs (appendix 8.3) to the parsimonious cost model in section 3. This step reveals that processing cost heterogeneity across FR’s is virtually imperceptible once raw product harvesting costs are included, i.e., raw product costs represent roughly 60\%-80\% of total production costs depending on species. We simplify our simulation model and assume FR’s share common costs parameters. Note that this assumption does not remove cost heterogeneity at the firm level since three of the 2016 groundfish firms own and manage more than one FR plant.\textsuperscript{20}

Our data do not allow us to estimate product-specific groundfish demand. We instead rely on demand analyses from previous literature. Asche et al. (2007) surveys whitefish seafood demand and conclude that, “in general the demand elasticities are either about -1.0 or more elastic.” A survey of 168 demand studies by Gallet (2009) reports a median own-price demand elasticity of -0.82 across all types of fish.\textsuperscript{21} The available evidence suggests that own-price demand for groundfish species, which are whitefish with copious substitutes, is likely to be relatively elastic, or at lest not

\textsuperscript{19}A question for future research is whether vertical coordination and production efficiency would increase in a counterfactual industry setting in which ownership limits are removed.

\textsuperscript{20}Our simulations assume firms that manage multiple FR’s distribute production equally to minimize firm costs.

\textsuperscript{21}A meta-analysis conducted in Gallet (2009) finds that fish demand is more elastic in regional as opposed to country-level market analyses, and when focus is on demand by U.S. consumers as opposed to consumers in other countries.
strongly inelastic. Since the design and efficacy of ownership limits vary with demand conditions we consider a range of elasticity scenarios in our simulations.

To help illustrate the potential for welfare loss due from strategic underproduction, we simulate equilibrium outcomes and welfare under different firm conduct scenarios. A first case, although illegal, assumes that groundfish firms collude. The outcome of collusion provides a reference against which outcomes under legal behavior can be compared. A second scenario assumes no IFQ regulation exists in the groundfish fishery. Here we assume groundfish firms play a non-cooperative Cournot groundfish production game. Finally, we solve the equilibrium of our two-stage quota-constrained production game. We consider a no-ownership-limit scenario, $s = 1$, and a welfare maximizing ownership limit scenario, $s^*$. We follow the current regulatory precept wherein ownership limits apply to individual entities, which we take to be the 10 firms in the 2016 groundfish fishery.

Aggregate quotas are set to 2016 values. Unless noted, the result that follow assume a groundfish demand elasticity equal to -0.82, which is the median value from the Gallet (2009) meta-analysis. All simulations assume the groundfish production technology that is non-joint with no cross species substitution/complementarity in product demand. We report results for two high profit margins species in our data, sablefish and petrale sole. To save space, abbreviated results are reported for lower-profit margin species, thornyheads and rockfish.

**Sablefish**

The 2016 sablefish AC was 6.90 m. lbs. The effective production quota after adjusting for processing waste is 5.65 m. lbs. If the IFQ regulation were not present and firms engaged in noncooperative Cournot competition, the equilibrium aggregate production of sablefish in 2016 is predicted to be 5.89 million pounds. Our simulation model predicts that total sablefish production would have exceeded the 2016 AC without the IFQ regulation. The model suggests that an industry cap on production is necessary to achieve sustainable management of sablefish.
Equilibrium aggregate production in the presence of the IFQ regulation but with no ownership limit is 3.34 m. lbs. This equilibrium involves strategic underproduction with quota hoarding by the largest firm. Cost inefficiency is present but is small, with $\Psi^C = 1.03$. Forgone consumer surplus on the other hand is substantial, resulting in welfare efficiency of $\Psi^W = 0.77$. Our calibrated model predicts that 23% of total surplus, mostly consumer surplus, could be lost due to tacit coordination and strategic underproduction of the sablefish AC.

To add further context, note that if groundfish firms were to collude in production, aggregate industry production of sablefish would be 3.26 m. lbs. In this case $\Psi^W$ declines further but by less than a percentage point. Thus, tacit coordination under a strategic underproduction equilibrium allows firms to nearly replicate the collusive outcome, through a channel of legal quota trade.

The IFQ ownership limit that maximizes sablefish welfare is $s^* = 0.14$. The industry equilibrium outcome under this limit produces the entire quota and thus consumer surplus, $\Psi^S = 1$, but does not attain full cost efficiency, although cost inefficiency is small $\Psi^C = 1.002$. Note that an ownership limit of $s^* = 0.14$ is binding for firms that manage multiple FR’s. This constraint prevents the industry from distributing total sablefish production equally across the 10 FRs, as would occur under collusion. This causes a value of $\Psi^C$ that is above unity.

**Petrale sole**

The 2016 AC of petrale sole is 5.81 m. lbs. The effective production quota after adjusting for processing waste is 3.08 m. lbs. The Cournot unconstrained equilibrium production level is 4.12 m. lbs. As with sablefish, our simulation model finds that a cap on total petrale sole production is necessary to achieve sustainable management goals.

The IFQ-constrained equilibrium aggregate production of petrale sole at $s = 1$ is 2.29 m. lbs. Cost and overall inefficiency are, respectively, $\Psi^C = 1.01$ and $\Psi^W = 0.84$. If groundfish firms were to collude in production, the model predicts industry aggregate production at 2.27 m. lbs. As with sablefish the strategic underproduction equilibrium nearly replicates full collusion.
The welfare maximizing IFQ ownership limit for petrale sole falls in the range, \( s^* \in [0.27, 0.47] \). This result is understood from figure 4 of section 5. For all \( s^* \in [0.27, 0.47] \), the equilibrium of the two-stage quota-constrained game involves production of the entire 2016 quota with \( \Psi_C = \Psi_W = 1 \). An ownership limit that exceeds the upper bound, i.e., \( s > 0.47 \) results in strategic underproduction and reduced consumer surplus. When \( s \) falls below its lower bound value, 0.27, the ownership limit constrains quantities of petrale sole that can be produced by the largest firm in the industry, which raises the total cost of production in the industry. Thus for \( s < 0.27 \), \( \psi_C > 1 \) which causes \( \Psi_W \) to decline.

**Other species**

Our empirical estimations for whiting costs do not map well to the two-parameter cost model of section 3. Application of firm behavior simulations to whiting is reserved for future work.

Analysis of Thornyheads and Rockfish species yield additional insights. First, equilibrium aggregate production in a Cournot quota-unconstrained game (no IFQ regulation) falls significantly below the effective annual quotas for these species. Results find that for Thornyheads, 1.68 m. of the 4.78 m. lb. AC is produced and for rockfish, 3.73 m. of the 15.85 m. lb. AC is produced. For these species, the IFQ regulation does not appear necessary for meeting stock conservation goals.

Despite underproduction of annual quotas, results indicate that the unconstrained Cournot outcome is nearly efficient. This result is explained by low profit margins and substantive economies of scale for these species. That is, own firm marginal profits remain relatively constant and positive over a wide range of production levels. Cournot quota-unconstrained aggregate production is large and close to the consumer-surplus-maximizing aggregate production level.

Interestingly, equilibrium play under an IFQ regulation, but with no ownership limits, results in significantly lower aggregate production and welfare. When \( s = 1 \), firms are able to tacitly coordinate production through quota trading. The result of lower industry production and increased industry profit. Welfare reduction for these two species is of similar magnitude and type; cost
inefficiency is small, but foregone consumer surplus is substantive, with $\Psi^W \approx 0.75$ for both species. Not surprisingly, the optimal ownership limit for Thornyhead and rockfish species is small, $s^* = 0.1$. This ownership limit results in an aggregate production equal to the Cournot quota-unconstrained quantity, and near full efficiency in the industry.

7 Conclusion

Concerns for market power in cap-and-trade (CAT) regulated industries have centered on monopoly and monopsony distortions in permit trading. This paper highlights a strategic underproduction channel that has been overlooked in the literature. We present a two stage game of Pareto efficient permit trading followed by a non-cooperative capacity-constrained production. Market power inefficiency emerges and is sustained in equilibrium through legal permit trading and tacit coordination. Firms’ CAT permit acquisitions act as commitments by individual firms to limit their own production. One or more firms acquire excess permit that remain unproduced. Underproduction raises the price of the industry output and increases profit. Underproduction of the regulated cap however reduces consumer surplus and can increase costs above minimum.

Our results show that total welfare generated in a strategic underproduction equilibrium is non-monotonic and discontinuous in permit ownership limits. Ownership limits, if appropriately set, prevent permit hoarding and forestall inefficient tacit coordination by firms. Ownership limits can redistribute production from lower cost to higher cost producers. Our results find that welfare loss under a poorly designed ownership limit policy is more likely when firm costs are heterogeneous and/or when the production technology exhibits increasing returns to scale.

We evaluate industry conduct and performance in the U.S. west coast groundfish fishery, which is currently regulated with individual fishing quotas (IFQs) and stringent limits on IFQ ownership. This analysis shows first that standard indices of market power can be misleading under CAT regulation. Large profit margins, as indicated by Lerner indices for example, can indicate non-competitive pricing or a binding cap under a CAT regulation.
A calibrated simulation model predicts industry outcomes in the absence of ownership limits that mirror full collusion, with underproduction of the regulated annual catch limits and reduced welfare, mostly foregone consumer surplus, in the range of 25% of the total welfare available in the groundfish fishery.

Our model and empirical investigations highlight need for alternative approaches to detect and regulate market power inefficiency in CAT industries. Lerner indices do not differentiate monopoly pricing from quota rent. While the former indicates inefficiency, the latter is a desired effect of the CAT regulation, i.e., in the case of a fishery, to internalize the shadow price of a scarce renewable resource. Detecting strategic underproduction in fisheries should be prioritized. Underproduction of allowable quotas is a common occurrence in the groundfish fishery and in IFQ-regulated fisheries throughout the world (Karp et al., 2024).

Our paper exposes important challenges in the design of a welfare-maximizing ownership limit policy within a CAT regulation. Lax ownership limits will not prevent strategic underproduction. Excessive limits can reduce productive efficiency by redistributing production away from the most cost efficient firms in an industry, and/or by preventing exploitation of scale economies. Welfare maximizing ownership limits vary with cost heterogeneity across firms, with the rate at which marginal costs increase, with the elasticity of final product demand, and with the size of demand relative to the cap chosen by the regulator. Measurement of each of these characteristics and their role in the design of ownership limit policies in CAT is recommended.
References


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8 Appendix

8.1 Permit trading in CAT programs

Acid Rain Program

The U.S. Acid Rain Program is the first nationwide emissions allowance trading program in the United States. It was initiated by Environmental Protection Agency (EPA) in an effort to reduce overall atmospheric levels of sulfur dioxide and nitrogen oxides.

Under the program, initial allowances are allocated to electric generating units which are mostly coal-fired power plants. In addition to these direct allocations, allowances can be bought or sold in a decentralized setting throughout the year or within a centralized auction that is held annually by the EPA.

The EPA tracks allowance holdings and records transactions. Data are publicly available (see Air Markets Program Data portal (https://ampd.epa.gov/ampd/). We collect all transaction records of Phase II period (2010-present). Based on transaction type information, we calculate for each year, the amount of allowance purchased at EPA auction vs the total trading volume from private transfers. Figure 8b summarizes transfers by type and transfer frequency date for 2010-20.

European Union Emissions Trading Scheme (EU ETS)

The EU ETS is the world’s first international emissions CAT program. It began in 2005, and currently operates in all EU countries (plus Iceland, Liechtenstein and Norway). The program limits emissions of approximately 10,000 installations in the power sector and manufacturing industry.

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22These regulations are listed in 40 CFR 73.10, Tables 1 and 2, which are based on a baseline fuel consumption and a rate of emissions (in lbs/million British thermal units). https://www.govinfo.gov/content/pkg/CFR-2011-title40-vol16/pdf/CFR-2011-title40-vol16-sec73-10.pdf

23Auctions began in 1993 and annually thereafter, usually the last week in March. Any individual, corporation, or governing body may participate as a bidder or a seller. Auction data reveals that the EPA itself is the major seller of emissions permits (in most year the sole seller) in the auction. Permit supply comes from the Auction Allowance Reserve, which is approximately 2.8 percent of the total annual allowances. The auction is a variant of a first price sealed bid auction. The auction helps ensure that new producer units have a public source of allowances beyond those allocated initially to existing units.
(a) Frequency

(b) Volume
Centralized auction play a prominent role in EU ETS’ initial allowances allocation. The centralized auction is held weekly, and has a larger number of participants than the U.S. Acid Rain Program.

The EU ETS Transaction Log database records all transfers into and out of individual entity accounts. We evaluated phase III (2013–2020) data. We identify and remove transactions between installations that are registered under a shared account. A summary of trading activity is reported in figure 9b. The data confirm that decentralized private transfers dominate trading activity both in terms of volume and frequency.

In phase III (2013–2020) auctioning became the default method of allocation, although free allocations continue to be used. In total, around 50% of total allowances were auctioned in 2013, with this number rising over the course of this trading period.

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24In phase III (2013–2020) auctioning became the default method of allocation, although free allocations continue to be used. In total, around 50% of total allowances were auctioned in 2013, with this number rising over the course of this trading period.

25https://ec.europa.eu/clima/ets/transaction.do
Individual fishing quota regulations of marine commercial fisheries commonly issue harvest rights to resource users in the form of perpetual ownership share of a total allowable catch. The Total allowable catch is chosen by the fishery management authority, usually annually, and may vary with changing estimates of stock abundance and/or other bioeconomic conditions. The quantity of fish that can be legally harvested by the share owner is determined as the share quantity, $s_i$, for agent $i$ times the allowable catch in the period, say $\bar{Q}_t$ in period $t$. Producer $i$ may then legally harvest $q_{it} = s_i \bar{Q}_t$ units of fish during year $t$. Some carryover from year to year is often allowed.

Trade in both shares and annual quantity units $q_{it}$ is common, occurs repeatedly and frequently, often at terms of trade that remain private information to trading partners. Trades can number in the thousands per year (Newell et al., 2005; Holland et al., 2015). Barter trading between traders with no permit price is common in some programs (Holland, 2013). Permit trading in centralized market settings, e.g., brokerage firms that post trading prices in professional magazines and facilitate quota exchange, occur but typically involve smaller adjustments to permit holdings of firms at prices that fluctuate within and across fishing seasons.

8.2 Proof of Proposition 1

Proof of part (i):

The proof follows from the Pareto efficiency requirement of the Nash Bargaining solution (Nash, 2016).

By the definition of the Nash bargaining solution, the equilibrium allocation $q^E$ maximizes the joint product of individual firm’s bargaining surplus’. If the allocation $q^E$ did not maximize industry profit, a Pareto improvement will exist such that the product of surplus’ would increase. Thus by contradiction, $q^E$ maximizes industry profit.

Proof of part (ii):

The linear demand and quadratic cost assumptions are given as:
• Price vector \( p : \mathbb{R}^M_+ \mapsto \mathbb{R}^M_+ ; p(H) = \alpha - BH, \) where \( \alpha \) and \( H = \sum_i h_i \) are \( M \times 1 \) vectors, and \( B \) is a \( M \times M \) positive definite matrix.

• Individual firm cost function \( c_i : \mathbb{R}^M_+ \mapsto \mathbb{R}_+ ; c_i(h_i) = h_i^T \theta_i + \frac{1}{2} h_i^T \Lambda h_i, \) where \( \theta \) is \( M \times 1 \) vector, \( \Lambda \) is \( M \times M \) positive semi-definite matrix.

Individually firm (player) profit can be written as:

\[
\pi_i(h) = h_i^T p(h) - c_i(h_i),
\]

\[
= h_i^T (\alpha - BH) - h_i^T \theta_i - \frac{1}{2} h_i^T \Lambda h_i,
\]

\[
= h_i^T (\alpha - \theta_i) - h_i^T (B + \frac{1}{2} \Lambda) h_i - h_i^T BH_{-i}.
\]

Each player takes \( h_{-i} \) as given, and solves the problem, \( \max_{0 \leq h_i \leq q_i} \pi_i(h_i, h_{-i}) \). Therefore, the constrained Cournot-Nash equilibrium can be characterized by the following system of conditions:

\[
\frac{\partial \pi_i}{\partial h_{im}}(h) - \lambda_{im} \leq 0; \quad h_{im} \geq 0; \quad \left( \frac{\partial \pi_i}{\partial h_{im}}(h) - \lambda_{im} \right) h_{im} = 0; \quad (q_{im} - h_{im}) \lambda_{im} = 0; \quad \lambda_{im} \geq 0,
\]

for \( \forall i, m \), where \( \lambda_{im} \) is the Lagrangian multiplier of constrain \( h_{im} \leq q_{im} \).

**Step 1: The second stage solution \( h(q) \) exists and is a function**

Following Laye and Laye (2008), we derive the lemma below.

**Lemma 1.** For any given \( q \), the following two statements are equivalent: 1) \( h(q) \) is an equilibrium
of second stage production subgame, 2) $h(q)$ is a maximizer of

$$F(h) = \sum_{i=1}^{I} h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] - \sum_{i=1}^{I} c_i(h_i)$$  \hspace{0.5cm} (12)$$

subject to the constraint $0 \leq h_{im} \leq q_{im}$, for all $i, m$, where $H_{-i} = \sum_{j \neq i} h_j$.

Proof. We have

$$F(h) = h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] + \sum_{j \neq i} h_j^T [\alpha - \frac{1}{2} B H_{-j} - Bh_j] - \sum_{i=1}^{I} c_i(h_i)$$

$$= h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] + \sum_{j \neq i} h_j^T [-\frac{1}{2} B H_{-j}] - c_i(h_i) + R_1(h_{-i})$$

$$= h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] + \sum_{j \neq i} h_j^T [-\frac{1}{2} B (H_{-i} + h_i - h_j)] - c_i(h_i) + R_1(h_{-i})$$

$$= h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] + \sum_{j \neq i} h_j^T [-\frac{1}{2} Bh_i] - c_i(h_i) + R_2(h_{-i})$$

$$= h_i^T [\alpha - \frac{1}{2} B H_{-i} - Bh_i] - \frac{1}{2} H_{-i}^T Bh_i - c_i(h_i) + R_2(h_{-i})$$

$$= h_i^T [\alpha - B H_{-i} - Bh_i] - c_i(h_i) + R_2(h_{-i})$$

$$= \pi_i(h) + R_2(h_{-i}),$$

where $R_1(h_{-i})$ and $R_2(h_{-i})$ collect terms containing only $h_i$.

Therefore, the Kuhn-Tucker conditions for program (12) can be written as

$$\frac{\partial \pi_i}{\partial h_{im}}(h) - \lambda_{im} \leq 0; \quad h_{im} \geq 0; \quad \left( \frac{\partial \pi_i}{\partial h_{im}}(h) - \lambda_{im} \right) h_{im} = 0;$$

$$q_{im} - h_{im} \geq 0; \quad \lambda_{im} \geq 0; \quad (q_{im} - h_{im}) \lambda_{im} = 0;$$

for all $i, m$, where $\lambda_{im}$ is the Lagrangian multiplier of constrain $h_{im} \leq q_{im}$. 54
Note that the above system of conditions is identical to the set of conditions in (11) that characterizes the second stage Cournot-Nash equilibrium.

$F$ is strictly concave (see section 8.2 below) w.r.t. $h$. Feasible production is

$$\Gamma(q) = \{(h_{11}, \ldots, h_{IM}) \mid 0 \leq h_{im} \leq q_{im}, \text{ for } \forall i, m\}$$

and is a convex set. Therefore, the maximizer of the above problem exists and is unique. Hence, for any given $q$, the Nash equilibrium of the second stage subgame $h(q)$ exists and is unique.

**Step 2: Continuity of $h(q)$**

Since the objective function $F(h; q)$ is continuous w.r.t. $h$ and $q$, the feasible $\Gamma(q)$ is a compact-valued and continuous correspondence. By Berge’s Maximum Theorem, the optimal policy correspondence $h(q) = \{h \in \Gamma(q) \mid F(h; q) = \max_{x \in \Gamma(q)} F(x; q)\}$ is nonempty, compact-valued, and upper hemi-continuous (w.r.t. the parameter $q$). From Step 1, $h(q)$ is single-valued, thus a function. Therefore, $h(q)$ upper hemi-continuous implies $h(q)$ continuous.

**Step 3: Existence of Equilibrium for the Two-Stage Game**

Since both $\Pi(h)$ and $h(q)$ are continuous, therefore the composite functions $\Pi(h(q))$ is also continuous.

Since the domain of $q$, $\Omega_Q = \{(q_{11}, \ldots, q_{IM}) \in \mathbb{R}_{+}^{I \times M} \mid \sum_i q_{im} = Q_m \text{ for } \forall m\}$, is compact, according to the Weierstrass Extreme Value Theorem, a maximizer $q^*$ of the continuous function $\Pi(h(q))$ exists. Thus, an equilibrium allocation $q^*$ of the two-stage game exists.
Strict concavity of $F$

The Hessian matrix of $F$ is

$$
\nabla^2 h F = \begin{bmatrix}
-2B - \Lambda & \cdots & -B & \cdots & -B \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
-B & \cdots & -2B - \Lambda & \cdots & -B \\
\vdots & \cdots & \vdots & \ddots & \vdots \\
-B & \cdots & -B & \cdots & -2B - \Lambda 
\end{bmatrix}
$$

$$= -I_I \otimes (B + \Lambda) - L_I \otimes B$$

where $I_I$ is the $I \times I$ unit matrix; $L_I$ is the $I \times I$ matrix of ones.

To show that the above matrix is negative definite, we introduce following lemma.

**Lemma 2.** If two matrices $M$ and $N$ are positive (semi) definite, then $M \otimes N$ is also positive (semi) definite.

**Proof.** Let $\lambda_i \ (i = 1, \ldots, m)$ be the eigenvalues of the matrix $M$. Let $\mu_j \ (j = 1, \ldots, n)$ be the eigenvalues of the matrix $N$. Then, the eigenvalues of $M \otimes N$ are $\lambda_i \mu_j \ (i = 1, \ldots, m, j = 1, \ldots, n)$. Since a matrix is positive (semi) definite if and only if all of its eigenvalues are (non-negative)positive, the positive (semi) definiteness of $M$ and $N$ translates into the positive (semi) definiteness of $M \otimes N$.

From Lemma 2, $I_I \otimes (B + \Lambda)$ is positive definite; $L_I \otimes B$ is positive semi-definite. Therefore, $\nabla^2 h F$ is negative definite.

### 8.3 U.S. west coast groundfish data and empirical analysis

This section presents data, estimation, and results of our empirical analysis of the U.S. west coast groundfish fishery. The results are summarized in section 6 in the main body of the paper.
Industry and data

Pacific whiting, hereafter whiting, is a pelagic species that inhabits the midwater-column along with some rockfish species. Whiting is co-managed by the Agreement Between the Government of the United States of America and the Government of Canada on Pacific Hake/Whiting of 2003. This agreement allocates 73.88% of the total allowable catch (AC) to the United States. This allocation is then divided between the mothership cooperative, the catcher-processor cooperative, the shore based individual fishing quota (IFQ) Program, and Pacific Coast treaty tribes. The shore-based IFQ Program receives 42% of the non-tribal US commercial share of whiting. The other major groundfish species are generally bottom dwellers and are managed as several separate limited entry fisheries, tribal fisheries, an open access fishery, as well as the IFQ program. There are currently 30 quota categories, defined by species, species grouping and/or catch area. Our model only considers the IFQ components of the whiting and non-whiting groundfish fisheries.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pac. Whiting</th>
<th>Sablefish</th>
<th>Thny.heads</th>
<th>Petr. Sole</th>
<th>Oth. Sole</th>
<th>Rockfish</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>77.578</td>
<td>7.584</td>
<td>2.076</td>
<td>2.659</td>
<td>9.585</td>
<td>2.037</td>
<td>101.519</td>
</tr>
<tr>
<td>2010</td>
<td>48.800</td>
<td>7.740</td>
<td>2.578</td>
<td>0.913</td>
<td>8.217</td>
<td>1.759</td>
<td>70.008</td>
</tr>
<tr>
<td>2011</td>
<td>136.723</td>
<td>6.721</td>
<td>1.472</td>
<td>0.959</td>
<td>4.831</td>
<td>2.506</td>
<td>153.212</td>
</tr>
<tr>
<td>2012</td>
<td>76.114</td>
<td>6.068</td>
<td>1.797</td>
<td>1.488</td>
<td>6.768</td>
<td>2.772</td>
<td>95.006</td>
</tr>
<tr>
<td>2013</td>
<td>143.283</td>
<td>5.418</td>
<td>2.412</td>
<td>2.806</td>
<td>7.229</td>
<td>2.665</td>
<td>163.814</td>
</tr>
<tr>
<td>2015</td>
<td>88.742</td>
<td>6.520</td>
<td>1.708</td>
<td>3.557</td>
<td>5.474</td>
<td>3.942</td>
<td>109.944</td>
</tr>
<tr>
<td>Ave.</td>
<td>101.135</td>
<td>6.485</td>
<td>1.996</td>
<td>2.406</td>
<td>6.865</td>
<td>2.785</td>
<td>121.672</td>
</tr>
<tr>
<td>Ave. Share</td>
<td>0.822</td>
<td>0.056</td>
<td>0.018</td>
<td>0.019</td>
<td>0.062</td>
<td>0.024</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Pac. Whiting</th>
<th>Sablefish</th>
<th>Thny.heads</th>
<th>Petr. Sole</th>
<th>Oth. Sole</th>
<th>Rockfish</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>30.066</td>
<td>37.729</td>
<td>5.051</td>
<td>2.600</td>
<td>17.015</td>
<td>3.402</td>
<td>95.865</td>
</tr>
<tr>
<td>2012</td>
<td>50.515</td>
<td>27.168</td>
<td>5.146</td>
<td>5.295</td>
<td>16.570</td>
<td>5.230</td>
<td>109.924</td>
</tr>
<tr>
<td>2013</td>
<td>70.884</td>
<td>20.146</td>
<td>5.679</td>
<td>8.669</td>
<td>17.316</td>
<td>5.550</td>
<td>128.244</td>
</tr>
<tr>
<td>2016</td>
<td>42.635</td>
<td>25.520</td>
<td>3.419</td>
<td>10.628</td>
<td>14.625</td>
<td>6.326</td>
<td>103.153</td>
</tr>
<tr>
<td>Ave.</td>
<td>49.809</td>
<td>28.173</td>
<td>4.708</td>
<td>7.316</td>
<td>15.894</td>
<td>5.437</td>
<td>111.337</td>
</tr>
<tr>
<td>Ave. Share</td>
<td>0.439</td>
<td>0.258</td>
<td>0.043</td>
<td>0.067</td>
<td>0.144</td>
<td>0.050</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Annual Production, 2009-16. Quantities are in millions of pounds. Revenue is in millions of 2016 dollars.

Table 2 reports annual production and revenue by major IFQ species. Whiting comprised 82.2%
of production and 43.9% of revenue during the 2009-16 data period. Whiting is a shorter-lived species with relatively high variability in recruitment and growth. Note that most of the variability in total annual production reported in table 2 derives from variability in whiting. Production and revenue of other groundfish species is relatively stable, although trends in quantities and revenue are apparent.

### Prices

<table>
<thead>
<tr>
<th>Year</th>
<th>P. Whit.</th>
<th>Sbl. fish</th>
<th>T. Heads</th>
<th>P. Sole</th>
<th>D. Sole</th>
<th>Rockfish</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.606</td>
<td>4.631</td>
<td>2.197</td>
<td>2.802</td>
<td>2.163</td>
<td>2.310</td>
</tr>
<tr>
<td>2010</td>
<td>0.705</td>
<td>5.149</td>
<td>1.985</td>
<td>3.123</td>
<td>2.226</td>
<td>2.193</td>
</tr>
<tr>
<td>2011</td>
<td>0.530</td>
<td>6.007</td>
<td>3.000</td>
<td>4.117</td>
<td>3.068</td>
<td>2.266</td>
</tr>
<tr>
<td>2012</td>
<td>0.741</td>
<td>5.247</td>
<td>3.441</td>
<td>4.417</td>
<td>2.907</td>
<td>2.382</td>
</tr>
<tr>
<td>2013</td>
<td>0.522</td>
<td>4.491</td>
<td>2.811</td>
<td>3.897</td>
<td>2.826</td>
<td>2.382</td>
</tr>
<tr>
<td>2014</td>
<td>0.538</td>
<td>5.092</td>
<td>2.754</td>
<td>3.779</td>
<td>3.215</td>
<td>2.099</td>
</tr>
<tr>
<td>2015</td>
<td>0.413</td>
<td>5.548</td>
<td>2.999</td>
<td>3.720</td>
<td>2.814</td>
<td>2.256</td>
</tr>
<tr>
<td>2016</td>
<td>0.453</td>
<td>5.785</td>
<td>2.392</td>
<td>3.676</td>
<td>2.612</td>
<td>1.987</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>P. Whit.</th>
<th>Sbl. fish</th>
<th>T. Heads</th>
<th>P. Sole</th>
<th>D. Sole</th>
<th>Rockfish</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.140</td>
<td>2.790</td>
<td>0.582</td>
<td>0.942</td>
<td>0.395</td>
<td>0.922</td>
</tr>
<tr>
<td>2010</td>
<td>0.093</td>
<td>2.846</td>
<td>0.615</td>
<td>1.304</td>
<td>0.361</td>
<td>0.715</td>
</tr>
<tr>
<td>2011</td>
<td>0.116</td>
<td>4.235</td>
<td>0.663</td>
<td>1.646</td>
<td>0.476</td>
<td>0.750</td>
</tr>
<tr>
<td>2012</td>
<td>0.148</td>
<td>3.313</td>
<td>0.660</td>
<td>1.602</td>
<td>0.472</td>
<td>0.994</td>
</tr>
<tr>
<td>2013</td>
<td>0.134</td>
<td>2.781</td>
<td>0.703</td>
<td>1.399</td>
<td>0.498</td>
<td>0.794</td>
</tr>
<tr>
<td>2014</td>
<td>0.121</td>
<td>3.381</td>
<td>0.712</td>
<td>1.225</td>
<td>0.495</td>
<td>0.689</td>
</tr>
<tr>
<td>2015</td>
<td>0.087</td>
<td>3.474</td>
<td>0.670</td>
<td>1.299</td>
<td>0.473</td>
<td>0.737</td>
</tr>
<tr>
<td>2016</td>
<td>0.075</td>
<td>3.737</td>
<td>0.687</td>
<td>1.286</td>
<td>0.462</td>
<td>0.575</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>P. Whit.</th>
<th>Sbl. fish</th>
<th>T. Heads</th>
<th>P. Sole</th>
<th>D. Sole</th>
<th>Rockfish</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.467</td>
<td>1.841</td>
<td>1.615</td>
<td>1.859</td>
<td>1.768</td>
<td>1.388</td>
</tr>
<tr>
<td>2010</td>
<td>0.611</td>
<td>2.302</td>
<td>1.370</td>
<td>1.818</td>
<td>1.865</td>
<td>1.477</td>
</tr>
<tr>
<td>2011</td>
<td>0.414</td>
<td>1.772</td>
<td>2.337</td>
<td>2.472</td>
<td>2.592</td>
<td>1.516</td>
</tr>
<tr>
<td>2012</td>
<td>0.593</td>
<td>1.934</td>
<td>2.782</td>
<td>2.815</td>
<td>2.435</td>
<td>1.388</td>
</tr>
<tr>
<td>2013</td>
<td>0.388</td>
<td>1.710</td>
<td>2.107</td>
<td>2.498</td>
<td>2.328</td>
<td>1.588</td>
</tr>
<tr>
<td>2014</td>
<td>0.416</td>
<td>1.712</td>
<td>2.042</td>
<td>2.554</td>
<td>2.720</td>
<td>1.409</td>
</tr>
<tr>
<td>2015</td>
<td>0.326</td>
<td>2.074</td>
<td>2.329</td>
<td>2.422</td>
<td>2.341</td>
<td>1.519</td>
</tr>
<tr>
<td>2016</td>
<td>0.378</td>
<td>2.048</td>
<td>1.706</td>
<td>2.390</td>
<td>2.150</td>
<td>1.412</td>
</tr>
</tbody>
</table>

Table 3: **Product and Ex-Vessel Average Prices: By major groundfish species.**

Production weighted average prices, by species, for final processed products and for raw fish deliveries (the exvessel price) are reported in table 3 for the 2009-16 data period. The product-over-exvessel price markup is reported in lower section of the table.26

26Prices are calculated as the quantity-weighted average across all FRs in each calendar year. Averages do not reflect within-species differences in product form. All prices are deflated to 2016 dollars using the gross domestic product implicit price deflator.
Sablefish earns the highest price per pound in the final product market. Petrale sole prices are second highest ranging from $2.80/lb. in 2009 and $4.42/lb. in 2012. Product prices are less for dover sole, thornyhead species, and rockfish species. Whiting earns the lowest price per pound. The price spike for pacific whiting in 2012 was likely the result of an unusually low harvest that year.

**Regulations**

The regulation limits the IFQ that is owned or controlled by a “particular individual, corporation, or other entity” (Fisheries Off West Coast States and Rule, 2010). Ownership limits are reported in figure 10.

**Revenue and cost**

<table>
<thead>
<tr>
<th></th>
<th>Revenue</th>
<th>Processing Expenses</th>
<th>Raw Fish Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>Whiting</td>
<td>3.348</td>
<td>5.327</td>
<td>1.153</td>
</tr>
<tr>
<td>Groundfish</td>
<td>4.136</td>
<td>4.622</td>
<td>2.719</td>
</tr>
<tr>
<td>Total</td>
<td>22.139</td>
<td>18.608</td>
<td>13.303</td>
</tr>
</tbody>
</table>

Table 4: **Revenue and Cost Summary.** N = 119. Values are in $2016 m.

Table 4 reports summary statistics for FR revenue and costs. Processing expenses are separated into variable and fixed expense categories. Variable expenses include custom processing, product additives, plant maintenance and cleaning, communication and office supplies, energy, packing materials, water, waste disposal, and, importantly, fish-line and managerial labor. Expenses in the fixed category include building, machinery, and processing equipment, and building rental costs.
Below is a table showing the shorebased IFQ program accumulation limits as specified at 50 CFR 660.140(d)(4)(i)(C). These values are used to calculate the amount of quota shares that can be transferred to a quota share account.

The **QS and IBQ Control Limits** are accumulation limits and are the amount of QS and IBQ that a person, individually or collectively, may own or control. QS and IBQ control limits are expressed as a percentage of the Shorebased IFQ Program's allocation.

<table>
<thead>
<tr>
<th>IFQ Category</th>
<th>QS and IBQ Control Limit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrowtooth flounder</td>
<td>10.0%</td>
</tr>
<tr>
<td>Bonito rockfish South of 40°10' N.</td>
<td>13.2%</td>
</tr>
<tr>
<td>Canary rockfish</td>
<td>4.4%</td>
</tr>
<tr>
<td>Chilipepper rockfish South of 40°10' N.</td>
<td>10.0%</td>
</tr>
<tr>
<td>Cowcod South of 40°10' N.</td>
<td>17.7%</td>
</tr>
<tr>
<td>Darkblotched rockfish</td>
<td>4.3%</td>
</tr>
<tr>
<td>Dover sole</td>
<td>2.6%</td>
</tr>
<tr>
<td>English sole</td>
<td>5.0%</td>
</tr>
<tr>
<td>Lingcod N. of 40°10' N. lat.</td>
<td>2.5%</td>
</tr>
<tr>
<td>Lingcod S. of 40°10' N. lat.</td>
<td>2.5%</td>
</tr>
<tr>
<td>Longspine thornyheads North of 34°27' N.</td>
<td>6.0%</td>
</tr>
<tr>
<td>Minor shelf rockfish North of 40°10' N.</td>
<td>5.0%</td>
</tr>
<tr>
<td>Minor shelf rockfish South of 40°10' N.</td>
<td>9.0%</td>
</tr>
<tr>
<td>Minor slope rockfish North of 40°10' N.</td>
<td>5.0%</td>
</tr>
<tr>
<td>Minor slope rockfish South of 40°10' N.</td>
<td>6.0%</td>
</tr>
<tr>
<td>Other flatfish</td>
<td>10.0%</td>
</tr>
<tr>
<td>Pacific cod</td>
<td>12.0%</td>
</tr>
<tr>
<td>Pacific halibut (IBQ) North of 40°10' N.</td>
<td>5.4%</td>
</tr>
<tr>
<td>Pacific ocean perch North of 40°10' N.</td>
<td>4.0%</td>
</tr>
<tr>
<td>Pacific whiting</td>
<td>10.0%</td>
</tr>
<tr>
<td>Petrale sole</td>
<td>3.0%</td>
</tr>
<tr>
<td>Sablefish North of 36° N.</td>
<td>3.0%</td>
</tr>
<tr>
<td>Sablefish South of 36° N.</td>
<td>10.0%</td>
</tr>
<tr>
<td>Shortspine thornyheads North of 34°27' N.</td>
<td>6.0%</td>
</tr>
<tr>
<td>Shortspine thornyheads South of 34°27' N.</td>
<td>6.0%</td>
</tr>
<tr>
<td>Splitnose rockfish South of 40°10' N.</td>
<td>10.0%</td>
</tr>
<tr>
<td>Starry flounder</td>
<td>10.0%</td>
</tr>
<tr>
<td>Widow rockfish</td>
<td>5.1%</td>
</tr>
<tr>
<td>Yelloweye rockfish</td>
<td>5.7%</td>
</tr>
<tr>
<td>Yellowtail rockfish North of 40°10' N.</td>
<td>5.0%</td>
</tr>
<tr>
<td>Non-whiting Groundfish Species</td>
<td>7.6%</td>
</tr>
</tbody>
</table>

Figure 10: Shorebased IFQ Program Accumulation Limits. Source: NOAA.
Expenditures on taxes and license fees are not included as they represent transfers to the government.

**Cost analysis**

This section presents our estimation of individual FR processing costs. The data available for this analysis is an *almost complete* panel of 15 FRs that processed the bulk of whiting and groundfish (89.8% of total revenues) during our 2009-16 data period. One FR is active for 7 of the possible 8 data years (2010-16). There are 119 observations on annual cost and production across multiple final products. Annual expenses are not differentiated by product. We therefore specify a multi-product cost function and estimate the costs attributable to individual products econometrically.

One consideration for our estimations is the potential of endogeneity of production quantities. We note that the extent to which FRs adjust production may be limited under the groundfish regulations, pre- and post-IFQ regime. Quota adjustments are limited particularly for IFQ that bind at the aggregate level, e.g., whiting, sablefish and petrale sole. Complementarity in harvest and thus deliveries to FRs imposes limits on adjustments to the mix of groundfish species produced. Natural variation in stock abundance due to exogenous growth and environmental conditions, e.g., see variability in whiting production in table 2) are a determining factor in annual FR production. Despite these considerations endogeneity of production quantities can bias the estimation. We therefore control for potential endogeneity bias in our estimations with FR-level constants and slope parameters to control for time-invariant, unobserved heterogeneity across FRs.

Groundfish FRs process a large number of products several of which are outside the focus of the current study, e.g., salmon, tuna, herring, and crab are processed by many FRs in our data. We separate products into three categories: (i) whiting, (ii) major groundfish species including sablefish, petrale sole, and an aggregate of thornyheads, dover sole and rockfish, and (iii) all other fish species that are not managed under the IFQ regulation. We specify a mixed econometric model that includes a parametric component that is similar to the specification used in section 3 for product categories (i) and (ii). A semiparametric component is specified for product category (iii).
We specify the following a latent class stochastic frontier model to accommodate heterogeneity across FRs (Greene, 2005):

\[ \ln C_{it} = \theta_i' X_{it} + g(Z_{it} | \tau) + v_{it}, \]
\[ \theta_i = \bar{\theta} + \Gamma \theta W_i, \]

where \( C_{it} \) is the cost of production for FR \( i \) in year \( t \). \( X_{it} \) includes production quantities from product categories (i) and (ii) and factor input prices. \( g(Z_{it} | \tau) \) is a semi-parametric function specified for production category (iii); \( g(\cdot) \) is specified as a second order polynomial function with nuisance parameter \( \tau \). \( v_{it} \) denotes the model error term.

We assume \( v_{it} | X_{it}, Z_{it} \sim N(0, \sigma^2_v) \), \( W_i | X_{it}, Z_{it} \sim N(0, I_K) \), \( K \) is the dimension of \( X_{it} \), and \( \Gamma \theta \) is an upper triangular matrix such that \( \Gamma' \theta \Gamma \theta \) is positive definite.

We observe a random sample on \((C_{it}, X_{it}, Z_{it})\). Our goal is to estimate \((\theta_i, \eta)\). We follow Greene (2005) and Train (2009), and use the simulated maximum likelihood method to estimate \( \theta_i \). Estimation of the common parameter vector \( \eta \) and nuisance parameters \( \tau \) follows standard least squares methods and is hereafter suppressed to simplify the presentation.

Denote \( C = \{(C_{i1}, \ldots, C_{iT})\}_{i=1}^N \) the vector of FR costs, \( X = \{(X_{i1}, \ldots, X_{iT})\}_{i=1}^N \) the vector of covariates, and \( W = (W_1, \ldots, W_N) \) the vector of unobserved firm specific characteristics. The conditional log-likelihood function is given by:

\[ L(C \mid X, W, \bar{\theta}, \Gamma \theta, \sigma^2_v) = \sum_{i=1}^N \sum_{t=1}^T \log f_v(C_{it} - (\bar{\theta} + \Gamma \theta W_i)^' X_{it} \mid W_i, \bar{\theta}, \Gamma \theta, \sigma^2_v), \]

where \( f_v(u) = \frac{1}{\sigma_v} \phi \left( \frac{u}{\sigma_v} \right) \), and \( \phi(\cdot) \) is the density function of the standard normal distribution. Since we do not observe \( W_i \), we cannot compute the above likelihood. We instead maximize the
unconditional log-likelihood:

\[
L(C \mid X, \theta, \Gamma_\theta, \sigma_v^2) = \sum_{i=1}^N \sum_{t=1}^T \log f_v(C_{it} - (\bar{\theta} + \Gamma_\theta W_i)'X_{it} \mid W_i, \bar{\theta}, \Gamma_\theta, \sigma_v^2) \phi(w_i) dw_i
\]

Even though the distribution \(\phi(w_i)\) of \(W_i\) is known, the integral is difficult to compute. For this reason, we maximize the simulated version of the unconditional likelihood. We draw a sample \(\{W_i^r\}_{r=1}^R\) from the distribution \(\phi(w_i)\), which is the multivariate standard normal distribution in our case. We call \(L_s\) the simulated likelihood function:

\[
L_s(C \mid X, \bar{\theta}, \Gamma_\theta, \sigma_v^2) = \sum_{i=1}^N \sum_{r=1}^R \left[ \sum_{t=1}^T \log f_v(C_{it} - (\bar{\theta} + \Gamma_\theta W_i^r)'X_{it} \mid W_i^r, \bar{\theta}, \Gamma_\theta, \sigma_v^2) \right].
\]

By maximizing the simulated maximum likelihood function, we obtain estimates of \(\hat{\theta}, \hat{\Gamma}_\theta\) and \(\hat{\sigma}_v^2\).

Finally, using Bayes’ rule we can estimate \(\theta_i\) as a weighted average of the \(\hat{\theta}_i^r\) as follows:

\[
\hat{\theta}_i = \sum_{r=1}^R \omega_i^r \hat{\theta}_i^r,
\]

where

\[
\omega_i^r = \frac{\exp \left( \sum_{t=1}^T \log f_v(C_{it} \mid W_i^r, X_{it}, \bar{\theta}, \hat{\Gamma}_\theta, \hat{\sigma}_v^2) \right)}{\sum_{r=1}^R \exp \left( \sum_{t=1}^T \log f_v(C_{it} \mid W_i^r, X_{it}, \bar{\theta}, \hat{\Gamma}_\theta, \hat{\sigma}_v^2) \right)}, \quad \text{and} \quad \hat{\theta}_i^r = \bar{\theta} + \Gamma_\theta W_i^r.
\]

**Wild Bootstrap estimate of a cost function with firm specific slope using simulated maximum likelihood (SML)**

Following estimation using SML, we obtain the residuals,

\[
\hat{v}_{it} = C_{it} - \hat{\theta}_i^r X_{it} - g(Z_{it} \mid \tau) \quad \text{for all } i \text{ and all } t.
\]
For each FR \( i \), we draw \( \omega_i \) equal to -1 with probability 0.5 and 1 with probability 0.5. We then construct the bootstrap sample \( C_{it}^* = \hat{\theta}_i'X_{it} + \omega_i\hat{v}_{it} \). We repeat the estimation algorithm for SML on \( (C_{it}^*, X_{it}, Z_{it}) \):

\[
C_{it}^* = \theta_i'X_{it} + g(Z_{it}|\tau) + v_{it}^*,
\]

\[
\theta_i^* = \bar{\theta}_i + \Gamma_\theta W_i,
\]

where \( v_{it}^* \mid X_{it} \sim N(0, \sigma_v^2) \), \( W_i \mid X_{it} \sim N(0, I_K) \). \( K \) is the dimension of \( X_{it} \), and \( \Gamma_\theta \) is an upper triangular matrix such that \( \Gamma_\theta' \Gamma_\theta \) is positive definite (Cholesky decomposition). We repeat this procedure \( B \) times and get \( \{ \hat{\theta}_i^* \}_{i=1}^B \), which we use to recover standard errors of our estimator.

The maximum likelihood estimation and bootstrap algorithm is programmed with Gauss software.

**Estimation and results:**

The model is estimated with category (ii) products that include sablefish, petrale sole, and combined production of dover sole, thornyhead species, and rockfish.\(^{27}\) We include the labor wage rate. Prices of other factors of are not available and are implicitly assumed to be constant throughout the data period. The logarithm of annual FR cost is specified as the dependent variable. FR-specific parameters \( \theta_i \) are specified for the models first-order slope parameters. To accommodate large differences in the production scale across FRs we specify a cubic functional form and impose common second and third order effects across all FRs. The wage rate is entered in logarithm form.

Point parameter estimates and bootstrap confidence intervals for the model in (13) are reported in table 5. Parameters are not easily interpreted. We therefore use the fitted cost model to calculate relevant economic effects, including marginal production costs, cost elasticities, and curvature of the cost function.

\(^{27}\)There are at most 8 annual observations for each FR. Estimates of firm-specific slope parameters for all species of groundfish is not possible. Dover sole, thornyhead sp. and rockfish ps. are aggregated linearly. The implicit assumption is that costs per pound of processed fish is the same for these species.
Parm. mean values ($\bar{\theta}$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>6.581</td>
<td>0.797</td>
<td>5.142</td>
<td>8.105</td>
</tr>
<tr>
<td>PWhit.</td>
<td>0.166</td>
<td>0.058</td>
<td>0.055</td>
<td>0.284</td>
</tr>
<tr>
<td>Sble.Fish</td>
<td>0.350</td>
<td>0.218</td>
<td>-0.088</td>
<td>0.776</td>
</tr>
<tr>
<td>Ptrl.Sole</td>
<td>0.216</td>
<td>0.359</td>
<td>-0.454</td>
<td>0.926</td>
</tr>
<tr>
<td>Oth.Grnd.F</td>
<td>0.205</td>
<td>0.194</td>
<td>-0.186</td>
<td>0.581</td>
</tr>
</tbody>
</table>

Model fixed parms.

<table>
<thead>
<tr>
<th>Term</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\sum h_i)^2$</td>
<td>-0.636</td>
<td>0.339</td>
<td>-1.337</td>
<td>-0.015</td>
</tr>
<tr>
<td>$(\sum h_i)^3$</td>
<td>0.007</td>
<td>0.004</td>
<td>-0.001</td>
<td>0.017</td>
</tr>
<tr>
<td>Wage</td>
<td>0.154</td>
<td>0.326</td>
<td>-0.512</td>
<td>0.734</td>
</tr>
</tbody>
</table>

Table 5: **Cost Function Parameter Estimates.**

Table 5 reports point estimates, bootstrap standard errors and 95% confidence intervals for parameter vector $\bar{\theta}$ and the empirical model’s fixed parameters, which include coefficients on total production entered quadratically and cubically.28

Point estimates of $\bar{\theta}$’s are positive for all species. The signs of the second and third order production terms are, respectively, negative and positive, suggesting that fitted costs may not be strictly increasing across the range of positive production quantities. Further investigation reveals that monotonicity and convexity of fitted costs is violated for whiting. This finding may reflect variation in the product mix and scale across FRs in our data.

95% confidence intervals for $\bar{\theta}$ include zero for sablefish, petrale sole, and for the thornyhead/Dover Sole/Rockfish group. Confidence intervals for the quadratic and cubic production terms also contain zero.

Production costs are increasing and concave in the labor wage, although the elasticity is not precisely estimated. The point estimate suggests that 10% increase in the wage rate will result in a 4.70% increase in FR cost.

**Economies of scale**

Confidentiality prevents us from reporting FR-specific cost elasticity estimates. Results indicate that current regulations do not allow IFQ ownership at quantities that are consistent with scale efficiency in the industry.

28 The estimate of $\hat{\Gamma}_\theta$ is not reported to save space.
Figure 11: Costs, marginal cost, and function curvature.

Figure 11 plots costs, marginal costs, and cost function curvature, which we calculate as the second derivative of the fitted function, across the range of product quantities observed in our data. Results are reported for four product forms. Horizontal axes are the 5’th through the 95’th production quantity quantiles for each product. Values on the vertical axes are $2016. The results hold all function arguments at sample mean values.

The fitted costs are not globally convex or strictly increasing in the quantity of whiting production. Differences between whiting- and non-whiting specialists explain these results.

The model suggests that sablefish marginal costs vary in the range of $0.764/lb. when production is at the 5’th quantile value (zero annual sablefish production), to $0.994/lb. when production increases to the 95’th quantile, 869,000 lbs. per year.

Results for petrale sole and for the other groundfish product grouping find that costs are close to linear in quantity produced. For petrale sole, marginal costs increase from $0.508 / lb. when
production is at the 5'th quantile value (zero annual production), to $0.528 /lb. when production increases to the 95'th quantile value, 267,000 lbs. per year.

For the other groundfish product group, marginal costs increase from $0.471 / lb. when production is at the 5'th quantile (zero annual production), to $0.540 /lb. when production increases to the 95'th quantile value, 1,076,000 lbs. annually.

### 8.4 Equilibrium derivation when many firms

Equilibrium quota ownership $q^*$ and harvest $h^*$ must satisfy:

- (i) Individual Rationality: For each $i$, given $q^*$ and $h^*_{-i}$, $h^*_i$ solves
  
  $$
  \max_{0 \leq h_i \leq q^*_i} \pi_i = p(h_i + H^*_{-i})h_i - c_i(h_i).
  $$

- (ii) Aggregate quota constraint: $\sum_i q^*_i = Q$.

- (iii) Ownership limit: $0 \leq q^*_i \leq sQ$.

- (iv) Industry optimality: $\Pi(h^*) \geq \Pi(h')$ for all candidate $(h', q')$ that satisfy above three constraints

### Algorithm Description

Finding the equilibrium using a grid search over candidate quota allocations becomes computationally impractical when the number of firms $I$ is large. The following reduces the dimensionality of the grid search problem to two dimensions. The algorithm has two steps: we obtain the unique $h^*(s)$; we then generate the set $\{q^*(s)\}$ that support $h^*(s)$.

**Step 1: Finding $h^*(s)$**

(1) Let $E \in \Omega_E = 2^I$ denote some subset of all players $I = \{1, 2, \ldots, I\}$, and $mc \in \mathbb{R}^I_+$ a non-negative scalar value of marginal cost. For given pair $(s, Q)$, define a vector-valued function
\[ f(h; E, mc) : \mathbb{R}_+^I \mapsto \mathbb{R}_+^I \] with \( i \)’th component,

\[
    f_i(h; E, mc) = \begin{cases} 
    \max \{0, \min \{br_i(h), sQ\}\} & \text{if } i \in E \\
    \max \{0, \min \{br_i(h), sQ, \frac{mc-\theta_i}{\eta_i}\}\} & \text{if } i \notin E
    \end{cases}
\]

where \( br_i(h) = (\alpha - \beta H_i - \theta_i)/(2\beta + \eta_i) \) is \( i \)'s best production response.

(2) Find the fixed point of \( f(h) \) for each \((E, mc)\) pair. Denote this fixed point, \( h_{E}(mc) : \Omega_{E} \times \mathbb{R}_+ \mapsto \mathbb{R}_+^I \), i.e. \( h_{E}(mc) = f(h_{E}(mc)) \).

Any non-negative individual rational harvest vector with elements that do not exceed \( sQ \) can be fully characterized by \((E, mc)\).

(3) Optimize industry profit over \( E \) and \( mc \), subject to two feasibility constraints

\[
\max_{E \in \Omega_{E}} \max_{mc \in \mathbb{R}_+} \Pi(h_{E}(mc)) \\
\text{s.t. } \sum_{i \in I} h_i(mc) \leq Q \\
\sum_{i \in B_{h}(mc)} h_i(mc) + (I - \vert B_{h}(mc) \vert)sQ \geq Q,
\]

where \( \vert B \vert \) is the size of set \( B_h = \{i \in I : h_i < br_i(h)\} \).

Feasibility constraints guarantee that \( h^* \) can be supported by a vector \( q^* \) that satisfies both the aggregate quota constraint \( \sum_i q_i = Q \) and ownership limit constraint, \( 0 \leq q_i \leq sQ \) in the next step.

(4) After \((E^*, mc^*)\) is found by grid search, let \( h^*(s) = h_{E^*}(mc^*) \)

**Step 2: Constructing \( q^*(s) \)**

Now we construct the vector \( q^*(s) \) based on previously constructed \( h^*(s) \) from the following steps.
1. Separate $I$ into two sets $B_{h^*}$ and $I \setminus B_{h^*}$, where $B_{h^*} = \{i \in I : h_i < br_i(h^*)\}$

2. Construct $q$ by choosing components for each set:

   • For $i \in B_{h^*}$, let $q_i^* = h_i^*$;
   
   • For $j \in I \setminus B_{h^*}$, \{q_j^*\} should satisfy:
     
     a) $h_j^* \leq q_j^* \leq sQ$, for $\forall j$; and
     
     b) $\sum_{j \in I \setminus B_{h^*}} q_j^* + \sum_{i \in B_{h^*}} h_i = Q$

By construction, quotas will bind for all players in $B_{h^*}$.

Finally, it can be shown the $h^*$ and $q^*$ obtained from above algorithm satisfy all requirements of the equilibrium.