INTERPRETATION OF ULTRASONIC SCATTERING MEASUREMENTS
BY VARIOUS FLAWS FROM THEORETICAL STUDIES

J. A. Krumhansl
Cornell University
Ithaca, New York

Let me begin by saying that this is part of a program which is very much complementary to and is helped by the experimental program; so I'll repeat, in part, some of the overview comments that Bruce Thompson made this morning.

We've regarded our role as one of trying to plug into the overall program those aspects of the theory of elastic wave scattering which can be developed in a utilitarian way in terms of modern analytical and computational techniques, in a form which I hope can be useful in signal processing, interpretation, design of experiment, etc.

To outline our point of view, I'm going to first give a survey of what we have been doing and then give some of the results. It's rather shocking to think of the amount of money that goes into an experimental program and its interpretation, if the interpretation is done in terms of acoustic or scalar wave scattering theories, when the differences from proper elastic theory can be as great as those which Bruce Thompson pointed to this morning.

Now, there certainly has been a tremendous amount done on elastic wave scattering. There are some methods, however, which have now become practical because of the advances in our understanding of an analytical techniques and computers. The work I'm reporting on has been done primarily by Jim Gubernatis of Los Alamos, Eytan Domany and others at Cornell, and myself. The experimental work we have interacted strongly with is that of Tittmann at Rockwell and Adler of Tennessee. We have received stimulus from the ARPA MRC, Materials Research Council program, and from Tony Mucciardi at Adaptronics, and from the Rockwell program in general.

As I said, our program objective is to be utilitarian, and utilitarian means not necessarily simple.

The diagnostics of ultrasonic flaw detection involve two aspects: where is the defect (the existence of a defect) and what is the defect? Perhaps the real justification of the extent to which we try to carry the theory is in this latter regard, doing everything we can to squeeze out whatever characteristic information we can from ultrasonic scattering data.

Let me briefly outline the theoretical methods. A traditional method with which most of us are familiar for problems of this sort is to use normal mode expansions. Now, in textbooks and teaching, this method is useful, but the geometries chosen are always spheres. Once you go beyond spherical harmonics you find that, if not hopeless, life is at least extremely difficult.

Programming some of the calculations is cumbersome, and non-intuitive. At least some of the training in modern theoretical physics has enriched the bag of tools which can be used, particularly with integral equation methods which have the advantage that one plugs in information which can be related rather directly to the experimental features of the situation in question. They are also subject to approximation techniques such as iteration- try something, improve it ;try it again, improve it - principle. With brute force large scale computers, one can hope to do some of these things inexpensively. Indeed, there are standard approximation methods that we use in nuclear scattering; Born approximation, variational methods, distorted wave methods and so forth, which provide a hunting license of sorts to see what one can do in the present context.

Our first year's program, 1975, was largely concerned with establishing the theoretical base; the Cornell Materials Science Center report number 2654 presents a summary of the general theory, and provides detailed computer results for the Born approximation. The Born approximation simply inserts incident field displacement and strain at appropriate points in the integral formula as needed. This really doesn't solve the integral equation, but it's a first step, and has been very useful as an exploratory device.

As the program has progressed we have tried to express the results of such a calculation concisely, and Jim Gubernatis has found that it's particularly useful to define something called an "f-vector", which will give complete scattering data. It depends directly on the changes in material parameters, $\Delta\rho$, and directly on the changes in the elastic constant, Δc. In addition, we require a knowledge of the displacement and strain fields in the flaw region . .

We are now in the process of using this approach to examine carefully exact limiting behavior for low frequency. By utilizing all the information present in the longitudinal, transverse, and mode converted scattering, material parameter changes can be determined--over and above any imaging or echo detection. Hopefully, microprocessors can eventually make the on-line analysis cheap and easy.

Now, I'd like to run through the main results we have obtained at Cornell during the past year. My colleagues have been Eytan Domany, Paul Muzikar, Steve Teitel, Dave Wood, and Jim Gubernatis, whose support came from Los Alamos.

1. General Summary of 1975-76 Research

Last year we presented an integral equation formulation of the ultrasonic scattering problem. We also compared the results of the Born approximation with exact results for spherical scatterers. From that study we learned much about the regions of applicability and validity of the Born approximation.

In this year's research we have continued the theoretical work[1], but placed special emphasis on attempts to make contact with the experimental situation for laboratory flaws prepared at Rockwell in determined geometries - (a step closer to the "real world"). In particular, we tried to identify some general features or indices that might be useful in evaluation of scattering data for NDT.

In the course of many fruitful interactions with the experimental groups at Rockwell and Tennessee we learned about the needs and the formats most convenient for them, and developed a library of computer programs for various scattering situations.

We have also investigated further approximations (static, quasi-static), which may be better in certain limits, and started to compute and compare them with exact and experimental data.

Finally, we have begun work to implement scattering theory for defects other than holes or inclusions, particularly scattering by flat cracks. We proceed with details of the studies.

II. Indices for NDT

We have considered scattering by defects of two shapes, spheroids and cylinders, to seek information that indicates the deviation of the scatterer from spherical symmetry. The geometry of both scatterers can be characterized by a ratio b/a (see Fig. 1).

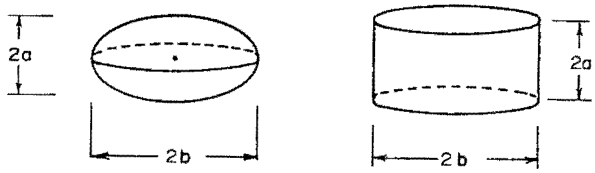


Figure 1. Cylindrical and spheroidal scatterers, characterized by b/a ratio.

Since we now know that the Born approximation (BA) is good for Al in Ti, we looked at Al inclusions in Ti. A sample of the numerous figures we generated is given in Fig. 2; in all these the incident wave is longitudinal, and along the axis of symmetry. After generating many of these, and recognizing that this abundance of information was conceptually unwieldy we looked for some features that seem general and physically plausible.

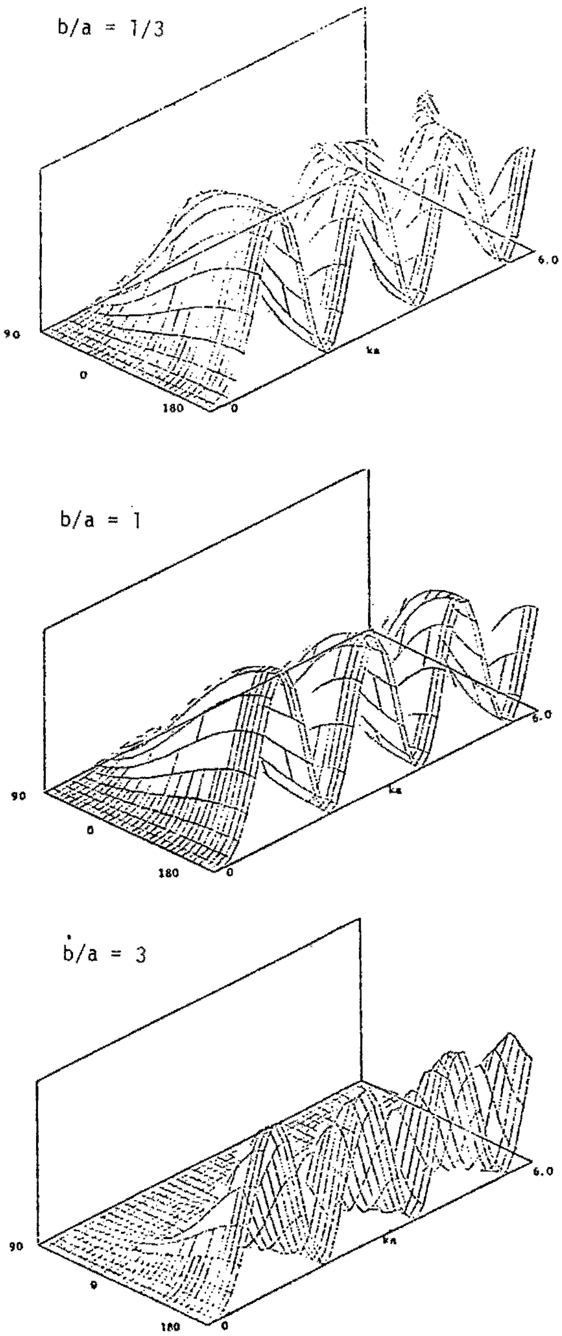


Figure 2a. Scattered (longitudinal) power for longitudinal wave incident along the symmetry axis of spheroidal scatterers (Al in Ti) of varying b/a ratio.

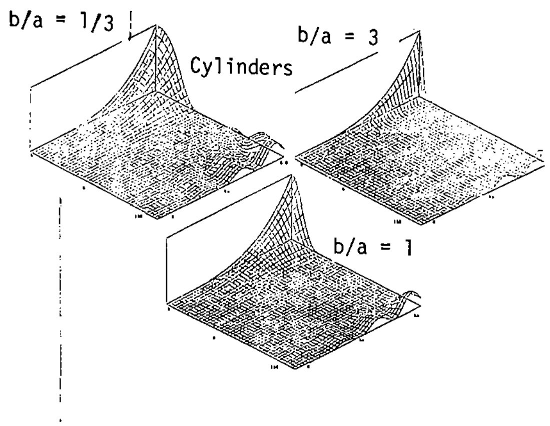


Figure 2b.. Scattered (longitudinal) power for longitudinal wave incident along the symmetry axis of cylindrical scatterers (Al in Ti) of varying b/a ratio.

II. 1) <u>Cylinder vs Spheroid.</u> The <u>backscattered</u> power vs k is shown for each in Fig. 3. These results still have to be checked experimentally.

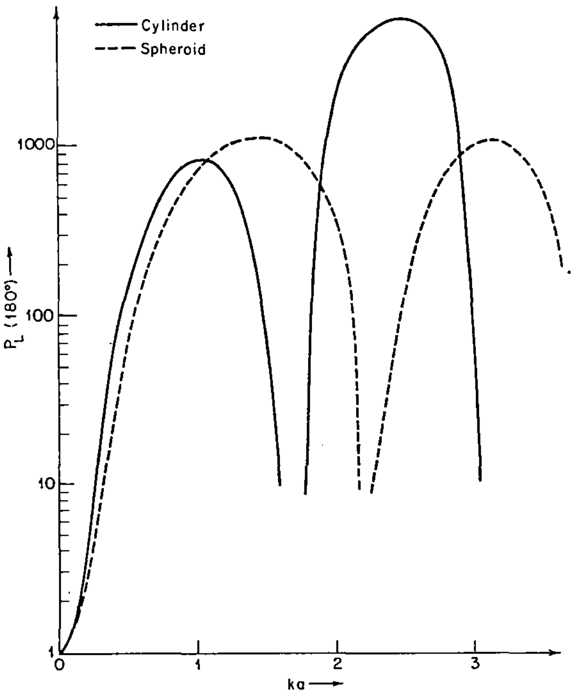


Figure 3. Back scattered longitudinal power, for longitudinal wave incident along axis of symmetry of cylindrical and spheroidal cavity.

II.2) <u>The b/a Ratio</u>. We found that on <u>the average</u> (taken over a range of k) the ratio of back to 90° scattering depends strongly on b/a, both for spheroids <u>and</u> cylinders. Not only is the averaging <u>usually</u> done experimentally in the transducer and circuitry, but also the averaging tends to wash out "accidental" computational resonances that obscure the detailed pictures. Thus, we studied the (k-averaged) back scattered power, P(180), to 90°, P(90) for Al in Ti- where BA is expected to work; and then "conjectured" it for cavities too (BA is not too bad for low ka$\sim$1 and 90 < θ < 180°). Results are shown in Fig. 4 for spheroids; we get similar results for cylinders. Thus, we conclude that given the orientation of a spheroidal/cylindrical scatterer, with longitudinal waves incident along the axis of symmetry, one can hope to determine the b/a ratio from a relatively simple measurement; the more oblate the object, the larger the ratio of averaged back/90° scattering.

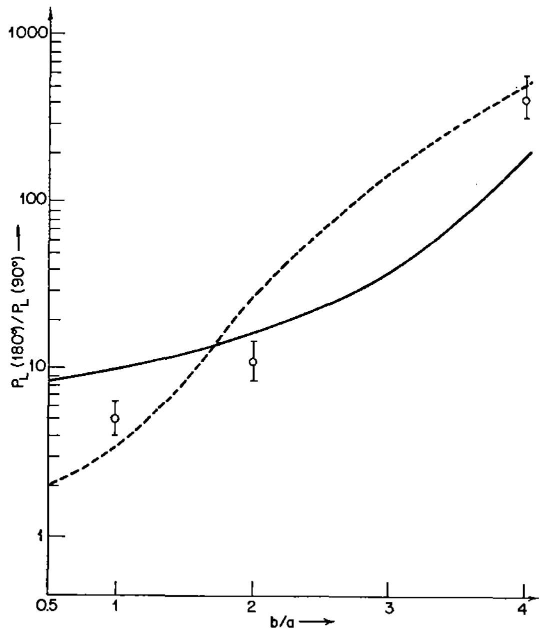


Figure 4. The ratio p (180/p (90) for longitudinal power with longitudinal wave incident along symmetry axis of spheroidal cavities in Ti, vs b/a of the scatterer. Uniform averages of 0 < ka < 1 (full lines) and 0 < ka < 2 (broken lines) were used. The circles are experimental results.

It should be emphasized that our analytical "result" is <u>not</u> intended to serve as a basis for <u>quantitative</u> comparison with experiment, but rather as a general, simple, qualitative feature which should be considered experimentally, particularly as a training criterion for computer-adaptive flaw identification procedures.

A similar investigation of similar transverse waves from longitudinal incident waves and for an incident transverse wave is planned. The experimental situation here has been looked at by Adler.

Mr. CRAIG BIDDLE (Pratt/Whitney): Is that against the side of the cylinder or against the end of the cylinder?

DR. KRUMHANSL: It's against the end of the cylinder along the axis--incident along the axis of the cylinder.

MR. BIDDLE: Flat bottom hole?

DR. KRUMHANSL: Right!

II. 3) Non-normal Incidence on Spheroid. As part of characterizing a spheroidal defect, one might want to determine its orientation. To this end, we considered the case of a longitudinal wave incident at an angle α to the axis of symmetry. See Fig. 5.

We first looked at the (experimentally) simplest situation; that of back scattering (single transducer experiment). Our results are summarized in Fig. 6.

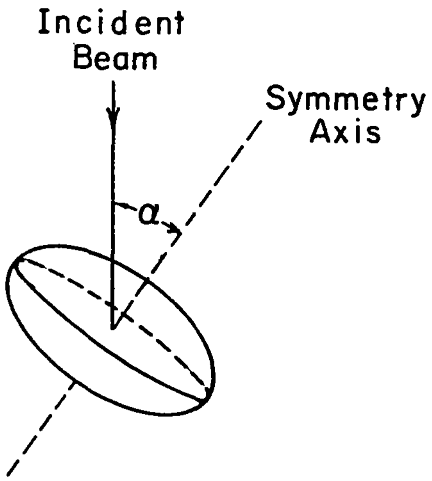
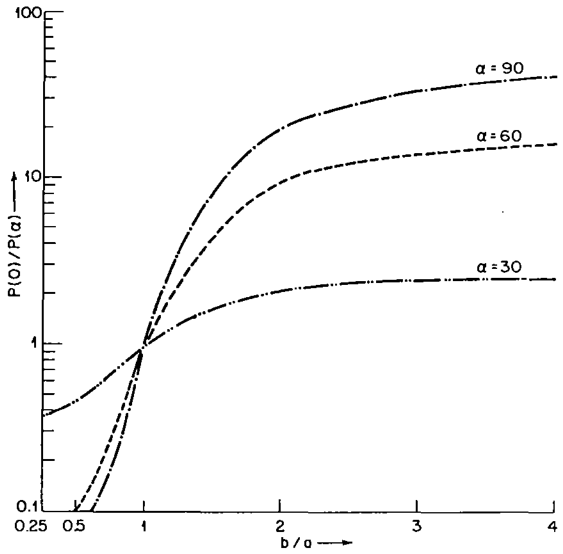
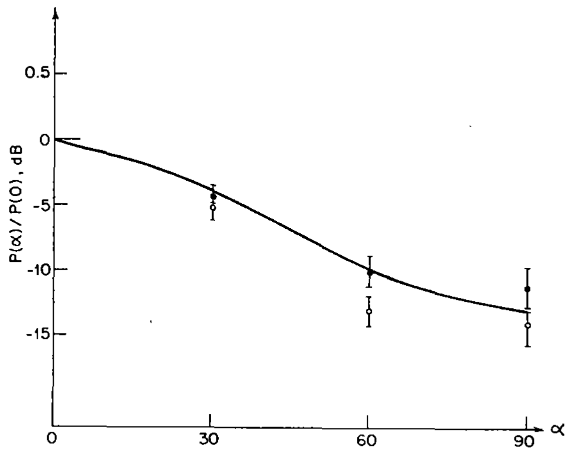


Figure 5. Scattering situation with incident beam at angle α off symmetry axis of scatterer.



(a)



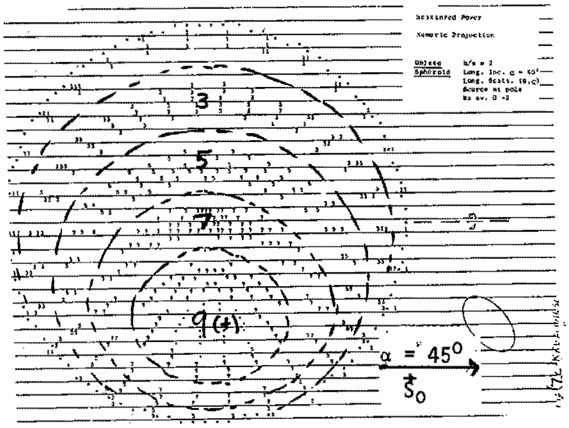
(b)

Figure 6. (a) Ratio of backscattered powers, p(0)/p(α) of longitudinal wave incident along axis of symmetry (α = 0) and off axis by angle α; Rockwell transducer characteristics were used for the frequency averaging. (Spheroidal cavity in Ti).

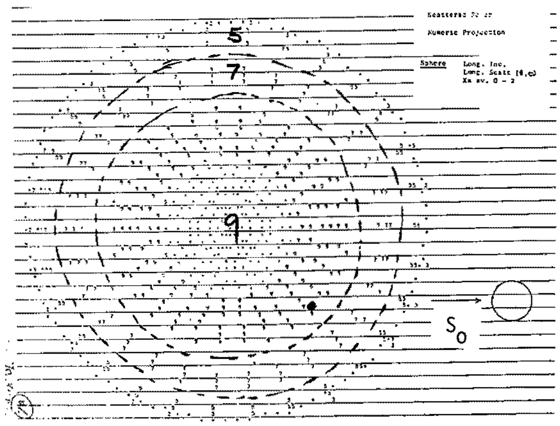
(b) Same for spheroidal cavity of b/a= 400 μ/200μ as a function of off symmetry angle α.

The agreement (for the oblate spheroid) with experiment is surprising. In any case, it seems that again we can say that the more oblate the scatterer, the higher the ratio of back scattering powers for normal incidence/incidence at α = o.

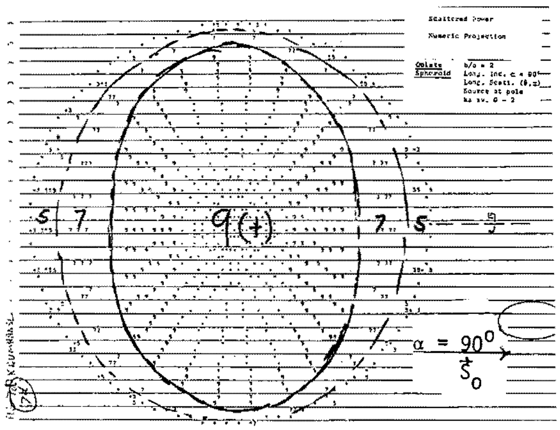
II.4) A Display Format. Using the computed data from these programs we have followed D. Thompson's suggestion for an efficient visual presentation of our results. Imagine an array of transducers on a hemisphere (generalization to plane is straight forward). The one located on the pole sends in a longitudinal signal; the scattered longitudinal power is now recorded by all the array, and the relative (to the maximum) power displayed by each transducer. The power ratio, for each receiving position, is then plotted by mercator projection. Some samples of pictures one gets in various scattering situations are shown in Figs. 7. The development of asymmetrical, as well as numerical, features with orientation changes is to be noted.



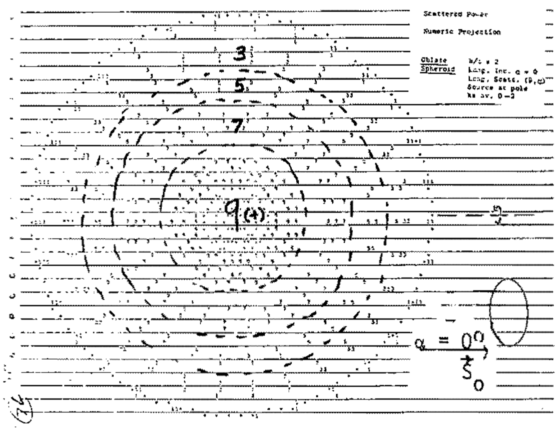
7(c) Oblate Spheroid



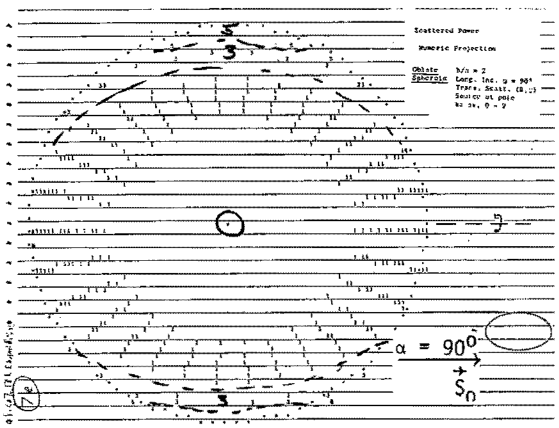
7(a) Sphere



7(d) Oblate Spheroid



7(b) Oblate Spheroid



7(e) Oblate Spheroid

Figure 7. Projection display of scattering in various directions.

III. Static and Quasi-Static Approximations

The integral equation for the scattered field by an elastic inclusion is

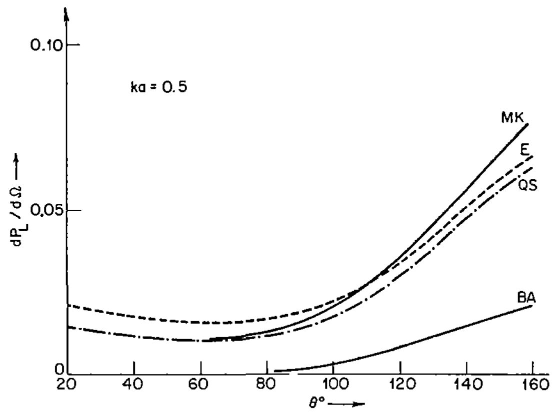
$$u_i^s(r) = u_i^o(\underline{r}) + \delta\rho\omega^2 \int_V d\underline{r}'\, g_{im}(\underline{r}-\underline{r}')u_m(\underline{r}') + \delta C_{ijkl} \int_V d\underline{r}' g_{ij,k}(\underline{r}-\underline{r}')u_{l,m'}(r') \quad (1)$$

In the far field limit[1] this expression has been reduced to a simple form; (with the explicit subsituation of the Greens function) u_i^s is determined by volume integrals of the displacement field and the strain field in the volume of the defect. The Born approximation consists of replacing these fields by the respective incident field. In a static approximation proposed by Mal and Knopoff[2] the displacement field is approximated by the incident displacement field at the center of the defect. As to the strain field, one considers the solution of a static problem, with uniform stress at infinity equal to the incident stress, i.e.:

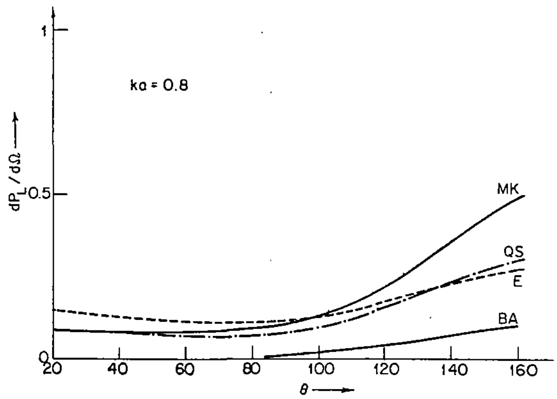
$$u_{m,n}(\phi) = i\left[k_n^o u_m^o + k_m^o u_n^o\right]$$

The solution of the static problem, (known for ellipsoids)[3] is then used in the integral. This static (or MK) approximation is exact in the long wavelength limit.

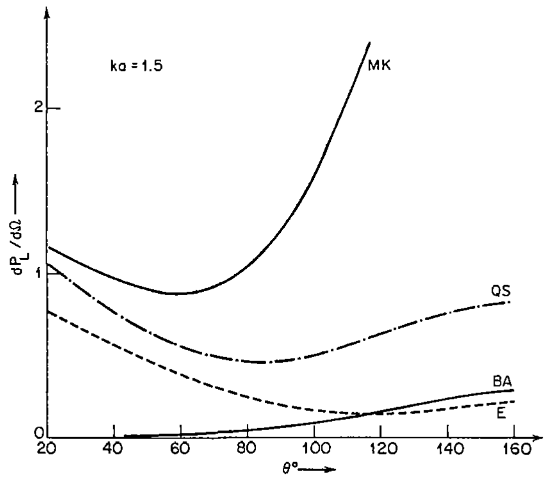
The quasi-static approximation, proposed by one of us, consists of allowing for spatial variation of the various fields inside the scatterer. Some of the results of these approximations for scattering by a spherical cavity are compared with the exact results in Fig. 8 (a,b,c,d.)



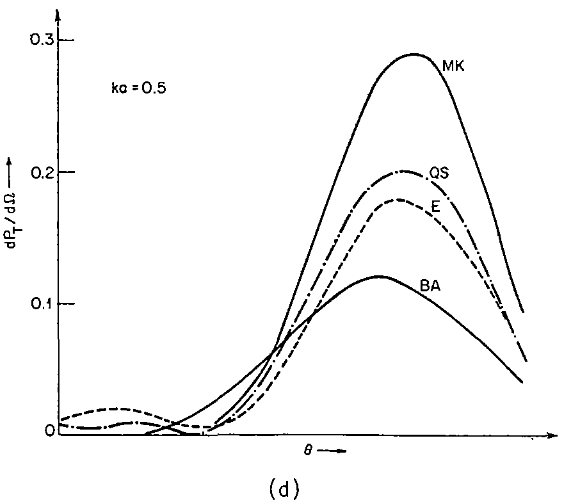
(a)



(b)



(c)



(d)

Figure 8. Comparison of exact (E), BA, Mal-Knopoff (MK) and quasi-static (QS) approximations for scattering of incident longitudinal wave by spherical cavity in Ti for various values of (ka).

The quasi-static approximation for scattering by a sphere, with some ad-hoc assumption about the variation inside the scatterer, yields an expression for u_i^s which is identical to one derived independently by E. R. Cohen[4]. However, our expressions can be easily generalized for scattering by spheroids and ellipsoids; those results will be presented elsewhere.

IV. Scattering of a Longitudinal Wave by a Stress-free Circular Crack.

While the volume integral equation turned out to be most useful in generating approximate solutions to scattering by volume defects, for "flat" cracks surface integral representations are more natural. An extensive survey of background has been given by Kraut[5].

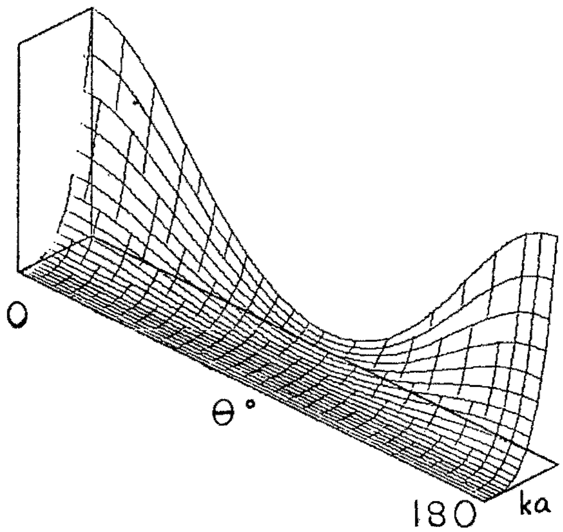
We have considered various known approximations, and also constructed a new approximation, expected to be good in the long wavelength limit.

In all these approximations the scattered field at point $\underline{r}$ is represented in terms of an integral over a surface Σ of the displacement and stress fields.

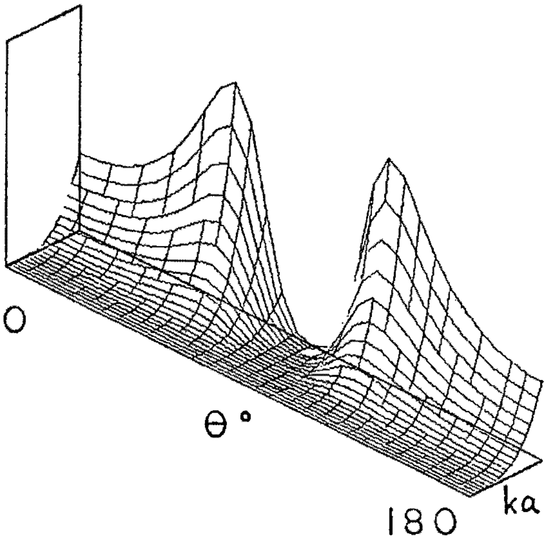
In Fig. 9 we display the results of scattering by a circular crack, using various approximations. Our approximations are inserted in the formula[5]

$$u_m^s(\underline{r}) = C_{ijkl} \int_S ds\, n_j \left\{ g_{im} \left[u_{k,l}^s \right]_-^+ - g_{km,l} \left[u_i^s \right]_-^+ \right. \qquad (2)$$

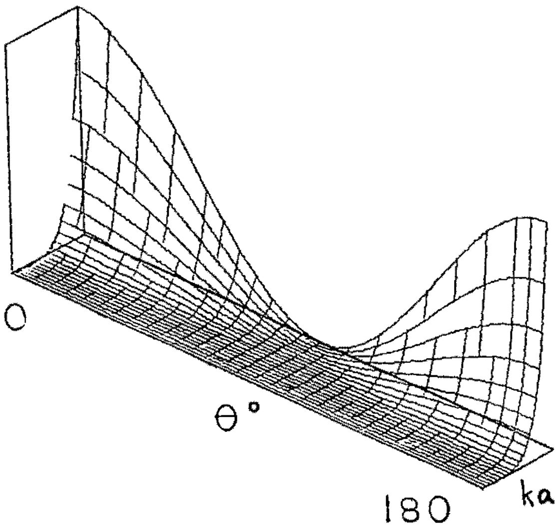
where S is the surface of the crack, u_i^s is the scattered displacement field, and $[\;]_-^+$ is the jump in the appropriate quantity. For a stress-free crack only the jump in the displacement field contributes to Eqn. (2). In additon to the simple Kirchoff choices, as an approximant to $[u_i^s]_-^+$ we used the solution to the problem of a circular crack under static uniform stress[6]. This approximation is "quasi-static" and is expected to be good at low k, i.e. long wavelength.



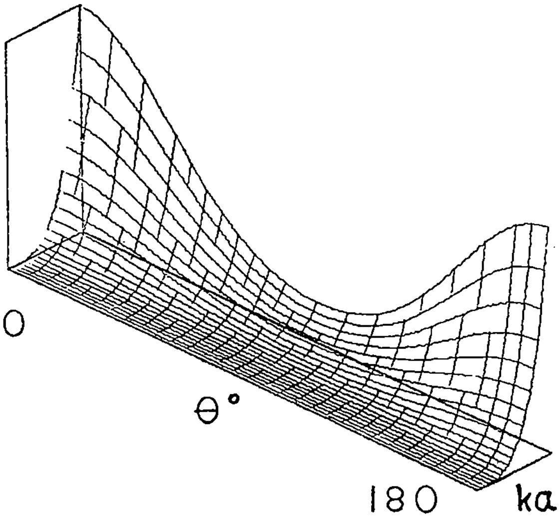
9(a)



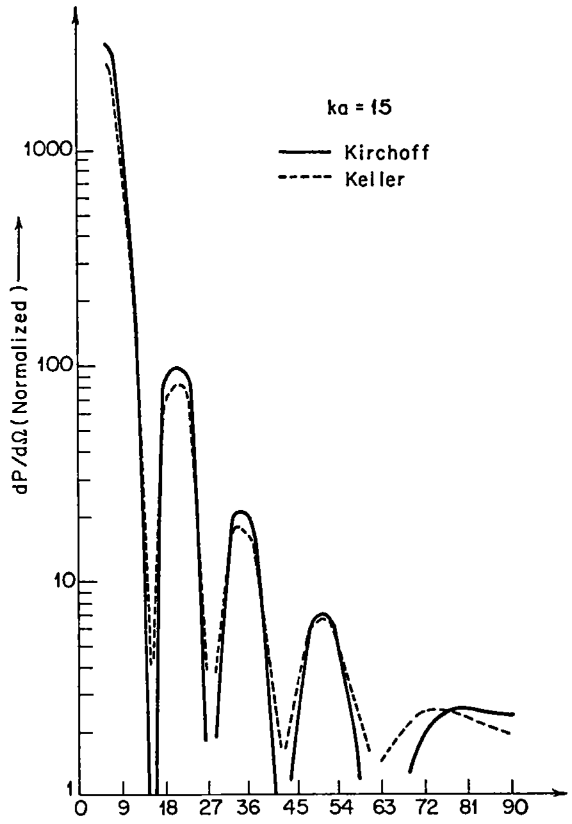
9(b)



9(c)



9(d)



9(e)

Figure 9. Scattered longitudinal power by stress free circular crack of radius a; incident longitudinal wave along axis of crack.

a) Kirkhoff condition on displacement jump.
b) Half-plane Greens function.
c) Comparison with Filipczynski (see Review by E. Kraut[5]).
d) Quasi-static
e) Kirkhoff vs. Keller for ka = 15.

The first programming of these studies has just been carried out; we expect to devote considerable attention to cracks, corners, etc. during the coming year.

Let me summarize, when your data does not allow you to direct imaging, or when you're in a region where there is a limitation on wavelength for example, in a lossy medium whose attenuation increases rapidly with frequency, then perhaps all of this rich additional detail can be used to squeeze estimates out where you're at the limit of other methods.

Thank you.

References

1. MSC Report).

J. E. Gubernatis, E. Domany, J. A. Krumhansl, and M. Huberman, Report #2654, Materials Science Center, Cornell University 1976.

2. A. K. Mal and L. Knopoff, J. Inst. Maths. Applis, 3, 376-387.

3. J. D. Eshelby, Proc. Roy. Soc. A241, 376 (1957).

4. E. R. Cohen, May 1976, Rockwell Report SC579.31R on Contract F 44620-74-C-0057.

5. E. A. Kraut, IEEE Transactions, SU-23, 162 (1976).

6. I. N. Sneddon, "Fourier Transforms," p. 490, McGraw Hill, 1951.

DISCUSSION

DR. EMMANUEL PAPADAKIS (Ford Motor Company): Good. We have three quarters of a minute. Let's have a question.

DR. LASZLO ADLER (University of Tennessee): Is it safe to say that for the back scattering region the Born approximation behavior is about the same as the exact calculations?

DR. KRUMHANSL: The Born approximation is really quite usable for the backward scattering in almost all cases, including cavities which present a complete discontinuity in elastic properties. The Born approximation is often remarkably good for all scattering angles for moderate material changes; e.g. computations cost about $5 or so, a really ridiculously cheap kind of thing compared to the cost of an experiment.

DR. PAPADAKIS: Just one, yes.

DR. GORDON KINO (Stanford University): If you now go through the electrostatic approximation which modifies the Born approximation, how does that compare for the sphere with the exact theory? Where does it begin to drop?

DR. KRUMHANSL: Well, we had one of those plots, Gordon, at ka equals unity. The "static" in MK could be called S; Mal and Knopoff where the first to use it, in a geophysics context. Our calculation gives an indication of how badly off it is for large scattering angles. The Born is quite wrong at small scattering angles, yes. That's the reason for exploring these other approximations systematically.

DR. PAPADAKIS: Thank you very much.