IMPULSE-RESPONSE METHOD TO PREDICT ECHO-RESPONSES FROM DEFECTS IN SOLIDS : A FIRST APPROACH

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INTRODUCTION

The ultrasonic pulse-echo method aims to use the measured echo-signals to characterize defects — their location, orientation and size. The solution of this inverse problem requires the availability of good models of the forward problem, that is an understanding of radiation and reception of ultrasonic waves by realistic transducers as well as of the processes of scattering by arbitrary defects. This in itself is very complicated and requires simplifications and approximations. This paper aims to extend a previous model dealing with the case of propagation in fluids [1,2]. Modeling the actual propagation in solids as a propagation in fluids is a simplification commonly made in the considerable literature. A solution for solids requires account to be taken of both compression and shear waves. Since the two types of wave propagate with different velocities, a solution for fluids is acceptable if the defect is sufficiently distant from the transducer, that is, if the arrival-times of the respective echoes are well separated in time. When this condition is not fulfilled, compression and shear waves as well as their mode-conversion at interfaces must be considered. The earlier solution for fluids has shown the paramount importance of transducer diffraction effects in the echo-forming mechanism. The solution explicitly used the impulse-response theory for calculating transducer diffraction effects both in radiation and reception. The first step to extend the work to solids was to dispose of a solution for the transient radiation in a solid medium. Such a solution has been developed. It is based on the approximation that Rayleigh and head waves contributions generated by the transducer are negligible at field-points of interest [3,4], i.e., the solution is suitable at field-points not too close to the interface where the transducer radiates. In using this solution for modeling the behavior of the transducer both in radiation and reception, the complete solution for radiation, scattering and reception will not apply for modeling echo-responses from defects close to or at the interface, e.g., surface breaking cracks. Despite this fundamental limitation, number of practical cases of pulse-echo method may be modeled.

Under basically the same assumptions, Weight [5] has developed a simple model for the case of a point-defect. The scattering by the defect is treated by an important approximation that cannot apply for defects of finite surface and complex geometry. At first, the solution for transient radiation in solids is recalled. Then, the model for transient radiation, scattering by a defect and reception is proposed with emphasis on the assumptions and approximations made in the derivation. The model is derived in a general form. Finally, we consider the

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simple case of a flat-bottom hole interrogated by a conventional disk transducer working in the thickness-mode. In this case, analytic evaluation of the general formulation is derived.

TRANSIENT RADIATION IN SOLIDS [3, 4]

The recently developed approximate model predicts the transient displacement field radiated by an arbitrary source of traction $\mathbf{T}(\mathbf{r}_R,t)$ directly applied on the plane free surface of an elastic half-space, $\mathbf{r}_R$ being a running point of the radiating surface R. It was derived under the approximation that wavefield coupling at the plane interface of the elastic half-space makes a negligible contribution at field-points not too close to the interface. If space and time variables are separable in the source term (piston source), i.e., if $\mathbf{T}(\mathbf{r}_R,t)$ can be expressed as $\mathbf{T}(\mathbf{r}_R,t) = \Gamma(\mathbf{r}_R)\ T(t)$, the i-th component of the displacement at a point $\mathbf{r}$ at time t can be approximated by a sum of convolution integrals of displacement impulse-responses with $T(t)$.

$$u_i(\mathbf{r},t) = T(t) * h_i(\mathbf{r},t) = T(t) * [h_i^L(\mathbf{r},t) + h_i^T(\mathbf{r},t) + h_i^{T->L}(\mathbf{r},t)], \qquad (1)$$

with,

$$h_i^L(\mathbf{r},t) = \iint_R \frac{\gamma_i \gamma_j}{\lambda + 2\mu}\,\Gamma_j(\mathbf{r}_R)\,\frac{\delta(t - |\mathbf{r} - \mathbf{r}_R| / c_L)}{2\pi |\mathbf{r} - \mathbf{r}_R|}\,dS_R, \qquad (2)$$

$$h_i^T(\mathbf{r},t) = -\iint_R \frac{\gamma_i \gamma_j - \delta_{ij}}{\mu}\,\Gamma_j(\mathbf{r}_R)\,\frac{\delta(t - |\mathbf{r} - \mathbf{r}_R| / c_T)}{2\pi |\mathbf{r} - \mathbf{r}_R|}\,dS_R, \qquad (3)$$

$$h_i^{T->L}(\mathbf{r},t) = \int_{c_T}^{c_L} \iint_R \frac{3\,\gamma_i \gamma_j - \delta_{ij}}{\rho_0\, c^2}\,\Gamma_j(\mathbf{r}_R)\,\frac{\delta(t - |\mathbf{r} - \mathbf{r}_R| / c)}{2\pi |\mathbf{r} - \mathbf{r}_R|}\,dS_R\,\frac{dc}{c}, \qquad (4)$$

where $\gamma_i = (r_i - r_{R_i}) / |\mathbf{r} - \mathbf{r}_R|$, δ_{ij} is Kronecker's delta, ρ_0 denotes the density of the propagation medium and λ and μ (Lamé's coefficients) its elastic constants. c_L and c_T are the compression and shear wave velocities, given by $c_L = [(\lambda+2\mu)/\rho_0]^{1/2}$ and $c_T = (\mu/\rho_0)^{1/2}$.

Throughout, the summation convention for repeated subscripts is followed. For sources of simple symmetry, analytic expressions can be derived using the same approach as that developed by Stepanishen [6] for calculating the Rayleigh integral. For a uniform disk thickness-mode transducer, analytic expressions of impulse-responses are given in [4] — see also [7] in this volume. For a uniform Y-cut crystal, the analytic solution is given in [4].

TRANSIENT SCATTERING BY A DEFECT MODELED AS A FREE SURFACE

We want now to study the scattering by a defect inside the solid half-space. For this, we follow step by step the derivation proposed for the same problem in the case of acoustic waves [1]. However, we limit the present study to the cases of defects that can be modeled as free surfaces (an opened crack, a void) at the surface of which traction vanishes, and to transducers working in the transmit-receive mode of active surface R directly coupled to the plane interface Π of the half-space V. We assume that the boundary ∂V of V can be decomposed into different parts (as shown in Fig. 1), $\partial V = R + [\Pi - R] + D + D' + \Sigma + \Psi$, D being the area of the defect seen from the transducer assumed to be a free surface, D' the shaded area of the defect, Σ and Ψ being arbitrary surfaces closing the volume V. Σ, is projected to infinity where elastodynamic Sommerfeld's condition is chosen [8]. D' and Ψ are assumed to be perfectly absorbent boundaries. Furthermore, we assume that the reception of the echoes from the defect does not start before the whole radiation process as seen from the

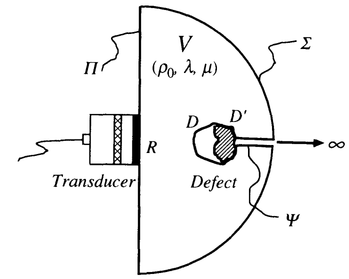

Fig. 1. Decomposition of the boundary ∂V of the volume of propagation V.

defect is completed. Taking account of the above hypotheses, the general integral formulation — see [9] for example — for the displacement at a point in V or on D simplifies into an implicit integral,

$$\alpha(\mathbf{r})u_n(\mathbf{r},t) = u_n^{inc}(\mathbf{r},t) - \int_{-\infty}^{+\infty} d\tau \iint_D u_i(\mathbf{r}_D,\tau)c_{ijkl}(\mathbf{r}_D)n_j G_{kn,l}(\mathbf{r}_D,t-\tau;\mathbf{r},0)dS(\mathbf{r}_D)\,, \tag{5}$$

where

$$\begin{aligned} G_{ij,k}(\xi,\tau;\mathbf{r},t) = {} & \frac{15\,\gamma_i\gamma_j\gamma_k - 3\gamma_i\delta_{jk} - 3\gamma_j\delta_{ik} - 3\gamma_k\delta_{ij}}{4\pi\rho_0}\frac{1}{r^4}\int_{r/c_L}^{r/c_T} \tau'\delta(t-\tau-\tau')\,d\tau' \\ & + \frac{6\,\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - \gamma_k\delta_{ij}}{\rho_0 c_L^2}\frac{\delta(t-\tau-r/c_L)}{4\pi r^2} - \frac{6\,\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - 2\gamma_k\delta_{ij}}{\rho_0 c_T^2}\frac{\delta(t-\tau-r/c_T)}{4\pi r^2} \\ & + \frac{\gamma_i\gamma_j\gamma_k}{\rho_0 c_L^3}\frac{\dot{\delta}(t-\tau-r/c_L)}{4\pi r} - \frac{\gamma_i\gamma_j\gamma_k - \gamma_k\delta_{ij}}{\rho_0 c_T^3}\frac{\dot{\delta}(t-\tau-r/c_T)}{4\pi r}\,. \end{aligned} \tag{6}$$

and where $\alpha(\mathbf{r}) = 1, 1/2$ for a field-point $\mathbf{r}$ inside V or on a smooth part of ∂V, respectively. $\dot{\delta}(t)$ denotes the time-derivative of a δ-function. $\mathbf{u}^{inc}(\mathbf{r},t)$ is the incident displacement at $\mathbf{r}$.

The displacement field on D is composed of the incident field radiated by the transducer and a term given by an implicit integral. The latter term expresses the wave-field coupling on D (which results of secondary diffraction phenomena as well as mode-conversion of bulk waves into surface waves propagating along D). Our approximation consists in neglecting these second-order phenomena so that the displacement field on D is simply given by

$$u_i(\mathbf{r}_D,t) = 1/\alpha(\mathbf{r}_D)u_i^{inc}(\mathbf{r}_D,t). \tag{7}$$

If the defect is locally smooth at $\mathbf{r}_D$, the displacement on D is twice the incident displacement. Our last assumption is to neglect again the wavefield coupling at the plane interface Π of the half-space to model the reception of the transducer. The transducer is sensitive to the instantaneous total velocity (for example, a thickness-mode transducer is sensitive to the normal component of the velocity integrated over the whole active surface, a Y-cut crystal is sensitive to one component of the velocity parallel to its active surface), and is not sensitive to its own incident field. Introducing Eq. (7) into Eq. (5), the last approximation leads to the following expression for the n-th component of the total velocity received by the transducer,

$$<v_n>(t) = -\iint_R \frac{\partial}{\partial t}\int_{-\infty}^{+\infty} d\tau \iint_D 2u_i^{inc}(\mathbf{r}_D,\tau)c_{ijkl}(\mathbf{r}_D)n_j G_{kn,l}(\mathbf{r}_D,t-\tau;\mathbf{r}_R,0)dS_D\, dS_R\,. \quad (8)$$

The integral formulation is mathematically explicit. It is worth noticing that mode-conversions of compression waves into shear waves (and vice-versa) occurring at the defect surface are taken into account in this formulation. They are expressed by the various sums over the subscripts i, j, k and l. The time integration being a simple convolution and after inversion in the order of integrals, Eq. (8) becomes

$$<v_n>(t) = -\frac{\partial}{\partial t}\iint_D \left[2\, c_{ijkl}(\mathbf{r}_D)n_j\, u_i^{inc}(\mathbf{r}_D,t) * \iint_R G_{kn,l}(\mathbf{r}_D,t\,;\,\mathbf{r}_R,0)dS_R\right] dS_D\,. \quad (9)$$

Introducing Eq. (1) in this expression, we can define an impulse-response $H_n^{TSR}(t)$ for the whole problem of transmission, scattering by a defect and reception so that

$$<v_n>(t) = T(t) * H_n^{TSR}(t), \quad (10)$$

with

$$H_n^{TSR}(t) = -\frac{\partial}{\partial t}\iint_D \left[2\, c_{ijkl}(\mathbf{r}_D)n_j\, h_i(\mathbf{r}_D,t) * \iint_R G_{kn,l}(\mathbf{r}_D,t\,;\,\mathbf{r}_R,0)dS_R\right] dS_D\,. \quad (11)$$

We want now to calculate this expression. The integration over R that signifies the reception can be calculated analytically in certain circumstances, namely, for the same transducers for which an analytic expression has been derived for the transient radiation [3,4].

TRANSIENT RECEPTION BY A TRANSDUCER COUPLED TO A SOLID

The problem is now to calculate integrals of the form

$$H_{kn,l}(\mathbf{r}_D,t) = 2\iint_R G_{kn,l}(\mathbf{r}_D,t\,;\,\mathbf{r}_R,0)dS_R\,, \quad (12)$$

$G_{kn,l}$ being given by Eq. (6). The subscript n depends on the kind of transducer used. For a transducer working in the thickness-mode (compressional wave transducer), we have $n = z$. For a shear wave transducer polarized in the direction x, we have $n = x$. Subsequently, we limit the study to the former case assuming a disc-shaped source of radius a. We also limit the study to the case of a normally aligned flat-bottom hole. Therefore, some of the nine integrals defined by Eq. (12) (n being fixed) vanish because the fourth-order tensor c_{ijkl} takes non-zero values only for certain combinations of i, j, k, l, due to the symmetries of the isotropic propagation medium expressed by,

$$c_{ijkl} = \lambda\, \delta_{ij}\, \delta_{kl} + \mu\, (\delta_{ik}\, \delta_{jl} + \delta_{il}\, \delta_{jk}), \quad (13)$$

and since $j = z$ and $i = r$ or z. Note that these limitations are not necessary conditions. Whatever the transducer used and the shape, orientation and size of the scatterer, the calculation of the general integral formulation given by Eq. (11) requires the calculation of integrals of the type of Eq. (12). As for the calculation of the radiation integrals [Eqs. (2-4)], we derive the solution for the transient reception of elastic waves using the same approach as that developed by Stepanishen [6] for calculating the radiation by a transducer in a fluid medium.

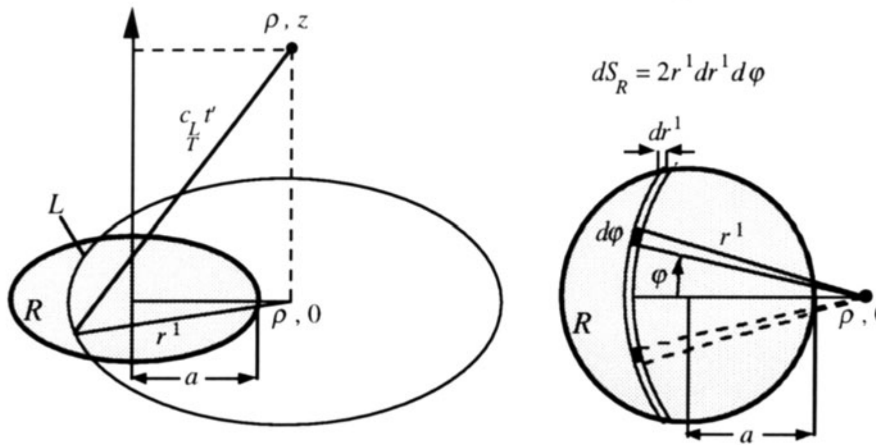

Fig. 2 Definition of the change of variables used in the derivation.

If one observes the waves arising from a point $\mathbf{r}_D$ (ρ,z) of the defect, at an instant t', the parts of the transducer surface receiving the z-component of the elastic particle velocity is the intersection of the spherical surface of radius ct' centered at the point $\mathbf{r}_D$, with the transducer surface R. For the second and fourth terms of Eq. (6), $c = c_L$. For the third and fifth terms, $c = c_T$. For the first term, c varies from c_T to c_L. The transducer being axisymmetric, we can derive the solution in the cylindrical co-ordinate system and consider only the radial and axial co-ordinates. Since $j = z$, integrals to be calculated are $G_{rz,z}$, $G_{zz,r}$, $G_{rz,r}$ and $G_{zz,z}$. Consider now the following change of variable (at a given time t') where φ is the angle describing half of the arc $L(\mathbf{r}_D, t')$ (taking account of the symmetry of the source) [see Fig.2].

$$ct' \equiv r = |\mathbf{r}_D - \mathbf{r}_R|,\ r^1 = \sqrt{c^2 t'^2 - z^2},\ \text{thus,}\ dS_R = 2r^1\, dr^1\, d\varphi = 2r\, c\, dt'\, d\varphi\ . \qquad (14)$$

Under this change of variables, the integration over φ is limited to $[0, \varphi_{lim}]$ and Eq. (12) is proportional to terms of the form

$$H_{kn,l}(\rho,z,t) \propto \iint_R f(t')g(\varphi)\, \frac{\delta\,(t-t')}{4\pi r}\, dS_R = \int_{t_{min}}^{t_{max}} \int_0^{\varphi_{lim}} f(t')g(\varphi)\, \frac{c\ \delta\,(t-t')}{2\pi}\, d\varphi\, dt'\ . \qquad (15)$$

The simple geometry of the transducer allows φ_{lim} to be calculated analytically. We make explicit use of the solution derived by Stepanishen and the value of φ_{lim} is tabulated in Table I. The direction cosines γ_i are functions of the new integration variables t' and φ,

$$\gamma_r = (c^2 t'^2 - z^2)^{1/2} \cos\varphi\ /\ ct' \ \text{and}\ \gamma_z = z\ /\ ct'\ . \qquad (16)$$

Table I Value of φ_{lim} as a function of the field-point position (ρ,z) and of ct.

φ_{lim}	$\rho = 0$	$0 < \rho < a$	$\rho = a$	$\rho > a$
$t < t_0$ or $t > t_2$	0	0	0	0
$t_0 \le t \le t_1$	π	π	$\pi/2\ (t_0{=}t_1)$	0
$t_1 \le t \le t_2$	0	$\cos^{-1}\left[\dfrac{c^2t^2-z^2+\rho^2-a^2}{2\rho\sqrt{c^2t^2-z^2}}\right]$	$\cos^{-1}\left[\dfrac{\sqrt{c^2t^2-z^2}}{2a}\right]$	$\cos^{-1}\left[\dfrac{c^2t^2-z^2+\rho^2-a^2}{2\rho\sqrt{c^2t^2-z^2}}\right]$

with $t_0\,(c) = z\ /\ c,\ t_1\,(c) = \sqrt{z^2 + (a-\rho)^2}\ /\ c,\ t_2\,(c) = \sqrt{z^2 + (a+\rho)^2}\ /\ c$.

Due to the presence of a δ-function, the time integration (a time-convolution) is trivial.

$$H_{kn,l}(\rho,z,t) \propto \int_{t_{min}}^{t_{max}} \int_0^{\varphi_{lim}} f(t')g(\varphi) \frac{c\,\delta(t-t')}{\pi}\, d\varphi\, dt' = \frac{c f(t)}{\pi} \int_0^{\varphi_{lim}} g(\varphi)\, d\varphi. \qquad (17)$$

The first term of Eq. (6) can be re-written in the form of an integral over wave speed as

$$\int_{c_T}^{c_L} \frac{15\,\gamma_i\gamma_j\gamma_k - 3\gamma_i\delta_{jk} - 3\gamma_j\delta_{ik} - 3\gamma_k\delta_{ij}}{\rho_0\, c^2} \frac{\delta(t - r/c)}{4\pi r^2} \frac{dc}{c}\,. \qquad (18)$$

Terms to be integrated depend on φ through the various products of direction cosines. There are terms proportional to γ_z, γ_r, $\gamma_r^2\gamma_z$, $\gamma_r\gamma_z^2$ or to γ_z^3. This makes necessary to calculate two intermediate integrals over φ which are trivial. We have,

$$\int_0^{\varphi_{lim}} \cos\varphi\, d\varphi = \sin\varphi_{lim} \quad \text{and} \quad \int_0^{\varphi_{lim}} \cos^2\varphi\, d\varphi = \frac{1}{2}\left(\sin\varphi_{lim}\cos\varphi_{lim} + \varphi_{lim}\right)$$

From Eq. (11), it can be seen that the terms proportional to u_z are $H_{zz,z}$ and $H_{rz,r}$ which are multiplied respectively by a factor $(\lambda+2\mu)$ and λ. We can group them as follows.

$$(\lambda + 2\mu)\, H_{zz,z} + \lambda\, H_{rz,r} = \lambda\, \Lambda_z + 2\mu\, M_z\,. \qquad (19)$$

In the same way, the terms proportional to u_r are $H_{zz,r}$ and $H_{rz,z}$ which are multiplied by a factor μ. We can group them as follows.

$$\mu\, H_{zz,r} + \mu\, H_{rz,z} = \mu\, M_r\,. \qquad (20)$$

Introducing the exact value of the integration over φ into Eq. (6) and taking account of Eqs. (18-20), Λ_z, M_z and M_r are given after some algebra by the following expressions which are the exact evaluations of the integrals given by Eq. (12).

$$\begin{aligned}
\rho_0\, \Lambda_z = & \int_{c_T}^{c_L} \left[\frac{15z^3 - 9z\,c^2t^2}{c^4t^4} \frac{\varphi_{lim}(c)}{2\pi} + \frac{15z\,(c^2t^2 - z^2)}{c^4t^4} \frac{\sin\varphi_{lim}(c)\cos\varphi_{lim}(c)}{2\pi} \right] \frac{dc}{c^2} \\
& + \frac{1}{c_L} \left[\frac{3\,z^3 - z\,c_L^2t^2}{c_L^4t^4} \frac{\varphi_{lim}(c_L)}{\pi} + \frac{3\,z\,(c_L^2t^2 - z^2)}{c_L^4t^4} \frac{\sin\varphi_{lim}(c_L)\cos\varphi_{lim}(c_L)}{\pi} \right] \\
& - \frac{1}{c_T} \left[\frac{3\,z^3 - 2z\,c_T^2t^2}{c_T^4t^4} \frac{\varphi_{lim}(c_T)}{\pi} + \frac{3\,z\,(c_T^2t^2 - z^2)}{c_T^4t^4} \frac{\sin\varphi_{lim}(c_T)\cos\varphi_{lim}(c_T)}{\pi} \right] \\
& + \frac{1}{c_L^2} \frac{\partial}{\partial t} \left[\frac{z^3 + z\,c_L^2t^2}{c_L^3t^3} \frac{\varphi_{lim}(c_L)}{2\pi} + \frac{z\,(c_L^2t^2 - z^2)}{c_L^3t^3} \frac{\sin\varphi_{lim}(c_L)\cos\varphi_{lim}(c_L)}{2\pi} \right] \\
& - \frac{1}{c_T^2} \frac{\partial}{\partial t} \left[\frac{z\,(c_T^2t^2 - z^2)}{c_T^3t^3} \frac{\varphi_{lim}(c_T) + \sin\varphi_{lim}(c_T)\cos\varphi_{lim}(c_T)}{2\pi} \right],
\end{aligned} \qquad (21)$$

$$\rho_0 M_z = \int_{c_T}^{c_L} \left[\frac{15z^3 - 9z\, c^2 t^2}{c^4 t^4} \frac{\varphi_{lim}(c)}{\pi} \right] \frac{dc}{c^2} + \frac{1}{c_L} \left[\frac{6z^3 - 3z\, c_L^2 t^2}{c_L^4 t^4} \frac{\varphi_{lim}(c_L)}{\pi} \right]$$

$$- \frac{1}{c_T} \left[\frac{6z^3 - 4z\, c_T^2 t^2}{c_T^4 t^4} \frac{\varphi_{lim}(c_T)}{\pi} \right] + \frac{1}{c_L^2} \frac{\partial}{\partial t} \left[\frac{z^3}{c_L^3 t^3} \frac{\varphi_{lim}(c_L)}{\pi} \right] - \frac{1}{c_T^2} \frac{\partial}{\partial t} \left[\frac{z^3 - z\, c_T^2 t^2}{c_T^3 t^3} \frac{\varphi_{lim}(c_T)}{\pi} \right], \quad (22)$$

$$\rho_0 M_r = \int_{c_T}^{c_L} \frac{(30z^2 - 6c^2 t^2)\sqrt{c^2 t^2 - z^2}}{c^4 t^4} \frac{\sin\varphi_{lim}(c)}{\pi} \frac{dc}{c^2}$$

$$+ \frac{1}{c_L} \left[\frac{2\,(6z^2 - c_L^2 t^2)\sqrt{c_L^2 t^2 - z^2}}{c_L^4 t^4} \frac{\sin\varphi_{lim}(c_L)}{\pi} \right] - \frac{1}{c_T} \left[\frac{3\,(4z^2 - c_T^2 t^2)\sqrt{c_T^2 t^2 - z^2}}{c_T^4 t^4} \frac{\sin\varphi_{lim}(c_T)}{\pi} \right]$$

$$+ \frac{1}{c_L^2} \frac{\partial}{\partial t} \left[\frac{2\, z^2 \sqrt{c_L^2 t^2 - z^2}}{c_L^3 t^3} \frac{\sin\varphi_{lim}(c_L)}{\pi} \right] - \frac{1}{c_T^2} \frac{\partial}{\partial t} \left[\frac{(2z^2 - c_T^2 t^2)\sqrt{c_T^2 t^2 - z^2}}{c_T^3 t^3} \frac{\sin\varphi_{lim}(c_T)}{\pi} \right]. \quad (23)$$

Now, the impulse-response for the complete problem of the radiation, the scattering and the reception of short pulses of ultrasound is given by a simple surface integral over the insonified part of the scatterer where h_z, h_r, Λ_z, M_z and M_r are analytically known. We have,

$$H_z^{TSR}(t) = -\frac{\partial}{\partial t} \iint_D \{h_z(\mathbf{r}_D,t) * [\lambda \Lambda_z(\mathbf{r}_D,t) + 2\mu M_z(\mathbf{r}_D,t)] + h_r(\mathbf{r}_D,t) * \mu M_r(\mathbf{r}_D,t)\} dS_D \ . \quad (24)$$

The computation of Eq. (24) is straightforward because of the compactness of the formulation and the introduction of analytic results for both the radiation and reception processes. Clearly, the expressions of the reception of echoes arising from a free surface given by Eqs. (21-23) are a little more complex than those for the transient radiation given in [7]. A full derivation of a solution for more general configurations would require similar evaluations of the $3^3 = 27$ integrals given by Eq. (12) — four of them being given herein— but there is no formal difficulty for doing it. It is worth noticing that terms proportional to a time-derivative in Eqs. (21-23) are of larger amplitude than the others.

EXAMPLE : THE FLAT-BOTTOMED HOLE (FBH)

To illustrate the capabilities of our new formulation, we show in Fig. 3 some comparisons of predicted and measured echo-responses from flat-bottomed holes interrogated by a disc thickness-mode transducer of 19-mm-Ø working both as a radiator and a receiver. Experiments are those shown in [5]. They arise from small FBH of 2 mm diameter at a range of 10, 15 and 25 mm. In [5], the FBH was modeled as a point defect. Here we integrate Eq. (24) over the actual finite size of the defect with a time-step of 6.66 ns (corresponding in the frequency domain to a sampling frequency of 150 MHz). Despite the simplifications made in [5], the discussion of the origin of the echoes measured from a FBH is very interesting. To summarize, the key point of the discussion is the demonstration of the paramount importance of transducer diffraction effects (in radiation and reception), leading to a complex echo-structure greatly varying with the range of the defect, even if the defect is a simple small FBH. The results shown in Fig. 3 were obtained by convolving the computed impulse-responses [Eq. (24)] with a signal "measured" from the published results in [5] (Fig. 12 in this reference). The accuracy of the model is well demonstrated in the simple cases considered.

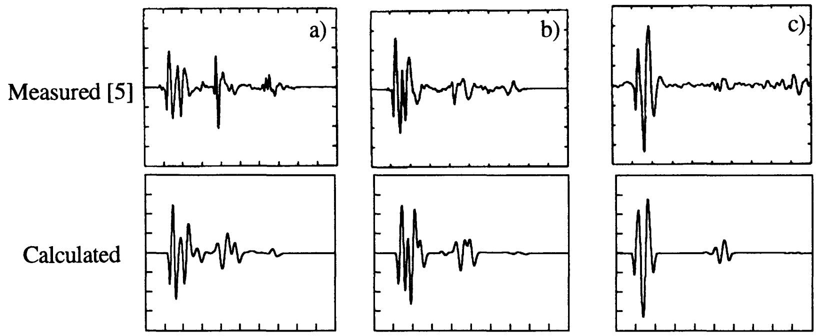

Fig. 3 Measured (in [5]) and calculated echo-responses from 2-mm-Ø FBH at a range of a) 10 mm, b) 15 mm, c) 25 mm [atten. 3 dB relatively to a) and b)]. (1 μs / div.).

SUMMARY AND FUTURE WORK

We have derived an approximate solution to the problem of radiation, scattering by a defect and reception of pulses of ultrasound in an elastic half-space. This solution allows the full three dimension and time dependent solution to be limited to a surface integral over the insonified area of the defect (modeled as a free surface) of intermediate analytic expressions for the transient radiation and reception by a realistic transducer. The first results have been obtained in the special case of a flat bottom-hole interrogated by a contact transducer working in the thickness-mode. They show clearly the paramount importance of transducer diffraction effects. Since the complete set of formulas for the transient reception has only been given in the case of a compression wave transducer, the paper is entitled "a first approach". However, the general integral formulation can be readily used to derive similar solutions in other cases. It can be also interesting to develop a similar model in the case of defects like hard inclusions modeled as rigid boundaries. This makes necessary to compute the traction radiated by the transducer at the defect surface. In reception, only the second-order Green's tensor will intervene, leading to simpler calculations for the transient reception than these shown in the present paper.

To fully demonstrate the accuracy of this model, it will be necessary to compare experiments and predictions for various defect and transducer geometries. We are attempting to conduct such experiments. The first comparisons shown in the present paper are encouraging. Moreover, the model for elastic waves is closely related to the previous model for acoustic waves [1] (fully validated by experiments [2]).

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