SUPPORT MINIMIZED NONLINEAR ACOUSTIC INVERSION WITH ABSOLUTE PHASE ERROR CORRECTION

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INTRODUCTION

The predominant factors which prohibit the inversion of acoustic scattering data for the purposes of flaw characterization are 1) limited angular access to the flaw, 2) limited temporal frequency signal bandwidth, and 3) lack of absolute phase information between individual measurements (zero of time problem). An additional complication which impedes the data inversion is the non-linear dependence of the scattering data on the scattering object. This problem must be handled by either linearizing the problem or by applying an iterative procedure which may have questionable convergence properties. An approach to data inversion is presented here which shows potential in overcoming the aforementioned difficulties. This approach compensates for the lack of data by constructing a solution which yields simulated scattering consistent with the measured data, while simultaneously minimizing a functional measure of the support (i.e. volume) of the flaw. Such an approach to limited data inversion has proven effective in limited view X-ray CT applications when reconstructing discontinuous boundary flaws such as cracks and inclusions [1, 2, 3]. The application presented here is by-and-large analogous to the X-ray CT application, except for the additional complication of the lack of absolute phase between measurements. This *zero-of-time* problem is handled here by treating the absolute phase of each measurement as a variable in the minimization of the flaw support.

Attention in this study is limited to the two-dimensional acoustic back-scatter problem, and thus at present represents an assessment of the conceptual feasibility. We do present an experimental result, however, which demonstrates the robustness of the approach. Future

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reduction of this approach to practice will require 1) the extension of the employed forward scattering model from acoustic to elastic wave scattering, and 2) an improvement in the efficiency of the forward scattering model (through better algorithms and better computers) to enable extension from two to three dimensions. These advancements are currently being pursued.

ACOUSTIC SCATTERING

The scattering from an acoustic scatterer embedded in an acoustic medium is assumed to be governed by the velocity Helmholtz equation

$$\nabla^2\phi + \frac{\omega^2}{c(x)^2}\phi = 0 \tag{1}$$

where $c(x)$ is the velocity of the wave propagation in the media at any given location. Obviously, for a medium containing an isolated inhomogeneity, $c(x)$ is constant everywhere except for the region containing the inhomogeneity. The change of variable $v(x) = c_0^2/c(x)^2 - 1$, where c_0 is the velocity of the homogeneous background medium, enables rearrangement of Eq. (1) to an equivalent compact source problem of the form

$$\nabla^2\phi + \frac{\omega^2}{c_0^2}\phi = -\frac{\omega^2}{c_0^2}v(x)\phi \tag{2}$$

The formal solution to the above problem yields a second kind integral equation for the total wave field $\phi(x)$.

$$\phi(x) = \phi^{inc}(x) - \frac{\omega_0^2}{c_0^2}\int v(x')\phi(x')G(x'|x)dx' \tag{3}$$

where $G(x'|x)$ is the Green's function for the Helmholtz equation. Given a scatterer potential $v(x)$, first, one has to calculate the field over the support of the scatterer and then calculate the scattered field outside the support of the scatterer using

$$\phi^{sc}(x) = -\frac{\omega_0^2}{c_0^2}\int v(x')\phi(x')G(x'|x)dx' \tag{4}$$

We proceed to discretize Eqs. (3) and (4) using an appropriate basis function $b(\cdot)$

$$\phi(x) = \sum_i \phi_i b_\phi(x - x_i) \tag{5}$$

$$v(x) = \sum_i v_i b_v(x - x_i) \tag{6}$$

Combining Eqs. (3), (4), (5) and (6), one obtains in matrix notation.

$$\sum_{j=1}^{N} G_{ij}v_j\phi_j = \phi_i^{inc} \ , \ i = 1, N \tag{7}$$

$$\sum_{j=1}^{N} G_{ij}^0 v_j\phi_j = \phi_i^{sc} \ , \ i = 1, M \tag{8}$$

where Eq. (7) is an $N \times N$ system of equations for the determination of N discrete values of ϕ over the support of v, and Eq. (8) is an $M \times N$ system for determining the scattered data ϕ^{sc} at M measurement points given N values of ϕ. The computational bottleneck in the inversion process is the inversion of the Eq. (7). Computationally efficient means of inverting Eq. (7) can be employed which exploit special properties of problem such as the convolutional property of the Green function. Furthermore, approximations to the inverse of the Eq. (7) can be effctively utilized up to the final stages of the inversion, since the scattering object is itself an approximation throughout most of the inversion process. Although important to the implementation of a practical algorithm, the details of the scheme used to invert Eq. (7) are secondary to the purposes of this paper, and will be deferred to future writings.

SUPPORT MINIMIZED INVERSION

The inverse problem of interest in this work deals with reconstruction of an acoustic scatterer given far field back-scattered field measurements at finite number of angles and temporal frequencies. It is not fully understood what constitutes a set of complete data for general nonlinear acoustic inversion. However, our previous work [4] has indicated that data sufficiency conditions for Born inversion are a conservative estimate of the data sufficiency conditions for nonlinear inversion. For Born inversion of a real valued object, data collected over 180 degrees and at all temporal frequencies is sufficient for a complete reconstruction of the object. Practical applications generally provide data at discrete points over limited angular range and a finite temporal frequency bandwidth about a center frequency. In order to compensate for the missing data and thereby regularize the underdetermined inversion problem, the inversion process must to be supplemented with additional information about the scatterer. In this work, additional information is provided in the form of a support penalty functional applied to the solution.

The choice of minimum support as a supplemental constraint assumes *a priori* that scattering is due to discontinuous boundary objects. The idea of using support constraint on the object has been used effectively in other applications, in which the support is fixed to an *a priori* prescribed region [6, 5]. For NDE applications, a procedural algorithm was proposed in [7] to find solutions that have small support, however, no explicit functional was introduced. The approach employed here does not *a priori* fix the allowed support, but rather minimizes the object support through optimization of the functional

$$\mathcal{S}(v) = \int \frac{|v|^\eta}{|v|^\eta + \epsilon^\eta} dx \tag{9}$$

where ϵ is a prescribed threshold which depends on the noise level expected in the final solution and η is a parameter which controls the sharpness of the penalty function transition from zero to one. By optimizing $\mathcal{S}(v)$ along with a measure of fidelity to the measured scattering data, one finds the smallest support object that is consistent with the measurements. Qualitatively stated, the support functional seeks to determine whether or not a non-zero object value v_i is essential at each x_i. If a non-zero value is essential for agreement with the measured data, a penalty $\simeq 1$ is assigned to that position; if not, a penalty $\simeq 0$ is assigned. The key feature of this penalty is that all non-zero values of v_i are assigned approximately the same penalty. A consequence of this behavior is that non-uniqueness among support minimized solutions may remain (i.e. multiple solutions with essentially the same support). Therefore, it is frequently desirable to supplement the support functional with additional functionals to establish a preferred condition on the non-zero object values. Applications of such additional functionals are demonstrated in refs. [2, 3, 4].

INVERSION OF LIMITED BACKSCATTERED MEASUREMENTS

A variational optimization is employed to simultaneously solve the systems of Eq. (7) and Eq. (8) for the variables ϕ and v. The optimization determines ϕ and v which

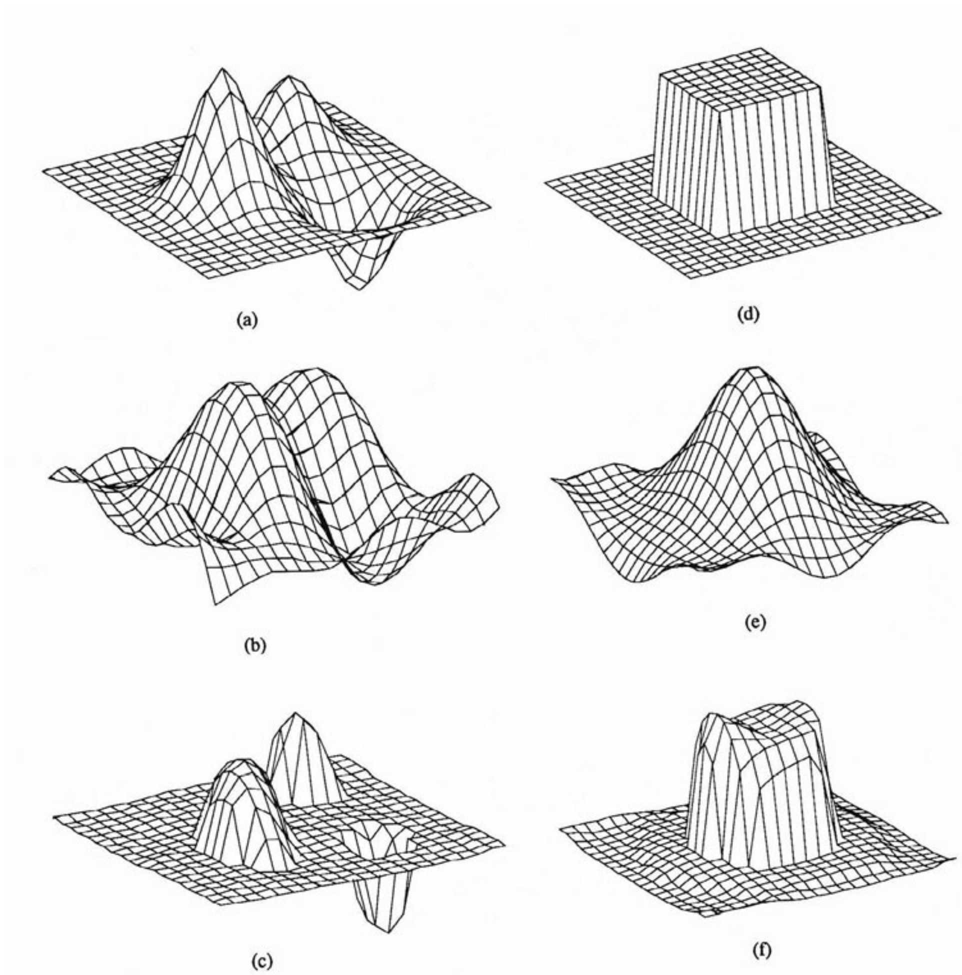

Figure 1. Limited 120° view-angle finite bandwidth inversion for two types of scatterers: a,d) True scatterer, b,e) inversion without using minimum support functional, c,f) inversion using minimum support functional.

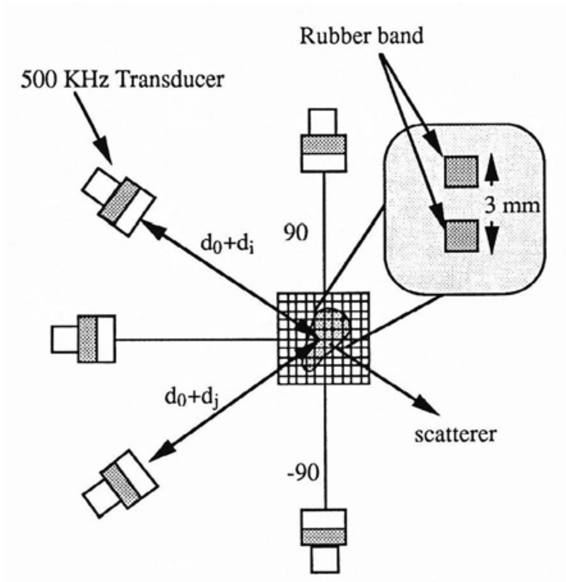

Figure 2. Diagram showing the set-up for the experiment (in actual experiment one transducer was used at different positions).

minimizes a combined measure of 1.) agreement between the measured data and simulated (via Eq. (8)) scattering data, and 2.) object support. The choice of the measure of agreement with scattering data is very important to the convergence rate of the iterative inversion. In this work, the scatterer is not weak enough to justify use of the Born approximation. However, the Born approximation can provide insight into the construction of an efficient measure of agreement with the scattering data. The total functional optimized in this work is

$$\mathcal{O}(v) = \sum_i \sum_j |\mathcal{W}_i(\phi_{ij}^{sc:f}(v) - \phi_{ij}^{sc:m})|^2 + \sum_m |v_m|^\eta / (|v_m|^\eta + \epsilon^\eta) \quad (10)$$

$$\mathcal{W}_i = \omega_i^{-3/2} \quad (11)$$

where the weighting $\mathcal{W}_i$ applies a filtering similar to that applied in a Born inversion. The functional $\mathcal{O}$ is optimized using a conjugate gradient search. It is noted that, due to the choice of wighting W_k, the gradient vector bears considerable resemblance to the Born inverse solution. Numerical experiments are presented which demonstrates the effectiveness of support minimized inversion on two different scatterers, depicted in Figs. (1a) and (1d). The first scatterer consists of three neighboring gaussian-shaped components, representing a 30% reduction in velocity, a 25% reduction in velocity and a 80% increase in velocity. This scatterer has continuously varying object boundaries. The second scatterer is a square, discontinuous boundary object representing a 30% reduction in velocity. Figs. (1 b) and (1 e) show the inversion without using the minimum support functional. Back scattered data was limited to a 120° view angle and 3 frequencies equally spaced over a range of $(1.13 < ka < 3.39)$ for Fig. (1b,c) and $(0.8 < ka < 2.4)$ for Fig. (1e,f) where a is the approximate radius of the circle circumscribing the object. The objects in both cases have a larger support than the true object and the object edges are not well defined. Figs. (1c) and (1f) show the result of a subsequent application of support minimization to the the same data set. The support threshold was set to 5% of the object's maximum and the sharpness parameter was set to $\eta = 4$. The object edges are much better defined and the support more closely approximates that of the true object. Comparison of the Figs. (1c) and (1f) demonstrates the performance of support minimization for compact discontinuous boundary scatterers. Note that the scatterer with a continuously varying boundary is reconstructed with a support smaller than that of the true scatterer and that the scatterer boundary displays a rapid transition. For the case of the discontinuous boundary scatterer, the support is almost the same as the true scatterer. The slight reduction in scatterer support is due to severe deficiency in the available data. Overall, the inversion results of Fig. (1c) and (1f) are more intelligible that those of Fig. (1b) and (1e). The support minimized solution allows the specification of lower bound on the flaw size. This could be a significant aid in the minimization of false detection since the support minimized solution will by definition be no larger than is absolutely required by the measured data.

ABSOLUTE PHASE ERROR CORRECTION

A significant problem in the practical implementation of any inversion scheme is the assignment of an absolute phase reference (i.e. a "zero-of-time") to individual measurements. This is particularly true in ultrasonic backscatter measurements where a single probe is mechanically (perhaps even manually) positioned at various angular orientations. Accurate inversion of the data requires knowing the probe position to within a small fraction of a wavelength, which, in practice, is not realistic to expect. This lack of absolute phase information is compensated for in our approach by treating the "zero-of-time" associated with each measurement as variables in the variational optimization of the support functional. In practice, recorded signals scattered from a flaw under different angular orientations are aligned visually to roughly correspond to a common origin within the component. Following this visual alignment, the remaining uncertainty in probe position will likely be within a few wavelengths or less. Corrections for the uncertainties in probe position are denoted d_i, corresponding to the uncertainty in the radial distance from the probe to the flaw at the i_{th} measurement position. The associated phase correction is incorporated into the functional $\mathcal{O}(v,d)$ as

$$\mathcal{O}(v,d) = \sum_i \sum_j |\mathcal{W}_i(\phi_{ij}^{sc:f}(v)e^{jkd_i} - \phi_{ij}^{sc:m})|^2 + \sum_m |v_m|^\eta/(|v_m|^\eta + \epsilon^\eta) \qquad (12)$$

The d_i's are treated as additional unknown variables, and are included in the total unknown vector along with the unknown variables v_i. After optimization is complete, the value of d_i indicates how much correction was applied to the i_{th} measurement position.

In order to test the robustness of the proposed algorithm, an experiment was set up as shown in Fig. (2). In this experiment a series of time signals were collected at finite number of angular positions. The objective of the experiment is to reconstruct the scatterer without an absolute knowledge of the zero-of-time. The only additional a priori information about the object is an assumed compactness. A total of 19 measurements were made uniformly spaced in a 180 degree view angle. Following visual alignment, the time signals were Fourier transformed and deconvolved to remove the receiver response. Spectral values at a finite number of frequencies around the 500 khz operating frequency were selected as input data to the inversion algorithm. Results presented here utilized 6 frequency components equally spaced in the 250 to 650 khz range. As seen in Fig. (3a), the reconstruction of the object without support minimization or absolute measurememt phase is not in agreement with the true scatterer geometry. Artifacts in Fig. (3a) indicate a far larger scatterer than actually exists. In Fig. (3b) the result of inversion with support minimization and absolute phase error correction is presented. The reconstruction artifacts have been removed. The separation of the two objects is in agreement with the actual separation. The amplitude of the scattering potential likely does not truely represent the rubber band because the variable velocity Helmholtz equation does not accurately represent scattering from rubber in water. In Fig. (4), correction values for the position of the transducer at each measurement position are given. These corrections are the side product of the phase corrected inversion with minimum support. Note that corrections to the nominal distance d_0 are almost symmeteric about the middle transducer at 0 degrees. This is expected since the object is symmetric about the 0 degree axis in Fig. (3). The visual alignment of the waveforms resulted in an approximately symmetric initial phase error.

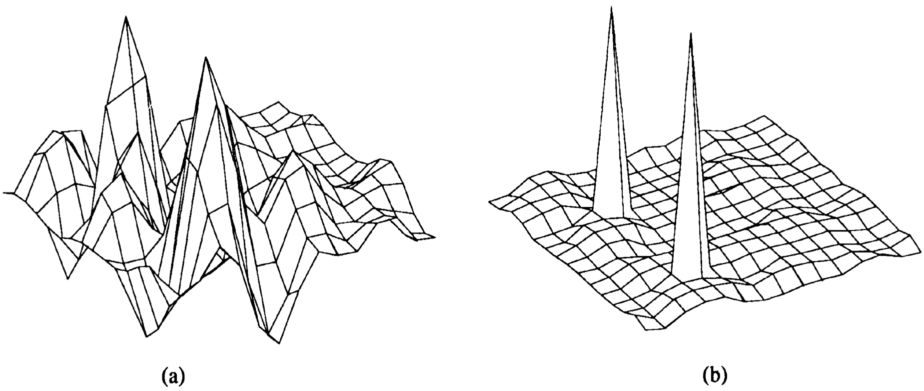

Figure 3. Results of inversion for scatter potential v using experimental data: a) without the minimum support functional and absolute phase corrections, b) with the minimum support functional and absolute phase corrections.

CONCLUSION

A robust algorithm is presented for nonlinear acoustic inversion using acoustic backscatter data limited in both spatial and temporal frequency domains. This algorithm produces a more accurate and intelligible result when applied to scattering data for which an *a priori* assumed compactness is justified. The algorithm compensates for incompleteness in the measured data through the minimization of a functional measure of object support. Examples of application to simulated backscatter measurements demonstrated the effectiveness in reconstructing severly limited data. A scheme for compensating for unknown absolute measurement phases was introduced. This scheme variationally determines the zero-of-time associated with each measurement position as part of the support minimization procedure. Application of this algorithm to experimental back scatter data yielded excellent reconstruction of the scatterer geometry assuming no known zero-of-time data. Work is currently under way to extend the algorithm to elastic media.

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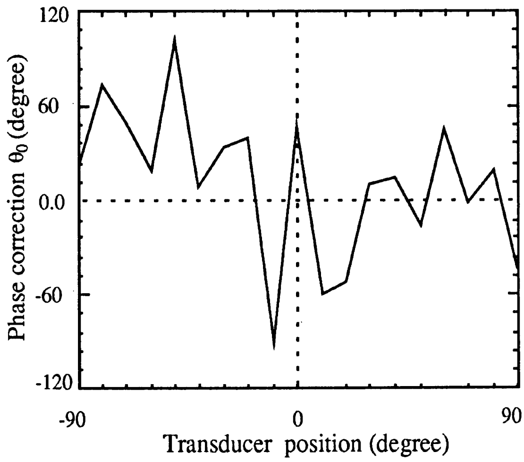

Figure 4. Phase corrections at center frequency of 500 khz ($\theta_0 = 360d_i/\lambda_0$).

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