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Radiative instability of quantum electrodynamics in chiral matter

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ABSTRACT

Modification of the photon dispersion relation in chiral matter enables $1 \to 2$ scattering. As a result, the single fermion and photon states are unstable to photon radiation and pair production respectively. In particular, a fast fermion moving through chiral matter can spontaneously radiate a photon, while a photon can spontaneously radiate a fast fermion and anti-fermion pair. The corresponding spectra are derived in the ultra-relativistic approximation. It is shown that the polarization of the produced and decayed photons is determined by the sign of the chiral conductivity. Impact of a flat thin domain wall on the spectra is computed.

1. Introduction

One of the macroscopic manifestations of the chiral anomaly of QCD is the emergence of the topological CP-odd domains in hot nuclear matter [1]. QED is coupled to these domains via its own chiral anomaly. This is represented by the triangular diagrams that involve two photon fields and the axial current generated by the topological fluctuations of the gluon field. The axial current rapidly increases with temperature which triggers a variety of non-trivial electromagnetic effects in quark–gluon plasma [2].

At a more fundamental level, the chiral anomaly makes photon effectively massive. Consequently, single photon and fermion states become unstable. Recall that photon radiation by a charged fermion in vacuum $f(p) \to f(p') + \gamma(k)$ and the cross-channel process of pair production in vacuum $\gamma(k) \to f(p') + \bar{f}(p)$ are prohibited by momentum conservation.[1] Indeed in the rest frame of one of the fermions $k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$. The right-hand-side never vanishes since $\varepsilon > m$, whereas in the left-hand-side $k^2 = 0$ [3].[2] In chiral matter, i.e. in a matter supporting the CP-odd domains, the chiral anomaly modifies the photon dispersion relation as [4–10][3]

$$k^2 = -\lambda \sigma_\chi |\boldsymbol{k}| , \tag{1}$$

where λ and $\boldsymbol{k}$ are photon helicity and momentum and σ_χ is the chiral conductivity [11–13]. This opens the $1 \to 2$ scattering channels, viz. the pair-production if $k^2 > 0$ and the photon radiation if $k^2 < 0$. Thus, single-particle states in chiral matter are unstable with respect to spontaneous radiation and decay. Moreover, in a matter with positive σ_χ, only the right-polarized photons with $\lambda = +1$ can be radiated, while only the left-polarized photons with $\lambda = -1$ decay and vice-versa in a matter with negative σ_χ.

The main goal of this paper is to compute the photon radiation and pair production spectra due to the modified photon dispersion relation in chiral matter. The fermions (and antifermions) are considered to be external particles propagating through the chiral matter. Their energy is assumed to be much larger than the medium ionization energy. The calculation method is borrowed from [15] and relies on several approximations: (i) photons and fermions are ultra-relativistic in the laboratory frame (the one associated with the matter), this means that $\varepsilon \gg m$, $\omega \gg \sigma_\chi$. Apart from making calculations significantly less bulky, this allows one to neglect the effect of the electromagnetic field instability in the infrared region as explained in Sec. 2. (ii) The matter is assumed to be spatially homogeneous and consisting of either one infinite CP-odd domain or of two semi-infinite domains separated by a flat domain wall.

The paper is structured as follows. In Sec. 2 and Sec. 3 the wave functions of an ultra-relativistic photon and fermion in chiral matter are derived. In Sec. 4 they are employed to compute the photon spectrum radiated by a fermion in chiral matter with constant σ_χ as well as in matter with two chiral domains separated by a thin flat domain wall. The cross-channel process of pair production is analyzed in Sec. 5. The results are summarized and discussed in Sec. 6.

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1 p, p' and k are four-momenta with the components $p = (\varepsilon, \boldsymbol{p})$, $p' = (\varepsilon', \boldsymbol{p}')$ and $k = (\omega, \boldsymbol{k})$.

2 In the limit of massless fermions, no production is possible as well, since in the only kinematically allowed configuration all particle momenta are strictly parallel.

3 In covariant form $k^2 = -\lambda\sqrt{(n \cdot k)^2 - n^2 k^2}$, where $n^\mu = \sigma_\chi \delta_0^\mu$ in the matter rest frame.

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2. Photon wave function

The CP-odd domains in the chiral matter can be described by a scalar field θ whose interaction with the electromagnetic field $F^{\mu\nu}$ is governed by the Lagrangian [4,16–18]

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} - \frac{c_A}{4}\theta \tilde{F}_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi\,, \tag{2}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$ is the dual field tensor and c_A is the chiral anomaly coefficient [19]. A working assumption of this paper is that the field θ is spatially uniform (apart from possible thin domain walls) and a slowly varying function of time. The "free" field equations of electrodynamics in chiral electrically neutral and non-conducting matter read

$$\nabla\cdot\boldsymbol{B} = \nabla\cdot\boldsymbol{E} = 0\,, \tag{3}$$

$$\nabla\times\boldsymbol{E} = -\partial_t\boldsymbol{B}\,, \tag{4}$$

$$\nabla\times\boldsymbol{B} = \partial_t\boldsymbol{E} + \sigma_\chi\boldsymbol{B}\,, \tag{5}$$

where $\sigma_\chi = c_A\dot{\theta}$ is the chiral conductivity [11–13]. (Often $\dot{\theta}$ is denoted by μ_5 and is referred to as the axial chemical potential [14]). In the radiation gauge $A^0 = 0$ and $\nabla\cdot\boldsymbol{A} = 0$ the vector potential obeys the equation

$$-\nabla^2\boldsymbol{A} = -\partial_t^2\boldsymbol{A} + \sigma_\chi(\nabla\times\boldsymbol{A})\,. \tag{6}$$

Its solution describing a photon moving along the z-direction with energy $\omega \gg k_\perp, \sigma_\chi$ is described by the wave function

$$\boldsymbol{A}^{(0)} = \frac{1}{\sqrt{2\omega V}}\boldsymbol{e}_\lambda\, e^{ik_z z - i\omega t}\,, \quad k_z = \omega\,, \tag{7}$$

where the polarization vector satisfies $\boldsymbol{e}_\lambda\cdot\hat{\boldsymbol{z}} = 0$. V is the normalization volume. It is convenient to use the helicity basis $\boldsymbol{e}_\lambda = (\hat{\boldsymbol{x}} + i\lambda\hat{\boldsymbol{y}})/\sqrt{2}$. To determine the effect of the chiral anomaly on the photon wave function, look for a solution in the form

$$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}}(\boldsymbol{e}_\lambda\varphi + \hat{\boldsymbol{z}}\varphi')\, e^{i\omega z - i\omega t}\,, \tag{8}$$

where φ and φ' are functions of coordinates slowly varying in the longitudinal (z) direction, viz. $|\partial_z\varphi/\varphi| \ll \omega$ and $|\partial_z\varphi'/\varphi'| \ll \omega$. The two unknown functions φ and φ' are required in order to account for the change of the photon polarization direction. The gauge condition yields a constraint

$$(\boldsymbol{e}_\lambda\cdot\nabla_\perp)\varphi + \partial_z\varphi' + i\omega\varphi' \approx (\boldsymbol{e}_\lambda\cdot\nabla_\perp)\varphi + i\omega\varphi' = 0\,. \tag{9}$$

Substituting (8) into (6) one obtains

$$\boldsymbol{e}_\lambda\left(-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi\right) + \hat{\boldsymbol{z}}\left(-2i\omega\partial_z\varphi' - \nabla_\perp^2\varphi'\right) = \sigma_\chi\left(\omega\lambda\boldsymbol{e}_\lambda\varphi - \boldsymbol{e}_\lambda\times\nabla\varphi - \hat{\boldsymbol{z}}\times\nabla_\perp\varphi'\right)\,. \tag{10}$$

Taking the scalar product of this equation with $\boldsymbol{e}^*_\lambda$ and using $\boldsymbol{e}^*_\pm\cdot\boldsymbol{e}_\mp = 0$ and $\boldsymbol{e}^*_\pm\cdot\boldsymbol{e}_\pm = 1$ produces

$$-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi = \sigma_\chi(\omega\lambda\varphi - i\lambda\partial_z\varphi) + i\lambda\boldsymbol{e}^*_\lambda\cdot\nabla_\perp\varphi'\,, \tag{11}$$

where we used the identity $\boldsymbol{e}_\lambda\times\hat{\boldsymbol{z}} = i\lambda\boldsymbol{e}_\lambda$. In view of (9) one can drop the small term proportional to φ'. Neglecting also $\partial_z\varphi$ in parentheses furnishes an equation for φ:

$$-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi = \sigma_\chi\omega\lambda\varphi\,. \tag{12}$$

Taking the scalar product of (10) with $\hat{\boldsymbol{z}}$ yields

$$-2i\omega\partial_z\varphi' - \nabla_\perp^2\varphi' = \sigma_\chi i\lambda(\boldsymbol{e}_\lambda\cdot\nabla)\varphi\,. \tag{13}$$

One can eliminate in (13) the term proportional to φ using the gauge condition (9). This furnishes an equation for φ' which is precisely the same as equation (12) obeyed by φ.

A solution to (12) can be written as

$$\varphi = e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\exp\left\{-i\frac{1}{2\omega}\int_0^z\left[k_\perp^2 - \sigma_\chi(z')\omega\lambda\right]dz'\right\}\,. \tag{14}$$

It follows from (9) that

$$\varphi' = -\frac{\boldsymbol{e}_\lambda\cdot\boldsymbol{k}_\perp}{\omega}\varphi\,. \tag{15}$$

Substituting (14) and (15) into (8) yields the photon wave function in the high energy approximation

$$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}}\boldsymbol{\epsilon}_\lambda\, e^{i\omega z + i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp - i\omega t} \times\exp\left\{-i\frac{1}{2\omega}\int_0^z\left[k_\perp^2 - \sigma_\chi(z')\omega\lambda\right]dz'\right\}\,, \tag{16}$$

where the polarization vector

$$\boldsymbol{\epsilon}_\lambda = \boldsymbol{e}_\lambda - \frac{\boldsymbol{e}_\lambda\cdot\boldsymbol{k}_\perp}{\omega}\hat{\boldsymbol{z}}\,. \tag{17}$$

Clearly, $\boldsymbol{\epsilon}_\lambda\cdot\boldsymbol{k} = 0$ up to the terms of order $k_\perp^2/\omega^2$ and σ_χ/ω. If the scattering process happens entirely within a single domain, then the chiral conductivity is constant. However, if a domain wall is located at, say, $z = 0$, than the chiral conductivity is different at $z < 0$ and $z > 0$. This is why a possible z-dependence of σ_χ is indicated in (16). Even though the boundary conditions on the domain wall induce a reflected wave, it can be neglected in the ultra-relativistic approximation [15,20].

It is seen in (1) that half of the infrared modes $|\boldsymbol{k}| < \lambda\sigma_\chi$ have $\mathrm{Im}\,\omega > 0$ implying exponential growth of the corresponding wave function with time. This infrared instability and its applications are discussed in many recent publications [2,8,21–36]. However, it is only tangentially related to the radiative instability discussed in this paper, even though both originate from the same dispersion relation. In particular, the infrared instability can be ignored in the ultra-relativistic limit $\omega \gg k_\perp \gg |\sigma_\chi|$ because equation

$$k_z \approx \omega - \frac{1}{2\omega}\left(k_\perp^2 - \lambda\sigma_\chi\omega\right) \tag{18}$$

has only real solutions.

3. Fermion wave function

The free fermion wave function ψ at high energy $\varepsilon \gg p_\perp, m$ can be obtained using the same procedure. Since it satisfies the Dirac equation we are looking for a solution in the form

$$\psi = \frac{1}{\sqrt{2\varepsilon V}}u(p)\phi e^{i\varepsilon z - i\varepsilon t}\,, \tag{19}$$

where $u(p)$ is a spinor describing a free fermion with momentum p and ϕ is a scalar function of coordinates. Substituting ψ into $(\partial^2 + m^2)\psi = 0$ and neglecting $\partial_z\phi$ compared to $\varepsilon\phi$ one obtains

$$2i\varepsilon\partial_z\phi + \nabla_\perp^2\phi = m^2\phi \tag{20}$$

with a solution

$$\phi = \exp\left\{ i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp - iz\frac{\boldsymbol{p}_\perp^2+m^2}{2\varepsilon}\right\}. \tag{21}$$

Thus, the fermion wave function is

$$\psi = \frac{1}{\sqrt{2\varepsilon V}}u(p)e^{i\varepsilon z - i\varepsilon t}\exp\left\{ i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp - iz\frac{\boldsymbol{p}_\perp^2+m^2}{2\varepsilon}\right\}. \tag{22}$$

4. Photon radiation

Modification of the photon dispersion relation in chiral matter makes possible spontaneous photon radiation $f(p)\to f(p')+\gamma(k)$. The corresponding scattering matrix element reads

$$S = -ieQ\int \bar\psi\gamma^\mu\psi A_\mu d^4x \tag{23}$$

$$= -ieQ(2\pi)\delta(\omega+\varepsilon'-\varepsilon)\frac{\bar u(p')\gamma^\mu u(p)\epsilon^*_\mu}{\sqrt{8\varepsilon\varepsilon'\omega}} \times\int_{-\infty}^{\infty}dz\int d^2x_\perp\,\phi^*_{p'}(z,\boldsymbol{x}_\perp)\varphi^*_k(z,\boldsymbol{x}_\perp)\phi_p(z,\boldsymbol{x}_\perp) \tag{24}$$

$$= i(2\pi)^3\delta(\omega+\varepsilon'-\varepsilon)\delta(\boldsymbol{p}_\perp-\boldsymbol{k}_\perp-\boldsymbol{p}'_\perp)\frac{\mathcal{M}}{\sqrt{8\varepsilon\varepsilon'\omega V^3}}, \tag{25}$$

where Q is the fermion electric charge. The wave functions φ_k and ϕ_p are given by (14) and (22) respectively with the subscripts indicating the corresponding momenta. The amplitude $\mathcal{M}$ is given by

$$\mathcal{M} = -eQ\,\bar u(p')\gamma^\mu u(p)\epsilon^*_\mu\int_{-\infty}^{\infty}dz \times\exp\left\{ i\int_0^z dz'\left[\frac{p'^2_\perp+m^2}{2\varepsilon'}-\frac{p^2_\perp+m^2}{2\varepsilon}+\frac{k^2_\perp-\sigma_\chi\omega\lambda}{2\omega}\right]\right\} \tag{26}$$

$$= \mathcal{M}_0\int_{-\infty}^{\infty}dz\exp\left\{ i\int_0^z\frac{q^2_\perp+\kappa_\lambda(z')}{2\varepsilon x(1-x)}dz'\right\}, \tag{27}$$

where we introduced notations $\mathcal{M}_0 = -eQ\,\bar u(p')\gamma^\mu u(p)\epsilon^*_\mu$, $x=\omega/\varepsilon$,

$$\boldsymbol{q}_\perp = x\boldsymbol{p}'-(1-x)\boldsymbol{k}_\perp, \tag{28}$$

and

$$\kappa_\lambda(z) = x^2m^2-(1-x)x\lambda\sigma_\chi\varepsilon. \tag{29}$$

The amplitude $\mathcal{M}_0$ can most efficiently be computed in the helicity basis using the matrix elements derived in [37]. Keeping in mind that at high energies $k^+ = xp^+$ (where $p^+=\varepsilon+p_z$, $k^+=\omega+k_z$), one obtains

$$\mathcal{M}_0 = -eQ\,\bar u_{\sigma'}(p')\gamma\cdot\epsilon^*_\lambda(k)u_\sigma(p) \tag{30}$$

$$= -\frac{eQ}{\sqrt{2(1-x)}}\left[xm(\sigma+\lambda)\delta_{\sigma',-\sigma} - \frac{1}{x}(2-x+x\lambda\sigma)(q_x-i\lambda q_y)\delta_{\sigma',\sigma}\right], \tag{31}$$

where $\sigma=\pm1$ and $\sigma'\pm1$ are the fermion helicities before and after photon radiation.

The transition probability can be computed as

$$dw = |S|^2\frac{Vd^3p'}{(2\pi)^3}\frac{Vd^3k}{(2\pi)^3} = |S|^2\frac{Vd^2p_\perp dp'_z}{(2\pi)^3}\frac{Vd^2q_\perp dk_z}{(2\pi)^3} \tag{32}$$

The cross section is the rate per unit flux V^{-1}, while the number of produced photons N is the cross section per unit area. Using the usual rules for dealing with the squares of the delta-functions and integrating over the phase space yields

$$dN = \frac{1}{(2\pi)^3}\frac{1}{8x(1-x)\varepsilon^2}\frac{1}{2}\sum_{\lambda,\sigma,\sigma'}|\mathcal{M}|^2d^2q_\perp dx, \tag{33}$$

where the sum runs over the photon and fermion helicities. Eqs. (33), (31), (27) give the spectrum of radiated photons. In the following subsections the explicit expressions for the photon spectrum are derived for a single domain and for two domains separated by a domain wall at $z=0$.

4.1. One infinite domain

Consider first an infinite chiral matter with constant chiral conductivity. Performing the integral over z in (27) yields

$$\mathcal{M} = 2\pi\mathcal{M}_0\,\delta\left(\frac{q^2_\perp+\kappa_\lambda}{2\varepsilon x(1-x)}\right). \tag{34}$$

The square of the delta-function in (34) is interpreted as the delta-function multiplied by $T/2\pi$, where T is the observation time. Namely,

$$|\mathcal{M}|^2 = 4\pi\varepsilon x(1-x)\delta(q^2_\perp+\kappa_\lambda)T|\mathcal{M}_0|^2. \tag{35}$$

The relevant intensive observable quantity then is the photon radiation rate W given by

$$\frac{dW}{dx} = \frac{1}{16\pi\varepsilon}\frac{1}{2}\sum_{\lambda,\sigma,\sigma'}|\mathcal{M}_0|^2\theta(-\kappa_\lambda), \tag{36}$$

where θ is the step-function. It follows from (29) that κ_λ is negative if $\lambda\sigma_\chi>0$ and

$$x < x_0 = \frac{1}{1+m^2/(\lambda\sigma_\chi\varepsilon)}. \tag{37}$$

Assume for definitiveness that $\sigma_\chi>0$. Then only the right-polarized photons with $\lambda>0$ are radiated. Using (34), (31) in (36) and performing the summations and the integration yields the density of spontaneously radiated photons

$$\frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x^2(1-x)}\left\{-\left(\frac{x^2}{2}-x+1\right)\kappa_+ + \frac{x^4m^2}{2}\right\}\theta(x_0-x) = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_\chi\varepsilon\left(\frac{x^2}{2}-x+1\right)-m^2x\right\}\theta(x_0-x), \tag{38}$$

$$\frac{dW_-}{dx} = 0. \tag{39}$$

Photon spectrum radiated in a matter with $\sigma_\chi<0$ can be obtained by replacing $W_\pm\to W_\mp$ and $\sigma_\chi\to-\sigma_\chi$. Note that since the anomaly coefficient $c_A\sim\alpha$, the spectrum (38) is of the order α^2.

The total energy radiated by a fermion per unit time is

$$\frac{\Delta\varepsilon}{T} = \int_0^1\frac{dW_+}{dx}x\varepsilon dx = \frac{1}{3}\alpha Q^2\sigma_\chi\varepsilon, \tag{40}$$

where the terms of order $m^2/|\sigma_\chi|\varepsilon$ have been neglected for simplicity. Thus, energy loss increases exponentially with time. It can be neglected only for time intervals much smaller than $\sim 1/|\sigma_\chi|\alpha$.

4.2. Two semi-infinite domains separated by a domain wall at $z = 0$

Suppose now that the chiral matter consist of two semi-infinite domains separated by a thin domain wall at $z = 0$. Performing the integral over z in (27) yields

$$\mathcal{M} = \mathcal{M}_0 \left\{ \int_{-\infty}^{0} dz\, e^{iz\frac{q_\perp^2+\kappa'_\lambda - i\delta}{2\varepsilon x(1-x)}} + \int_0^{\infty} dz\, e^{iz\frac{q_\perp^2+\kappa_\lambda + i\delta}{2\varepsilon x(1-x)}} \right\} \quad (41)$$

$$= 2\varepsilon x(1-x)\mathcal{M}_0 \left\{ \frac{-i}{q_\perp^2+\kappa'_\lambda - i\delta} - \frac{-i}{q_\perp^2+\kappa_\lambda + i\delta} \right\}, \quad (42)$$

where the values of κ_λ at $z < 0$ and $z > 0$ are denoted by κ'_λ and κ_λ respectively and $\delta > 0$ is inserted to regularize the integrals. Plugging (42), (31) into (33) and performing summation over spins yields the radiation spectrum

$$\frac{dN}{d^2q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x} \left\{ \left(\frac{x^2}{2} - x + 1 \right) q_\perp^2 + \frac{x^4 m^2}{2} \right\} \times \sum_\lambda \left| \frac{1}{q_\perp^2+\kappa'_\lambda - i\delta} - \frac{1}{q_\perp^2+\kappa_\lambda + i\delta} \right|^2 . \quad (43)$$

The spectrum peaks at $q_\perp^2 = -\kappa_\lambda$ and/or $q_\perp^2 = -\kappa'_\lambda$ provided that $\kappa_\lambda < 0$ and/or $\kappa'_\lambda < 0$ respectively. In the limit $\kappa_\lambda \to \kappa'_\lambda$ the results of the previous subsection, provided that the square of the delta functions is treated as explained after (34). Let us also note that when $q_\perp^2 + \kappa_\lambda = 0$ in (41), the second integral equals $T/2$, which implies that we have to identify $\delta = 4\varepsilon x(1-x)/T$ (the same result is of course obtained using the first integral).

Away from the poles, one can neglect δ in (43). The resulting spectrum coincides with the spectrum of the transition radiation once κ_λ's are replaced by $\kappa_{tr} = m^2x^2 + m_\gamma^2(1-x)$, where m_γ is the effective photon mass [15,20]. Unlike the spontaneous radiation, the transition radiation is not possible in a uniform matter. Indeed, the amplitude (34) vanishes because $\kappa_{tr} > 0$. Another key difference between the transition and spontaneous radiation is that the former has a finite classical limit $\hbar \to 0$, while the later one does not. The spontaneous radiation spectrum (44), (45) is a purely quantum effect that vanishes in the classical limit $\hbar \to 0$. This is of course not surprising at all because it originates from a quantum anomaly.

Integral over the momentum $q_\perp$ in (43) is dominated by the poles at $q_\perp^2 = -\kappa_\lambda$ and $q_\perp^2 = -\kappa'_\lambda$. There are two distinct cases depending on whether σ_χ and σ'_χ have the same or opposite signs. Consider first $\sigma_\chi > 0$ and $\sigma'_\chi > 0$. In this case the photon spectrum is approximately right-polarized. Keeping only the terms proportional to $1/\delta$ one obtains

$$\frac{dW_{++}}{dx} = \frac{\alpha Q^2}{8x^2(1-x)\varepsilon} \left[\left(\frac{x^2}{2} - x + 1 \right) |\kappa_+ + \kappa'_+| + \frac{x^4 m^2}{2} \right] \times \theta(x_0 - x)\theta(x'_0 - x), \quad (44)$$

where the double plus subscript indicates that the helicity is positive in both domains. The maximum energy fraction taken by the photon x_0 is defined in (37); x'_0 is the same as x_0 with σ_χ replaced by σ'_χ. Consider now $\sigma'_\chi > 0$ and $\sigma_\chi < 0$. The integration gives

$$\frac{dW_{+-}}{dx} = \frac{\alpha Q^2}{8x^2(1-x)\varepsilon} \left\{ \left[\left(\frac{x^2}{2} - x + 1 \right) |\kappa'_+| + \frac{x^4 m^2}{4} \right] \theta(x'_0 - x) + \left[\left(\frac{x^2}{2} - x + 1 \right) |\kappa_-| + \frac{x^4 m^2}{4} \right] \theta(x_0 - x) \right\}. \quad (45)$$

Clearly, photons radiated to the left of the domain wall ($z < 0$) are right-polarized, while those radiated to its right ($z > 0$) are left-polarized.

5. Pair production

Momentum conservation prohibits the spontaneous photon decay $\gamma(k) \to \bar{f}(p) + f(p')$ in vacuum. However, in chiral matter this channel is open due to the chiral anomaly. This is the cross-channel of the photon radiation computed in the previous section. The scattering matrix is now given by

$$S = i(2\pi)^3 \delta(\omega - \varepsilon' - \varepsilon)\delta(\boldsymbol{k}_\perp - \boldsymbol{p}_\perp - \boldsymbol{p}'_\perp) \frac{\mathcal{M}}{\sqrt{8\varepsilon\varepsilon'\omega V^3}}, \quad (46)$$

where

$$\mathcal{M} = -eQ\,\bar{u}(p')\gamma^\mu v(p)\epsilon_\mu \int_{-\infty}^{+\infty} dz \exp\left\{ i \int_0^z \frac{\tilde{q}_\perp^2 + \tilde{\kappa}_\lambda(z')}{2\omega x(1-x)} dz' \right\}. \quad (47)$$

We introduced new notations $x = \varepsilon'/\omega$,

$$\tilde{\boldsymbol{q}}_\perp = \boldsymbol{p}'_\perp - x\boldsymbol{k}_\perp, \quad (48)$$

and

$$\tilde{\kappa}_\lambda(z) = m^2 + (1-x)x\lambda\sigma_\chi\omega. \quad (49)$$

Notice that the sign in front of the second term is opposite to that of (29). Using the matrix elements listed in [37] one obtains

$$\mathcal{M}_0 = -eQ\,\bar{u}(p')\gamma^\mu v(p)\epsilon_\mu \quad (50)$$

$$= -\frac{eQ}{\sqrt{2x(1-x)}} \left[-m(\sigma+\lambda)\delta_{\sigma',\sigma} + (2x - 1 - \lambda\sigma)(\tilde{q}_x + i\lambda\tilde{q}_y)\delta_{\sigma',-\sigma} \right]. \quad (51)$$

5.1. One infinite domain

In the case the entire matter is a single domain with constant σ_χ, integration over z produces the delta function similar to the one in (34). Substituting the amplitude (47) into (33) and performing summation over spins yields the photon decay rate

$$\frac{dW}{dx} = \frac{\alpha Q^2}{4x(1-x)\omega} \left\{ \left(x^2 + (1-x)^2 \right)(-\tilde{\kappa}) + m^2 \right\} \theta(-\tilde{\kappa}). \quad (52)$$

The condition $\tilde{\kappa} < 0$ is satisfied if $\lambda\sigma_\chi < 0$, $\omega > 4m^2/|\sigma_\chi|$ and $x_1 < x < x_2$ where

$$x_{1,2} = \frac{1}{2}\left(1 \mp \sqrt{1 - \frac{4m^2}{|\sigma_\chi|\omega}} \right). \quad (53)$$

Thus, if $\sigma_\chi > 0$, then only left-polarized photons with $\lambda = -1$ can produce a pair, whereas the right-polarized photons cannot decay at all. The corresponding spectrum is

$$\frac{dW_-}{dx} = \frac{\alpha Q^2}{4\omega} \left\{ \left(x^2 + (1-x)^2 \right)\sigma_\chi\omega + 2m^2 \right\} \theta(x_2 - x)\theta(x - x_1), \quad (54)$$

$$\frac{dW_+}{dx} = 0. \quad (55)$$

In a domain with $\sigma_\chi < 0$ only right-polarized photons decay. The corresponding spectrum is obtained by replacing $W_\pm \to W_\mp$ and $\sigma_\chi \to -\sigma_\chi$ in (54) and (55).

5.2. Two semi-infinite domains separated by a domain wall at $z = 0$

The calculation in the case of chiral matter consisting of two semi-infinite domains separated by a thin domain wall at $z = 0$ is analogous to that in Sec. 4.2. The result is

$$\frac{dN}{d^2\tilde{q}_\perp dx} = \frac{1}{(2\pi)^3}\frac{1}{8x(1-x)\omega^2}\frac{1}{2}\sum_{\lambda,\sigma,\sigma'}|\mathcal{M}|^2 \tag{56}$$

$$= \frac{\alpha Q^2}{4\pi^2}\left\{\left(x^2+(1-x)^2\right)\tilde{q}_\perp^2 + m^2\right\} \times \sum_\lambda \left|\frac{1}{\tilde{q}_\perp^2 + \tilde{\kappa}'_\lambda - i\delta} - \frac{1}{\tilde{q}_\perp^2 + \tilde{\kappa}_\lambda + i\delta}\right|^2, \tag{57}$$

where the values of $\tilde{\kappa}_\lambda$ at $z < 0$ and $z > 0$ are denoted by $\tilde{\kappa}'_\lambda$ and $\tilde{\kappa}_\lambda$ respectively. Replacing $\tilde{\kappa}_\lambda \to m^2 - m_\gamma^2 x(1-x)$ yields the transition pair production spectrum [15,20]. Integration over $\tilde{\boldsymbol{q}}_\perp$ gives

$$\frac{dW_{--}}{dx} = \frac{\alpha Q^2}{16x(1-x)\omega}\left\{\left(x^2+(1-x)^2\right)|\kappa'_- + \kappa_-| + m^2\right\} \times \theta(x - \max(x_1, x'_1))\theta(\min(x_2, x'_2) - x), \tag{58}$$

if $\sigma'_\chi > 0$ and $\sigma_\chi > 0$ and

$$\frac{dW_{-+}}{dx} = \frac{\alpha Q^2}{16x(1-x)\omega}\left\{\left[\left(x^2+(1-x)^2\right)|\kappa'_-| + \frac{m^2}{2}\right] \times \theta(x - x'_1)\theta(x'_2 - x) + \left[\left(x^2+(1-x)^2\right)|\kappa_+| + \frac{m^2}{2}\right]\theta(x - x_1)\theta(x_2 - x)\right\}, \tag{59}$$

if $\sigma'_\chi > 0$ and $\sigma_\chi < 0$. Here $x'_{1,2} = 1/2 \mp \sqrt{1/4 - m^2/|\sigma'_\chi|\omega}$. Eqs. (58) and (59) clearly indicate that in a domain with positive/negative chiral conductivity most pairs are produced by left/right-polarized photons.

6. Summary and discussion

The main result of this paper is that a free charged fermion moving through chiral matter *spontaneously* radiates electromagnetic radiation. This indicates instability of the single-fermion states. We derived the radiation spectrum in two cases: when the matter is a single CP-odd domain, given by (38)–(39), and when it consists of two such domains separated by a flat thin domain wall, given by (43). The photon polarization is determined by the sign of the chiral conductivity: if it is positive/negative, the radiation is right/left-polarized. The cross-channel process of spontaneous photon radiation is spontaneous pair production by a real photon. This indicates instability of the single-photon states. We computed the fermion spectrum and found that in a domain with positive/negative chiral conductivity only left/right-polarized photons decay, see (54), (55).

The rate of energy loss by single-particle states is found to be proportional to $\alpha\sigma_\chi$. Since the temporal evolution of chiral conductivity has much higher rate of σ_χ, it seems plausible that it can play an important role in the long-time dynamics of the radiative instability and perhaps even tame it. This is a problem that deserves further investigation.

In electromagnetic plasma, the existence of the CP-odd domains would trigger the radiative instability causing radiation of photons of a certain polarization, and decay of photons of opposite polarization. These processes tend to polarize the plasma within a domain. Since the $2 \to 2$ scattering as well as transitions due to spatial inhomogeneities [38] are also of order α^2 the question of whether there is an equilibrium polarization of electromagnetic field requires further investigation.

Despite the radiative instability, the Maxwell–Chern–Simons effective theory (2) is a useful tool to study macroscopic effects of the chiral anomaly if σ_χ is a sufficiently small compared to the typical energy scales. This is the case in the quark–gluon plasma where σ_χ is about two order of magnitudes lower than the plasma temperature [39,40]. Thus, convoluting (38) with the Fermi–Dirac distribution over the phase space gives photon spectrum spontaneously radiated by quarks in quark–gluon plasma. Since the spectrum is proportional to α^2 it gives only a minor contribution to the total photon spectrum radiated by the plasma.

The chiral anomaly of QCD can also be described by an effective theory similar to (2). Thus the color fields in the quark–gluon plasma exhibit much the same radiative instability as the electromagnetic field. This effect may have a significant impact on the quark–gluon plasma phenomenology.

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