Evaluation of the Heat Pulse Ratio Method for Measuring Soil Water Flux

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ABSTRACT

Soil water flux is an important hydrologic parameter, yet few techniques for measuring it in situ are available. Here we evaluate the heat pulse ratio method for measuring water flux. We conducted heat pulse measurements of flux in packed columns of sand, sandy loam, and silt loam soil. Water fluxes were calculated from the data following both a traditional temperature increase difference method and a new temperature increase ratio method. Both methods yielded similar estimates of flux, agreeing to within 0.84 cm h^{-1} on average. The low flow detection limits for both methods were also similar and ranged from 0.1 to 0.4 cm h^{-1}. However, the ratio method was superior in that it permitted simpler calculations, reduced the number of required parameters by four, and exhibited two to three times greater precision. We found strong linear relationships ($r^2 \geq 0.98$, standard error $<$ 0.4 cm h^{-1}) between estimated and imposed water fluxes up to 40 cm h^{-1}. However, the slopes of these relationships were less than one, ranging from 0.739 for the sand to 0.224 for the sandy loam. These slopes indicate that the sensitivity was less than predicted by the standard conduction–convection model. We have not discovered the cause of these errors, but we did find that the errors could not be explained by increasing the magnitude of the conduction term in the model as has been previously suggested. Instead, the errors could be explained by reducing the magnitude of the convection term. This finding can help direct future research efforts to improve the accuracy of the ratio method.

Soil water flux can vary in time and space by many orders of magnitude and is a principal variable in subsurface chemical transport, ground water hydrology, and the soil water balance. Measurements of water flux in soil and other porous geologic materials are difficult to obtain, and the lack of suitable measurement techniques complicates the study of many important research problems. For these reasons, scientists continue to search for more effective methods to measure water flux in situ. Recent encouraging developments include the automated equilibrium tension lysimeter (Brye et al., 1999; Masarik et al., 2004), the controlled-suction period lysimeter (Kosugi and Katsuyama, 2004), the vadose zone fluxmeter (Gee et al., 2003, 2002), and heat pulse methods for measuring water flux (Hopmans et al., 2002; Mori et al., 2003; Ren et al., 2000; Wang et al., 2002).

Heat pulse methods for measuring water flux are based on measuring the convective transport of a heat pulse introduced by a small heater. If the relationship between the water flux and the convective transport of heat is known, then flux can be determined. Traditionally, heat pulse methods have relied on the difference between temperature increases at points downstream and upstream of the heat source as the indicator of water flux (Byrne et al., 1967, 1968; Kawanishi, 1983; Melville et al., 1985; Ren et al., 2000). However, the mathematical form of the relationship between flux and the temperature increase difference is complicated (Kluitenberg and Warrick, 2001; Ren et al., 2000). This complexity is an obstacle to the implementation of heat pulse methods. Wang et al. (2002) proposed that the ratio of temperature increases at points downstream and upstream of the heat source would serve as a better indicator of water flux. Wang et al. (2002) showed theoretically that using the temperature increase ratio would result in greatly simplified data analysis if the temperature sensors were equidistant from the heat source. However, few data with which to evaluate the Wang et al. (2002) theoretical finding have been reported (Mori et al., 2003). If the heat pulse ratio method suggested by Wang et al. (2002) can in fact enable precise and accurate in situ monitoring of water flux, then this method will be useful in a wide range of hydrologic research endeavors.

The main objective of this paper is to provide an empirical evaluation of the ratio method. Specifically, soil water flux estimates from the ratio method will be compared with those from the traditional temperature increase difference method, and the relative merits of each method will be considered. Also, flux estimates from the ratio method will be compared with independent flux measurements, and the differences between the two will be examined.

THEORY

The type of heat pulse sensor used in this study consists of three stainless steel needles embedded in an epoxy body. Each of the outer two needles contains a thermocouple at its midpoint, and the full length of the center needle is occupied by an electrical resistance heater. The sensor is connected to an external data logger and power supply. The sensor is embedded in soil through which water flows, such that the plane in which the needles lie is parallel with the flow direction and the axes of the needles are perpendicular to the flow direction. During a measurement, the power supply is turned on for a few seconds generating a heat pulse at the center needle. That heat pulse is transferred by conduction and convection to the

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Abbreviations: DTD, dimensionless temperature increase difference; MDTD, maximum dimensionless temperature increase difference.

temperature sensors where the resulting temperature increase is recorded as a function of time.

To formulate relationships for interpreting this temperature increase data, we begin with the conduction–convection equation for heat transfer. For porous media with water moving uniformly through it in the x direction, the two dimensional conduction–convection equation is commonly written as

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - V\frac{\partial T}{\partial x} \qquad [1]$$

where T in our case is temperature increase (not absolute temperature) (K), t is time (s), α is the thermal diffusivity of the porous media (m^2 s^{-1}), V is the heat pulse velocity (m s^{-1}), and x and y are the spatial coordinates (Marshall, 1958). In our experiments x was typically positive downward.

The heat pulse velocity is related to the water flux, J, by

$$V = J\frac{C_w}{C} \qquad [2]$$

where C_w is the volumetric heat capacity of water and C is the volumetric heat capacity of the porous media (J m^{-3} K^{-1}). Note that J (m^3 m^{-2} s^{-1}) is the volume rate of transport of water per unit area of porous media in the y–z plane. It is the product of the volumetric water content and the pore water velocity in the x direction.

The measured temperature increases are, in theory, described by the analytical solution to Eq. [1] for the case of pulsed heating of an infinite line source parallel to the z-axis and located at $(x, y) = (0,0)$. That solution is

$$T(x, y, t) = \frac{q}{4\pi\lambda}\int_0^t s^{-1}\exp\left[-\frac{(x - Vs)^2 + y^2}{4\alpha s}\right]\mathrm{d}s \qquad 0 < t \leq t_0 \qquad [3a]$$

$$T(x, y, t) = \frac{q}{4\pi\lambda}\int_{t-t_0}^t s^{-1}\exp\left[-\frac{(x - Vs)^2 + y^2}{4\alpha s}\right]\mathrm{d}s \qquad t > t_0 \qquad [3b]$$

where λ is the thermal conductivity of the porous media (W m^{-1} K^{-1}), q is the heating power (W m^{-1}), t is time since the initiation of the heat pulse (s), and t_0 is the duration of the heat pulse (s) (Ren et al., 2000).

Ren et al. (2000) developed a theoretical relationship between dimensionless temperature increase difference (DTD) and water flux. The DTD is defined by

$$DTD = \frac{4\pi\lambda}{q}(T_d - T_u) \qquad [4]$$

where T_d and T_u are the downstream and upstream temperature increases. During a heat pulse measurement DTD varies with time. If the temperature sensors are equidistant from the heater, DTD starts at 0, increases during the heating period, reaches a maximum value after the heating period ends, and then gradually decays back to 0. The maximum value attained by DTD during the measurement is referred to as the MDTD. Mathematically, the MDTD is expressed as

$$MDTD = \int_{t_m - t_0}^{t_m} s^{-1}\left\{\exp\left[-\frac{(x_d - Vs)^2}{4\alpha s}\right] - \exp\left[-\frac{(x_u + Vs)^2}{4\alpha s}\right]\right\}\mathrm{d}s \qquad [5]$$

where t_m is the time that the maximum temperature difference occurs (s), x_d is the distance from the heater to the downstream needle (m), and x_u is the distance from the heater to the upstream needle (m). To determine water flux based on the measured MDTD, Eq. [5] must be solved implicitly to determine the heat pulse velocity. Then, the heat pulse velocity can be converted to water flux by Eq. [2]. This temperature increase difference method will be referred to as the MDTD method.

As an improvement on the MDTD method, Wang et al. (2002) proposed that the ratio of temperature increases downstream and upstream from the heater be used as the indicator of water flux. They began by inserting x_d and x_u into Eq. [3] to obtain integral expressions for T_d and T_u. These expressions were differentiated with respect to time, and then the ratio of the derivatives was calculated. For the special case of $x_d = x_u$, the ratio of the derivatives is independent of time, permitting straightforward integration. This procedure leads to the result

$$J = \frac{\lambda}{x_0 C_w}\ln\left(\frac{T_d}{T_u}\right) \qquad t > 0 \qquad [6]$$

where x_0 is the needle spacing (m).

The requirement that $x_d = x_u$ is difficult to fulfill in practice due to minor variations in the construction of heat pulse sensors. For the case of $x_d \neq x_u$, Wang et al. (2002) showed theoretically that T_d/T_u is time dependent but approaches a constant value as $t \rightarrow \infty$. For large times the relationship between flux and T_d/T_u is approximated as

$$J = \frac{2\lambda}{(x_d + x_u)C_w}\ln\left(\frac{T_d}{T_u}\right) \qquad t >> t_0 \qquad [7]$$

Equations [6] and [7] show that water flux is linearly related to ln (T_d/T_u). This simple linear relationship removes one obstacle that has hindered implementation of heat pulse methods, that is, the previously implicit relationship between flux and sensor response. This new analysis is also attractive because it is computationally simple and it eliminates the need to know q, α, t_m, and t_0. The use of Eq. [7] to estimate water flux will be referred to as the ratio method.

MATERIALS AND METHODS

Experiments

The heat pulse sensors used in this study were based on the design of Ren et al. (1999). The sensors consisted of three 1.3-mm diam. stainless steel needles protruding 4 cm from an epoxy probe head and lying parallel in a common plane separated by 6 mm. The center needle contained an electrical resistance heater and the outer two needles contained chromel-constantan thermocouples.

The measurement system for the heat pulse sensors consisted of a datalogger (21x, Campbell Scientific Inc., Logan, UT)[1], a thermocouple multiplexer (AM16/32, Campbell Scientific Inc., Logan, UT), a multiplexer for the heating circuits (AM416, Campbell Scientific Inc., Logan, UT), a 1Ω current-sensing resistor (VPR5, 0.1% tolerance, Vishay Resistors, Malvern, PA), a 0.5 amp direct-current relay (R42-1D.5-6, NTE Electronics, Bloomfield, NJ), and a direct current power supply (Model 1635, B & K-Precision, Maxtec International Corp., Chicago, IL). The heating power was typically 50 to 60 W m^{-1}, and t_0 was 15 s. The temperatures of the downstream and upstream needles of each sensor were measured before heating and one time per second for 100 s after the initiation

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[1] Mention of trade names or commercial products in this article is solely for the purpose of providing specific information and does not imply recommendation or endorsement by the U.S. Department of Agriculture.

Table 1. Particle-size distribution, organic matter content, bulk density, and saturated hydraulic conductivity (K_{sat}) for the three soils.

Soil type	Particle size Sand	Silt	Clay	Organic matter content	Bulk density	K_{sat}
	%			g kg^{-1}	Mg m^{-3}	cm h^{-1}
Sand	**92**	**5**	**3**	**8.0**	**1.52**	**45.3**
Sandy loam	**66**	**23**	**11**	**23**	**1.32**	**40.6**
Silt loam	**20**	**54**	**26**	**11**	**1.20**	**2.87**

of heating. For each sensor the distances from the heater to the upstream and downstream needles were determined by performing heat pulse measurements with the sensor immersed in water stabilized with agar (6 g L^{-1}) to prevent convection (Ochsner et al., 2003).

We performed laboratory experiments with packed columns of sand (Hanlon series; coarse-loamy, mixed, superactive, mesic Cumulic Hapludolls), sandy loam (Clarion series; fine-loamy, mixed, superactive, mesic Typic Hapludolls), and silt loam soil (Ida series; fine-silty, mixed, superactive, calcareous, mesic Typic Udorthents). The particle-size distribution, organic matter content, bulk density, and saturated hydraulic conductivity for each soil are listed in Table 1. The particle-size distribution was determined by the pipette method (Gee and Or, 2002), and organic matter content was estimated by loss on ignition (Nelson and Sommers, 1996). The soils were air-dried, ground, and sieved to pass a 2-mm screen. The air-dry soil was then packed into 10-cm diam., 20-cm long polyvinylchloride columns. During packing a heat pulse sensor was positioned 5 cm from the bottom of each column. The sensor was centered radially in the column, so the entire sensor body was surrounded by soil. The cross-sectional area of the sensor body was 1.8 cm^2. For comparison, the cross-sectional area of the soil column was 82 cm^2. The sensor body occupied only 2.2% of the column cross-sectional area, so the effect of the sensor body on the water flow was small. We included the entire sensor in the column to mimic the actual application of this method in situ. The sensor leads exited the column through a hole in the side. The plane of the sensor needles was vertical. The finished soil columns were 15 cm high, leaving 5 cm of the PVC column available for ponding water on top of the soil. During the experiments a Mariotte bottle was used to maintain a constant head at the top of the columns. At the bottom of the columns a perforated plate permitted outflow across the entire cross-sectional area into a small water-filled chamber with a single outlet port. By changing the elevation of a water-filled Tygon tube connected to this outlet port, we were able to control the head drop across the column and the water flux.

After packing the air-dry soil into the columns, the columns were flushed with CO$_2$ to displace the ambient air in the soil pores and enhance subsequent saturation. Then, the soil columns were slowly saturated from the bottom with 5 mmol CaCl$_2$ solution. The solution also contained 0.06% formaldehyde by weight to reduce microbial activity over the course of the experiments. After the columns were saturated, the soil thermal properties were determined from heat pulse measurements under no-flow conditions (Bristow et al., 1994). The thermal properties of the soils are listed in Table 2. Theoretical estimates of C (Kluitenberg, 2002) were calculated based on the bulk density and organic matter content, assuming 100% saturation and particle density of 2.65 Mg m^{-3}. The theoretical and measured values of C agreed to within 4% suggesting that the sensor calibration was valid (i.e., no deflection of the needles occurred). The saturated soil columns were subjected to water fluxes ranging from 0.10 to 37.2 cm h^{-1}. The outflow from each column was collected and weighed using a laboratory balance (± 0.01 g precision) to determine the flux. At each imposed flux, heat pulse measurements were obtained for each sensor four times.

Table 2. Thermal diffusivity (α), thermal conductivity (λ), and volumetric heat capacity (C) for the saturated soil columns. Means and (standard deviations) of eight measurements.

Soil type	α	λ	C	Theoretical C
	10^{-6} m^2 s^{-1}	W m^{-1} K^{-1}	MJ m^{-3} K^{-1}	MJ m^{-3} K^{-1}
Sand	**0.773 (0.028)**	**2.17 (0.044)**	**2.81 (0.057)**	**2.90**
Sandy loam	**0.570 (0.016)**	**1.70 (0.027)**	**2.98 (0.039)**	**3.09**
Silt loam	**0.364 (0.016)**	**1.15 (0.038)**	**3.17 (0.036)**	**3.18**

Data Processing

The temperature increase versus time data from the heat pulse measurements were analyzed using the MDTD method and the ratio method. The maximum value of the difference between T_d and T_u was converted to MDTD using Eq. [4] and measured values of λ and q. The time at which this maximum difference occurred (t_m) was also identified. Known values of MDTD, t_m, t_0, x_d, x_u, and α were then used along with Eq. [5] to estimate the heat pulse velocity. Equation [5] was solved implicitly for heat pulse velocity using the Wijngaarden-Dekker-Brent iterative technique (Press et al., 1989). This was accomplished by first writing Eq. [5] in the form (Kluitenberg and Warrick, 2001)

$$MDTD = \exp\left(\frac{Vx_d}{2\alpha}\right)\left[W\left(\frac{x_d^2}{4\alpha t_m}, \frac{Vx_d}{2\alpha}\right) - W\left(\frac{x_d^2}{4\alpha(t_m - t_0)}, \frac{Vx_d}{2\alpha}\right)\right] - \exp\left(-\frac{Vx_u}{2\alpha}\right)\left[W\left(\frac{x_u^2}{4\alpha t_m}, \frac{Vx_u}{2\alpha}\right) - W\left(\frac{x_u^2}{4\alpha(t_m - t_0)}, \frac{Vx_u}{2\alpha}\right)\right] \quad [8]$$

where W is the well function for leaky aquifers, defined as (Hantush, 1964, p. 321)

$$W(u, \beta) = \int_u^\infty z^{-1}\exp(-z - \beta^2/4z)\mathrm{d}z \quad [9]$$

The integral in Eq. [9] was evaluated using the series approximation approach suggested by Kluitenberg and Warrick (2001). Values of heat pulse velocity obtained in this manner were converted to estimates of flux using Eq. [2].

For the ratio method, the average value of T_d/T_u from 40 s $\leq t <$ 50 s was computed. These values were used along with x_d, x_u, and λ in Eq. [7] to calculate water flux. Since the upstream and downstream needles were not precisely equidistant from the heater, the temperature increase ratios asymptotically approached constant values. For the soils, sensors, and fluxes in this study the temperature increase ratios were near their asymptotic values by t = 40 s. At later times for large values of α and water flux, the upstream temperature increase can become too small to maintain acceptable precision. Under those conditions, precision might be improved by increasing the heating power, but in this study we kept the heating power constant. The sampling window used in this study was chosen empirically based on our limited experience and may not be appropriate in all situations. Note that the center of the sampling window was at $t \approx 3\ t_0$.

To quantify the effect of unequal needle spacing in the ratio method, estimates of heat pulse velocity were also obtained using

$$\frac{T_d}{T_u} = \frac{\displaystyle\int_{t-t_0}^{t} s^{-1}\exp\left[-\frac{(x_d - Vs)^2}{4\alpha s}\right]\mathrm{d}s}{\displaystyle\int_{t-t_0}^{t} s^{-1}\exp\left[-\frac{(x_u + Vs)^2}{4\alpha s}\right]\mathrm{d}s} \quad t > t_0 \quad [10]$$

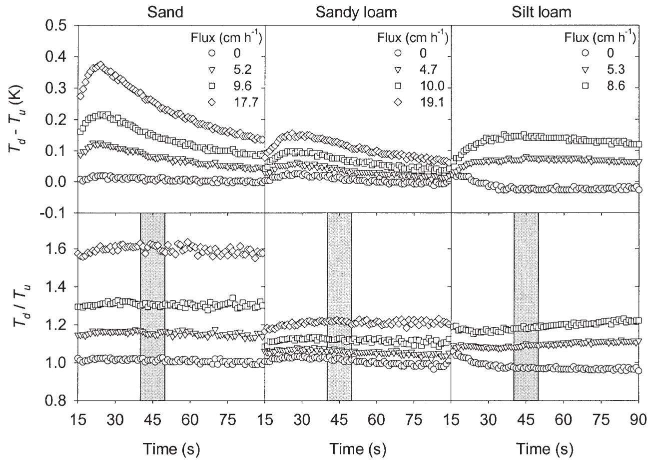

Fig. 1. Samples of measured temperature increase differences (top panel) and temperature increase ratios (bottom panel) in sand, sandy loam, and silt loam soil for water fluxes from 0 to 20 cm h^{-1}. The shaded regions in the bottom panel indicate the time window over which the temperature increase ratio was averaged.

which follows from Eq. [3], the complete analytical solution. Specifically, Eq. [10] was used to estimate heat pulse velocity for known values of t, t_0, x_d, x_u, α, and the value of T_d/T_u corresponding to time t. This approach is similar to the ratio method in that T_d/T_u is used to estimate heat pulse velocity (and subsequently water flux). It differs, however, in that the calculations are based on the complete analytical solution instead of Eq. [7], the approximate solution. Thus, it yields "exact" estimates of heat pulse velocity that account for nonequidistant probe spacing. For each heat pulse measurement, Eq. [10] was evaluated using the measured value of T_d/T_u at $t = 45$ s along with the other necessary parameters. Equation [10] was solved implicitly for heat pulse velocity using the same approach that was employed for solving Eq. [5].

RESULTS AND DISCUSSION

Raw Data

Measured Values of $T_d - T_u$ and T_d/T_u

Samples of the $T_d - T_u$ and T_d/T_u data for a few flow rates are shown in Fig. 1. The $T_d - T_u$ data in Fig. 1 show that as water flux increases the maximum value of $T_d - T_u$ increases. This is consistent with the results of Melville et al. (1985) and Ren et al. (2000). At low flow rates in the silt loam where $x_d > x_u$, $T_d - T_u$ was negative for most of the measurement period, and the minimum rather than the maximum value of DTD was used in Eq. [5]. The T_d/T_u data exhibit some time dependence, but for much of the measurement period T_d/T_u is relatively stable, and the value of T_d/T_u increases as the flux increases. These T_d/T_u data are consistent with the theoretical predictions of Wang et al. (2002).

MDTD and ln(T_d/T_u) as Functions of Water Flux

In Fig. 2a, the measured values of MDTD are plotted against the values of flux determined by collecting the outflow from the columns. The MDTD increased linearly with water flux for all three soils in the measured range of fluxes. Ren et al. (2000) also observed a linear relationship between measured MDTD and flux. The choice of pulsed rather than continuous heating (e.g., Byrne et al., 1968; Kawanishi, 1983) gives rise to the linearity of the MDTD versus flux relationship in the upper portion of this range. Numerical integration of Eq. [3] indicates that if the sensors in the present study were heated continuously, the response of the sensors would be markedly nonlinear above 10 cm h^{-1} (Fig. 3). With pulsed heating the sensor response is expected to be increasingly nonlinear if the flux exceeds 40 cm h^{-1}, but this may be of little concern in natural settings.

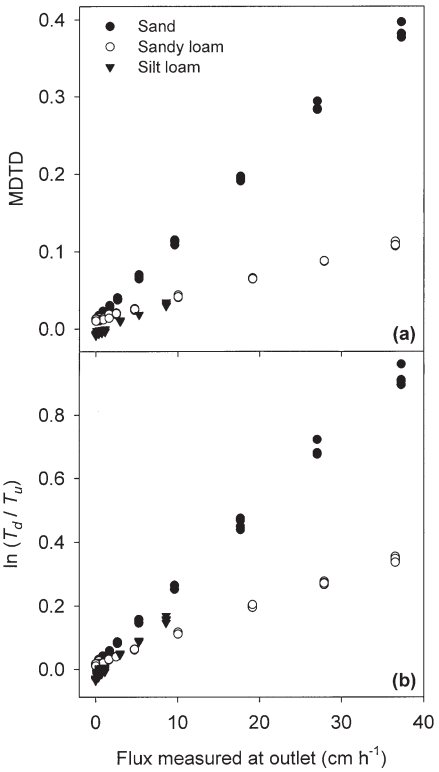

Fig. 2. Measured values of (a) maximum dimensionless temperature increase difference (MDTD) and (b) ln (T_d/T_u) as functions of water flux through the soil columns.

In Fig. 2b the measured values of ln(T_d/T_u) are plotted against the outflow water flux measurements. For all three soils linear relationships exist between the measured values of ln(T_d/T_u) and the measured values of water flux. These data verify the theoretical prediction of a linear relationship between water flux and ln(T_d/T_u) made by Wang et al. (2002).

Comparison of MDTD Method and Ratio Method Water Flux Estimates

One to One Comparison

The flux estimates from the MDTD method are plotted against the flux estimates from the ratio method in Fig. 4. The estimates from these two methods are similar and the data follow the 1:1 line. The mean differences between flux estimated by the ratio method and flux estimated by the MDTD method were 1.19, 0.78, and −0.51 cm h^{-1} for the sand, sandy loam, and silt loam, respectively. The mean absolute difference across all three soils between fluxes estimated using these two

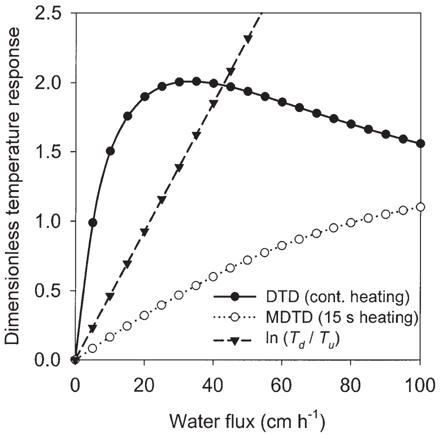

Fig. 3. Theoretical dimensionless temperature responses versus water flux for continuous heating (filled circles), for a 15-s heat pulse using the maximum dimensionless temperature increase difference (MDTD) method (open circles), and for a 15-s heat pulse using the ratio method (filled triangles). Thermal conductivity was 1.5 W m^{-1} K^{-1}, volumetric heat capacity was 2.5 MJ m^{-3} K^{-1}, needle spacing was 6 mm, and for the continuous heating scenario dimensionless temperature increase difference after 3600 s is shown.

methods was 0.84 cm h^{-1}. These differences are due to the combination of errors in each method and cannot be attributed solely to either method.

Precision

The ratio method resulted in more precise measurements of water flux than did the MDTD method. At each imposed flux four repetitions of the heat pulse measurements were performed in each soil. The ratio and MDTD methods were used to estimate soil water flux for each of these four repetitions, and the standard deviation (SD) of the flux estimates were calculated following the procedure of Dixon (1986) for small sample sizes. The numbers in Table 3 are the average SD of water flux estimates from the ratio and MDTD methods for different ranges of water flux. Generally, the SD of water flux is lower for the ratio method than for the MDTD method. The only exception is at fluxes > 10 cm h^{-1} in the sand. At fluxes < 10 cm h^{-1}, the SD of repeated ratio method flux estimates was 0.08 cm h^{-1} averaged across all three soils; for MDTD method flux estimates it was 0.22 cm h^{-1}. Averaged across all fluxes and soils, the SD values for repeated water flux estimates were 0.15 cm h^{-1} for the ratio method and 0.25 cm h^{-1} for the MDTD method. The ratio method exhibited precision that was two to three times greater than the MDTD method. Recall that the measured values of T_d/T_u were averaged over 10 s for the ratio method whereas the MDTD method relied on a single measurement of T_d and T_u. This difference contributed to the greater precision of the ratio method.

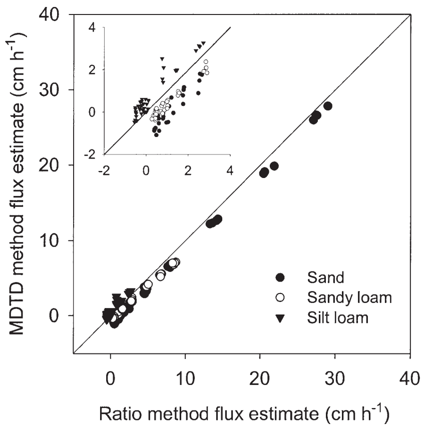

Fig. 4. Comparison of water flux estimated by the maximum dimensionless temperature increase difference (MDTD) method with water flux estimated by the ratio method. The inset expands the scale at the low end.

Table 3. Standard deviation (SD) of water flux estimates from the ratio and maximum dimensionless temperature increase difference (MDTD) methods for different ranges of water flux.

Flux measured at outlet	SD for ratio method			SD for MDTD method		
	Sand	Sandy loam	Silt loam	Sand	Sandy loam	Silt loam
	cm h^{-1}					
0 to 1	0.07	0.09	0.08	0.32	0.16	0.13
1 to 10	0.12	0.05	0.09	0.37	0.19	0.21
>10	0.72	0.15	n/a	0.56	0.21	n/a

Sensitivity at Low Fluxes

The MDTD and ratio methods exhibited similar sensitivity for detecting low flow rates. A *t* test assuming equal variances (Hayslett, 1968) was used to identify the lowest values of imposed water flux at which the mean of four heat pulse flux estimates was significantly greater than the mean of four heat pulse flux estimates in the absence of water flow. This procedure resulted in estimates of the low flow detection limit of water flux for the conditions of this experiment (Table 4). The numbers in parentheses in Table 4 show the probability that the two means are not significantly different. The low flow detection limits range from 0.10 to 0.40 cm h^{-1} and are similar for the ratio method and the MDTD method. When converted to V, these low flow limits range from 3.7×10^{-7} to 1.5×10^{-6} m s^{-1}. Wang et al. (2002) estimated the theoretical low flow detection limits for V to be between 1×10^{-7} and 1×10^{-6} m s^{-1}, and our measurements are consistent with that estimation. For all but one of the cases shown in Table 4, no lower flows were attempted and the actual low flow detection limit may be slightly lower than indicated. Detection limits should be lower for sensors with greater distance between the needles or for porous media with lower λ (e.g., unsaturated soils).

Table 4. Low flow detection limits for the ratio and maximum dimensionless temperature increase difference (MDTD) methods.

Soil type	Ratio method detection limit	MDTD method detection limit
	cm h^{-1}	
Sand	0.19 ($P < 0.001$)†	0.19 ($P < 0.028$)†
Sandy loam	0.40 ($P < 0.078$)	0.28 ($P < 0.018$)†
Silt loam	0.10 ($P < 0.001$)†	0.10 ($P < 0.001$)†

† No lower fluxes were attempted.

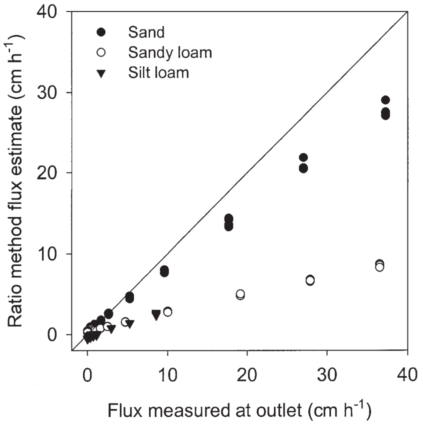

Fig. 5. Water flux estimated by the ratio method versus water flux measured at the column outlet.

Overall Superiority of the Ratio Method

Disadvantages of the ratio method are the error introduced by unequal needle spacing and the lack of a theoretically defined sampling time. The data show that the effects of these disadvantages were relatively small as evidenced by the agreement between the MDTD and ratio method flux estimates. These two methods also exhibited similar sensitivity for detecting low flow rates. However, the ratio method was computationally simpler, eliminated the need to know q, α, t_m, and t_0, and resulted in more precise water flux estimates.

Comparison of Ratio Method Flux Estimates and Outflow Flux Measurements

Accuracy of the Ratio Method

The heat pulse ratio method estimates of water flux are plotted in Fig. 5 against values of water flux measured at the outlet of the soil column. Strong linear relationships exist between the estimates of flux and the measured flux up to 40 cm h^{-1}. Ideally, the plot of estimated versus measured flux would have a slope of one and an intercept of zero, but the experimental results show slopes of less than one and small non-zero intercepts. The slope and intercept from linear regression of the estimated versus measured water flux for each soil are listed in Table 5, along with the standard error (SE) and r^2 for the regression. The slopes range from 0.739 for the sand to 0.224 for the sandy loam, and the intercepts range from -0.364 cm h^{-1} for the silt loam to 0.584 cm h^{-1} for the sand. The linearity and precision of the estimated versus measured water flux relationships are reflected in the r^2 values, which are all ≥ 0.98 and the SE values, which are all <0.4 cm h^{-1}.

Table 5. Slopes, intercepts, standard errors (SE), and coefficients of determination (r^2) from linear regression of water flux estimated using the ratio method versus the measured outflow from the columns.

Soil type	Slope (S)	Intercept	SE	r^2
		cm h^{-1}		
Sand	**0.739**	**0.584**	**0.377**	**0.998**
Sandy loam	**0.224**	**0.435**	**0.121**	**0.998**
Silt loam	**0.342**	**−0.364**	**0.137**	**0.980**

The slopes, visible in Fig. 5 and quantified in Table 5, are less than one indicating that the ratio method was less sensitive to water flux than predicted. The discrepancies were large for the sandy loam and silt loam, which had slopes 78 and 66% lower than expected. These errors arise from some significant, unknown mismatch between theory and experiment. Research to discover and correct the source of this discrepancy is ongoing. For now, it appears that empirical calibration will be needed to obtain accurate water flux measurements in medium textured soils. In contrast, the ratio method is reasonably accurate in sand and may be useful without additional calibration.

Mori et al. (2003) employed the ratio method to calculate water flux from heat pulse measurements in Tottori Dune sand using a multi-function heat pulse probe. Measurements were performed under steady-state saturated flow conditions and transient unsaturated flow conditions. The data show good agreement between estimated and measured flux under saturated conditions in the range of 4 to 40 cm h^{-1}. The data points lie on both sides of the one-to-one line, in contrast to our results in which the estimated flux was consistently less than the measured flux for the Hanlon sand in this range. Still, the data of Mori et al. (2003) provide further evidence that the heat pulse ratio method can be used to effectively determine water flux in saturated sand.

Under transient unsaturated conditions, Mori et al. (2003) were not able to make a direct comparison between flux estimated by the ratio method and flux measured at the column outlet. During transient flow, water flux changes with depth, and the heat pulse sensor was located 5 cm above the column outlet. Therefore, the flux at the sensor was not equal to the flux measured at the column outlet. Mori et al. (2003) used a numerical model to simulate water flux at the sensor and found that the simulated fluxes were extremely sensitive to the estimated soil hydraulic properties. Little, if any, relationship existed between the simulated water flux and that estimated by the ratio method. These results highlight the need for evaluation of the heat pulse ratio method under steady-state unsaturated flow conditions.

Explanations for Non-Zero Intercepts

The non-zero intercepts in Table 5 resulted primarily from the unequal spacing between the upstream and downstream temperature sensing needles. The intercepts were positive for the sand and sandy loam because the downstream needles were slightly nearer to the heater than the upstream needles (Table 6). The silt loam exhibited a negative intercept because the upstream needle was slightly nearer to the heater than the downstream needle. The exact flux estimates obtained using Eq. [10] differed from the ratio method flux estimates by an amount that was relatively constant for each soil. The mean differences between the ratio method flux estimates and the exact flux estimates were 0.609, 0.462, and -0.470 cm h^{-1} for the sand, sandy loam, and silt loam, respectively. These numbers represent the average error introduced by the unequal spacing of the temperature sensing needles. Note that these errors are similar to the intercepts shown in Table 5.

Further Testing

To examine the reproducibility of the slopes shown in Table 5 we repeated the experiment with two additional columns of the silt loam soil. A different sensor was used in each column of the silt loam. Table 6 gives the sensor and soil column pairings, and the sensor calibrations. The slopes of the linear regressions between the ratio method flux estimates and flux measured at the column outlet were 0.390 for silt loam Column II and 0.282 for silt loam Column III. These slopes bracket the slope of 0.342 shown for the original silt loam column in Table 5. For silt loam Column III, we also ran an experiment with flow upward rather than downward. The regression slope for the upward flow was 0.270, so the effect of flow direction was minimal.

Previous Observations of Lower than Expected Sensitivity to Water Flux

Before considering possible explanations for the low slopes observed in this study, we note that previous related work has shown similar discrepancies between measured heat transfer in porous media and that predicted by Eq. [1]. In particular, the theoretical influence of convective heat transfer is generally greater than the measured influence. For example, Ren et al. (2000) found that theoretical values of MDTD were generally greater than measured values of MDTD with sand giving the closest agreement, followed by clay loam, and then sandy loam. They listed four possible causes for these discrepancies: (i) failure of the thermal homogeneity condition as water flux increases, (ii) invalidation of the infinite line source representation of the heater with increasing water flux, (iii) flow distortion by the sensor needles, and (iv) systematic flow nonuniformity within the soil columns. Feldkamp (1996) found that theoretical results overpredicted the response of his heated-cylinder ground water velocimeter by about a factor of 2.5. He did not propose an explanation, but stated that thermal dispersion was not a likely cause in his case. Melville et al. (1985) found that standard heat transfer theory overpredicted the magnitude of the temperature difference between downstream and upstream positions and also predicted the maximum temperature difference would occur earlier than observed. They suggested that the errors might arise from the use of a model for a moving heat source in a stationary medium when in reality the heat source is stationary and the water is moving. In three out of four cases, the sensors of Byrne et al. (1967, 1968) exhibited smaller than predicted temperature differences between upstream and downstream positions. They suggested distortion of the heat and water flow fields by the sensor or water flow bypassing the sensor as possible explanations. Theoretical overpredictions of sensor response are not the exception but rather the rule in thermal techniques for measuring water flux. Heat pulse techniques have also been applied for measuring sap flow, and the uncorrected heat pulse measurements typically underestimate sap flow by a factor of 50% or more (Green et al., 2003). This implies theoretical overpredictions of sensor response of similar magnitude to those observed in geologic media. In the case of sap flow, these errors are often attributed to disruption of flow in the vicinity of the sensor needles (Green et al., 2003).

Table 6. Sensor and soil column pairings with associated downstream (x_d) and upstream (x_u) calibrated spacings. The number in parentheses following the calibrated spacing is a label to identify the needles of the sensor. Sensors were calibrated before each use. The maximum change in calibration between uses was 1.7% for Sensor 16 Needle 3.

Column	Sensor	x_d	x_u
		mm	
Sand	**12**	**6.10 (1)**	**6.28 (3)**
Sandy loam	**16**	**5.90 (1)**	**6.04 (3)**
Silt loam	**12**	**6.25 (3)**	**6.12 (1)**
Silt loam II	**2**	**5.77 (3)**	**6.06 (1)**
Silt loam III	**16**	**5.94 (3)**	**5.87 (1)**

Identifying the Appropriate Type of Correction Factor

The linearity and strength of the relationship between estimated and measured water flux suggest that the form of Eq. [6] is correct, but some correction factor is needed to account for the low slopes. An empirical correction factor can be introduced as follows

$$\frac{a}{b} = \frac{1}{S} \qquad [11]$$

and

$$J = \frac{a}{b}\frac{\lambda}{x_0 C_w}\ln\left(\frac{T_d}{T_u}\right) \qquad [12]$$

where a and b are constants and S is the slope of the linear regression between estimated and measured water flux for each column (Table 5). The slope of water flux estimated by Eq. [12] versus the outflow water flux measurement would be one. Rearranging Eq. [12] and making use of Eq. [2] leads to

$$\frac{T_d}{T_u} = \exp\left(\frac{bVx_0}{a\alpha}\right) \qquad [13]$$

Equation [13] describes the temperature increase ratio predicted by a modified form of Eq. [1]:

$$\frac{\partial T}{\partial t} = a\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - bV\frac{\partial T}{\partial x} \qquad [14]$$

Now if we choose $a = 1/S$ and $b = 1$, then Eq. [14] can be referred to as an "enhanced conduction" model. If instead we chose $a = 1$ and $b = S$, then Eq. [14] can be referred to as a "reduced convection" model. Either way, the slope of flux estimated by Eq. [12] versus the outflow flux measurements would be one. However, the temperature increase versus time curves would be quite different depending on how the correction factor is chosen.

We calculated temperature increase versus time curves for the enhanced conduction model and the reduced convection model and compared the results of each to our T_d and T_u data. The modeled temperature increase curves were generated by numerically integrating Eq. [3]. We

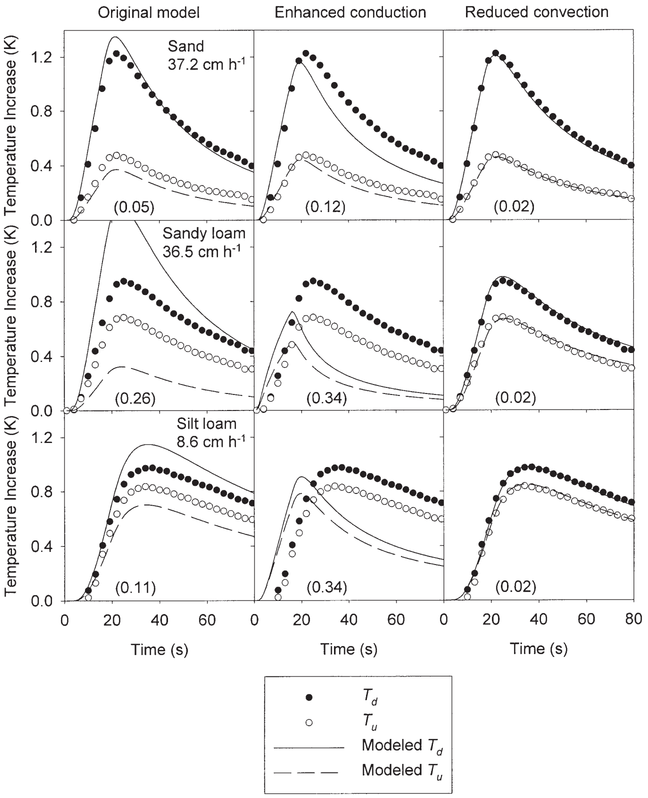

Fig. 6. Measured temperature increases at the highest water flow rate for sand, sandy loam, and silt loam soil and modeled temperature increases using the original model ($a = b = 1$), an enhanced conduction model ($a = 1/S$), and a reduced convection model ($b = S$). Root mean square error (K) is shown in parentheses at the bottom of each pane.

inserted the calibrated values for x_d and x_u, set $y = 0$, multiplied α and λ by a, and multiplied V by b. Figure 6 shows the comparisons between modeled and measured temperature signals for all three soils at the highest imposed flux where disagreement between the original model and the measured data was the greatest. The reduced convection model predicts temperature signals that agree very well with the measured data, in contrast to the original model and the enhanced conduction model. The enhanced conduction model leads to underprediction of the magnitude and time of arrival of the peaks of the temperature signals. The time of peak arrival is inversely related to the effective thermal diffusivity ($a\alpha$), so any model that increases the value of the effective thermal diffusivity must lead to earlier predictions of the time of peak arrival. The enhanced conduction model is an insufficient explanation for these data, and the disagreement between the original model and the measured temperature signals can apparently be attributed to overprediction of the convective heat transfer around the sensor. The physical explanation for this overprediction remains uncertain. Still, from this analysis we conclude that the correction factor to account for the low slopes in Fig. 5 should be applied to the convection term in Eq. [1] and not the conduction term.

Sisodia and Helweg (1998) used a thermal dispersion model to represent the performance of a "heat sense flowmeter" at water fluxes from 142 to 663 cm h^{-1}. Similarly, Hopmans et al. (2002) hypothesized that the theoretical overprediction of MDTD observed by Ren et al. (2000) was due to the failure of Eq. [1] to account for thermal dispersion. The thermal dispersion model is a more elaborate form of the enhanced conduction model. It increases the value of the effective thermal diffusivity as water flux increases. The thermal dispersion model predicts earlier arrival of the peaks than does the original model shown in Fig. 6. The original model accurately predicts the time of arrival of the temperature increase peaks, so the thermal dispersion model would underpredict the time of arrival of the temperature increase peaks. Like the enhanced conduction model, the thermal dispersion model is an insufficient explanation for these data.

The physical basis for the apparent reduced convection is currently unknown, and there are several possible explanations to consider. For example, in Eq. [3] no accounting is made for the finite length and diameter of the sensor needles, and this deficiency is a potential source of error. However, the best available analysis suggests this leads to errors of <3% in the water flux estimates (Hopmans et al., 2002). Also, the fluid and solid phases are assumed to always maintain local thermal equilibrium. The validity of this assumption has not been demonstrated. Furthermore, it is certain that the water follows a three dimensional path around the soil particles and the needles, but the model only considers one dimensional water flow. Finally, we join previous researchers in recognizing the possibility of bypass flow. If the water flow in our columns bypassed the soil surrounding the sensor, the convective heat transfer would be reduced. Such internal flow nonuniformity would be difficult to detect but our use of disturbed (i.e., homogenized) saturated soil perhaps reduced the likelihood of bypass flow. In any case, the similarity between our results and other published data indicates that the behavior we observed is not atypical.

CONCLUSIONS

We conclude that the ratio method for processing heat pulse data to determine water flux is more effective than the MDTD method. Both methods led to similar estimates of flux, but the ratio method exhibited greater precision, was computationally simpler, and reduced the number of required parameters by four.

As in previous studies, the effect of water flux on the measured temperature signals was overpredicted by the commonly applied heat transfer theory. In this study the measured sensitivity to flux ranged from 26% less than predicted for the sand to 78% less than predicted for the sandy loam. We have not discovered the physical cause or causes of these relatively large errors; however, we did find that the errors could not be explained by increasing the conduction term in Eq. [1]. Instead, the errors could be explained by reducing the convection term. This finding can help direct future research efforts to improve the accuracy of the ratio method.

Our results highlight several research needs. At a basic level, a need exists to understand why the effect of convective heat transfer is often less than theoretically predicted. This phenomenon has been observed in many studies where heat pulse techniques were used to measure fluid flow in porous media. Concerning the ratio method for measuring water flux, our findings point to the need to develop a practical and reliable way to determine the correction factor in the reduced convection model. Another need is to minimize the intercept errors caused by unequal needle spacing. Although these errors were 0.6 cm h^{-1} or less in this study, they could be important in low flow measurement applications. Finally, there is a need to evaluate the performance of this method in unsaturated flow conditions.

Overall, we have shown that the heat pulse ratio method proposed by Wang et al. (2002) is a clear improvement on previous heat pulse methods for measuring water flux. Our data also show that with appropriate correction factors it should be possible to obtain accurate flux measurements in the range from 0.1 to 40 cm h^{-1}. The theoretical or empirical determination of these corrections factors is an important remaining challenge facing users of the ratio method.

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REFERENCES

Bristow, K.L., G.J. Kluitenberg, and R. Horton. 1994. Measurement of soil thermal properties with a dual-probe heat-pulse technique. Soil Sci. Soc. Am. J. 58:1288–1294.

Brye, K.R., J.M. Norman, L.G. Bundy, and S.T. Gower. 1999. An equilibrium tension lysimeter for measuring drainage through soil. Soil Sci. Soc. Am. J. 63:536–543.

Byrne, G.F., J.E. Drummond, and C.W. Rose. 1967. A sensor for water flux in soil. “Point source” instrument. Water Resour. Res. 3:1073–1078.

Byrne, G.F., J.E. Drummond, and C.W. Rose. 1968. A sensor for water flux in soil. 2. “Line source” instrument. Water Resour. Res. 4:607–611.

Dixon, W.J. 1986. Extraneous values. p. 83–90. *In* A. Klute (ed.) Methods of soil analysis. Part 1. 2nd ed. Agron. Monogr. No. 9. ASA and SSSA, Madison, WI.

Feldkamp, J.R. 1996. Theoretical analysis of an impervious, heated-cylinder ‘groundwater velocimeter’. International Journal for Numerical and Analytical Methods in Geomechanics 20:815–830.

Gee, G.W., and D. Or. 2002. Particle-size analysis. p. 255–293. *In* J.H. Dane and G. C. Topp (ed.) Methods of soil analysis. Part 4. SSSA Book Series No. 5., SSSA, Madison, WI.

Gee, G.W., Z.F. Zhang, and A.L. Ward. 2003. A modified vadose zone fluxmeter with solution collection capability. Vadose Zone J. 2:627–632.

Gee, G.W., A.L. Ward, T.G. Caldwell, and J.C. Ritter. 2002. A vadose zone water fluxmeter with divergence control. Water Resour. Res. 38:doi:10.1029/2001WR000816.

Green, S.R., B.E. Clothier, and B. Jardine. 2003. Theory and practical application of heat pulse to measure sap flow. Agron. J. 95:1371–1379.

Hantush, M.S. 1964. Hydraulics of wells. Adv. Hydrosci. 1:281–432.

Hayslett, H.T. 1968. Statistics made simple. Doubleday, New York.

Hopmans, J.W., J. Simunek, and K.L. Bristow. 2002. Indirect estimation of soil thermal properties and water flux using heat pulse probes measurements: Geometry and dispersion effects. Water Resour. Res. 38:doi:10.1029/2000WR000071.

Kawanishi, H. 1983. A soil-water flux sensor and its use for field studies of transfer processes in surface soil. J. Hydrol. (Amsterdam) 60:357–365.

Kluitenberg, G.J. 2002. Heat capacity and specific heat. p. 1201–1208. *In* J. H. Dane and G. C. Topp (ed.) Methods of soil analysis. Part 4. SSA Book Series No. 5. SSSA, Madison, WI.

Kluitenberg, G.J., and A.W. Warrick. 2001. Improved evaluation procedure for heat-pulse soil water flux density method. Soil Sci. Soc. Am. J. 65:320–323.

Kosugi, K., and M. Katsuyama. 2004. Controlled-suction period lysimeter for measuring vertical water flux and convective chemical fluxes. Soil Sci. Soc. Am. J. 68:371–382.

Marshall, D.C. 1958. Measurement of sap flow in conifers by heat transport. Plant Physiol. 33:385–396.

Masarik, K.C., J.M. Norman, K.R. Brye, and J.M. Baker. 2004. Improvements to measuring water flux in the vadose zone. J. Environ. Qual. 33:1152–1158.

Melville, J.G., F.J. Molz, and O. Guven. 1985. Laboratory investigation and analysis of a ground-water flowmeter. Ground Water 23:486–495.

Mori, Y., J.W. Hopmans, A.P. Mortensen, and G.J. Kluitenberg. 2003. Multi-functional heat pulse probe for the simultaneous measurement of soil water content, solute concentration, and heat transport parameters. Vadose Zone J. 2:561–571.

Nelson, D.W., and L.E. Sommers. 1996. Total carbon, organic carbon, and organic matter. p. 961–1010. *In* D. L. Sparks (ed.) Methods of soil analysis. Part 3. SSSA Book Series No. 5. SSSA, Madison, WI.

Ochsner, T.E., R. Horton, and T. Ren. 2003. Use of the dual-probe heat-pulse technique to monitor soil water content in the vadose zone. Vadose Zone J. 2:572–579.

Press, W.H., B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling. 1989. Numerical recipes in pascal. The art of scientific computing. Cambridge University Press, New York.

Ren, T., K. Noborio, and R. Horton. 1999. Measuring soil water content, electrical conductivity, and thermal properties with a thermo-time domain reflectometry probe. Soil Sci. Soc. Am. J. 63: 450–457.

Ren, T., G.J. Kluitenberg, and R. Horton. 2000. Determining soil water flux and pore water velocity by a heat pulse technique. Soil Sci. Soc. Am. J. 64:552–560.

Sisodia, S., and O.J. Helweg. 1998. Modeling the heat sense flowmeter. J. Irrig. Drain. Eng. 124:148–151.

Wang, Q., T.E. Ochsner, and R. Horton. 2002. Mathematical analysis of heat pulse signals for soil water flux determination. Water Resour. Res. 38:doi10.1029/2001WR1089.