

STRUCTURE CHARACTERIZATION WITH THERMAL WAVE IMAGING

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INTRODUCTION

Thermal imaging is a technique of recent interest for the nondestructive evaluation of materials. This method attempts to characterize the internal structure of a sample (perhaps to locate flaws—cracks, bubbles, corrosion, etc.) by using its surface temperature response to an external heating. Some recent work on this subject is detailed in [2], [3], [4] and [6].

In this paper the problem of detecting and identifying the location, size and shape of an unknown internal void in a planar domain using thermal methods is examined. The void could represent a defect in the material, or it could be a feature which is supposed to be present, e.g., a conduit, whose location or geometry is to be assessed. The focus is on the case in which the thermal stimulus, an applied heat flux at the boundary of the sample, is a periodic point heat source. Separating the temporal and spatial variables leads to an inverse or domain identification problem for an elliptic equation. This is solved with an optimization approach and uses a boundary integral equation formulation to approximate the heat conduction problem.

MATHEMATICAL MODEL

The two-dimensional sample under consideration is shown in Figure 1. A periodic point heat source is applied on the top sample surface. The resulting periodic temperature response on the top is then measured and used to locate and identify the void D^* . Since the heating is periodic, the temperature $T(t, x)$ can be written as

$$T(t, x) = \operatorname{Re}\{e^{i\omega t}T(x)\} \quad (1)$$

where t denotes time, $x = (x_1, x_2)$ is the spatial variable and the periodic heating

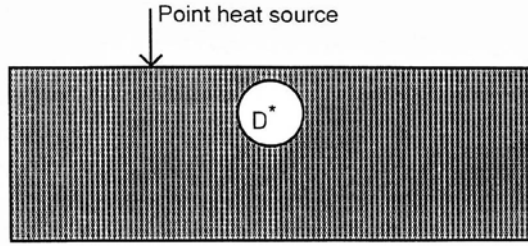


Figure 1. Sample geometry and heating.

has frequency $\frac{\omega}{2\pi}$. The spatial part of the solution, $T(x)$, is assumed to satisfy

$$\begin{aligned} \alpha \nabla^2 T - i\omega T &= 0 \quad \text{in the sample} \\ k \frac{\partial T}{\partial n} &= g \quad \text{on the boundary} \end{aligned} \quad (2)$$

where α is the sample thermal diffusivity, k the sample thermal conductivity and g is the heat flux at the boundary. In our case $g = \delta_P$ where P denotes the point at which the input heat flux is applied. The problem under consideration can be stated as follows: Given measurements of the solution T to equation (2) on the boundary of the sample, identify the void D^* . Note that the function T is complex-valued, consisting of a real or in phase component and imaginary or out of phase component.

We will assume that the void D^* can be described by finitely many parameters, $\vec{q} = (q_1, \dots, q_m)$ and that the boundary of D^* is reasonably smooth, e.g., has finite curvature everywhere. One approach to finding the unknown void is to use an optimization approach. Specifically, suppose $T_i, i = 1, \dots, n$ are measurements at points x_i of the boundary temperature of the sample with void D^* . One can then seek an estimate of D^* by finding that region $D(\vec{q})$ which minimizes the quadratic functional

$$J(\vec{q}) = \sum_{i=1}^m |T_q(x_i) - T_i|^2 \quad (3)$$

where $T_q(x)$ is the solution to equation (2) with $D = D(\vec{q})$ replacing D^* . In short, one varies the vector \vec{q} (and so $D(\vec{q})$) to obtain the best fit to the measured data T_i .

One of the drawbacks to the optimization approach is that minimizing the functional (3) requires many repeated solutions to the heat conduction problem with varying regions D . It is thus highly advantageous to have a means of rapidly solving the heat conduction problem. Moreover, standard methods for minimizing $J(\vec{q})$ also require the derivatives of J with respect to \vec{q} . For these reasons the method of boundary integral equations was chosen for solving equation (2). If the sample without the void D is denoted by Ω then the function $T(x)$ satisfies the

integral equation

$$-\frac{1}{2}T(x) + \int_{\partial(\Omega \setminus D)} T(y) \frac{\partial G(x, y)}{\partial n_y} dS_y = \frac{1}{k} \int_{\partial(\Omega \setminus D)} G(x, y) g(y) dS_y \quad (4)$$

for each $x \in \partial(\Omega \setminus D)$ where $\frac{\partial}{\partial n_y}$ is the normal derivative in the y variable and dS_y is surface measure. The function $G(x, y)$ is a fundamental solution or Green's function for the operator $\nabla^2 - \frac{i\omega}{\alpha}$. Such a function is given by

$$G(x, y) = -\frac{1}{2\pi} \left(\ker\left(\sqrt{\frac{\omega}{\alpha}}r\right) + i\text{kei}\left(\sqrt{\frac{\omega}{\alpha}}r\right) \right) \quad (5)$$

where $r = |x - y|$ and $\ker()$ and $\text{kei}()$ are the Kelvin functions. For a derivation of the boundary integral equation see [5], chapter 3.

There are a variety of methods for solving the equation (4). We have chosen Nyström's method; see [1] for more details. The boundary integral equation offers several advantages for the current identification problem. They are:

It reduces the dimension of the problem by one; the two-dimensional heat equation is reduced to a one-dimensional integral equation.

It only solves for the temperature $T(x)$ where its value is needed, on the surface of the sample, with a corresponding increase in speed.

Allows simultaneous computation of $\frac{\partial T}{\partial \bar{x}}$; these derivatives satisfy the integral equation (4) but with a different right hand side, thus much of the work involved in solving (4) can be re-used to solve for these derivatives.

The method is easy to program, stable and accurate.

One drawback to the boundary integral equation technique is that it is easily applied only to linear equations with constant coefficients.

SIMULATIONS AND STRATEGY

Optimization approaches have a pitfall, namely, the possibility of getting caught in a local minima which is not a global minima. The following example illustrates this point. The sample is taken to be a rectangular aluminum block with length 1.27 cm and height 0.32 cm. The point heat source has a power of one watt and is at a frequency of 3.0 Hz. The "true" void D^* is circular, centered at x_1, x_2 coordinates (0.9 cm, 0.24 cm) where the lower left corner of the sample is (0,0). The void radius is 0.06 cm. The heat source was applied near D^* , at x_1 coordinate 0.9 cm as illustrated in Figure 2. Equation 4 was solved using Nyström's method to yield the temperature $T(x)$ solving equation 2 on the boundary of the sample; the solution was obtained at 40 points on the top surface, denoted by $T_i, i = 1, \dots, 40$. The "prospective" void, D , also a circle, was chosen to have the same x_2 coordinate and radius as D^* but the x_1 coordinate was varied from 0.15 cm to 1.15 cm. For each x_1

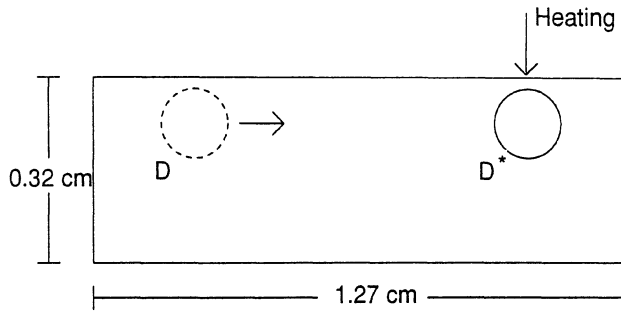


Figure 2. Set up for computation of least-square functional.

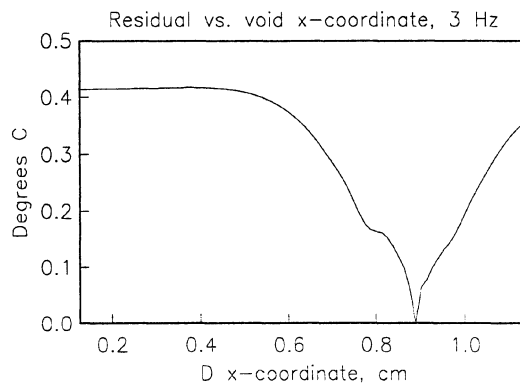


Figure 3. Least-squares functional.

coordinate the functional $J(q)$ was computed and the x_1 coordinate of D versus $J(q)$ graphed to illustrate the nature of the least-squares functional, as shown in Figure 3.

As can be seen in Figure 3, the residual J is zero when the x_1 coordinates of D and D^* coincide. However, any optimization technique requires an initial guess at the parameters. In the present case if one chose an initial guess for the x_1 coordinate of D which was far from the correct coordinate (e.g., between 0.1 and 0.5), then the optimization routine would likely be unable to adjust the initial guess to find the correct x_1 coordinate. The residual curve is nearly flat in this region; in fact, it slopes slightly away from the correct value. Thus, a poor initial guess would probably not converge to the correct parameters for the void, particularly in the presence of noise. Moreover, if the heating is applied far from the true void then computational experiments show that the least-squares functional typically has many local minima which are not global minima; it is essential that the heat source be close enough to “illuminate” the void.

For this reason, the strategy of applying a single heat source, but in multiple locations over the length of the sample, was adopted. By examining the temperature response at the sample surface as the heat source location changes, the void x_1 coordinate can be more accurately located. Computational experiments indicate that both the in and out of phase components should peak as the heat source moves over a subsurface void. The graphs below illustrate this phenomena for the out of phase component of the temperature. The simulated sample is as in Figure 2, with circular void centered at x_1 coordinate 0.635 cm, x_2 coordinate 0.16 cm and radius 0.12 cm. The heating is at 3 Hz and the heat source is applied in 9 different equispaced locations, moving from right to left. Position 1 is $3/4$ of the way along the length of the sample, position 9 is $1/4$ of the way along the length and position 5 half way, directly over the void. Note that the response peaks when the heat source passes over the void.

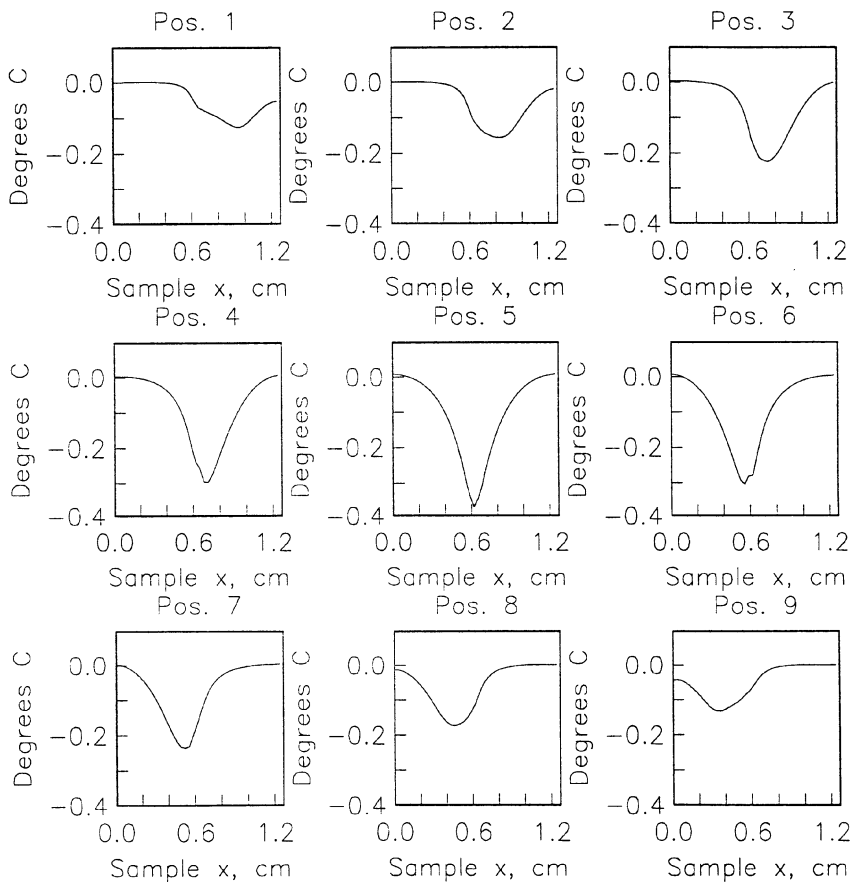


Figure 4. Out of phase response for varying point heat source locations.

EXPERIMENTAL RESULTS

Figure 5 depicts the experimental configuration used to collect thermal data for testing the numerical method. The laser heating source was periodic with a cycle of 0.5 seconds on, 3.5 seconds off. An infrared camera, sensitive in the 8-12 micron range, measures the sample's thermal response at a rate of 15 frames per second. By doing a Fourier transform, one can recover the sample thermal response for frequencies from zero to 7.5 Hz, corresponding to the solution to equation (2). An aluminum block 1.27 cm in length, 0.32 cm in height and 0.20 cm deep was used as the sample. The infrared response was averaged over the z-direction of the top face to provide an approximation to a two-dimensional model. The void is cylindrical with radius 0.12 cm, x coordinate 0.635 cm and y coordinate 0.16 cm. As in the previous computational example, the heating source is applied at 9 equi-spaced points on the top surface.

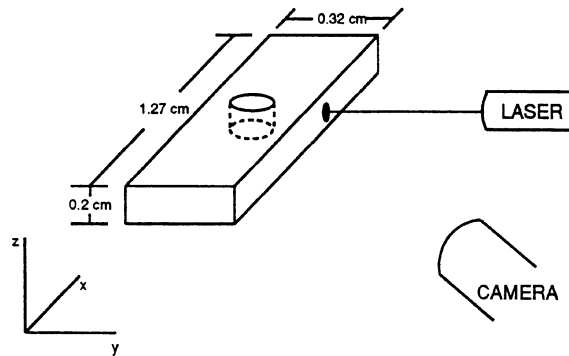


Figure 5. Experimental configuration.

As an example we use the temperature response at 0.94 Hz. The averaged top surface out of phase response for each heating position is shown in Figure 6. It is clear that the peak response occurs for source number 5. The top surface data for source 5 is then used to recover an estimate of the void by performing the minimization of equation (3), using a Levenberg-Marquardt algorithm as outlined in [7]. One twist: since the experimental data contains no information about the scale of the response, all computational and experimental data is re-scaled to a common scale, in this case, an RMS value of 1.0 across the sample top surface. This means that the optimization is attempting to fit the shape of the temperature response without regard to its magnitude. The actual and recovered estimates of the void are shown in Figure 7. The estimated void has radius 0.11 cm and center at (0.65 cm, 0.20 cm).

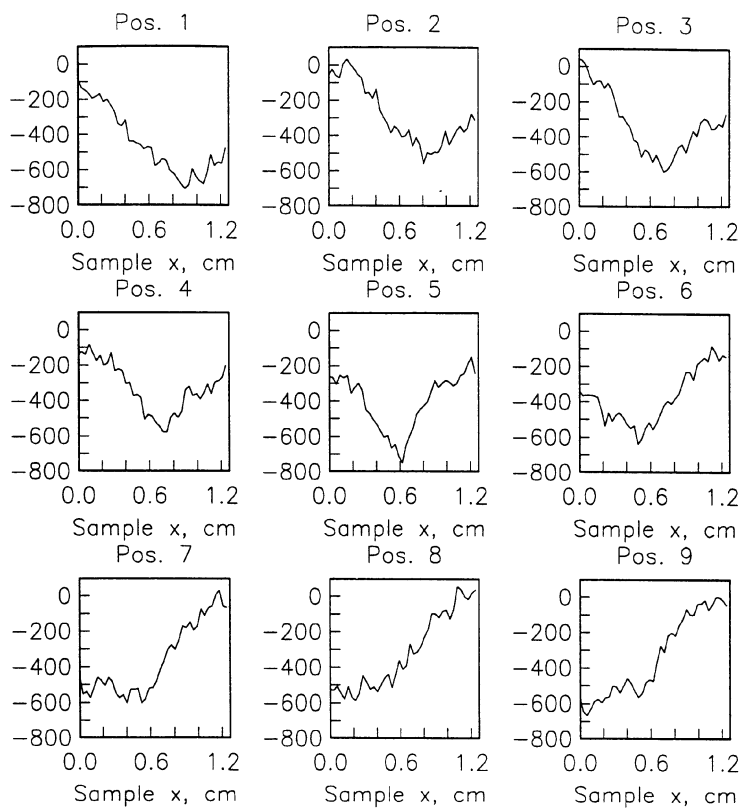


Figure 6. Out of phase experimental response for varying point heat source locations.

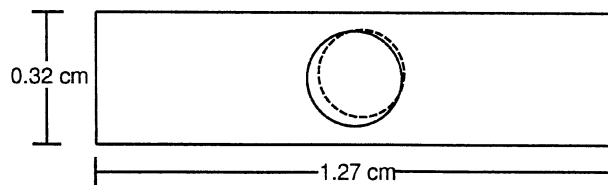


Figure 7. Actual and recovered voids.

CONCLUSION

An algorithm based on a least-squares/optimization approach using a boundary integral method to solve the model heat conduction problem has been demonstrated for locating voids in a sample. As is common with optimization approaches, a reasonable initial guess at the void is needed. Much more experimental data remains to be analyzed, including an analysis of the method's resolution for voids of varying size, depth and shape. This may also include the development of a full three-dimensional boundary integral model for the heat conduction problem.

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