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Modeling of micromagnetic Barkhausen activity using a stochastic process extension to the theory of hysteresis

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Recent work by Bertotti [IEEE Trans. Magn. **MAG-24**, 621 (1988)] and others has shown that it is possible to model the micromagnetic Barkhausen discontinuities at the coercive point using a two-parameter stochastic model. However, the present formulation of the model is restricted to limited regions of the hysteresis curve over which dM/dH is approximately constant and when dH/dt is held at a constant rate. A natural extension of this model is to take the basic result, in which the level of Barkhausen activity in one time period is related to the activity in the previous time period, and increment it by a small amount which is dependent on the differential permeability. The extension of the model proposed here uses the theory of ferromagnetic hysteresis to determine the differential permeability at any point of the hysteresis loop. The Barkhausen activity is then assumed to vary in proportion to the differential permeability. The resulting model allows the Barkhausen sum of discontinuous changes in magnetization to be modelled around the entire hysteresis loop, leading to an important generalization of the basic model.

I. INTRODUCTION

The Barkhausen effect arises from the discontinuous changes in magnetization under the action of a continuously changing magnetic field. The theoretical description of this phenomenon is known to be difficult, but progress has been made by Bertotti[1] in which the Barkhausen events are described by the stochastic motion of a domain wall under the action of an applied field and subject to a random potential inside the material.

The approach taken by Alessandro *et al.*[2] can be used to describe the process by a set of equations that are equivalent to the those used to describe Brownian motion. In that case the position of a particle in a given time interval is correlated with its position in the previous time interval, even though the motion occurring in the time interval is random in both direction and magnitude. The same approach can be applied to the number of Barkhausen events in a given time interval t_n; these will be correlated with the number of events in the previous time interval t_{n-1}.

In the original formulation[2] this was handled by requiring that $dB/dt=$const, which can be managed over limited regions of the magnetization curve when dB/dH is constant and the field is changed at a constant rate with time, $dH/dt=$const. In the present work it is shown how this procedure can be extended to the entire hysteresis loop provided dB/dH can be found at each point. This can be described using the theory of hysteresis,[3,4] so that variable dB/dH can be taken into account. In fact, it is easily shown that the model can be extended further to take into account the number of Barkhausen events in time intervals of different length.

II. RELATIONSHIP BETWEEN BARKHAUSEN EVENTS AND CHANGES IN BULK MAGNETIC PROPERTIES

The rate of change of magnetization with time $\dot{M}$, which is constant in the analysis of Alessandro *et al.*,[2] is determined by the differential susceptibility χ' and the rate of change of field with time, $\dot{H}$,

$$\dot{M}=\left(\frac{dM}{dH}\right)\left(\frac{dH}{dt}\right)=\chi'\dot{H}. \quad (1)$$

It is an assumption of the present model that the level of Barkhausen activity in a given time interval is proportional to $\dot{M}$,

$$\frac{dM_{JS}}{dt}\propto\chi'\dot{H}, \quad (2)$$

where M_{JS} represents the Barkhausen activity in terms of the "jump sum" as discussed by Swartzendruber *et al.*[5,6] and is basically of the product of number of events N and the average Barkhausen jump size $\langle M_{disc}\rangle$ over a given time interval. More precisely, we can write the discontinuous changes in magnetization in any time period as

$$M_{JS}=\sum m_i=N\langle M_{disc}\rangle,$$

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where m_i are the individual changes in magnetization. It follows that the Barkhausen jump sum M_{JS} in a given period Δt is proportional to the total change in magnetization ΔM in that period,

$$M_{JS}=\gamma\Delta M=\gamma\left(\frac{dM}{dt}\right)\Delta t$$

$$=\gamma\left(\frac{dM}{dH}\right)\left(\frac{dH}{dt}\right)\Delta t=\gamma\chi'\dot{H}(\Delta t) \quad (3)$$

where γ is simply a coefficient of proportionality ($0\leqslant\gamma\leqslant1$), which represents the ratio of discontinuous magnetization change to total magnetization change.

The remaining question is to find out how the Barkhausen emission events, as represented by the "jump sum," in a given time interval correlate with the events of the previous time interval. If we assume that $\langle M_{disc}\rangle$, the average jump size, is the same in two consecutive periods, then this correlation amounts to a correlation of the number of events $N(t_n)$ and $N(t_{n-1})$ in consecutive time periods.

III. BARKHAUSEN EFFECT MODEL

The Barkhausen activity in a given period is related to Barkhausen activity in the immediately preceding period, by incrementing N by an amount ΔN which is random, but is also small compared with N. If we begin by considering $N'(t_n)=dN(t_n)/dM$, the number of Barkhausen events per unit change of M in a given time period t_n, then this can be described by an equation of the form

$$N'(t_n)=N'(t_{n-1})+\Delta N'(t_{n-1}). \quad (4)$$

The number of Barkhausen events is a random process in which the probability of a Barkhausen event occurring at any given location is low, but there are a large number of locations. The uncertainty in the number of Barkhausen events from one measurement period to the next will be $\sqrt{N}$ where N is the total number of events. This is the standard deviation of a Poisson distribution. Therefore, we can incorporate this uncertainty into the statistics using the relation

$$\Delta N'(t_{n-1})=\delta_{rand}\sqrt{N'(t_{n-1})}, \quad (5)$$

where δ_{rand} is a random number in the range $-1\leqslant\delta_{rand}\leqslant+1$.

The Barkhausen jump sum in the interval between t_{n-1} and t_n will be the sum of all discontinuous changes in magnetization in that time interval, which on average will be

$$M_{JS}(t_n)=\langle M_{disc}\rangle N(t_n)=\langle M_{disc}\rangle\int_{t_{n-1}}^{t_n}dN(t). \quad (6)$$

Thus,

$$M_{JS}(t_n)=\langle M_{disc}\rangle\int_{t_{n-1}}^{t_n}\left(\frac{dN}{dM}\right)\chi'(H)\dot{H}\,dt. \quad (7)$$

This can be converted into the jump sum rate dM_{JS}/dt by differentiation,

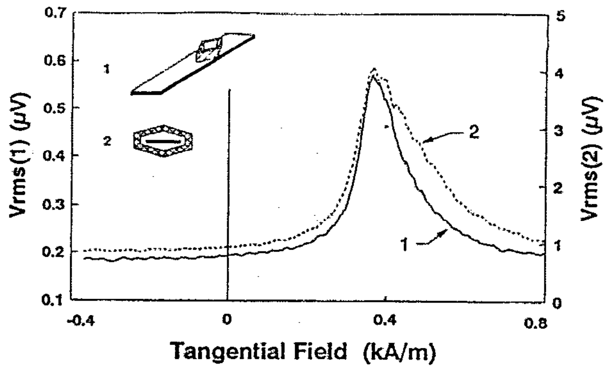

FIG. 1. Experimental Barkhausen signal amplitude after Swartzendruber and Hicho (Ref. 6).

$$\frac{dM_{JS}(t_n)}{dt}=\langle M_{disc}\rangle\chi'\dot{H}N'(t_n), \quad (8)$$

$$\frac{dM_{JS}(t_n)}{dt}=\langle M_{disc}\rangle\chi'\dot{H}[N'(t_{n-1})+\delta_{rand}\sqrt{N'(t_{n-1})}]. \quad (9)$$

If dH/dt is kept constant, and assuming $\langle M_{disc}\rangle$, the average value of the changes in magnetization per Barkhausen jump remains constant,

$$\frac{dM_{JS}(t_n)}{dt}\propto\chi'[N'(t_{n-1})+\delta_{rand}\sqrt{N'(t_{n-1})}]. \quad (10)$$

This is the key equation in the extension of the model. It allows the number of Barkhausen events in time period t_n to be related to the number of events in time period t_{n-1} by a random increment $\Delta N(t_{n-1})$ in $N(t_{n-1})$, which is small enough to ensure that $N(t_n)$ is correlated with $N(t_{n-1})$. The equation can also take into account changes in differential susceptibility χ' and in the rate of change of magnetic field dH/dt.

IV. RESULTS

The Barkhausen emission voltage, which is determined by the jump sum rate dM_{JS}/dt, has been investigated as a function of magnetic field H by several authors, including Swartzendruber *et al.*[5] and Theiner and Altpeter.[7] In these cases dM_{JS}/dt exhibits a peak at the coercive field and decays away to zero at higher field strengths. An example from the recent work of Swartzendruber and Hicho is shown in Fig. 1.

The results obtained with model equations used in the present work are shown in Figs. 2, 3, and 4. It can be seen from these results that the features of the Barkhausen spectrum are described well by this model, and furthermore the effects of stress on Barkhausen spectrum, as reported by Jiles, Garikepati, and Palmer[8] can be modeled, using the dependence of differential permeability on stress given by Jiles.[9]

V. CONCLUSIONS

This work describes an extension of the stochastic process model of the Barkhausen effect. The Barkhausen jump

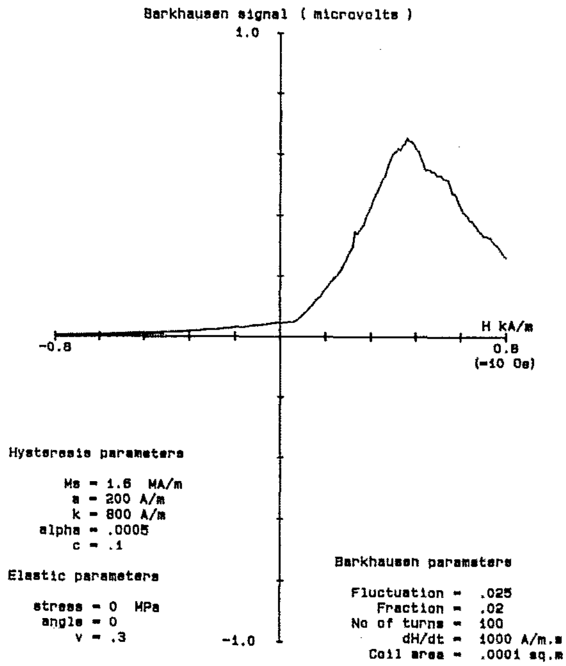

FIG. 2. Modelled Barkhausen signal amplitude under zero stress.

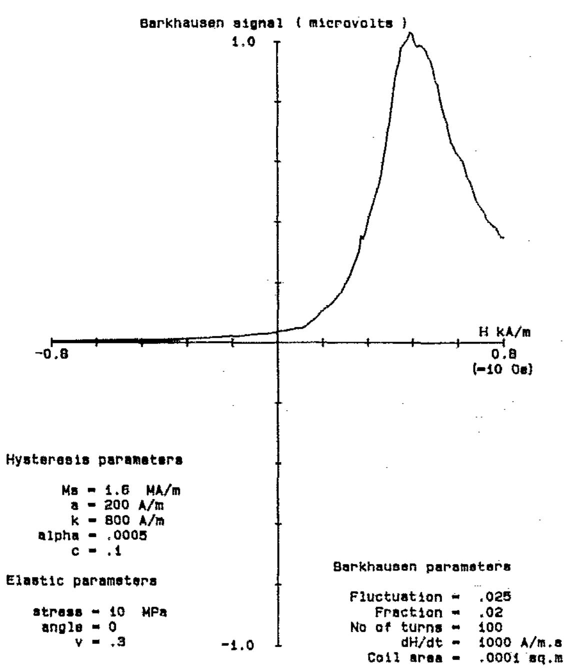

FIG. 3. Modelled Barkhausen signal amplitude under 10 MPa tension.

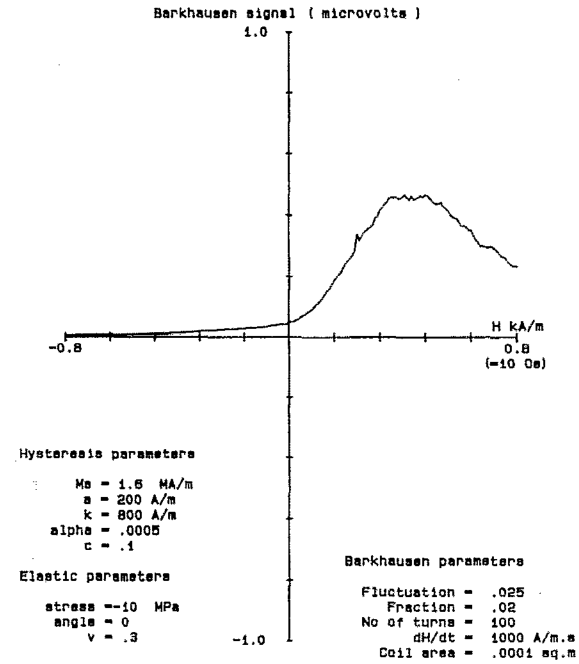

FIG. 4. Modelled Barkhausen signal amplitude under 10 MPa compression.

sum is shown to be proportional to the differential permeability and the rate of change of field. Therefore, the model can be used to describe the Barkhausen effect over ranges where both χ' and $\dot{H}$ vary.

The Barkhausen event rates per unit change in magnetization in two successive time intervals are correlated, yet the difference between them is a small random number. The size of this difference is determined by the square root of the number of counts per unit change in magnetization in the previous time interval.

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