

POWER AND EMPLOYMENT*

by

Roy Gardner

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In any economic system, but especially in centrally planned economies, there exists an important interaction between the political structure and the economic outcome. In a society like that of the Soviet Union, there are considerable differences in power among the various segments of society. These differences in power lead to the existence of economic classes, not unlike those in capitalist economies.¹ The difference in economic outcome in such circumstances can be striking. During the 1920's the Soviet Union experienced substantial unemployment among the industrial labor force, with unemployment rates exceeding 10%. At the same time, an industrial worker or employee who belonged to the Communist Party had between 1 and 4% chance of being unemployed.² This paper constructs an explanation of this fact.

The model constructed portrays the spoils system, first referred to by Senator Marcy of New York in 1832: "To the victor belong the spoils." We consider a game played on two levels. The game on the first level describes the struggle among groups for political power. The game on the second level describes how a coalition which wins the political struggle distributes jobs. Spoils here means access to jobs, as controlled by a winning coalition. The solution of each game is given by the Shapley value, which measures a player's expected value from playing in the game. The Shapley value provides insight into many questions of economics,³ and its role here is to formalize the connection between political power and the distribution of employment.

This decomposition of the employment allocation process into games on two levels is valid whenever the political and economic systems are closely

linked, as in the Soviet Union. Besides distributing goods like jobs, however, one can also imagine reverse spoils systems which distribute bads like involuntary armed service. If resistance to the draft is futile, there is a duality between the spoils system and its reverse: the probability of getting a job and the probability of avoiding the draft are the same, when the unemployment rate equals the draft quota. In this way, one simple model explains two important politically-based forms of privilege in the Soviet Union: access to jobs and avoidance of armed service.

I. Model of the Spoils Game

Society N consists of n agents, numbered 1 through n . A coalition S is any subset of N .

The political structure of society is described by the set of winning coalitions W . A winning coalition rules the society if it forms. It seems reasonable to impose the following three conditions on the political structure. First, the society itself is winning, N belongs to W . This is one expression of the Pareto condition. Second, if the coalition S is winning, the countercoalition $N-S$ is not. Two groups cannot both rule simultaneously. Finally, if S is winning, any coalition that contains S is winning. A winning coalition does not lose strength by growing larger. A coalition is minimal winning if it contains no subset which is itself winning. Minimal winning coalitions cannot afford any defections and still rule. Such coalitions turn out to play a special role in the model.

An individual player i has a veto if he belongs to every winning coalition. No coalition can win without him. Player i is a dictator if he is himself a winning coalition. This constitutes the strongest form of veto power. There can be at most one dictator, but there can be several players with veto power. When $W = \{N\}$, and only the grand coalition rules, then every player has a veto power.⁴

The characteristic function game on the political level, v_I , takes the form

$$v_I(S) = \begin{cases} 1 & \text{if } S \text{ is winning} \\ 0 & \text{otherwise.} \end{cases}$$

Player i 's marginal contribution to coalition S is the difference between

what S can do with as opposed to without him:

$$v_I(S \cup \{i\}) - v_I(S).$$

If S is losing with and without player i, then his marginal contribution is $0-0 = 0$. If S is winning both with and without player i, then his marginal contribution is $1 - 1 = 0$. If S wins with player i and loses without him, then his marginal contribution = $1 - 0 = 1$. In the latter case, player i is called pivotal.

The Shapley value of player i in a game v, $\phi v(i)$, is the expected value of player i's marginal contribution. This expectation is taken over random orders of the set of players, with all random orders equally likely. For the game v_I just defined, the Shapley value is the probability that a player is pivotal. The greater a player's Shapley value, the more likely he is to count in a winning coalition. For instance, $\phi v(i) = 1$ when i is a dictator. On the other hand, when a player i is never pivotal, his Shapley value is zero. $v_I(i)$ varies within these extremes, with total power equalling one:

$$\phi v_I(N) = \sum_{i=1}^n \phi v_I(i) = 1.$$

For these reasons, the Shapley value is often interpreted as a measure of political power.

The game at the economic level, v_{II} , portrays the allocation of job slots in the economy. Suppose each job is indivisible, each player can fill at most one job, and that there are k jobs in total available to the economy, $k \leq n$. The number of available jobs is exogenous to the power structure.

Regardless of which winning coalition forms, it cannot increase or decrease the job total. Within this limitation, a winning coalition S with s members can divvy up jobs as it sees fit. There are two possibilities. One is that $k < s$, in which case S can guarantee its members k jobs. The other is that $k \geq s$, in which case S can guarantee all its members jobs. On the other hand, a losing coalition cannot assure its members of any jobs. If each match of a coalition member and a job counts for one, then a coalition playing the spoils game can assure its members of v_{II} jobs, where

$$v_{II}(S) = \begin{cases} \min(k,s) & \text{when } S \text{ is winning} \\ 0 & \text{otherwise.} \end{cases}$$

II. Job Probabilities for the Spoils Game

One of the bad things about unemployment is that it tends not to be distributed evenly. The main idea of this paper is that the uneven distribution of employment is a direct reflection of power differences. To see this, we need to take a close look at the Shapley value for the economic level game ϕv_{II} . This again represents player i 's expected marginal contribution to a coalition. Alternatively, the Shapley value is the probability that player i gets a job.

To begin with, there are three possible marginal contributions. If S loses with and without i , then i 's marginal contribution is zero. He is merely contributing excess labor to a losing, hence jobless, coalition. If S is winning, but $k \leq s$, then i 's marginal contribution is again zero. He is bringing labor to a winning coalition, but all job slots are already filled. If S is winning and $s < k$, then i 's marginal contribution is one. His marginal contribution of labor now fills a slot. Finally, if S loses without i and wins with i , then i is pivotal and his marginal contribution is $\min(k, s+1)$. All the members of S owe their jobs to i 's having joined. One attaches probabilities to these possibilities via the random order hypothesis, taking into account the power structure defined at the political level. Player i 's expected value from playing the employment game is simply his probability of getting a job, based on his political role.

The interpretation of the Shapley value just given requires that $\phi v_{II}(i) \leq 1$. This need not always be the case, but violations can be handled by Shapley's

device of λ -transfers.⁵ This is a way of formalizing the interpretation of $\phi v_{II}(i) > 1$ as meaning that i is sure to get a job. Suppose that i 's filling a job slot counts for λ_i instead of 1. Let S be winning. Then i 's marginal contribution is λ_i if S has slots available and $\lambda_i - 1$ otherwise. If i is pivotal, his marginal contribution is $s + \lambda_i$. Define the game v_{II_λ} to reflect this:

$$v_{II_\lambda}(S) = \begin{cases} \min(s, k) & \text{if } S \text{ is winning and } i \text{ not in } S \\ \lambda_i + \min(s-1, k-1) & \text{if } S \text{ is winning and } i \text{ in } S \\ 0 & \text{otherwise.} \end{cases}$$

Finally, λ_i is determined by the condition

$$1 \cdot \lambda_i = v_{II_\lambda}(i).$$

Player i 's expected marginal contribution in the game v_{II_λ} is λ_i times his probability of getting a job, where the latter probability is 1.

We will adopt a notation to handle both these Shapley values. Let $x_i(k)$ denote the probability that player i has a job, given that the number of jobs is k . The vector $x(k)$ of job probabilities as given by the Shapley value is thus the main theoretical interest of the paper. Indeed, we shall trace out the path of $x(k)$, as k varies from 0 to n . Several observations about $x(k)$ can be made immediately. For $k = 0$, then $x_i(k) = 0$. When there are no jobs, there is no probability of getting a job for any player i . For $k = 1$, $x_i(1) = \phi v_I(i)$. The rationing of the only job in the economy identically reflects the distribution of political power. For $k = n$, $x_i(k) = 1$ for all i . When there are jobs for everybody, then everyone gets a job, regardless

of power differentials. The in-between cases, $2 \leq k < n$, require more work.

Job rationing is uniform when $x_i(k) = k/n$ for all i . Not surprisingly, this occurs when the distribution of power is even. Take for example the 3-player majority-rule game. Any coalition with two or more players is winning. The distribution of power is $(1/3, 1/3, 1/3)$. When there are two jobs available, each player's expected marginal contribution is $1/3(2)$, since each pivots with probability $1/3$ and a coalition with two members fills two jobs. The resulting $x(k)$ is depicted in figure 1a.

Even for 3-player games, there can exist substantial power differentials. Suppose any majority is winning, as long as it includes player 1. Then player 1 has veto power, although he is not a dictator. The distribution of power is $(4/6, 1/6, 1/6)$. The probabilities of employment are depicted in figure 1b. Player 1 is much more likely to be employed, and in fact is sure of a job when $k = 2$.

Powerless players can also arise when $n = 3$. Suppose that any majority that contains both players 1 and 2 is winning. Both 1 and 2 have a veto power, while player 3 is powerless. The latter is never pivotal. The distribution of power then is $(1/2, 1/2, 0)$. The only time player 3 has a chance for a job is when there is full employment. Players 1 and 2 are sure of jobs when $k = 2$. Figure 1c depicts this set of outcomes $x(k)$. Thus, one already sees in the case of 3-player games the dramatic impact of power differentials on the probability of being employed. We now turn to some results concerning $x_i(k)$ for general n .

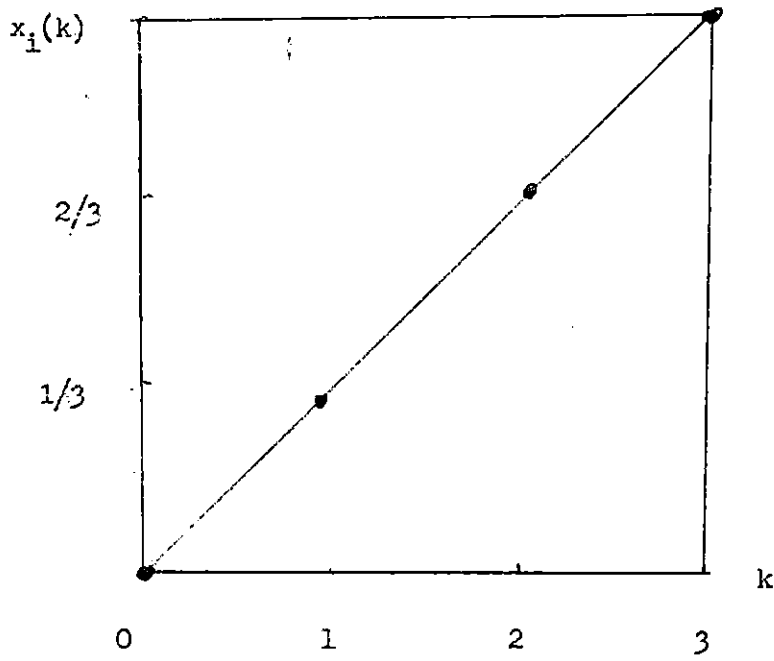


Figure 1a. Probability of employment for the 3-player, majority-rule game.

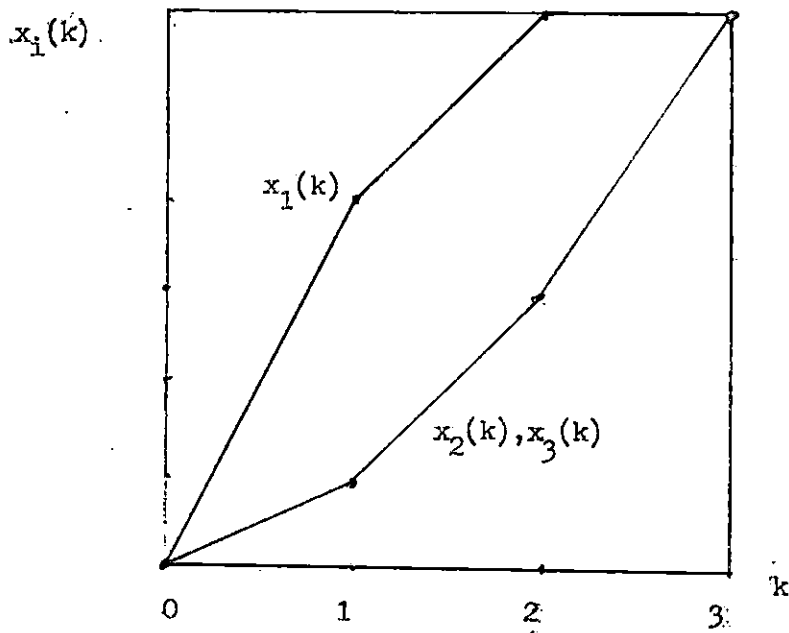


Figure 1b. Probability of employment for the 3-player, majority-rule game, with 1 veto player.

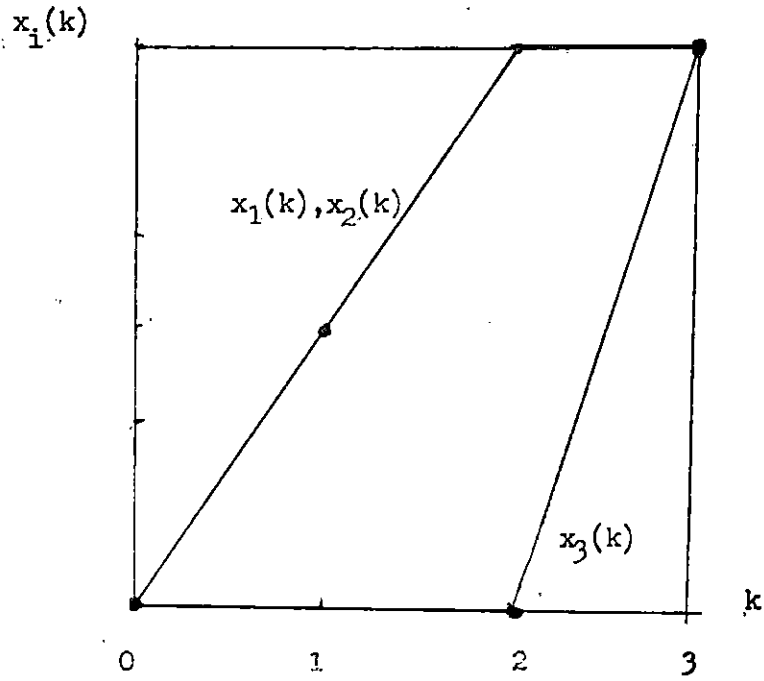


Figure 1c. Probability of employment for the 3-player majority rule game with 2 veto players.

Proposition 1. Suppose player i has veto power. Then there exists k^* such that $x_i(k) = 1$ for all $k \geq k^*$. An upper bound for k^* is given by \underline{s} , the size of the smallest minimal winning coalition.

Proof. Let c be the number of players with veto power. If $\underline{s} = n$, then every agent has veto power, and $c = n$ as well. Since every agent is equally powerful, $x_i(k)$ is given by uniform rationing. The critical $k^* = \underline{s}$ in this case.

Suppose then that $c \leq \underline{s} < n$. Let $k = \underline{s}$; we wish to show that $x_i(\underline{s}) = 1$, for i having veto power. It suffices to show that $\phi_{v_{II}}(i) \geq 1$, when $k = \underline{s}$. A veto player i can never join a coalition which is already winning. The only time his marginal contribution differs from zero is when he is pivotal. A veto player's expected marginal contribution must be at least as great as the product of \underline{s} and his probability of being pivotal:

$$\phi_{v_{II}}(i) \geq \underline{s} \phi_{v_I}(i).$$

Now if any coalition of size \underline{s} containing all c veto players is winning, then the power of an individual veto player i is given by⁶

$$\phi_{v_I}(i) = 1/c - ((\underline{s}-1)!(n-c)!)/(n! (\underline{s}-c-1)!c).$$

Then $\underline{s} \phi_{v_I}(i) \geq 1$, since

$$(\underline{s}-c)! n! \geq \underline{s}! (n-c)! \quad \text{implies}$$

$$(\underline{s}-c) \geq (\underline{s}! (n-c)!)/(n! (\underline{s}-c-1)!) \quad \text{implies}$$

$$\underline{s} c \phi_{v_I}(i) \geq c.$$

Finally, suppose not all players without veto power can figure in the smallest

sized minimal winning coalition. This makes a veto player even more likely to pivot. Thus, for employment $k = \underline{s}$, and any game v_I with c veto players, $x_i(\underline{s}) = 1$. Since the same argument will hold for any $k > \underline{s}$, we have shown that \underline{s} is the desired bound for k^* .

Proposition 2. Suppose player i is powerless. Then $x_i(k) = 0$ for $k \leq \underline{s}$.

Proof. The only case in which a powerless player makes a positive marginal contribution is when he joins a winning coalition and the size of that coalition $s < k$. If $k \leq \underline{s}$, then powerless player i never makes a positive marginal contribution and his expected marginal contribution $\phi^{v_{II}}(i) = 0$.

Comparing these two results, one can see that when employment is power determined; all players with veto power will be certain to have jobs before any powerless agent has even a positive probability of a job. This is reminiscent of such constructions as Marx's industrial reserve army.⁷ The powerless, like an industrial reserve army, are only called into the production process as a last resort.

For players i in between the extremes of veto power and powerlessness, $x_i(k)$ will in general be an increasing function of k . To start with, $x_i(1)$ will be positive. There are two ways in which increases in k will raise i 's job prospects. First, i 's marginal contribution when he is pivotal will rise as k in $\min(k, s)$ rises. Second, when i joins a winning coalition it is more likely that $s < k$, and he makes a positive marginal contribution. For players in this power range, $x_i(k)$ may never reach 1 before k reaches n . If $x_i(k)$ does reach 1, it will be after $k = \underline{s}$.

These two possibilities can be illustrated with a 4-player game. Suppose the minimal winning coalitions are $\{1,2\}$ and $\{1,3,4\}$. Player 1 has a veto but the other players do not. Player 2 is somewhat stronger than 3 or 4, the distribution of power ϕv_I being $(7/12, 3/12, 1/12, 1/12)$. Player 1, according to Proposition 1, is sure of being employed when $k > 2$. Player 2 is sure of being employed when $k = 3$. The two weakest players are sure to be employed when $k = 4$. Figure 2a depicts this result. The weakest player in the game will not be sure of employment until $k = n$. Somewhat stronger players have a better chance of being sure of employment as k approaches n .

A situation very useful in applications is one in which no player has veto power, no player is powerless, and individual power can take on exactly two values. This covers for instance societies with two political classes, one more powerful than the other. The simplest illustration of this situation is the 4-player game with minimal winning coalitions $\{1,3,4\}$ and $\{2,3,4\}$. No individual has veto power, but either individual 1 or 2 is more likely to be pivotal than individual 3 or 4. The distribution of power ϕv_I equals $(1/3, 1/3, 1/6, 1/6)$. The employment probabilities for this case are depicted in figure 2b. Note that no player is sure of employment until $k = n$, although a more powerful player is always more likely to be employed than a less powerful player.

For n large enough in the 2-class model, the job probabilities $x_i(k)$ are approximately linear in k until a neighborhood of $k = n$ is reached. Indeed, this latter is an exact result if one uses a model with a continuum of players. Thus, for mass situations where there is a single significant political

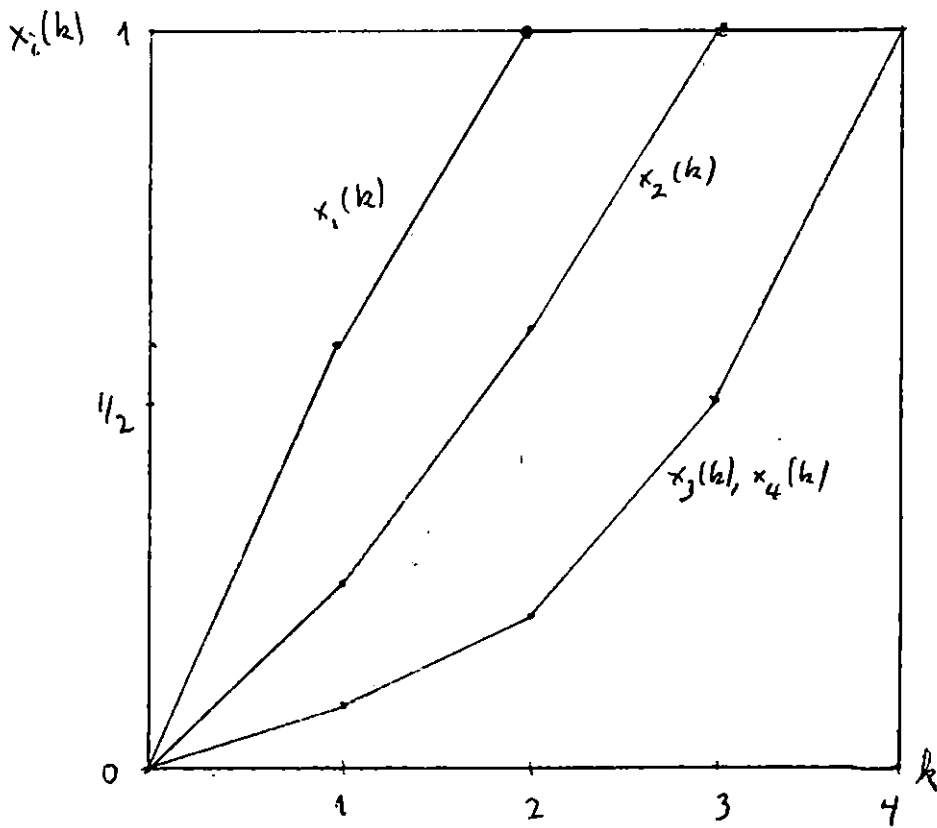


Figure 2a. Employment probabilities, $\{1,2\}$ and $\{1,3,4\}$ minimal winning.

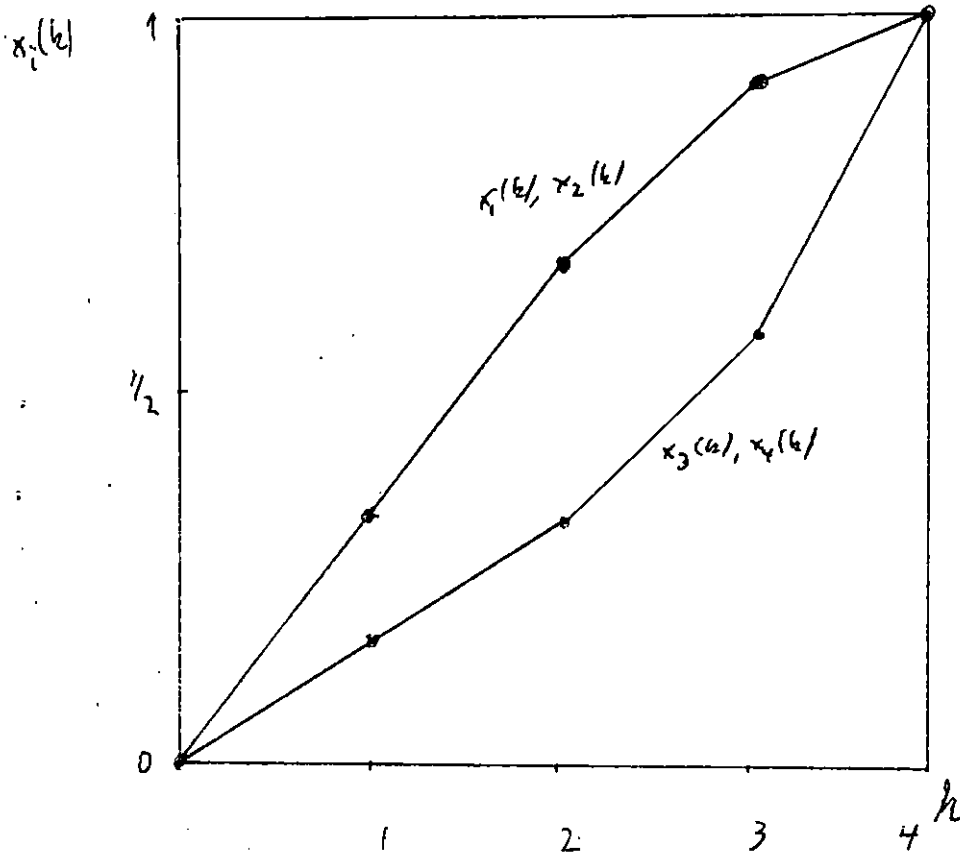


Figure 2b. Employment probabilities, $\{1,3,4\}$ and $\{2,3,4\}$ minimal winning.

difference, employment probability differentials can be used via linear estimation to infer political power differentials.

To illustrate the use of this technique, consider the industrial sector of the Soviet Union during the 1920's. Here was a 1-party state, with the major political difference being whether one belonged to the party or not. According to Rigby, a worker who belonged to the Communist party stood a 1/100 chance of being unemployed in 1925 and a 1/25 chance of being unemployed in 1927. The latter was a year of rather greater overall unemployment. Unemployment rates among non-party member workers were substantially higher. From data in Rigby, Nove, and official Soviet sources,⁸ the following employment picture emerges:

| <u>All Workers</u> | | |
|---------------------------------------|--------------------------------|-----------------------------------|
| <u>year</u> | <u>Total</u> (in thousands) | <u>Employed</u> (in thousands) |
| 1925 | 10,750 | 9,800 |
| 1927 | 12,900 | 11,500 |
| <u>Workers Belonging to the Party</u> | | |
| 1925 | 725 | 718 |
| 1927 | 1,046 | 1,004 |
| <u>Workers not so Belonging</u> | | |
| 1925 | 10,025 | 9,082 |
| 1927 | 11,854 | 10,496 |

Under the linearity hypothesis, one has

$$x_i(k) = a(i)k, \text{ for } i = 1 \text{ (Party) and } 2 \text{ (Nonparty)}$$

Normalizing by n removes the effect of the larger n in the second observation (1927):

$$x_i(k/n) = a(i)k/n.$$

Both formulations assume that the distribution of power is the same in the two years. The data are plotted in figure 3. Being in the party evidently makes a worker more powerful than otherwise. Indeed, by estimating the ratio $a(1)/a(2)$, one can quantify this difference in power. One has the estimates $a(1) = 1.016$ and $a(2) = .938$, hence a ratio of 1.08. Being a party member made a worker about 8% more powerful than his non-party cohort. This seemingly small relative advantage would be enough to account for his substantially better chances of finding a job. There is no Tammany Hall in Moscow, but the Soviets knew the game Senator Marcy had in mind.

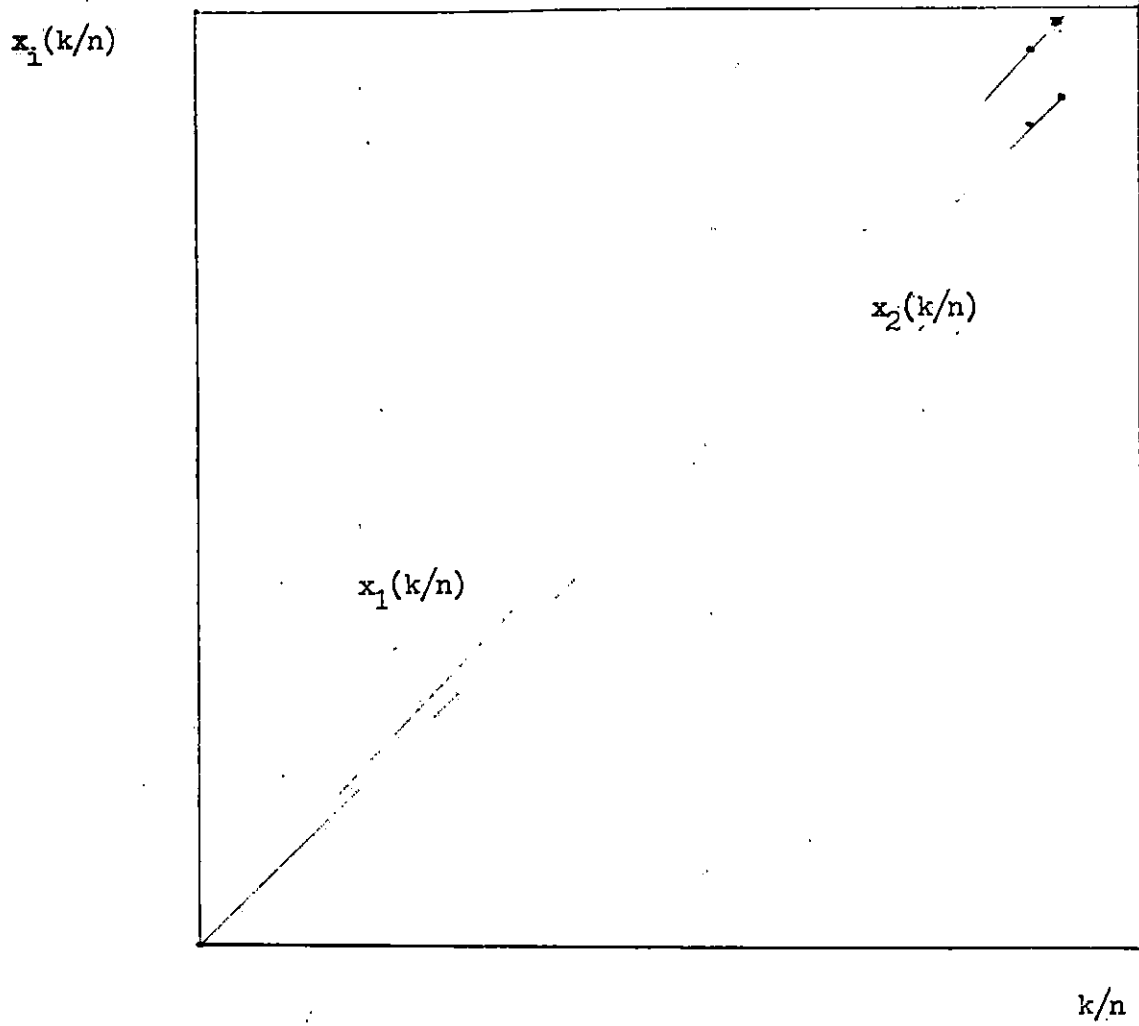


Figure 3. Employment Probabilities, Party/Nonparty, USSR, as functions of the employment rate.

III. Extensions of the Spoils Game

The aim of this section is to show how the Spoils System can serve as a model for other distributive mechanisms, in particular for those that distribute bads. Consider the distribution of involuntary service in the armed forces, the Draft System. We shall show that knowing the probability that someone gets a job under the Spoils System is tantamount to knowing the probability that someone avoids the draft under the Draft System.

Suppose as before that there are n agents, each of whom is liable for military service. The disutility of serving is -1 for any agent; not serving has 0 disutility. Let the draft quota be k' , $0 \leq k' \leq n$. This quota can be exceeded, although it is not optimal to do so. In any event, it must be met. The draft quota is exogenous to the political structure. The representation of the latter in terms of winning coalitions and the game v_I is as before. The argument for the level-II economic game v'_{II} runs as follows. If resistance to the draft is futile, then a coalition which is not winning cannot control the draft. Therefore, it cannot prevent its members from being drafted. For a losing coalition then, $v'_{II}(S) = -s$, the number of its members. If S is winning, it is an entirely different story. S is then in charge of the draft, and it can draft agents from outside its own ranks to fill the draft quota. If $k' \leq n-s$, the number of agents not in S , then S fills the quota completely with outsiders and none of its members have to serve: $v'_{II}(S) = 0$. If $k' > n-s$, then the membership of S has to make up the difference: $v'_{II}(S) = -(n-s-k')$. Summarizing the above discussion, one has

$$v'_{II}(S) = -\min(0, n-s-k') \text{ if } S \text{ is winning} \\ -s \text{ otherwise.}$$

Let $x'_i(k')$ denote the probability that player i avoids the draft when the draft quota is k' . The following proposition relates the probability of avoiding the draft in the game v'_{II} to the probability of getting a job in the game v_{II} .

Proposition 3. $x'_i(k') = x_i(k)$, when $k' + k = n$.

Proof. Consider the following linear transformation of v'_{II} :

$$v+(S) = v'_{II}(S) + s.$$

By the linearity of the Shapley value,

$$\phi v+(i) = \phi v'_{II}(i) + 1.$$

At the same time, $v+$ can be written as

$$v+(S) = \min(s, n-k') \text{ if } S \text{ is winning} \\ 0 \text{ otherwise.}$$

Putting $n-k' = k$, $v+$ corresponds to the Spoils System game v_{II} . In particular,

$$\phi v+ \Big|_{n-k'} = \phi v_{II} \Big|_k \text{ for } k'+k = n.$$

Let $x+(i)$ represent the Shapley value allocation for the game $v+$. Then i 's probability of getting drafted times the disutility of serving =

i 's probability of getting drafted times $(-1) = \phi v_{II}(i) \Big|_{n-k'} = x+(i)-1$.

Hence, one can interpret i 's probability of being drafted when the draft quota is k' with $1-x+(i)$. As just seen,

$$x+(i) = x_i(n-k').$$

Thus, the probability of avoiding the draft, $x'_i(k')$, satisfies

$$x'_i(k') = 1-(1-x+(i)) = x_i(n-k') = x_i(k)$$

when $n-k' = k$, as was to be shown.

Proposition 3 shows that all the results of the Spoils System have counterparts in the Draft System. When power is evenly distributed, the probability of avoiding the draft is the same for all. When power is unevenly distributed, the more powerful stand a better chance of avoiding the draft. Moreover, there is a critical draft quota, below which no veto player stands a chance of being drafted, but above which a powerless player is sure to be drafted. The dividing line in these two cases is the draft quota $= n-\underline{s}$, where \underline{s} is the smallest winning coalition.

When resistance to the draft is not futile, then the strategies involved become considerably more complicated and the above results must be modified. A simple illustration will show the difficulties involved. Suppose $n=3$, with $\{1,2\}$ and $\{1,3\}$ the minimal winning coalitions. When there is no resistance to the draft, then the probabilities of being drafted are shown in figure 4a, which is dual with figure 1b. Now suppose that a draftee

can refuse to serve at a cost $c < -1$. This is worse than serving, but nevertheless gives the draft resister a threat against the coalition controlling the draft, since one of its own members will have to take his place at a disutility of -1 . Suppose that the quota $k' = 1$. Then $v'_{II}(N) = -1$. Consider the winning coalition $\{1,2\}$. They threaten to draft agent 3, who counters with a threat of his own, to resist the draft. If both threats are carried out, the utilities to the winning and losing coalitions are -1 and c respectively. Since $v(N)$ represents the compromise level of aggregate disutility, the Nash solution to the above bargaining problem would propose the disutility compromise $-1-c/2$ and $c/2$ respectively. These correspond to the Nash-Harsanyi characteristic function values for $v'_{II}(\{1,2\})$ and $v'_{II}(\{3\})$ when there is draft resistance. The same argument holds for $\{1,3\}$ and $\{2\}$. When comparing $\{2,3\}$ versus $\{1\}$, neither is winning. However, since all are liable to the draft, in the absence of a compromise the corresponding disutilities would be $-1, -1$. The Nash bargain in this case is $-1/2, -1/2$.

For the specific value $c = -1.5$, figure 4b shows the draft avoidance probabilities for all values of the draft quota. Notice the effect of draft resistance on these probabilities. Players 2 and 3 are less likely to be drafted than in the no-resistance case. Indeed, as the cost of resistance approaches -1 , so that resistance is no more costly than serving, the threat to resist is completely effective and all power differentials disappear. On the other hand, as the cost of resistance becomes higher,

Probability of Avoiding the Draft

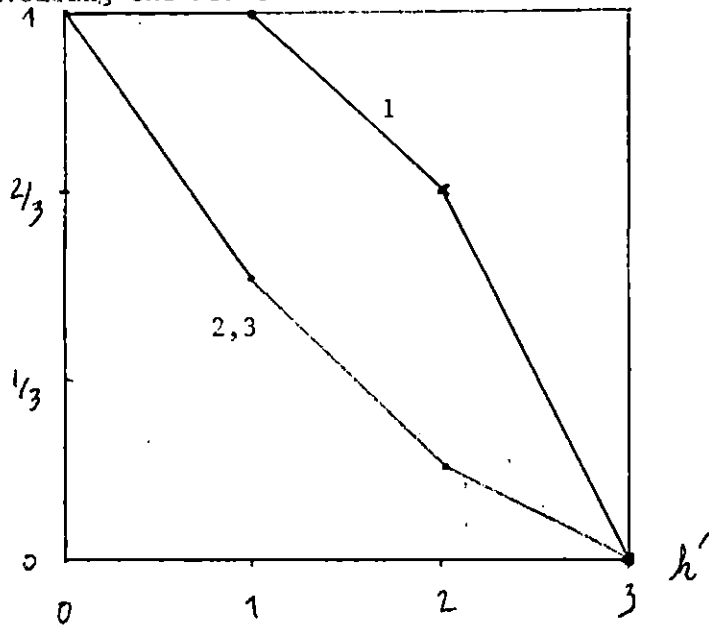


Figure 4a.
Probability of avoiding the draft,
{1,2} and {1,3} minimal winning.

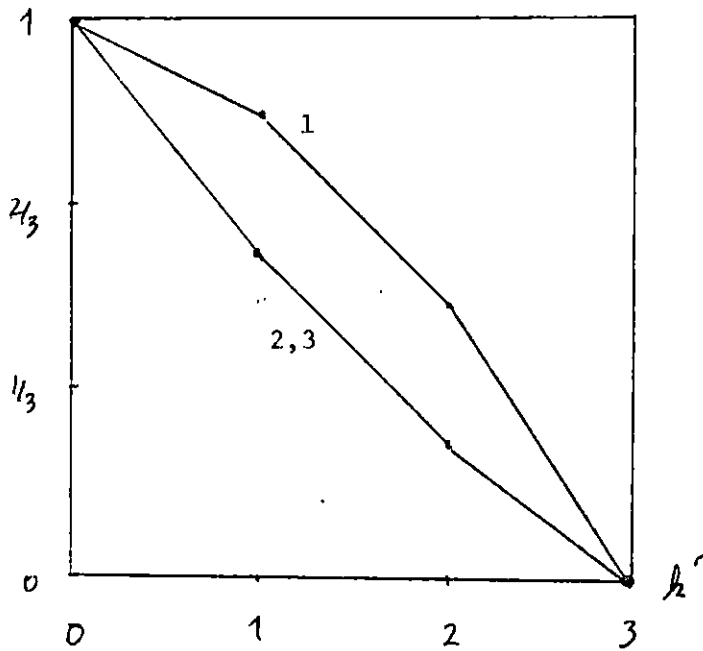


Figure 4b.
Probability of avoiding the draft,
with resistance costs = -1.5,
{1,2} and {1,3} minimal winning.

the threat to resist loses its credibility. In the limit, one is back in the no-resistance case.

As another example of the versatility of the Spoils model, we consider its implications for the marginal productivity theory of distribution. Suppose that the production technology is linear in labor L , $f(L) = L$, and that each agent has one unit of labor to supply. Only winning coalitions have access to the production technology. If output is perfectly divisible, v_{II} satisfies

$$v_{II}(S) = \begin{cases} s & \text{if } S \text{ is winning} \\ 0 & \text{otherwise.} \end{cases}$$

According to the marginal productivity theory of distribution, each agent should receive 1 unit of output, the marginal product of his labor. Now if the distribution of political power is even, then this will indeed follow from Shapley value considerations. But if political power is unevenly distributed, then an even distribution of the output is not indicated. A final 4-player example will make the point. Player 1 has a veto, coalitions $\{1,2\}$ and $\{1,3\}$ are minimal winning, and player 4 is powerless. The distribution of power $\phi v_I = (4/6, 1/6, 1/6, 0)$. The disparities in power show up in the distribution of output $\phi v_{II} = (25/12, 9/12, 9/12, 5/12)$. Not a single agent is paid his marginal product. The veto player gets paid the most, the powerless player the least. The veto player can certainly expect to exploit his political power for economic gain. Great disparities of power jeopardize any theory of distribution which ignores them, a point which proponents of the marginal productivity theory have perhaps underestimated

in the past. It would seem that the marginal productivity theory is most attractive when political power is uniform.¹⁰

IV. Conclusion

This paper has highlighted the role of power differentials in the distribution of jobs and the incidence of the draft. A number of restrictive assumptions have been made in order to make this role clear. The three most restrictive are the following. First, people and jobs have both been assumed uniform. Moreover, each agent has been assumed to get the same utility from each job. A more general model would allow for differences in jobs and workers.¹¹ One would conjecture that the more powerful the agent, the better the job he would get. Second, the structure of power has been assumed fixed. However, it is possible that the economic outcome of one period will change the distribution of power in the next period. Thus, unemployment in the Soviet Union in the 1920's helped transform the entire political structure of that period, leaving Stalinist dictatorship in its wake.¹² Third, the level of employment and the draft quota have both been assumed to be exogenous to the political structure. Here there is the possibility that a coalition will win on a platform to change these features of the economy.¹³ A model that did justice to these three phenomena would have to be much more complicated than any presented in this paper. Whatever the complexity encountered, political power will still manifest itself in the economic outcome. To the winner go the spoils.

FOOTNOTES

1. This is argued most forcefully by G. Bettelheim [3] .
- 2.. See the data quoted in Rigby [12, pp. 129-130] .
3. An early example is provided by Shapley and Shubik [14] . Aumann and Shapley [3] contains a comprehensive survey of this field. Gardner [9] contains an example of political-economic interaction.
4. An important discussion of veto power and the Shapley value is contained in Brown [6] .
5. See Shapley [13] for details.
6. This formula was pointed out to me by D. Blair.
7. See Marx [10, pp. 395-402] .
8. See Rigby [12] , Nove [11, chapter 4] and [15] .
9. Details of the strategic reasoning involved here are discussed more fully in Aumann and Kurz [2] .
10. The most general result for convergence of the Shapley value to Competitive Equilibrium is found in Aumann[1] .
11. For instance as discussed by Crawford and Knoer [7] .
12. Gardner [8] considers the electoral group choice dynamics of this situation.
13. A recent contribution in this direction is Brito and Intriligator [5] .

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