

EXTERNAL ECONOMIES AND DISECONOMIES
IN ECONOMIC DEVELOPMENT WITH REFERENCE
TO CANADA

by

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PREFACE

External economies and diseconomies considered jointly will, in this thesis, be called externalities. An externality, for the moment, may be defined as an effect on the cost structure of a firm which is not attributable to the action of that firm. Such effects may raise or lower costs. The following question is to be considered. Do externalities affect economic development and if so, in what manner? Incidental to this inquiry it is necessary to examine whether externalities are likely to advance or retard the rate of economic growth.

There are two views. On the one hand there are those who regard externalities as important. This group includes, to name only a few, Scitovsky (192), Chenery (29), Colin Clark (36), Hirschman (95), Islam (103), and Schumpeter (189). (Only Schumpeter and Hirschman argue that externalities systematically promote economic development; the others apparently believe the converse.) On the other hand, a group of economists including Fleming (66), Stockfish (205), Meade (137), Knight (116), and Young (236) appear to regard externalities, at least in pure competition, as so insignificant that it is difficult to find any realistic and convincing examples. Moreover, there is confusion over whether externalities should be defined as attributable to a general range of market imperfections (including fewness of competitors), as is believed by Bator (9), or as attributable almost exclusively to dynamic factors as is believed by Ellis and Fellner (55). Thus, although a large number of articles concerned with externalities have been published, the subject is still somewhat confused.

In this study, the problems have been approached on two fronts. First it seemed necessary to examine the relationship of externalities to economic development using theory. This is done in chapters one to five. Problems are delimited in chapter one, while chapter two provides a review and synthesis of the literature--though this review is more concerned with externalities than with economic development.

The evolution of externality concepts has been closely related to both socialist thinking and welfare economics, as a study of Pigou's Economics of welfare (159) or Baumol's Welfare economics and the theory of the state (12) will confirm. Chapter three, therefore, is an attempt to apply modern welfare concepts to the problem. This procedure is more useful than might at first be imagined, because it helps a good deal in resolving definitional problems. In fact the definitions throughout, reflect the analysis of chapter three.

Chapters four and five attempt to relate the externality concepts reviewed and extended in earlier chapters to problems of economic development. It must be admitted that this section is more closely related to economic growth (as opposed to development) than is desirable and that in most places the argument proceeds little beyond comparative statics. But it does develop meaningful concepts for statistical examination.

The second approach to externalities in development has been empirical. This part of the study follows concepts developed in the earlier theoretical part, but it is only partly a parallel study. It is much narrower in scope and much more detailed. But even so it is cast in terms of large aggregates so that the econometric procedure involves a number of assumptions and estimates that are not as neat as one might desire.

The empirical part involves, first, an examination of how frequent and important are economies of scale; second, a method for estimating marginal costs to the industry of increments in output is developed; third, an attempt is made to estimate the extent of pecuniary externalities for various Canadian industries and to determine whether these accrue in such a manner that industrial expansion can be self-financing.

The data on which this empirical work is based is not of uniformly high quality and the results must be interpreted with caution. But it may be concluded that, if prices to the industry are assumed constant, falling costs (in the industry) are fairly common. Pecuniary external economies (funds which are potential sources of internal financing) vary a good deal from one industry to the next and these variations do not appear always to correspond to capital intensity or to capital longevity. To the extent that they do not, the capital market is called upon to redirect the flow of investment funds.

It is possible to conclude that economic growth may be less than is achievable because of externalities. However, because development implies an increase in welfare, it is not clear that externalities do, in fact, interfere with economic development. It is clear that they might under certain circumstances, but that is a different matter. Pecuniary externalities are much easier to treat than technological externalities and one may even go some distance toward quantification of the former. But quantification of technological externalities appears remote.

I. INTRODUCTION

A. Background of Externalities in Economic Development

1. Purpose

This study is an attempt to clarify the relationship between externalities and economic development. It is not clear whether externalities are important in the study of growth and development, nor is it clear whether externalities are more important in developed than underdeveloped regions, and finally it is not clear whether the presence of externalities impedes development and growth or actually aids development and growth.

All these problems need to be examined both theoretically and, where possible, empirically. But while externalities have been the subject of a considerable volume of literature, and while economic development has recently been extensively investigated, there has been relatively little work devoted to the manner in which the two are related. This situation prevails, despite frequent reference in the development literature to external economies and diseconomies.

Externalities have previously been examined primarily within a theoretical framework so that empirical aspects have generally been ignored. Therefore, it is one purpose of this study to examine particularly those aspects of the relationship between externalities and economic development that can or might be quantified.

The purpose is not to advance an additional theory of economic development nor to see if there are previously unexamined causes of underdevelopment. Rather this study provides an examination of how externalities influence some of those parameters that are already generally accepted as

important causes of or restraints to economic development.

2. Definitions

Externalities are particularly difficult to define in a meaningful manner, and the subject deserves careful attention. This consideration is reserved until after chapter two (a literature review) and chapter three (on welfare economics) so that the matter may be more fully treated.

Definitions are given here without explanation. In the following, an externality will mean: any change in cost or revenue in a firm A which is caused or made possible by some action taken by a firm B in such a manner that A might reasonably offer B positive or negative compensation for the action, but when no such compensation takes place (See chapter four). A shorter, less precise definition will sometimes be used: an externality is involved whenever socially relevant considerations are not included in the plans of decision-making bodies because the decision-making bodies have no incentive to consider such effects.

In order to avoid problems of monopolistic competition, it will generally be assumed that B was ignorant of, or indifferent to, the effect of its actions on A. In general, too, attention is focused on costs (rather than on revenues) brought about by B's action. Such shifts are called cost externalities, while shifts in revenues are called revenue externalities. And, any change in profits brought about in this manner are called profit externalities. The literature differentiates pecuniary and technological external economies, but this frequently appears gratuitous. It is useful, however, to refer to externalities that result from the growth of the whole region as scale externalities.

In all cases it is necessary, for reasons to be examined in chapter four, to use these terms in application to increments. It is B's new action, not just B's existence or a continuation of a former trend in B's action, that brings about the externality. For more explicit definitions (particularly of costs, profits, and revenues) see chapter four.

Economic growth means a permanent increase in real per capita income.

Economic development is defined in the same manner as economic growth except that when the word "development" is used, an increase in welfare as well as in per capita real income will be implied. This general definition has the advantage of enabling specific discussion of particular theories of development.

Maximum rate of economic development means the highest rate of economic growth which is consistent with increasing social welfare.

If economic growth is to take place in a closed economy* there must exist an endowment of unused productive factors or techniques or alternatively, a new source of such resources. Consequently, an underdeveloped economy may be defined as an economy which possesses unused or inefficiently used productive factors which, were they put to use or to better use with the available state of technology, would bring about an increase in real income per capita as well as an increase in welfare. Alternatively, and dynamically, an economy is underdeveloped whenever the rate of flow of technological discoveries is greater than the rate of adoption of such discoveries. It must be provided that this discrepancy results

*Problems of international trade are to be ignored. This does not imply advocacy of autarky; rather it implies only an inability to deal with the whole universe of problems. For a consideration of how externalities relate to trade see Watanabe (230).

in a rate of change of real income per capita that is less than a maximum and that social welfare is suboptimum.* The former may be called static underdevelopment and the latter dynamic underdevelopment. It should be observed that neither state necessitates nor precludes the other.

These definitions in no way confine the use of the word "underdeveloped" to situations that involve relatively undeveloped economies. Underdeveloped, as used here, does not imply comparative poverty or underconsumption; it implies a stage or rate of economic advance that is less than the possible stage or rate (See chapter three).

Here, meaningful implies that the statement or hypothesis could be disproven (if it is false) by some imaginable set of observations made on real phenomena. A short interesting history of the concept is to be found in Ayer's *Language, truth, and logic* (5).

General equilibrium, except where otherwise defined, implies a situation in which the marginal equivalence criteria of welfare theory are met by the freely operating market mechanism (that is, without government intervention).

Market failure involves provision of a bundle of commodities or set of prices by the laissez-faire system that are inappropriate to welfare maximization because a feasible alternative bundle of goods or set of prices is preferable by a Pareto optimum criterion or by a social welfare criterion. Market failure may be dynamic (involving inappropriate time

*"Real income" because it is possible that a shift of prices in favor of commodities produced in abundance will be confused with development. Such a shift in a competitive economy reflects shifting consumer tastes and so corresponds to an increase in welfare. In an imperfectly competitive economy it may only reflect the changing pattern of competition. It is not likely that a price index could satisfactorily take care of both situations.

rates of change) or static (involving inappropriate mixtures at one point in time) or it may be both.

A Pareto optimum is a situation in which it is not possible to increase X's welfare without some sacrifice of Y's welfare, where X and Y are any two individuals in the society.

The duality theorem is one of the by-products of the ideological controversy between socialist and laissez-faire economists. Briefly, the duality theorem means that in order to achieve maximum welfare using a Pareto optimum criterion, it is always necessary that rates of exchange exist; and while under socialism these rates of exchange can assume the form of shadow prices, under capitalism they correspond to the real prices of the business world if these are determined by an ideally competitive system. Duality only applies where all functions are smoothly convex, where tastes and individual welfare functions are independent, where production functions are only related through the market mechanism (that is, where no external economies or diseconomies exist), and where there are, in each industry, a large number of firms. If these conditions are not approximately realized, the competitive exchange ratios lose their normative significance, and it is necessary either to revert to positive economics or to assume some social welfare function.

Market failure can occur whenever duality fails, but it should be emphasized that by no means all market failures may be attributed to externalities. Most frequently, when market failure becomes evident, imperfections of price-making are attributed to incomplete knowledge or to excessive individual power (the monopoly problem).

Ideal levels of any parameters of economic development will be

called norms of development.

B. Situations That Give Rise to Externalities

1. Examples

The causes of underdevelopment are numerous. Some of these, for example psychological or religious taboos, are probably best considered outside of economic analysis. Others are attributable to what some might judge as unfortunate economic or social institutions. But here only one cause is of interest: that associated with market failure of a type which results from divorcing of socially relevant considerations from the plans of the decision-making body or bodies.

The types of market failure which will be of concern in this study are examined in chapter three. Here, in order to more completely introduce the subject matter, some short examples are given.

1. Industrial safety devices cannot, in general, be provided by laborers. But monopsony power may be used to force laborers to assume avoidable risks without full compensation. The following table is illustrative.

Table 1 indicates that workers were bearing a major share (65.4 percent) of costs of injury. Thus employers' costs were lower and the injury rate no doubt higher (because of absence of safety devices) than under full compensation. But, unfortunately, there appears to be no economic theory which can be used to determine if such a situation promotes or retards economic development. An externality is involved because costs are shifted to the workers by the employers who, in turn, are the only ones capable of installing safety devices. Costs are incurred by

Table 1. Extent of compensation for on-the-job injuries in Massachusetts 1935^a

Extent of disability	Percent compensated
Fatal	14.8
Permanent total	75.4
Major permanent	32.4
Minor permanent	39.2
Temporary	54.9
Total	34.6

^aSource: Adapted from Kapp (109, Table 1, p. 62).

one group, benefits by another.

2. Ignorance of consumers and producers may result in tradition directed consumption and production patterns as Riesman *et al.* (165) have pointed out. But informing the public, through advertising, for example, will not be undertaken by private corporations unless advertising effects can be made specific in response. Moreover, a competitive price for information, because it would reflect society's values before the fact, cannot be of much value in determining the worth of that information (See chapter four). Galbraith (69) has treated such difficulties, but the most interesting theoretical discussion is by Haavelmo (78) who considers certain social institutions as a possible prerequisite to achievement of a desired goal. In the absence of such institutions involuntary economic decisions may result.

Evidently market failure is involved in this example, but it is not clear that such types of failure should be included in an analysis of externalities. The subject is more fully treated in chapter two.

3. When bees freely fertilize fruit blossoms and orchards freely provide bees with nectar, when valuable inventions are not patented and consequently are used by others without payment, and when roads, built for exploration by oil companies, are used by others for their own purposes, market failure is involved. In these cases costs are borne by one group or individual and part of the benefits accrue to others. An externality therefore exists.

2. Alternative economic systems

Because discussions of the failures of the laissez-faire system are likely to be emotionally charged, any criticism of any capitalist institution may be construed as a frontal attack on the whole of western capitalist democracy. It is therefore necessary to point out that complete central planning or complete socialism is not necessary to eliminate possible existent failures. This point is well made by the World economic survey, 1959 (215, p. 6).

"Though the objective of economic growth now dominates public policy thinking in all countries, in almost none of the advanced industrial countries has this objective been defined in terms of comprehensive policy for growth. In part this stems from a tendency to identify long-term economic policy with economic planning, which, it is feared, may endanger the system of private enterprise. As is evident, however, from the experience of a number of industrial countries, most notably France, Italy and Japan, even concrete long-term plans for economic growth need not carry any connotation of 'controls'; they may instead serve a highly useful purpose in the context of a private enterprise economy in providing a comprehensive framework for the harmonization of economic policy."

It is possible that central planners are able to take more account of the indirect costs and benefits that comprise externalities and therefore are in a superior position to promote economic growth. Comparison of growth rates in centrally-planned economies and capitalist countries (compare Tables 1.1 and 3.4 in World economic survey, 1959 (215) indicates with $p < 0.001$ that growth rates so different could be random samples from a population $N(\bar{X}, \sigma^2)$. Again comparison of the growth rates of investment given in Table 3.4 of World economic survey, 1959 (215) indicates with $P < 0.001$ that such rates of change of investment could be random samples from a population $N(\bar{X}_1, \sigma_1^2)$.

But little can be gained from such crude comparisons. It does seem possible (especially after examining the Polish and Russian investment criteria (215)) that centrally-planned economies are able to take more account of externalities than are capitalist economies. If so, differences in growth rate suggest that the existence of externalities retards, rather than promotes, economic development. Most of this study is devoted to examining evidence related to that hypothesis.

II. EVOLUTION OF THE CONCEPT

A. External and Internal Economies

The immediate purpose of this chapter is to trace the development of the concept "external economy" from its introduction by Marshall to the present. There are three reasons for this procedure. First, it is desirable to use definitions that involve developments of, and not large changes in, previous concepts. Second, observation of some of the errors and weaknesses of previous analyses have had a definite effect on the direction this study has taken. Third, it is not clear, without first examining the previous literature, what development economists mean when they use external economies and diseconomies in their analyses.

1. The shifting definitions

According to general equilibrium assumptions,* firms that experience falling costs will expand so that either monopoly elements result or all economies of scale disappear--whichever happens first. Therefore, in order to avoid considerations of monopolistic competition Marshall (134) developed three tools: 1. the short life of firms, 2. the falling demand curve, and 3. external economies.

Marshall's definitions of external economies are conceptually

*By "general equilibrium model" will be meant a model that involves, explicitly or implicitly, the assumption of ubiquitous competition, or, at any rate, the assumption that all marginal equivalence requirements of welfare theory are satisfied.

clear.* His analysis relates to realistic situations. But his definitions are somewhat too vague to provide a basis for quantification; moreover, it now seems better to deal directly with problems of monopolistic competition instead of attempting to use competitive models in imperfectly competitive situations.

It would therefore be understandable if definitions of external economies were altered somewhat. In fact, some definitions have been altered, but some still relate very closely to those used by Marshall.

A few examples will illustrate. Vakil and Brahmanand (223, p. 201) use the following implicit definition. Economic development is stimulated by economic integration.

"This in turn requires considerable investment in services which provide external economies like banking and credit agencies, insurance services, savings institutions, marketing services and storage facilities, information and educational facilities, technical training, provision of long-term capital and so on."

Williamson and Buttrick (233, p. 157) use a similar implicit definition which runs in terms of economies caused by social overhead facilities. Similarly, Bauer and Yamey (11, p. 244) note that assistance to some industries is not necessarily required even though:

"It is suggested that assistance is necessary because of the absence in underdeveloped countries of common facilities such as developed transport and financial systems, a tradition of industrial skill, and other external economies which in developed countries are enjoyed even by new enterprises and those engaged in new lines of activity."

*A. Marshall (134). See pages 167, 265, 266, 269, 271, 273, 278, 314, 316, 317, 320, 441, 460, 468, 479.

Three definitions are offered. External economies are economies "which arise out of the collective organization of the district as a whole" (p. xiii), "those dependent on the general development of the industry" (p. 266), and "Those which result from the general progress of the environment", (p. 441). He does not treat external diseconomies.

A seemingly paradoxical definition, quite unlike those provided by Marshall, is given by Leibenstein (119, p. 107).

"External economies usually refer to the fact that as an industry expands, costs for all firms within the industry are reduced, although no firm within the industry is any more efficient than it was previously."

A definition similar in part to the one used in this study is provided by Baron (8, p. 190). Investment in social overhead has a beneficial effect.

"This effect is referred to as 'external economies' which arise whenever the operation of one enterprise facilitates (cheapens) the establishment or the conduct of another."

These definitions, even the one given by Leibenstein, are clearly descendants of Marshall's definitions. But in non-perfectly competitive situations the difference between costs and revenue is not always clear. Advertising, product differentiation, good will, etcetera can affect both cost and revenue. Thus, the old definition in terms of costs is not entirely unambiguous except in competitive situations. For example, under imperfect competition an external effect that increases demand may be considered to reduce costs because the firm that has been so beneficially influenced may sell the same amount as before at the same price as before with lower outlays on advertising.

Consequently, as Scitovsky (192) has pointed out, there has been a tendency among more recent authors to define (hardly ever explicitly) external economies in terms of effects on profits rather than on costs. Rosenstein-Rodan (172) justifies this approach by arguing that where the capital market is imperfect, pecuniary external economies as well as technological external economies (that is, effects on profits as well as

effects on productive efficiency) can lead to market failure.

2. Meaningful definitions

A set of definitions advanced by Fleming (66) is in this tradition but is somewhat too complex to be simply dealt with here. Essentially, Fleming defines externalities as the divergence between private and social costs or private and social revenue. These, presumably, are intended to be meaningful definitions, but their use involves a difficulty of great fundamental importance. How can social cost or social revenue be defined?

Not too long ago it seemed that a simple solution existed. Any deviation from the marginal equivalence criteria of welfare theory could be defined as the result of some private but not social cost or revenue. This was unambiguous as long as welfare theory could tell exactly in which direction, and by how much, it would be necessary to change prices or output in order to reach the social ideal. In fact, this cannot be accomplished without reference to a social welfare function. (See chapter three.) Too, it appears from the Lipsey-Lancaster (127) exposition of the theory of the second best that where there are major imperfections (such as monopoly elements) in the system, application of particular general equilibrium norms to such other imperfections as externalities may lead away from, rather than toward, the social ideal. Galenson and Leibenstein (70), using a different argument, arrive at the same conclusion.

Thus, it is not clear in any particular case what social costs or social revenues mean. And any definition making use of these concepts is not likely to be of much help in quantification of externalities.

3. Definitions and measurement

Unfortunately, there does not appear to be any definitions which provide for measurement of external economies. Indeed, a fundamental difficulty arises in connection with measurement. If an external influence be regarded as a factor having positive or negative productivity, how can it be quantified? Physical units are, of course, not adequate. But factor evaluation based on productivity leads to difficulties of imputation if there are non-constant returns to scale. This difficulty is a familiar one in production theory. There is apparently no solution.

Thus, it is evident that any meaningful definition of externalities which can at once provide the basis for policy and for quantification must fulfill two requirements. 1. It must provide a method of evaluating external influences without attempting to do this by imputing total values of externalities from productivity theory. 2. It must relate to some welfare model; moreover, it is almost necessary that such a welfare model explicitly treat income distribution.

B. Importance of Economies of Scale

For two reasons, the theoretic development of the externality concept has not received much recent attention. This is because, first, theories of monopolistic and imperfect competition were put forward and seemed to comprise a more legitimate if less general method of approach, and second, because empirical evidence on economies of scale appeared to indicate that these were not as significant as had been thought by the classical economists. The classical economists tended to treat all non-agricultural pursuits as subject to decreasing costs, but this belief

conflicted with results of empirical studies (see Table 2), and so was given up. But it has recently been revived.

1. Some evidence

A large part of the examination of economies of scale followed the pioneer work of Douglas and Cobb. A tabular summary is provided by Leser (123), and a reduced version is included in Table 2.

With the exception of three studies, one on agriculture in the U.S.A.; one on manufacturing industries in the U.S.A., and one on the whole U.S. economy, there appears to be little evidence in Table 2 of increasing returns to scale. Quite the contrary, roughly constant returns to scale, appears to be the rule. Confronted with this impressive array of evidence it is no wonder that economists lost interest in economies of scale, whether of the internal or external variety.

But it is not clear that these studies provide valid measures of economies of scale. Consider a simple test. The exponent of capital for all the above studies averages 0.42. Thus, if there is a law of production having generality it would be expected that in less developed areas (having relatively much less capital and much more labor) the capital-output ratio would be considerably smaller than in developed countries.* But, in fact, as indicated by Table 3 capital-output ratios are surprisingly constant and do not appear to be (except in the case

*This follows from the function. For example, if, in general, $Q = aL^{0.58}C^{0.42}$ there is constant returns to scale. But holding labor constant and decreasing capital one percent will result in a decrease of output equal to only 0.42 percent. Therefore, output/capital should be larger where there is relatively less capital, and the capital/output ratio should be smaller.

Table 2. Results of fitting Cobb-Douglas functions^a

Subject	Date ^b		Sum of exponents ^c
Agriculture in U.S.	1920	1947	0.85; 0.87; 0.55; 3.67
Coal mining; U.K.	1923	1953	1.08; 0.00-1.02
Manufacturing; U.S.	1889	1922	0.93-0.96; 0.94-1.01
Manufacturing; Australia	1910	1937	1.07; 0.98; 0.98-1.01; 0.90-0.99; 0.99
Manufacturing; Canada	1923	1937	0.98-1.01
Manufacturing; New Zealand	1915	1939	0.91; 0.96
Manufacturing; South Africa	1937	1938	0.98; 1.02
Manufacturing; U.K.	1924	1930	0.85-0.90
Manufacturing; U.S.	1920	1940	1.95-2.27
Whole economy; U.S.	1921	1941	2.47
Whole economy; 24 countries	1937	1957	0.88; 0.90; 1.01; 1.06

^aSource: Leser (123). A Cobb-Douglas function is of the form $\text{Output} = aX_1^{b_1} X_2^{b_2} \dots X_n^{b_n}$, where the X_i ($i=1\dots n$) are factors.

^bOnly earliest and latest dates for each study are given; this does not imply that each or any study extended over the period shown.

^cOnly results not requiring the exponents to sum to one are shown. This leaves 25 studies. If the sum of exponents > 1 there is increasing returns to scale.

of Mexico and India) related to the stage of development.

Now as a first approximation, the constant capital-output coefficients presented in Table 3 suggest the exponent of capital in a Cobb-Douglas function equals 1.0. This leads to the conclusion that either the exponent of labor is roughly zero and there is constant returns to scale; or the exponent of labor is positive and there is increasing returns to scale as Leser (123) has pointed out. Perhaps this is an excessively

Table 3. Capital-output ratios^a

Country	Period	Coefficient
Argentina	1913	5.8
Australia	1913	5.5
Italy	1913	4.4
U.S.A.	1913	4.3
Japan	1913	3.6
U.S.A.	1889	3.0
U.S.A.	1909	3.4
U.S.A.	1919	3.8
U.S.A.	1939	3.3
Avg. 30 countries	?	3.0-4.7
Mexico	?	1.5
India	?	1.5

^aSource: Tinbergen (213, p. 72).

naive conclusion. Certainly it appears simple when it is compared with attempts to explain constant capital-output ratios such as that made by Kaldor (108). But even a simple approach like this leads to doubt respecting the universal nature of constant returns.

The extremely aggregate data used in the studies reported in Table 2 is another source of difficulty. The more aggregative the approach, the less justification there is for assuming that capital prices are independent of total output and therefore the less conclusive are the results (123). And this is by no means the only difficulty encountered in such studies. Other difficulties are examined in chapter six in connection with equations fitted for this study and reported there.

Recent empirical work appears to indicate that increasing returns is a considerably more common phenomenon than would be concluded from results of the Cobb-Douglas studies presented in Table 2. Thus, it seems unfortunate that attention was diverted from study of increasing returns. (The subject did receive a thorough examination in welfare theory, but most of the work suffers from a tendency to ignore income distribution (153).) Recent studies are relevant to the empirical part of this thesis and are examined in chapter six. They do not belong in a consideration of the historic evolution of concepts, but anyone interested in a summary might page ahead to the tables and graphs of chapter six.

C. Technological Development as Related to Scale of Output

Whether there are, at any one time, either untapped scale externalities or internal economies of scale, is not the only relevant question. Development itself may involve technological change so that economies of

scale are generated by the very process of economic growth. This line of thought goes back to Adam Smith (196). He claimed that, "The division of labour is limited by the extent of the market," and he provided some interesting and convincing illustrations in an attempt to prove it.*

While it often happens that increases in productivity are positively associated with time, it seems evident that Smith thought of the division of labor as being initiated by increases in market size. This was not only because existent technological means were indivisible and so required a large market before being brought into use; but also because technological discoveries and labor skills were stimulated by expanding and expanded markets.**

There is modern evidence to support Smith's contention. A study of Table 4 leads to the conclusion that frequency of performance begets labor efficiency if not invention.

These learned skills, incidentally, may result in an external diseconomy to the firm responsible for them, if other firms are able to lure

*See Smith (196, Book 1, Chapter 3).

**H. Leibenstein (119) has argued that this proposition is only sometimes true. He presents two diagrams in which the ordinate represents (division of labor). He then presents two isoquants in each figure in such a manner that the minimum capital per man expansion path moves to the right (increased specialization with increased output) in figure 1, but to the left (decreased specialization with increased output) in figure 2. Smith is only correct, he argues, if the situation is as represented in figure 1. Smith was, however, concerned with productivity in total and not just productivity of capital. But were he concerned only with capital it seems unlikely he would reach the conclusion implied by Leibenstein's figures where it pays up to some point to specialize but beyond that point to reduce the amount of specialization already achieved. Leibenstein thinks figure 2 must be admitted as a possibility because large indivisible multi-purpose machines may exist. They may, but are not likely to be more efficient than large indivisible single-purpose machines.

Table 4. Progress ratios of eight products, 1946-1951^a

Product	Progress elasticity ^b	Progress ratio ^c	Adjusted correlation coefficient
Semi-automatic turret lathe #1	-.2868	18.1	.965
Semi-automatic turret lathe #2	-.4114	24.8	.975
Automatic machine tool #1	-.2996	18.8	.890
Automatic machine tool #2	-.3370	20.8	.915
Automatic machine tool #3	-.2595	16.5	.837
Automatic machine tool #4	-.2868	18.1	.856
Textile machine	-.2774	17.5	.889
Multi-purpose construction machine	-.3233	20.0	.906

^aSource: Hirsch (94, Table 1, p. 139).

^bThis is the average percent change in labor requirement associated with a one percent increase in cumulative output.

^cThis is the average percent decline in labor requirements after a doubling of cumulative output.

the skilled workers away. The progress ratios shown in Table 4 would seem to suggest not only falling average costs but falling marginal costs as well.

There is additional evidence along the same line. For example, Hirsch (94) reports an "eighty percent curve" has been found by the air force to approximately describe labor requirements in air-frame manufacture. That is, doubling the number of frames ordered reduces by 20 percent the amount of labor required for each. Certainly Table 4 leads

to the conclusion that the efficiency of labor is limited by the size of the market; it appears that Smith was right.

1. Trends toward monopoly

But while the position that large scale makes additional division of labor possible has been treated as plausible enough, it has never seemed congruent with the assumptions of competition. If advantages of scale are available, why do firms not combine to achieve them until either the advantages of scale are lost or monopoly results?*

Theories of oligopoly provide one answer, Stigler (203) has provided another. Some processes, he contends, are carried on by a firm subject to increasing, others to decreasing costs. When all processes are considered, therefore, the firm may be subject to increasing, decreasing, or constant costs. Naturally if the industry remains competitive, firms must be subject to constant or increasing costs even though certain processes exhibit decreasing costs. This hypothesis may be given empirical content using data prepared by Schuman and Alpert (185).

It is evident from Table 5, that full advantage of scale is obtained at a somewhat lower level of output in pumps than in motors, and that there would be a lowering of capital costs so far as pumps are concerned

*The question is attributable to Knight (116). Piero Sraffa (201) has argued that the concept (I.R.T.S.) may be rescued under competition by assuming that the result of decreasing costs is not lower supply price but improved quality of product. Clapham (35) doubted the validity of dividing industries into a group exhibiting increasing, and a group exhibiting decreasing returns. The controversy is reviewed by Ellis and Fellner (55).

Table 5. Exponent n in the relationship $C=kS^n$ for components of a water pumping plant ($\text{Cost}=k (\text{output})^n$)

Flow gpm. (output)	Pump n	Motor n	Total plant materials ^a n
1	0.5	0.1	0.2
10	0.2	0.2	0.2
100	0.2	0.6	0.3
1,000	0.9	0.9	0.8
10,000	1.1	1.0	1.0
100,000	1.2	1.0	1.2
1,000,000	1.2	1.0	1.2

^aSource: Schuman and Alpert (185, p. 495).

if ten or more plants were established to pump the million gallons. The same, however, is not true of motors. Stigler (203) contends that firms would like to sell or subcontract those activities subject to increasing costs (pumps) in order to concentrate on those subject to decreasing costs (motors). But the former may be too small to support a special fabricator, or too small to obtain needed market information, or too closely related in a physical sense (as above) to make this vertical disintegration possible. Expansion of the industry might make it possible that such activities be undertaken by separate firms and even, after a while, produced competitively. Thus, expansion of the market allows vertical disintegration and tends to make it possible for each process to be

carried on at least-cost scale.* A similar proposition is advanced by Austin Robinson (167).

Evidently this situation involves an economy of scale. It is not an economy internal to any firm since activities originally performed by one firm are transferred to others. Therefore, it is appropriate to call this type of economy a negative cost externality (because costs are lowered), or a scale externality.

It remained for Young (235) to relate such economies to economic development. He reinterpreted Smith's theorem to read: the inducement to invest is limited by the size of the market. Schumpeter (189), on the other hand, appears to have regarded technological change itself as subject to something like decreasing costs. When one innovator manages to break through restraints and establish a new process he makes it easier for those following to duplicate his feat.** Each innovator makes the next innovation easier and less costly.

2. Economies of scale and balanced growth

The modern formulation of the Smith-Young line of thought involves

*This represents an economy external to individual firms. It is further argued that indivisibilities of management will prevent firms from expanding so that each produces under decreasing cost conditions, that is, firms twice as large would be inefficiently run. Firms only slightly larger would be more efficiently run. But this fails to explain why larger firms (those with corporate, rather than single proprietorship form) do not expand until decreasing cost tendencies are overcome by the increasing cost tendencies associated with indivisible management.

**This may be compared with Marshall's third definition of an external economy, i.e., an economy which results from the general progress of the environment (134, p. 441). Marshall, however, had in mind economies in the cost of production and not economies in the cost of development itself.

policy implications and the theory of balanced growth. Rosenstein-Rodan (172,173) and Nurkse (149, 150), combining hypotheses of increasing returns to scale and inelastic demand for exports, have advocated a simultaneous, balanced expansion in output of all products. This "Big Push" as Rosenstein-Rodan (172) named it, appears to have some historical justification in what Rostow (174) has called the "take-off into self-sustained growth." But only if outputs are balanced will there be a demand for every product, because effective demand must ultimately derive from production as Islam (103) has observed. Supply creates its own demand only if output is balanced. But, Kindleberger (114) has noted that the important word "balanced" is not satisfactorily defined.

Rosenstein-Rodan and Nurkse concentrated mainly on production of, and demand for, final consumer goods. The situation leading to balanced growth proposals was described as a horizontal external economy because all products were at the same level in the production process. Strangely enough, Rosenstein-Rodan felt he needed to justify use of the term external economy by describing how costs of output would be lower under balanced growth due to smaller marketing risks.

Recent treatments by Islam (103), Ohlin (151), and Scitovsky (192) incorporate vertical external economies into the theory. These arise due to the expansion of demand for intermediate products. In the later treatments expansion of demand is stressed and all or most processes are assumed subject to internal economies of scale.

Balanced growth theories have been carefully criticized by Fleming (66). He concludes that, in general, they fail to consider external diseconomies arising from factor shortage, and that these will become (at

some level of output) larger than the external economies deriving from balance. But, at least to some extent, this criticism misses the point. Output is to be expanded anyway (and so there will be factor shortages). The question is, should output be balanced or specialized? Activity analysis done by Chenery (29) and Haldi (82) appears to indicate that the answer depends on the extent of increasing returns in important industries, whether increasing returns are, or are not, a general phenomenon, and the effect of specialization on the terms of trade.

But the balanced growth theory has generally been treated as a subject related to, but separate from, the remainder of the externality analysis. This practice will be maintained. Balanced growth is a policy deserving mention. But it has received satisfactory treatment elsewhere-- particularly by Chenery (29) and Haldi (82), and it need not be of concern here. It is useful to return, therefore, to the more basic question of how external effects influence costs and revenue within the firm.

D. Cost Curves and Rents

Generally non-constant returns to scale within the firm are attributed to some variant of the law of proportionality or to physical constants such as the relationship between area and circumference or volume and surface. It is convenient, however, to call all such physical phenomena indivisibilities.*

*To use Joan Robinson's (168) example, were every factor as divisible as sand, every producer could have all the advantages of large scale industry. Chamberlin (27) objects to such loose definitions of divisibility. His approach is more correct, as is demonstrated by Boulding's Plea analogue, though it does not lend itself so readily to analysis.

1. The number of cost curves

A significant indivisibility may cause increasing or decreasing costs as output expands.* Two types of indivisible factors affect supply. First, those that are indivisible from the point of view of the firm (management, machinery, etc.) but not from the point of view of the industry; and, second those that are indivisible from the point of view of the industry (highways, dams, trade journals, etc.).** Thus, indivisibilities bring about at least two kinds of cost curves. But even in the absence of indivisibilities and rents there are three more average cost curves. They are average cost as output is expanded: 1. at the extensive margin, 2. at the intensive margin, and 3. by means of some combination of 1 and 2. Moreover, rising costs give rise to rents so that there corresponds to each of the above average cost curves, a curve drawn marginal to the average cost-including-rent curve. Thus, there are a very large--if not confusing--number of cost curves.

In *Wealth and welfare* (159, pp. 172-179) Pigou introduced a diagram illustrating two rising cost curves. The lower, labelled S_1 , he defined as a supply curve of the ordinary type. S_2 was described as "the difference made to aggregate expenses by the production of one more unit." He concluded that the intersection of S_2 (and not S_1), with the demand curve

*A factor is significantly indivisible if, in the relevant range (while there are still enough firms that the situation remains competitive), it is not possible for a firm to equate the factor price with the marginal value product of the factor. The inequality may lie in either direction.

**The latter implies, to paraphrase Lerner (122), that at the equilibrium output of the industry the marginal value product to the industry of the indivisible factor is not equal to factor cost.

provided the ideal output, and that, therefore, a system of bounties and taxes was required to obtain this ideal. His statement initiated a debate over the realism of externalities (55).

It is not clear which of the curves described above he intended to imply by S_2 . S_1 , apparently, was a curve of costs internal to the firm; S_2 was a curve of total costs (including both internal costs and external costs). Young (236), however, argued that the difference between S_2 and S_1 (the external costs) could only be nominal (i.e., rent) and so did not represent a real social cost.

The reasoning, briefly, is as follows. Private costs rise or fall because of indivisibilities and rents. But, if factors are indivisible from the point of view of the industry (for example, roads, dams, etc.) there will result one sort of imperfection or another or else such factor indivisibility is not significant. (The imperfect case was not, unfortunately, under discussion--at least not by Young.) It is for this reason that a fall in factor prices with increased factor demand could not take place under competition. If factors are indivisible from the point of view of the firm, expansion in demand will, after some point, be met by a proliferation of firms rather than by an increase in output per firm. But this will not involve an increase in costs other than rental costs. Thus, indivisibility (in competition) does not result in rising or falling supply curves. There remains only scarcity as a cause of increasing cost, and this gives rise to Ricardian or transfer rents. Consequently, increasing private costs can only be attributable to rents. And since rents are not social costs but merely transfers, the intersection of S_1 with

the demand curve provides the ideal output. Pigou gradually gave ground until, in the fourth edition, his argument is relegated to the situation in which rents are paid to foreign factor sellers.* As will be seen, he need not have given up quite so much of the argument if he had been willing to give up some of the assumptions of perfect competition.**

2. Pigou was partly right

Suppose, to use Young's (235) example, a subway is proposed but it appears to the subway company that revenues will be less than costs. However, the subway would cause values of contiguous real estate to rise. It is possible that by including this appreciation in value of real estate in the profit calculation, the subway proposal becomes profitable. But the company might not be able to buy the real estate without grossly affecting market values. In this case the subway will not be built. Apparently the private cost-revenue calculation cannot be made to correspond to social costs and revenues even when there are no restraints on the amount of capital the subway company might borrow.

Consider another example. Several companies are planning to mine a previously-worked and now flooded submarine coal pit. The first company to start work must pump out all the water and thus increase its private costs to the point where the venture is unprofitable. It remains

*This position was similar to that taken by Graham (74,75).

**An external diseconomy such as air pollution could hardly be compensated for by increased (rent) charges of air owners to all customers. This would have been a reductio ad absurdum of the competitive model, but such an escape seemed to be blocked by those who questioned the very existence of external economies and diseconomies. These included Clapham (35) and Robertson (166).

unprofitable unless the decreased costs of all subsequent firms (who now do not have to pump water) is allotted to it. All firms will avoid starting work first. The development will be put off indefinitely unless the firms can be integrated or unless they can come to a gentleman's agreement.

If these are the sorts of situations Pigou had in mind (as seems likely) then his primary error was in choosing a competitive framework for his analysis. His examples were easily shown by Robertson (166), Young (236), and Knight (116) to involve non-competitive assumptions. It was, for this reason, alleged that his analysis was unrealistic (116).

Further, in general equilibrium analysis it is assumed that the capital market is perfect so that rents, accruing as net revenues at various inelastic nodes within the economy, can be mobilized and logistically allocated by a frictionless system to where they are most needed. But these assumptions do not appear very useful in studies of development. Rather the competitive assumptions appear to lead to models involving stationary statics.

3. Types of models

More recently, there has been a tendency to move away from consideration of perfectly competitive situations. But the trend toward dealing with more practical situations has not come about without a great deal of difficulty. The nature of this difficulty may be illustrated following Knight (116). Knight finds Pigou's analysis (involving the overcrowding of a smooth road as compared to a rough road) weak because:

"...the assumptions diverge in essential respects from the facts of real economic situations. The most essential feature of competitive conditions is reversed, the feature namely, of the private ownership of the factors practically significant for production" (116, p. 586).

Evidently, real economic situations to Knight are synonymous with competitive conditions. The fact that the ownership of roads is really public was not in dispute, but "real economic situations" involved treating roads as though they were privately owned.

It now appears odd that such an effort should have been made to retain the competitive model. So long as it was used, no firm could be subject to increasing returns to scale and any economies of scale were, as a consequence, external economies. Once imperfect or monopolistic competition is introduced, however, it is quite reasonable to treat most or all economies as internal to some firm. The definitions thus become much less strained, but this is even now not always apparent as is evidenced by the paradoxical nature of Leibenstein's definition (see page 12 above).

Thus, it gradually became clear that one of the more important difficulties involved in the debate over the realism of externalities was the nature of economics itself. It was always possible to make the competitive output a social ideal by introducing more and more stringent definitions of competition. And while in its extreme form the model was interesting, it was hardly related to the solution of any significant problem of the real world. There has been, however, a gradual return

toward realistic models.* And with this return, attention has been redirected to problems of income distribution.

E. Can Pecuniary Externalities Result in Market Failure?

1. Financing economic activity

There exists now and there was, even in Marshall's time, evidence that some firms and indeed some industries were producing in regions of falling average cost. This leads, in a general equilibrium model (that is, under marginal cost pricing), to an immediate and intractable difficulty. How shall the deficit (MC is less than A C) be financed? And this question can only be answered if the normative income distribution is known. As a matter of fact as Oort (153) has observed, decreasing returns gives rise to the same questions about income distribution; however, in that case, welfare implications of income distribution can at least temporarily be put aside. (If MC is greater than A C, firms do not require a subsidy to stay in business even if MC = price.) The question of income distribution has generally not received enough consideration in welfare analysis, and the most important consideration of the question now appears to derive from students of economic development.

2. Pecuniary externalities are alleged to cause market failure

It has become customary in literature concerned with economic development to refer to external economies whenever the profits of one firm

*Thorbecke, associate professor of economics at Iowa State University, has pointed out in a private communication that some economists regard the realism of the underlying assumptions as largely irrelevant provided the model has predictive capacity. In the case above, the model did not have predictive capacity (smooth roads do become crowded) nor was it realistic (roads are not privately-owned).

are decreased by the action of another firm (192). It is implicit that such situations result in an allocation of resources that fails to correspond to a maximum rate of growth, and it is no doubt because of the ubiquitous inter-dependence of profits that externalities are so frequently thought to be a cause of difficulty.*

This is rather surprising because general equilibrium theory suggests that the price system does its best work in allocation of goods to alternative investments. And pecuniary externalities do not lead to market failure in the general equilibrium model. The equilibrating mechanism of general equilibrium works as follows. (Let capital letters stand for firms and lower case letters for their products.) Relatively large profits in any industry C leads to additional investment in output of c; this is followed by a lower c price and a consequent elimination of the excess profits.** Evidently this mechanism is thought to fail, but it is not clear exactly why. Capital rationing is mentioned by Rosenstein-Rodan (172) and imperfections of information and competition are implicit elsewhere.

Apparently, it is felt that excess profits either do not lead to additional investment and output (for example, because of monopoly restraints on entry or because of indivisibilities), or that additional

*See Rosenstein-Rodan (172); Nurkse (150, p. 13), the "real-income effect"; Arndt (3, p. 210), "...external economies are due to the fact that the investor is, for one reason or another, unable to appropriate to himself the whole of the yield of the investment"; Hirschman (95, p. 70-72), and especially Eckstein (50).

**In the meantime someone has obtained extra profits, but as mentioned before, in general equilibrium theory this is ignored because it involves a problem of income distribution.

investments and output do not lead to a fall in the rate of profit (for example, because the industry is subject to decreasing costs). Scitovsky (192) suggests a third possibility: one industry C takes some action (for example, it lowers its product price) which gives rise to relatively large profits in industry B (which, for example, uses c as a factor); then investment in B may stimulate C to extend even further its original action. This action and reaction may continue for some time, and until an equilibrium is reached it appears that there will not be an ideal allocation of resources. Integration of the two industries or even co-ordination of plans would provide superior resource allocation. Eckstein (50) provides a similar argument.

External economies as used in economic development appear now to mean a shift in profit caused by an outside influence. This change in terminology* should not be too surprising. The effect of one industry's purchases on the scale and therefore unit cost (and sometimes selling price) in other industries have been recognized for some time.** But what has not frequently been recognized (except by students of development) is the possibility that pecuniary externalities may result in market failure. Still, interdependence of profits is not a sufficient condition for market failure. (Profits are interdependent in general equilibrium too.) Why, then, do development economists think market failure is involved?

*Originally, as noted above, the terms "external economies" and "external diseconomies" were applied only to costs and not to revenues.

**Viner (228) called these pecuniary external economies, but they were not considered to result in market failure until about the time of Scitovsky's important paper (192). See also Stockfish (205).

A shift in the rate of profit within one or several industries brought about by a shift in price, falling costs, or technological changes provides the economic system with a signal to transfer resources from some industries to others. This temporary disequilibrium in profit rates provides--in the classical case--shortlived windfall gains and shortlived windfall losses. Moreover, it is possible to treat this disequilibrium state as without a time dimension so that windfall gains and losses equal zero. At any rate, these gains and losses are the cost of the signal. They persist no longer than is necessary to bring about the appropriate resource shifts. They are like a flow of electricity in the wire to a signal light at a railroad crossing--entirely necessary for the signal's proper operation. One certainly would not want to consider the size of such windfalls as a reflection of the system's failure. Rather they are in the neo-classical tradition, incidental to the system's success.

This is satisfactory so long as the theory makes use of a general equilibrium model. But the real world is not always so flexible. What happens if resources fail to move because of inertia, capital rationing, scarcity of entrepreneurs, lack of information, indivisibilities, government intervention or monopoly constraints? (These reasons for failure of adjustment receive more adequate treatment in chapter five.) Then the signal goes on flashing but eliciting no response. It is as though a light signals the presence of a train but there are no automobiles in view and none is expected.

Still, in a general equilibrium model it is not extra profits that involve failure; the system's failure is involved only to the extent that

tangible resources are not appropriately shifted. But this conclusion reflects the systematic neglect, by general equilibrium theory, of changes in income distribution. Actually, a change in profits or rents is not irrelevant. Changes in income distribution (particularly between firms) do count. Fundamentally, this is because of the inevitable imperfection of the capital market. When the capital market is imperfect asset expansion may have to be financed out of internal funds. And the economic system may fail to expand in an appropriate manner if internal funds do not accrue in an appropriate manner.

F. Summary

This chapter has provided a partial description of the evolution of that aspect of economic research which concerns itself with market failure in economic development. References in other chapters help to provide a more complete picture.

Several points have been stressed. These include: the importance of income distribution among firms; the difficulty of defining externalities in the absence of a satisfactory normative model; and the difficulty of measuring externalities because of the imputation problem.

The next chapter provides an examination of the welfare aspects of externalities, and in the following chapter the definition of externalities is examined in connection with a simple model of economic development.

III. NORMS AND ASSUMPTIONS

A. Capital Formation

1. Introduction

In this chapter an attempt is made to indicate how externalities in economic development relate to modern theories of welfare economics. It turns out that welfare theory plays a significant role in the definitions used in this thesis as well as the obiter dicta and implicit definitions found in development literature. Difficulties involved in dynamic welfare economics are considered in the first section. Welfare implications of the various externalities are considered in the second section.

2. Ideal rate

Suppose some omniscient central planner knows the exact form of all production functions, the relevant welfare functions of all existent and unborn persons, and suppose that there is no difficulty regarding the manner in which individual welfare functions at any one time enter the social welfare function.* In these circumstances it is still not possible to find the ideal rate of saving, investment, and capital formation.

This is because, even given these extreme assumptions, the ideal rate of capital formation depends on how much the welfare of future

*In this thesis social welfare functions except where otherwise specified are assumed to be of Paretian type. If one persons's welfare increases ceteris paribus, the social welfare increases.

generations* is to count. The rate of capital formation influences the inter-temporal distribution of consumer goods. Then if future generations dominate the social welfare function, the ideal present rate of capital formation must be high. While if generations farther away than some time horizon are ignored, it will be unnecessary to make allowance for the prior formation of capital equipment for them.

In practice, the welfare functions of future generations are not known; nor are they independent of the present level of capital formation. Even the size of future populations probably depends on the present rate of capital formation. This complicates the matter still further.

It might be argued that the future is so uncertain that little attention should be given to the welfare of persons who might be born in several hundred years. This does not change the basic problem but only narrows it in time. For if such distant future people are not to count it is still necessary to allocate weights to persons within the immediate future.

In fact, it is rather difficult to believe that sufficient weight was given, by decision-makers of the past, to future (now present and future) needs. Carter and Williams (25) believe the United States has made profligate use of her natural resources. Table 6 does not necessarily corroborate their view, but it does raise some important questions respecting inter-temporal decision-making.

It should be noted that this sort of approach also has its critics.

*Here future generations means the existent community at some time beyond the present. This might mean tomorrow or it might mean 100 years hence.

Table 6. Approximate degree of exhaustion of economically available mineral resources in the United States as of 1944^a

Mineral	Percent of original remaining
Magnesium	100
Nitrogen	100
Bituminous coal and lignite	99
Salt	99
Phosphate rock	93
Potash	92
Molybdenum	91
Iron ore	68
Natural gas (proven reserves)	65
Anthracite	65
Sulfur	59
Fluospar	56
Antimony	44
Petroleum (proven reserves)	42
Copper	40
Zinc	33
Tungsten	30
Manganese	30
Bauxite	28
Vanadium	28
Chromium	21
Chromite	Small
Gold	20
Lead	17
Silver	17
Mercury	3
Cadmium	Small
Platinum	Small
Asbestos	Small
Nickel	Small
Tin	Small

^aSource: Kapp (109, p. 273).

Scott (193) has noted that the conservation question might be better put as: which is it most important to leave for posterity; natural resources, or capital equipment manufactured from those resources. But if this is the relevant question, the record still appears blemished.

It is quite reasonable to suppose, for example, that future generations (people 20 to 50 years hence) will have a use for trees and forests. Yet according to U.S.D.A. sources (see Kapp, 109), management of timber is frequently poor and sometimes destructive. Table 7 is illustrative.

Table 7. Timber cutting practices in the U.S.A., 1948^a

Ownership characteristics	Character of cutting (percent)					Total
	High-order ^b	Good ^c	Fair ^d	Poor ^e	Destructive ^f	
Private	1	7	28	56	8	100
Public	8	59	19	13	1	100
Size of holding						
Small	11	69	19	1	0	100
Medium	6	37	32	24	1	100
Large	3	44	10	41	2	100
All lands	3	20	25	46	6	100

^aSource: Kapp (108, p. 275).

^bWill maintain quantity and quality of yields.

^cLeaves land in control of desirable species.

^dMaintains a reasonable stand of growing timber.

^eLimited means of natural reproduction, no growing timber.

^fLand left without timber and without means of reproduction.

Again it might be argued that since the welfare functions of future generations cannot be known the only persons who should count are those whose welfare functions are known. The only persons who should count are the present generation of choice-making adults. But this argument rests on a false premise. The welfare function of existent persons is not completely known, nor is the welfare function of future persons completely unknown.

True, uncertainty makes it necessary to use a different set of weights than would otherwise be used. But it does not eliminate the weighting problem.

There is one possible way within the new welfare economics to avoid the difficulty posed above. That is, the method of inter-temporal over-compensation. Since capital formation influences the inter-temporal distribution of consumer goods, it presumably operates so that persons at one time may benefit while persons at another time may lose. Those who gain may be able to more than compensate the losers.

For example, the large scale of investment required just before take-off into self-sustaining growth is associated with full employment but a scarcity of consumer goods.* The beneficiaries of this postponed consumption are largely those who inhabit the country after take-off.

Is it possible to say that take-off into self-sustaining growth involves achievement of some desirable goal? Application of a Paretian-type welfare function indicates that such a statement is not possible, because of the shortage of consumer goods prior to take-off.

*For a description of historical stages in economic growth see Rostow (174).

However, under some circumstances the beneficiaries may over-compensate the losers. Then take-off would be a desirable goal. But, in this situation, overcompensation involves transfer of funds through time. And compensation must be paid.* Thus, it becomes important to know whether funds can or cannot be transferred through time.

Under some circumstances the inter-temporal transfer of funds for overcompensation is possible. International borrowing and lending provide just this service to both creditor and debtor nations. But it may frequently happen that such international transactions are not feasible. In this case welfare functions which specify desirable inter-temporal distribution of income are necessary (see the next section: Welfare implications).

The inability of the new welfare economics,** under even a very extreme set of assumptions, to define an ideal rate of capital formation is discouraging. The new welfare economics is generally able to provide criteria for the determination of certain interesting necessary conditions for an ideal under some strong set of comparatively unrealistic assumptions. Here that is not possible. The ideal rate of capital formation can not be determined unless a considerable segment of future populations are assumed away entirely.

Consequently, in economic development, it appears necessary to stress

*There seems still to be some doubt that compensation must be paid as Reder (164) goes to considerable trouble to prove. But it is self-evident so long as a Paretian-type of welfare function is used.

**The new welfare economics is a term here applied to that body of welfare economics which is able to avoid inter-personal comparisons.

problems of positive economics. But this provides a special difficulty for the approach taken here. Traditionally, external economies and diseconomies have been related to normative general equilibrium analysis. An alleged external economy or diseconomy was only concluded to be a real external economy or diseconomy if it resulted in interference with the usual marginal conditions of general equilibrium analysis. But these marginal conditions were also necessary for welfare maximization. Consequently, external economies and diseconomies were of normative significance.

But this procedure cannot be retained in a consideration of externalities in economic development. There, externalities cannot be defined so that they have necessary normative significance because few normative statements may be made. Consequently, the definitions and procedure used by those who treat external economies and diseconomies in economic development do not represent a development of those concepts from former uses.*

B. Welfare Implications of a Simple Model

In the previous section attention was focused on the requirements for normative statements using the weak criteria of the new welfare economics. In this section externalities are related to the usual requirements for welfare maximization in order to highlight and clarify the difficulties. This procedure is also of use in comparing the value judgments required to specify an inter-temporal W function with the value judgments required to specify a W function in statics.

*It seems likely that this is the reason why economists concerned with economic development have been loath to define the terms external economy and external diseconomy.

Assume the following. Two homogeneous factors, labor L and capital C , in fixed supply may be used to produce two homogeneous products, apples A and nuts N . The two production functions $A = f(L_A C_A)$ and $N = g(L_N C_N)$ exhibit independence, constant returns to scale, and diminishing returns. Two persons, Jones J and Smith S , have ordinal utility functions $U_J = h(A_J N_J)$ and $U_S = i(A_S N_S)$ sensitive only to own consumption. What conditions define the maximum social welfare?*

Figure 1, p. 45 provides an Edgeworth box diagram in which any point defines six quantities. These are the amounts of L and C used in production of A and N , respectively, and the total output of A and N . The lower left corner is the apple origin, the upper right corner is the nut origin. Isoquants (marked ii), by the assumption of diminishing returns, are convex to their origins. The dimensions of the box are given by the amounts of L and C available.** The marginal rate of substitution of factors is given by the slope of the isoquants.

In this situation it is necessary, for a Pareto optimum, that the ideal output point, Δ^1 , occurs where the marginal rate of factor substitution in A production and in N production is equal. That is, isoquants must be tangent. If isoquants were not tangent it would be possible to have more A without sacrifice of N , or more N without sacrifice of A . In Figure 1 all such points of tangency are connected by the line hh .

*There is much to be gained by starting with this simple, almost traditional model. If it seems elementary one can only plead that things get complicated soon enough.

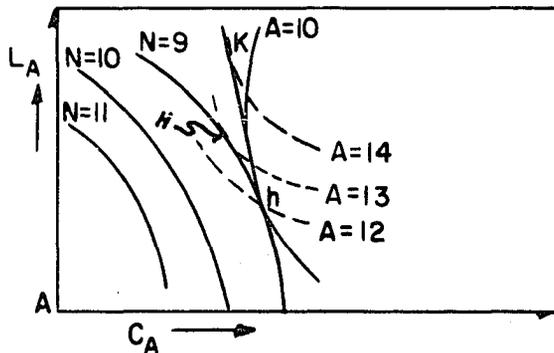
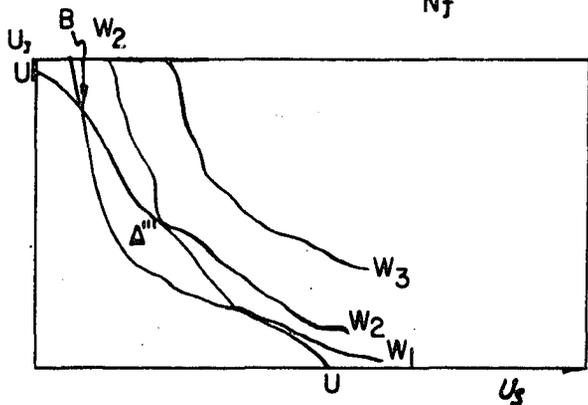
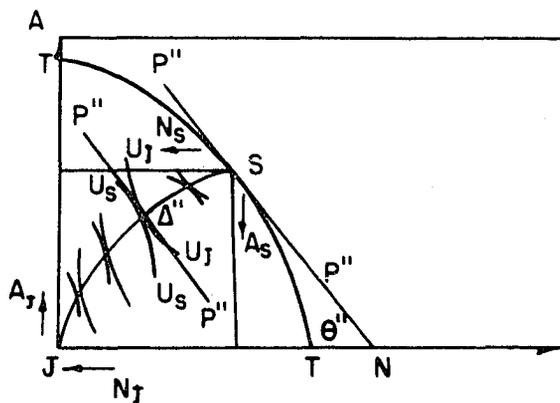
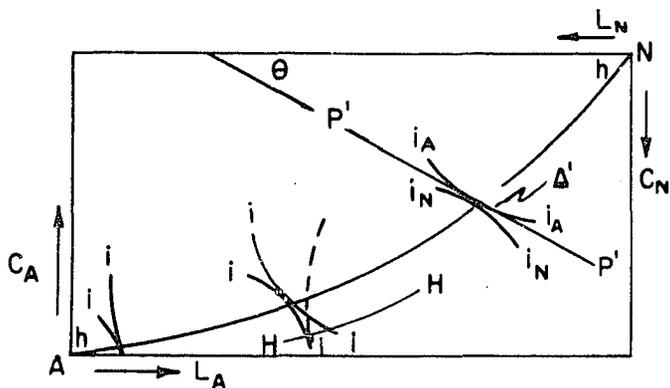
**Elastic factor supplies are difficult to fit into graphic analysis for this reason.

Figure 1. Tangency of isoquants

Figure 2. Production possibilities

Figure 3. The social welfare function

Figure 4. Interdependent isoquants



Let apples be fixed at some level A^1 (not marked), then the maximum amount of nuts, N^1 , which can be obtained is given by the nut isoquant which is tangent to the A^1 isoquants. Alternatively the amount of nuts could be fixed at N^1 in which case the maximum amount of apples would be A^1 . It is possible to use this procedure to obtain, in apple-nut space, a single curve which indicates for each level of one product the maximum available amount of the other product. Such a curve is labelled TT in Figure 2, p. 45.

At any point S on TT a new Edgeworth box diagram may be constructed. Let S represent the origin of Smith's ordinal utility map and let J represent the origin of Jones' ordinal utility map.

It is necessary for welfare maximization that the ordinal utility curves of the two persons be tangent. If not, it would be possible to move either Jones or Smith to a higher ordinal utility curve without reducing the ordinal utility of the other person. The line SJ traces the locus of all such points. But at the ideal point, Δ'' , it is also necessary that the subjective MRS curves be equal in slope to the slope of TT at S. If this were not true it would be possible to move northwest or southeast a marginal unit on TT (in effect transforming apples into nuts, or nuts into apples) in such a manner as to make one person better off without making the other worse off. For example, both consumers may be just willing to exchange one apple for one nut. And in production, by shifting resources, it may be possible in effect to trade one nut for one and one-half apples. Then, if both exchanges were made consumers would be no worse off, but there would be half an apple left over. This

could be used to increase the utility of either Smith or Jones.

For each box diagram such as the one in Figure 2 it is possible to proceed as follows. For any level of U_J obtain the maximum level of U_S . In this manner it is possible to obtain for each point on TT a curve in $U_J U_S$ space which indicates the maximum level of U_J for each level of U_S . Now, one may draw such a curve for each point on TT and obtain the envelope.*

This envelope is plotted in Figure 3, p. 45 as the line UU. The scales of the diagram in Figure 3 may expand or contract so that it is not possible to say anything about the shape of UU. But UU defines an infinity of points which are all efficient in the Paretian sense. To proceed further it is necessary to have a Bergson-Samuelson social welfare function defining the relative weights to be given to U_S as opposed to U_J (14, 180). Let such a function be WW. Δ''' therefore defines the society's bliss point.**

Implicit in this maximization of welfare procedure is the determination of prices. Let Δ''' correspond to Δ'' , correspond to Δ' . Producers will use the minimum cost factor combination, that is, the factor combination which equates ratios of marginal factor productivities with ratios of factor costs. Then at Δ' (Figure 1), factor prices must be as represented by the price line P^1P^1 . Similarly, in Figure 2, producers will not produce the S combination unless the product price ratio

*Corresponding to each point on TT there may be only one point on the JS curve that exhibits the same slope as TT. Plotting the $U_J U_S$ values at such a point yields a point on the envelope in $U_J U_S$ space.

**This terminology appears to originate with Samuelson. When there is danger of using words having emotive value he seems to find it useful to use instead a word which is ludicrous.

(confronting producers) is $P^i P^i$. And consumers will not consume the Δ^i combination unless the product price ratio (confronting consumers) is again $P^i P^i$. The WW function defines the ideal distribution of goods and therefore (in the absence of saving) provides information respecting relative factor ownership.* In the absence of saving, by Euler's theorem:

$$1. L P_L + C \tan \theta^i P_L = A P_A + N \tan \theta^i P_A.$$

This equation defines a ratio between the factor prices and the product prices. Consequently all the price ratios are known. The absolute level of prices is not known but all quantities may be fixed without it.**

This completes the analysis and provides a simple, if not very rigorous, proof of duality.

It is interesting to note the role played by WW. The Pareto optimum can only provide assistance in finding UU. But this is not sufficient. An infinity of points (the area between UU and $W_1 W_1$) which are suboptima by the Pareto criterion, are superior by the WW criterion to a point such as B (Figure 3) which satisfies all Paretian requirements. In practice this means that even the simplest recommendations cannot be made unless one knows whether the effect on income distribution is or is not desirable. It was only by ignoring this question that general equilibrium

*Factors must be owned so that for Smith:

$$L_S P_L + C_S \tan \theta^i P_L = A_S P_A + N_S \tan \theta^i P_A.$$

A similar equation could be written for Jones but it is not independent of the above. But two equations are required to determine L_S and C_S (and, by subtraction L_J and C_J). Therefore various factor ownership mixtures will do.

**All of this may be found in elementary texts. Bator (10) provides the most concise graphical treatment. The 2 x 2 x 2 case is analogous to the n factor, m product, g person case. Using calculus instead of graphs the same price ratios may be found.

analysts were able to give externalities normative significance. The ultimate consequences for a theory of development have already been considered. Normative statements are not possible. In stationary statics the situation is not nearly so difficult. We may not know the shape of WW or everyone may tender his own idea of how WW should look. We may not know whether to obtain WW by a voting procedure or by a paternalistic government. But at least we know what it means: national income should be distributed equally, is a meaningful statement so long as it is confined to the current situation.* Moreover, the marginal conditions will hold respecting any WW. But the crux of the matter is that in statics distribution is only one of many problems and so its treatment can be postponed, but in development, inter-temporal distribution is the very essence of the problem.

To facilitate consideration of changes which take time, let Figure 1, Figure 2, and Figure 3 be horizontal slices through a vertical time dimension. Position on this time axis may be indicated by a subscript (for example, A_{t+1} or A_{t-1}). It is convenient to let the origin of the time axis be the time-slice depicted in Figure 1, Figure 2, and Figure 3, and to denote this by the absence of a subscript. Finally, let the subscript T (for example, A_T) represent the path of the variable through time. For the moment assume $W_T W_T$ is meaningful. In this situation it may be demonstrated that achievement of Δ_T ^{'''} implies the existence, at any time, of the price and tangency conditions demonstrated above.

Naturally there are other conditions to be met in order for the

*Actually, there is some question about even this. Does income mean money income only, or does it also include psychic income:

system to track any Δ_T solution. Now each producer must be perfectly informed respecting the instantaneous rate of change of all prices and must react so that it is a matter of indifference to him how further increments of investment are scheduled. This condition alone results in an efficient (one of many) program of capital accumulation.

Let $P_1(t)$ be the price of commodity 1 at time t , then all producers know with certainty each $P_i(tX_1)$, $i=1, 2, \dots, n$. Let $r_1(t)$ be the net earnings of commodity 1 over and above maintenance and replacement expenses, during the t -th period so that $r_1(t)/P_1(t)$ is the own-rate of interest of commodity 1. The condition above may be interpreted to mean:

$$2. \quad \frac{r_1(t)}{P_1(t)} + \frac{P_1(t+1)}{P_1(t)} = \frac{r_2(t)}{P_2(t)} + \frac{P_2(t+1)}{P_2(t)} = \frac{r_i(t)}{P_i(t)} + \frac{P_i(t+1)}{P_i(t)} = r_0 + 1.$$

Here the 0-th commodity is numeraire so that r_0 is the money rate of interest. If equation two does not hold it will be possible for arbitragers to change from one kind of investment to another and thereby make a certain profit. Equation two, therefore, is in the invisible hand tradition (48). But these conditions though they enable the system to track some Δ_T are not sufficient for the system to track the particular Δ_T''' solution.

1. Types of market failure in a welfare model

It has been implicitly assumed that all points of tangency occur within the Edgeworth box diagrams. This is not necessary. When tangency occurs outside the diagrams a combination on the bound may be chosen. This causes inequalities to creep into the mechanics of defining necessary

conditions, but introduces no difficulties in finding a maximum of welfare. For example, in Figure 2 the slope of TT may not be equal to the slope of subjective MRS at any feasible output. This implies specialized production of either apples or nuts.

a. Indivisibilities The assumed homogeneity of factors and products together with constant returns to scale and diminishing returns, produced smooth convex curves. Indivisibilities if not too large result in linear segments and sharp corners. But linear programmers have demonstrated how such situations may be handled. Larger indivisibilities can cause increasing or decreasing returns to scale. This is a difficulty of more substance. Under decreasing returns to scale the ideal factor prices (given by P^1P^1 in Figure 1) times their respective factor quantities do not generate enough income for purchase of the product at the ideal product prices (given by $P''P''$ in Figure 2). If the difference accrues as entrepreneurial profits, then entrepreneurship as well as factor ownership must be distributed in a manner consistent with the ideal income distribution.

Increasing returns to scale, on the other hand, result (when factors are paid their MVP) in larger incomes to factor owners than expenditure on products. Worse, it may result in non-convexities in transformation functions (TT of Figure 2).^{*} If TT were non-convex at S, the Δ'' configuration would not be maintained by price $P''P''$ because S would

^{*}Increasing returns to scale in the output of apples may be more than compensated by decreasing returns to scale in the production of nuts so that TT remains convex. If both products are subject to increasing returns to scale, it is still possible that TT will be convex because of factor substitution under diminishing returns. Such difficulties are less serious when there are a large number of producers. See chapter five under non-convexity.

represent a minimum rather than a maximum profit position for producers. S could be maintained by a system of ad valorem sales taxes. But this might not be consistent with maintaining Δ'' . If not (and this would be the usual case), there is no price appropriate to both sides of the market. Producers must receive prices different from those paid by consumers. Moreover, under increasing returns to scale average costs are falling and $MC < AC$. Therefore, competitive producers sustain continued losses. Alternatively, if producer output is so small that losses are not realized, the marginal conditions are not met.

Indivisibilities also result in tendencies toward monopoly. Monopoly results in some producer control over prices. Thus, for example, if apples are produced by a monopoly and nuts are produced competitively, $P''P''$ in Figure 2 may be tilted counterclockwise. This may imply an income distribution closer to or farther from the ideal Δ''' . Monopoly may result in excess capacity (failure to use the full Edgeworth box of Figure 1). This problem might be set aside by assuming full employment, but it is not clear in dynamics what full employment means.

b. Interdependence The assumed independence of production functions precludes the existence of technological external economies (228). If this assumption is withdrawn, say in apple production, the production function for apples may take the form $A = f^1(L_A C_A L_N C_N)$ or even $A = f^2(L_A C_A L_N C_N N)$. It is possible to trace out new isoquants for Figure 1 on these assumptions because at any point the quantity of each independent variable is known. Let the dotted curve in Figure 1 be such an isoquant. The locus of socially ideal factor proportions must

go through the points where such dotted apple isoquants are tangent to nut isoquants. Part of such a curve is shown as HH.

If, however, the apple producers do not pay for the effect of L_N , C_N and N on apple output they will continue to equate the ratios of marginal productivity of factors for which they pay with the price ratio. That is, they continue to seek out points along hh.

The result is failure to achieve UU in Figure 3. Moreover, at any achieved point on hh, apple producers obtain more or less output than that for which they have budgeted. They obtain windfall gains or losses.* There are three possible measures of the extent of such external economies and diseconomies: 1. the worth to apple producers of the increment in apple production; 2. the cost to apple producers of purchased factors which could be used to substitute for the unpaid factors; 3. the amount which such unpaid factors could be worth to apple producers who used them under existing prices to maximize output.**

Refer to Figure 4, p. 45. Solid curves are (hypothetical) isoquants of the type $A = f(L_A C_A)$. Dotted curves are isoquants of the type $A = k(L_A C_A L_N C_N N)$. Profit maximizing apple producers seek the point h. They expect output to be 10 and output is really 12. There is an

*After a time expectations may become so reconciled to such isoquant shifts that no one is able to estimate the position of the original isoquants. Could an orchardist estimate the level of yield in a world devoid of bees?

**The amount which such unpaid factors would be worth to society under the Δ''' configuration of inputs, might also be used as a fourth measure. But this is extremely difficult to obtain.

external economy under measure one of 2 Pa.* But apple producers might attempt to substitute labor for capital along the price line hk. If prices did not change apple producers could reach the point k, and there would be an external economy (measure three) of 4 Pa. But, unless the nut isoquant is linear along hk, factor prices will change.**

Apple producers would presumably discontinue any shifts that led to isoquants lower than $A = 12$. Nut producers would presumably discontinue shifting toward less intensive use of labor if they found such shifts leading to nut isoquants lower than $N = 9$.*** The result would be production at point H where the external economy = 3 Pa.† H should not be confused with Δ' . H is only one of a series of such tangency points, while Δ' is the socially ideal point of tangency.

*Under constant returns to scale and competition the external economy under measure two will also be worth 2 Pa.

**The possibility of linear segments, corners and nonconvexity in dotted isoquants is not troublesome so long as producers respond only to private production functions (solid curves).

***But it seems unlikely that producers are so wise. In fact the usual n firm situation confronting them is one in which recognition of the true isoquants (the dotted ones) is hopelessly complicated.

†Index number problems have been assumed away. Actually P_A and P_N will change. But this model only provides relative prices so it is not possible to determine how much. Even when hypothetical as well as actual values of A , N , P_A and P_N are known all four would likely be different in the two situations (h and H). This gives rise to grave index number difficulties. The Hicks (91) overcompensation criterion could presumably be used to find out whether a social economy or social diseconomy existed. For example, (letting superscript, h, stand for the situation as it would exist in the absence of the external economy or diseconomy and superscript, a, stand for the actual situation) if $\sum p^a q^a > \sum p^h q^h$ and $\sum p^h q^h < \sum p^a q^a$ then a social external economy exists.

If H were achieved, there would be no external economies by a Pareto criterion. But if Δ^1 were not achieved, there would be an external economy or diseconomy by the social welfare criterion. Probably enough has been said to indicate the difficulties attendant in measures other than measure one. Unfortunately measure one does not relate to the social ideal Δ^1 . In fact it is possible if $N = g''(L_N C_N L_A C_A A)$ (that is, if there are external economies in nut production), that HH will fall on hh and in fact that Δ^1 be at h. This unlikely chance occurrence involves private external economies and diseconomies, but no failures of the price system. The unfortunate fact is that Δ^1 may be in any direction from h. Consequently, sound recommendations cannot be made in the absence of full knowledge of both total production functions. And a measure of externalities that has normative significance requires the same information respecting both total production functions plus information respecting how producers actually react to the free factor.

If the assumed independence of consumers' utility functions is withdrawn, difficulties such as those just analyzed in connection with Figure 1 arise in Figure 2. The analysis is parallel and may be omitted here.

c. Social goods Despite introduction of time, this model is still excessively static. Where does L and C come from? So far they have just been assumed to come into existence at just the right moment in the specified amounts. If the labor force is allowed to grow exogenously, and if $C_{t+2} = N_{t+1}$, the model becomes more interesting. The Edgeworth box of Figure 1 now expands in a vertical direction by an

amount equal to the production of nuts in the previous time period. It expands east by the amount of growth of the labor force. Incentives for nut purchases (that is, investment) reflect individual preferences for future income. When any individual can buy in $t+1$ only a small proportion of the nut output, his income in $t+2$ presumably must reflect this purchase.*

Yet we may require an income distribution that does not reflect rewards for investment (for example, equality of income). In this situation Δ_t''' can only be achieved by interference with the laissez-faire mechanism. The tangency of subjective MRS curves in Figure 2 is not consistent with achievement of Δ_t''' . Anyone who is foolish enough to buy nuts provides for a very slight increase in his own and everyone else's future income. This uncompensated gain received by all other persons may be termed an external economy.

It is possible, with fairly major manipulation to obtain the JS locus from Figure 2. The returns to be derived from nut purchases (social goods) accrue equally to both Jones and Smith. Then add the MRS of Jones at each level of nuts J_N to the MRS of Smith at S_N (where $S_N = J_N$). Do the same for MRS curves having origins at S. Connect all points of tangency of such summed MRS curves and the resulting locus of points is JS. Δ'' will be somewhere along JS.** J_N and S_N have nominal signif-

*That is, he must believe that the size of his income at $t+2$ is positively affected by his purchase of nuts at $t+1$. But if the society's income is equally distributed, this is manifestly untrue. Individuals would not invest. Then when income is equal or nearly equal, provision of investment takes on attributes of a social good.

**This matter is more fully treated in chapter five where the reasons why this technique may be used are set forth in detail.

icance only. Actually, U_S and U_J depend on the amount of N and not on its distribution.

But this procedure may be somewhat disconcerting. If subjective MRS curves depend on WW , then in what sense may WW be obtained outside of the system? Does not WW have also to be determined simultaneously with the price and quantity parameters? It does, and this means that externalities cannot be normatively defined except when everything else is known; WW , MRS , U_J , U_S , L , C , and production possibilities.

d. Technological changes Technical advances involve potential increases in productivity. If technical advances appear like manna from heaven, it may be possible to treat them by using Figure 1 and simply relabeling isoquants with every change in time period. This would not be very satisfactory, as most likely the slope of isoquants will also be affected. But surely such developments are closely related to presently used inputs, outputs and techniques. If this is the case, technical advances depend (in Figure 1) on the position of A and N producers. If producers remain at Δ^i for one time period isoquants must be relabeled and slopes changed in one manner, while if producers remain for one period at any other point in Figure 1 isoquants must be relabeled and slopes changed in a different fashion. Moreover, a technological breakthrough may require that producers remain at one point for an unspecified (long) time.*

*Slices (on the apple production surface) vertical to the LC axis and through some point Δ^i and the A origin may have almost any curvature. Such a slice involves A on the ordinate and various levels of some fixed proportions of L and C on the abscissa. If such curves are obtained on the assumption that corresponding to each point on the production function, the LC set of inputs have been held constant through the gamut of time, the curve may be non-convex.

Take Δ^1 and any second best point in Figure 1 at $t=0$. Production for one period at either position involves shifts in technical potentials. In both cases choose the Δ_{t+1}^1 production combination for the next period. Continue in this manner. After a number of periods the second-best initial choice may correspond to the best rather than the second-best final choice. (There seems to be no reason why the two should converge.) Thus, when allowance is made for technical advance, Figure 1 fails to provide h^1h^1 --the dynamic Pareto optimum production locus. This result derives from uncertainty respecting where or when technical advance may be made. Again the consequence is that externalities can at best be given only quasi-normative significance.

e. Irreversibilities As has been noted above, technological changes make it difficult to specify production norms. Discontinuous change may be treated by using linear programming methods, but irreversible changes are more difficult. One can cross a stream on an ice floe by a leap which propels the floating ice to the other side. Presumably, one can get back the same way. But the decision whether or not to cross would become much more difficult if the ice only moved in one direction, or if one could not tell in advance whether return would or would not be feasible. A wrong decision may be catastrophic.

Yet, the world is characterized by irreversible change. Mines become exhausted, soils erode, wage rates and consumer tastes become institutionalized, and overripe crops do not become less ripe. Some changes are reversible, of course. Machinery may be replaced, and even good or bad work habits can be changed, given enough time and determination.

Introduction of irreversibility into the model involves more than making allowance for derivatives that are of different magnitude depending on the direction of approach to the limit. From one direction or the other derivatives may not be defined. Socialist planners might treat such phenomena the same way they treat kinks and linearities in programming a solution. But unfortunately the solution obtained by the decentralized decision makers of the business community is not likely to correspond. To achieve the norm requires that they move to the solution from the right direction and stop at the right time. No approach involving successive approximations will do. Tatonnement does not work.

In Figure 2 imagine each subjective MRS curve to be marked with an arrow indicating necessary direction. It may happen that the Δ'' solution is not obtainable because it is found at a point of tangency of a MRS curve for Jones and a MRS curve for Smith which involve proceeding in different directions.* Moreover, though Δ''' might be feasible in the sense that both MRS curves involve, at the point of tangency, moving in the same direction it still may not be possible to obtain the Δ'' position by means of the laissez-faire system. Achievement of a position from which Δ'' could be approached may be impossible under free enterprise.

Unfortunately the matter is too important to be sloughed over. But it does not fit very conveniently into the simple welfare model used here. It is evident that Δ_T' may not be achieved under a free enterprise

* Δ'' might be maintained if the society were there at the beginning but be unobtainable if the society were not there at first.

system because of insufficient provision for irreversible changes such as soil erosion. That is, inter-temporal benefits or costs may accrue without the possibility of inter-temporal compensation. A complete system of forward prices cannot solve this problem because it makes no provision for inter-temporal transfer of funds.*

f. Equilibrium difficulties So far in this chapter it has been possible to avoid most considerations of uncertainty. But, while this is not a study of uncertainty, it is largely a study of the consequences of failure to integrate plans and failure to integrate accounts. And without uncertainty, failure to integrate plans and accounts would not likely be of great importance. Uncertainty may be related to the welfare considerations of Figure 2. The Δ'' configuration is obtained at price $P''P''$. But relatively higher nut prices may give rise to a gross shift toward more nut output and less apple output (to the southeast in Figure 2). The price may shift beyond $P''P''$ to a relatively high price for apples and relatively low price for nuts. Producers may again over-respond, and so on. The hog cycle provides an ideal example of this sort of response. But business cycles appear to be related, and are certainly much more important in studies of economic development.

Conceptually, a perfect system of forward prices would solve this difficulty (192). But in the absence of such a system it may sometimes be useful to relate changes in the output of one firm to changes in cost

*It might seem that future sales of (say) land of a certain quality would provide present land owners with both funds and incentives necessary to avoid soil erosion. But funds (if any) received by present owners of land sold under futures contracts come from contemporary purchasers and not from persons at the time of future sales.

and revenue in each other firm. Were this possible (and were plans based on such a calculation) the cyclical equilibrium difficulties might be avoided. This matter receives more thorough consideration in chapter five.

C. Theory of the Second Best

The difficulties involved in a normative definition of externalities have now been extensively treated. Some difficulties exist even in static perfect competition (the distribution problem) but in a dynamic world of imperfect competition, prescriptions based on general equilibrium criteria simply do not have normative significance. Such prescriptions are very likely to be wrong.

As an example, consider the problem of maximizing output in a single, unintegrated, successive-stage, productive process having one final product sold to consumers. Firms are A, B, C, Z; where A sells raw materials to B and Z sells to final consumption. There is an equal degree of monopoly so that markup $r = \frac{MVP}{MC} > 1$ is everywhere constant. Withdrawal of a small amount w of a productive factor from stage i reduces the output of that stage by rw . The succeeding stage is deprived of a factor worth rw and its product falls by r^2w . Losses cumulate so that withdrawal of factors worth w from i , reduces the output of the Z^{th} industry by $w(r)^{Z-i}$.

It follows that output will be larger if productive services are taken out of later stages and put in earlier stages. Intermediate products will be priced too high relative to unprocessed factors so that unprocessed factors tend to be too much devoted to later stages of the

production process.

Any policy which encourages additional fragmentation will make the matter worse. In fact, the ideal would appear to be a single vertically integrated monopoly. Movements toward the competitive ideal are not desired, unless it is possible to go all the way so that $r = 1$ in every industry.* Thus, given monopoly elements that cannot be removed, the norms of general equilibrium are no longer to be desired.

This conclusion has been made perfectly general by Lipsey and Lancaster (127). Given a constraint on the ability of the system to achieve one of the general equilibrium norms, the others no longer have normative significance.

For example, consider the problem of decreasing cost industries. Costs may decrease because of economies of scale or because of technological innovation. Price cannot equal marginal cost (either the cost of the last unit produced in a decreasing cost industry or the cost of production of the most advanced firm in a technologically advancing industry) or some firms will sustain continued losses. What, then, should be the pricing policy of constant - cost government controlled industry?

The general equilibrium norms no longer apply. In fact, the best that such a government controlled industry can do may be to set its price above marginal cost by a proportion r less than that used by the decreasing cost industry.**

*For a more extensive treatment see McKenzie (136).

**This conclusion follows from section y of the Lipsey and Lancaster article (127), but applies only to a rather simple model.

D. Summary

In this chapter necessary conditions for defining and achieving a maximum of welfare position in a very simple dynamic model have been examined. It has been noted that ordinarily no normative statements may be made. The achievement of a Pareto optimum is necessary but not sufficient for achievement of a maximum of welfare position. But a Pareto optimum is neither necessary nor sufficient for an improvement in welfare. Consequently, the Paretian conditions are not adequate criteria (in an imperfect world) for normative judgments.

It is possible to assume the existence of a social welfare function and to examine how certain external economies and diseconomies bring about failure to achieve the bliss position. But it is probably not very meaningful to assume the existence of an inter-temporal social welfare function. If not, the elimination of all unrewarded external effects cannot be recommended. By some hypothetical social welfare function an infinite number of points exhibiting externalities are superior to an infinite number of points exhibiting no externalities.

True, if bliss is to be achieved by the system of laissez-faire it is ordinarily necessary to eliminate both external economies, and external diseconomies, just as it is necessary to eliminate Pareto sub-optima. But when everything is imperfect, the elimination of an external economy or external diseconomy, just like the elimination of Pareto sub-optima, does not necessarily imply an improvement. For this reason it is convenient to give the word externality only positive significance.

IV. DEFINITIONS AND A MODEL

A. Difficulties of Definition

It has been demonstrated in chapter three that the practice of defining externalities with respect to normative general equilibrium is not satisfactory. It has broken down for two reasons: 1. the requirements of general equilibrium are unrealistic because they fail to make allowance for existent unalterable imperfections (this is the problem of the second best); and 2. because general equilibrium theory fails to include an adequate treatment of income distribution.

The latter difficulty might not be too great in a society having no intermediate manufacturing. If some people get less than a subsistence wage, the social security system will look after them. And output of all final goods and services will adjust to the demand vector. But when intermediate producers are brought into the picture the situation becomes more complex. Adjustment to final demand may require output from an intermediate producer who cannot stay in business without a subsidy. If so, income distribution (among firms) can be ignored only at the cost of a good deal of realism.

For these reasons, it is not useful to define externalities in relation to the general equilibrium norms. The alternative implicitly chosen by development economists is to relate externalities to profits. The following definitions are in that tradition.

But before proceeding it is in order to repeat a word of warning. Non-constant returns to scale and fixed factor proportions give rise to difficulties in imputing the surplus, or deficit, product to individual

factors. There are no satisfactory solutions to this problem. For example, finding the value of land from its productivity is ordinarily not possible. The same difficulty arises when defining and attempting to measure externalities. Under non-constant returns to scale or fixed factor proportions the total value of an external influence on output cannot be found.*

B. Definitions to be Used

The following definitions and analysis are intended to prepare the way for the empirical work in subsequent chapters. The definitions are slanted toward pecuniary externalities because these are the easiest to quantify. It would be desirable to more extensively treat technological change, and problems of entry. But these effects are very difficult to quantify. They are studied in the literary and theoretical chapters two and five, but receive scant attention in this chapter, or in chapter six. It should be emphasized that the model developed here is based on effects that are thought to be important. But they are not thought to be the sole cause or even the most important cause of failure to achieve economic development.

Let capital letters represent firms and lower case letters products. In the following, "externality" will mean: those changes in cost or revenue in any firm B, caused or made possible by a small change in output undertaken by a firm A, when it would have been reasonable for B to have offered compensation (positive or negative) to encourage or prevent A's action, but when no such compensation has taken place. "Caused or

*This important difficulty is not treated by Kahn (107) or by Fleming (66).

made possible", is to be interpreted as follows: If additional revenue in B is made possible by A's action, B may have to undertake some zero cost action in order to obtain the additional revenue. Where B's appropriate action has associated costs the externality is to be interpreted as net of such costs. Where B can avoid the cost increasing consequences of A's action by some move having associated costs, then the externality is to be interpreted as equal to the cost of avoiding the consequences of A's action. However, "made possible" does not include an output change.

It is convenient at this point to introduce two other terms. In the following, marginal revenue externality will mean that algebraic change in marginal revenue of firm A (at initial profit maximizing output) which is caused or made possible by a small change in output undertaken by another firm B. It will be convenient to use the symbol $MRE_{A/B}$ for the marginal revenue externality obtained by A due to the change in output of b.

The term marginal cost externality will be used as the exact cost analogue of marginal revenue externality. The symbol $MCE_{A/B}$ means: that algebraic change in marginal costs of firm A (at the initial profit maximizing output) which is caused or made possible by a small increase in the output of b. Both MRE and MCE may be either positive or negative. It is possible to define average and total revenue (or cost) externalities in the same manner as the marginal cost and marginal revenue externalities defined above.*

$$\begin{array}{ll}
 *TR_A = f(b) & TRE_{A/B} = \Delta TR_A \text{ due to } \Delta b \\
 MR_A = \frac{dTR_A}{da} & MRE_{A/B} = \frac{d\left(\frac{dTR_A}{da}\right)}{db} = \Delta MR_A \text{ due to } \Delta b
 \end{array}$$

Measurement is clearly another matter (see chapter six).

These definitions still leave freedom to distinguish between the long and the short run, and they have the advantage of relating in a familiar manner concepts which it is desired to study, that is change in profit and externalities. Thus, some Δb has as a consequence a $TCE_{A/B}$ and a $TRE_{A/B}$ so that profit is increased by at least $(TRE_{A/B} - TCE_{A/B})$.

$$\text{Then, 1. } \Delta \Pi_{A/B} = (TRE_{A/B} - TCE_{A/B}).$$

In general, the amount of the externality generated by any firm will be a function of that firm's output. Smoke nuisance is related to industrial output, neighborhood enjoyment to the size of Smith's lawn, factor prices to the amount of derived demand, et cetera. However, the particular form of this function must vary a good deal depending on circumstances. Therefore, it is convenient to proceed by using an example. Cost externalities will be treated first, then revenue externalities will be considered, and finally both will be treated at once.

As an example, think of the external-internal economy* (externality) in A which is involved when there are falling production costs and increasing merchandising costs. $TCE_{A/B}$ depends on the effect of Δb on production and merchandising expenditures in A. Assume constant prices, a closed economy, two firms, perfectly elastic factor supplies, that both a and b are consumer goods, and that production functions are not interdependent. These assumptions make $TCE_{A/B} = 0$ as far as production costs are concerned, but merchandising expenditures in A may still be influenced by Δb because of income effects and because a and b may be

*This term is attributable to Robertson (166) and is defined as the decrease in average costs (in a firm subject to decreasing costs) attributable to an increase in the size of the market.

related in consumption.* If Δb moves national income over a relatively small range one would expect income effects to remain approximately constant, that is, to be independent of b so long as investment is not too lumpy. Income effects bring about a downward movement in MC_A . That is, $MCE_{A/B}$ tends to be negative (provided a is not an inferior good).

But, though income effects may be approximately constant, it is clear from the law of diminishing marginal rates of substitution, that, if a and b are complements or substitutes, $TCE_{A/B}$ will not be linear when plotted as a function of b . If a and b are substitutes, $TCE_{A/B}$ will be concave, if complements $TCE_{A/B}$ will be convex from below.**

Figure 5, p. 71 is a graphical representation of the latter relation.*** In order to have B produce output OZ rather than zero or output OY , A would ordinarily be willing to pay a bribe per unit of time

*In this case the activity of B either reduces A 's merchandising expenditure (costs) or increases demand for A 's product (increases revenue). Thus revenue externalities and cost externalities are confounded, and this is one reason why the single term externality is preferable to the two terms external economy and external diseconomy when dealing with imperfect competition.

** Perhaps it is worth stressing that the purpose in this section is the development of a set of definitions and not the development of any solutions to externality problems. The example is convenient because it has already been extensively treated.

*** Were iso-cost curves (of a) drawn in a diagram with output of b on the ordinate and output of a on the abscissa, they might be concave to the ordinate. Iso-cost curves of b might be similarly drawn so that the Edgeworth box diagram could be used to provide an ideal (in the limited sense that advantage is taken of all cost externalities) price ratio of a and b , and output of b for any given output of firm A .

just less than n dollars.* And to keep B producing output OZ rather than some output larger than OY, firm B may be willing to pay much more. The line oyr (a mirror image of the total cost of output Q_a curve) is a line such that its vertical distance from the abscissa provides the maximum bribe A might reasonably pay for any output b rather than have $b = 0$ or $b = oy$.** This maximum bribe will be called pay-off. Note that $MCE_{A/B}$ is given by multiplying the slope of the line oyr by minus unity. The reason is as follows: $TCE_{A/B}$ is the difference made to total cost in A; the slope of $TCE_{A/B}$ is therefore change in the slope of TC_A or, symbolically, $MCE_{A/B}$.

But firm A might be a firm which would make continued losses were it not for the total externalities it receives. Then the bribe A could pay would not equal the pay-off.*** In the following, pay-off will only be considered potential if there are sufficient net revenues (in A) after the change in B that it could be covered.

Potential pay-off will tend to be larger, then, if A produces the output corresponding to maximum net revenue; it will tend to decrease in both directions from this level of output. But net revenue may be large

*The amount of this potential bribe will, of course, be tempered by conditions of profit in industry A. It may be that A would go out of business rather than pay any bribe at all. The bribe cannot be paid, however, or the situation would no longer be one to which the word "externality" could be applied.

**It is possible to obtain a curve showing the maximum amount A might pay to have B produce output oz rather than any non-zero level of output q , by dropping the curve oyr until it intersects the abscissa at q , then taking the vertical height of oyr at oz.

***Actually this provides a fundamental difficulty in determining pay-off. Either pay-off is dependent on profits or else it corresponds to no actual flow.

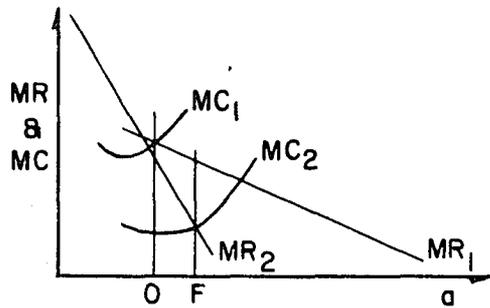
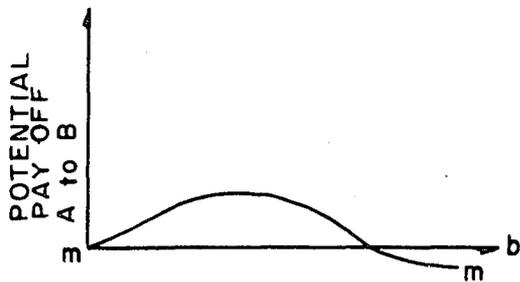
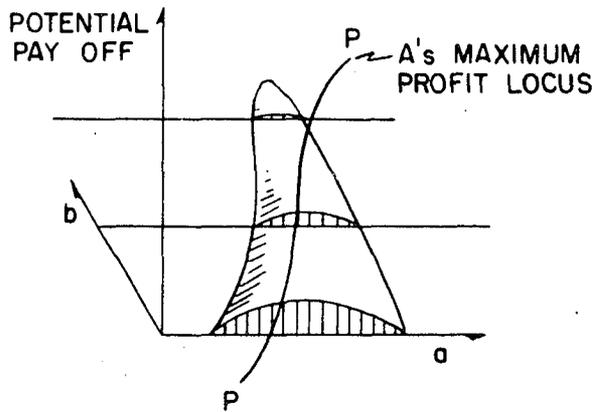
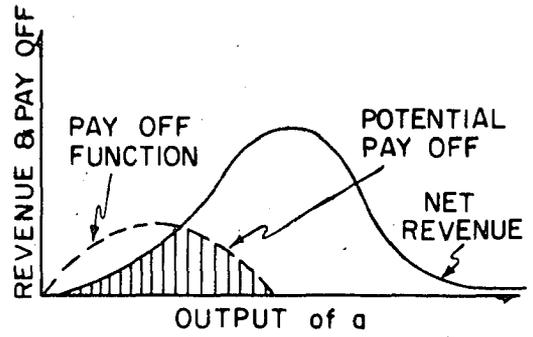
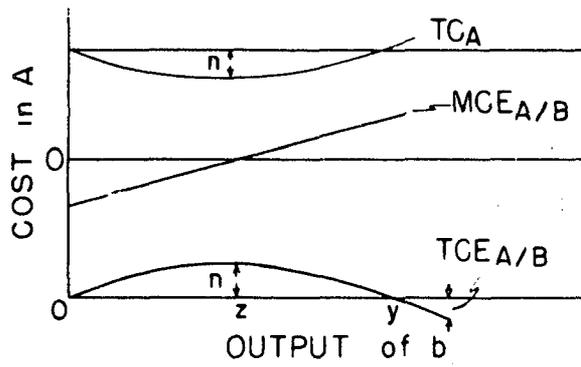
Figure 5. Total costs compared to total cost externality

Figure 6. Pay-off and potential pay-off functions

Figure 7. Maximum profit locus

Figure 8. Potential pay-off A to B

Figure 9. Shifts in equilibrium output



enough that it does not (in any relevant range) constrain potential pay-off. Even in this situation, however, pay-off is a function of a output. For if A were a very large firm any change in unit selling costs would involve much larger shifts in profit than were A a very small firm.

Figure 6, p. 71 illustrates how pay-off and potential pay-off may be related to output of a.

In Figure 5 the potential pay-off was related to output of a; in Figure 6 it was related to output of b. Evidently potential pay-off is a function of both a and b, and it should be possible to show this in a three-dimensional diagram. This is done in Figure 7, p. 71. Here potential pay-off means the maximum amount A would rationally pay B to have B produce an output corresponding to the b axis rather than zero.

A curve relating potential pay-off to the locus of maximum profit in A can be used. This locus P P (Figure 7) is defined as A's maximum profit expansion line over the range of variable output in B. That is, as b increases, A is assumed to produce its maximum profit output. Taking potential pay-off along A's maximum profit expansion line makes it possible to relate potential pay-off from A to B as a function of b. This is done in Figure 8, p. 71.

Note that the line mm in Figure 8 differs from the line oyr in Figure 5, since in Figure 8 A has adjusted to the maximum profit position after the change in b. It will be convenient to refer to pre-equilibrium and post-equilibrium potential pay-off.

Where only two firms are involved and where all externalities are included in the calculation as functions of b and where pay-off is not

constrained by revenue, it is possible, in the general equilibrium tradition, to obtain marginal social costs. Marginal social costs of b equal $MC_b + MCE_{A/B}$. It should be remembered, however, that $MCE_{A/B}$ refers to the pre-equilibrium situation (that is to Figure 5).

Is it correct to say that where the curve mm (Figure 8) is a maximum the output of b is, as far as A is concerned, the ideal output? Clearly, the answer to this question must be in the negative, for while this is, as far as A's cost is concerned, the ideal output of b, B's output also may affect A's revenue, and nothing has been said about that.

The procedure has been to obtain marginal social costs. Presumably marginal social revenue can be obtained in much the same way. Thus, output in any firm when marginal private cost equals marginal private revenue may be compared with the output which equates marginal social cost and marginal social revenue.

If initially, $MPC = MSC$ in both industries, and since MPC in A is shifted as a consequence of Ab, MSC in A must be similarly shifted or in the final situation, MSC will not equal MPC. But MSC of a can be shifted only by an externality generated in A. Consequently (if MSC is to remain equal to MPC), $MCE_{A/B}$ must equal $MCE_{B/A}$.*

This approach may have some appeal to those used to general equilibrium theory. But unfortunately pay-off in the general case is likely to be constrained by lack of revenue so that potential pay-off \neq pay-off.

*This applies only when there are but two firms. Where there are more, the requirement that MSC equals MPC involves $\sum_i MCE_{A/i} = \sum_i MCE_{i/A}$ for any firm A. However, whether this condition does or does not hold in any industry is likely to be extremely difficult to determine.

In this case marginal social costs and marginal social revenue cannot be found and perhaps cannot even be realistically defined. But this difficulty can be avoided, if one does not insist on obtaining social costs and revenue, because the largest change which is ever likely to be experienced involves the introduction of a new industry. When firm B enters into production in the economy there is caused or made possible in A changes in cost and revenue such that marginal revenue in A may be said to have changed from its original position by a vertical amount $MRE_{A/B}$ and marginal cost by a vertical amount $MCE_{A/B}$. MRE and MCE vary with the level of output in A; they are schedules. Therefore, the new (post-B-entry) marginal revenue curve in A is the old marginal revenue curve plus $MRE_{A/B}$. The new profit maximizing output in A is given by the intersection of these two new curves. The situation is illustrated in Figure 9, p. 71.

Profit maximizing output is originally O and finally F. In this case F represents a larger output than O, but this clearly need not be the case because both marginal revenue and marginal cost may be either higher or lower than formerly. This is one way of relating externalities to output.

It must not be concluded, however, that the intersection at F represents the general equilibrium ideal where $MSC = MSR$. Figure 9 represents only the new equilibrium output of firm A.

There is an alternative approach. The effect of Δb on $TCE_{A/B}$ and $TRE_{A/B}$ could be measured if only all other firms would hold their output constant (so that they are not generating new externalities) until the

measurement is completed. But real situations must correspond to this very roughly at best. Or, a system of equations might be set up and solved simultaneously for the TCE's and TRE's or for profit externalities.

Δb gives rise to $\Delta\pi_{A/B}$, $\Delta\pi_{C/B}$, $\Delta\pi_{N/B}$,

where,

$$2. \Delta\pi_{I/B} = TRE_{I/B} - TCE_{I/B}.$$

Moreover, it is possible to think of a small change in output of each industry, A, B, N, giving rise to changes in profits in each other industry. Thus:

$$f \begin{bmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta n \end{bmatrix} = \begin{bmatrix} - & \Delta\pi_{B/A} & \Delta\pi_{C/A} & \cdots & \Delta\pi_{N/A} \\ \Delta\pi_{A/B} & - & \Delta\pi_{C/B} & \cdots & \Delta\pi_{N/B} \\ \Delta\pi_{A/C} & \Delta\pi_{B/C} & - & \cdots & \Delta\pi_{N/C} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \Delta\pi_{A/N} & \Delta\pi_{B/N} & \Delta\pi_{C/N} & \cdots & - \end{bmatrix}$$

The elements of this matrix, the $\pi_{I/J}$'s are externalities. This completes the definition of externalities. Problems of measurement are treated in chapter six.

C. A Model of Economic Development

For the present it is desirable to formulate an extremely simple model of economic development. Choosing a useful and simple model is not

as difficult as it might seem because most models of economic development involve different functional forms of a fairly small number of variables which they have in common. One of these variables (investment) is usually required to carry a substantial share of the load.

The Keynesian investment equations are interesting, but one of the reasons why the Keynesian models are not as useful in studies of development as elsewhere involves his treatment of investment (113). Keynes lets the amount of investment and the interest rate be determined by schedules of demand for and supply of investment funds. These schedules are presumed to be independent, one reflecting time preference of consumers, the other anticipated yield on tangible capital. But when allowance is made for saving by firms and for investment in knowledge (intangible capital) the schedules lose their independence.*

Actually, investment funds do not come entirely from the consumer sector. Businesses also save. To the extent that they do, investment and saving are not independent. Business savings increase when investment opportunities are good and decrease when investment opportunities are bad in a manner that has very little to do with the interest rate.**

*Further, the marginal efficiency of capital depends on investment in two ways. First, investment opportunities once exploited may disappear as Keynes noted, but second, investment funds may be devoted to research and invention. Thus, the marginal efficiency of capital may move up or down with additional investment. The manner of employment of the investment funds determines which will occur.

**According to Meyer and Kuh (139, p. 189): "As with most previous regression analyses of investment behavior it was found that interest rates do not exert an important influence on the rate of investment." The regression coefficient between corporate income and corporate income retention in the U.S.A., 1915-1953, is reported by Dobrovolsky (47) to be as high as 0.97. But there was a good deal of variation among firms of various sizes, the smaller firms exhibiting more volatile changes in percent retention.

Businesses save out of profits, and business saving is substantial.*

It was natural perhaps that Keynes, a student of the stock market, would develop a theory so closely related to the proper functioning of that market. Actually, the stock market as a means for distribution of savings to positions of maximum yield is a most imperfect instrument. All small firms, and some whole industries (such as agriculture) which are made up of small firms, have virtually no direct access to it.**

1. Internal financing

Table 8 provides information respecting saving in the American economy. It is evident from Table 8 that business saving provides a relatively large, though variable, fraction of gross saving. Internal funds are more important in some industries than others, as is indicated in Table 9. Data on the relative importance of internal financing during various phases of the business cycle is provided in Table 10.

*Denison (46) reports that about 78 percent of gross investment is financed out of corporate income. Forty-three percent of gross investment is financed by capital consumption allowances and 35 percent by undistributed corporate earnings.

** Indirect access through the banking system exists. But banks tend to be conservative lenders, suitable as a source of operating capital, but not very well suited to providing investment capital. In Ontario about \$400 millions are needed annually to make possible the normal succession of farm ownership. None of this comes from the stock market and no more than 25 percent is available from the banking system (152).

Table 8. Sources of saving in the U.S.A., 1929-1957^a

Source	1929	1933	1939	1946	1950	1955	1957
	(billions of current dollars)						
Personal saving (includes unincorporated businesses)	4.2	-0.6	2.9	13.5	12.6	17.5	20.7
Corporate saving	2.9	-4.6	0.5	2.4	8.6	10.1	7.9
Net saving	7.1	-5.2	3.4	15.9	21.2	27.6	28.6
Govt. deficit	-1.0	1.4	2.1	-4.1	-8.2	-2.9	-7.7
Statistical discrepancy	-0.3	-1.0	-1.1	-2.0	-0.7	-1.0	-0.7
Depreciation	8.6	7.2	7.8	10.7	19.1	32.0	37.7
Gross saving	15.7	2.0	11.2	26.6	40.3	59.6	66.3

^aSource: Villard (227, p. 100).

Table 9. Relation of internal sources of funds and new investment in physical assets by manufacturing industry U.S.A., 1946-1953^a

	Billion of dollars	Ratio of internal funds to plant and equipment outlays	Ratio of internal funds to plant and equipment outlays plus increase in value of inventories
All manufacturing corporations	72.6	1.02	0.73
Food	6.7	1.07	0.76
Textiles	4.3	1.17	0.88
Paper	3.4	1.17	0.97
Chemicals	7.2	0.87	0.71
Petroleum	10.0	0.64	0.58
Rubber	1.4	1.37	0.91
Other nondurables	4.4	1.57	0.84
Stone, clay, glass	2.4	1.10	0.90
Basic and fabricated metals	12.5	1.05	0.80
Machinery, excluding electrical	6.3	1.43	0.78
Electric machinery	3.2	1.24	0.63
Transportation equipment	6.5	1.07	0.63
Other durables	4.3	1.26	0.70

^aSource: Duesenberry (49, p. 105).

Table 10. Relative importance of internal financing in large manufacturing corporations U.S.A., 1900-1954^a

Years Trough to trough	Internal financing			External financing		
	Undistributed profits	Depreciation	Total	Short term	Long term	Total
	(percent)			(percent)		
1900-1910 ^b	39.1	31.3	70.4	2.4	27.2	29.6
1914-1919 ^c	43.4	16.8	60.1	26.2	13.7	39.9
1919-1921	57.2	41.7	98.9	-46.9	47.9	1.1
1921-1924 ^{c,d}	36.3	42.4	78.7	-6.0	27.3	21.3
1924-1927	47.2	44.1	91.3	-0.8	9.5	8.7
1927-1932	33.6	79.7	113.3	-20.7	7.4	-13.3
1932-1938	12.3	67.2	79.5	13.9	6.6	20.5
1938-1946 ^c	35.0	40.9	75.9	21.5	2.6	24.1
1946-1949	42.0	26.3	68.3	15.0	16.7	31.7
1949-1954 ^c	35.2	30.5	65.7	23.3	11.0	34.3

^aSource: Creamer *et al.* (40, Table 44).

^bNot necessarily the average over one cycle.

^cUnderlying data cover a somewhat different period.

^dData for the period 1920-1923 show total internal financing to equal 138.3 percent.

Finally, Table 11 provides evidence of correlation between investment and sources of internal funds.

Table 11. Annual averages of partial correlations between investment and other variables^a

Year	Profits	Depreciation expense
1946	0.007	0.083
1947	0.073	0.066
1948	0.217	0.093
1949	0.266	0.259
1950	0.210	0.280

^aSource: Meyer and Kuh (139, p. 118).

Evidently there was, over the 1946-1950 period, a growing tendency toward internal financing as the correlation coefficients between investment and profits and investment and depreciation expense exhibit an apparently steady increase. This trend is consistent with findings of Creamer *et al.* (40) that the amount of internal financing tends to move in the opposite direction to the business cycle.

Profits then are associated with investment. In practice, this is not only because profits may be used to finance investment, but because profits enhance expectations and so encourage investment. It may also be because investment leads to profit. Thus, it seems reasonable to hypothesize that the relation between profits (π) and investment (I)

may be put as:

$$3. I_A = f(\pi_A, X_A^1, X_A^2, \dots, X_A^n), \text{ where } X^1 \dots X^n \text{ are}$$

other unknown influences.* Conceptually, it is possible to find changes in A's output that are induced by those pecuniary externalities arising out of B's change in output. Thus:

$$4. a = g(I_a, E),$$

where E includes the effect of capital intensity, degree of monopoly, relative flow of investment funds to output increasing as opposed to cost reducing investment, price elasticity of demand, et cetera. So that:

$$5. a = \frac{\partial a}{\partial I_a} dI_a \text{ from equation 4 and,}$$

$$6. dI_A = \frac{\partial I_A}{\partial \pi_A} d\pi_A \text{ from equation 3.}$$

Then the change in A's output that is a consequence of B's act is:

$$7. \Delta a = \frac{\partial a}{\partial I_a} \cdot \frac{\partial I_a}{\partial \pi_a} \cdot \Delta \pi_{A/B}.$$

This model is useful since it stresses the effect of externalities on investment. The matter is given empirical attention in chapter six.

2. Innovation

Respecting investment and innovation,** there have been two major types of approaches so that the literature is discernibly bimodal. On

*For a review of empirical evidence respecting the influence of profits on investment see Meyer and Kuh (139, appendix to chapter 2 and chapters 8 and 12).

**Innovation is the inauguration of a new process or product together with the invention of that process or product.

the one hand there is Adam Smith (196), Schumpeter (186), and Carter and Williams (26). On the other hand, there is Ricardo and Malthus (Roll, 171), Keynes (113), and Jewkes et al. (105).*

The former group treat innovation as important and endogenous. To Adam Smith the division of labor resulted in growth of both invention and science. Specialized philosophers who knew no single task well, but who stimulated the cross fertilization of ideas were present or anticipated. Their inventions encouraged expansion of output and instigated additional division of labor.

To Schumpeter innovation was the cornerstone. Investment expenditure might be important but it was not basic. Perhaps it is not too great an exaggeration to say that innovation was the cause of development (189), its periodic difficulty resulted in business cycles (186), and its institutionalization doomed the capitalist system (187).

Carter and Williams (26) empirically demonstrate how innovation is becoming institutionalized. The same institutional tendencies are noted by Jewkes et al. (105, p. 156 and 239). Innovations to Carter and Williams are no longer the result of flashes of intuition. Firms invest in innovation. They have research departments, so that invention is both the product of investment and the stimulant to investment. This is not to say that all inventions are attributable to research departments of firms. Such is, in fact, far from the case according to Jewkes et al. (105). Meyer and Kuh (139), in turn, find that research departments and

*These lists are certainly not exhaustive, J. M. Clark (37), for example, might well be included in the former group.

innovating firms may be starved for investment funds with which to carry on their innovation creating processes.* Typically firms and industries which are technologically the most progressive spend substantial sums on research and discovery.

Some idea of trends in research expenditure within various industries is obtained from Table 12.

It is, in practice, very difficult to separate research (towards invention) from development (towards sales) expenditure and most quoted figures, including those in Table 12, combine the two.

Substantial sums are spent on "research" in the United States. Table 13 provides some idea of the magnitudes and trends involved.

*Meyer and Kuh (139) found during the period of their study (1954-1956) that about 14 percent of firms were handicapped in their full use of science and technology by lack of funds. These firms were typically growing so fast that they could not retain enough profits to finance their own expansion. Others have considered the profits of firms and the availability of funds closely related. Compare: "...it does not matter whether we speak of 'growth' or 'profits' as the goal of a firm's investment activities." Penrose (157, p. 30).

"Payout, under an ideal dividend policy in a growth situation, should not exceed the minimum amount necessary to maintain the market position and integrity of existing debt and equity issues and of issues contemplated in the near future." Quinton, quoted by Penrose (157, p. 30).

"The earning of profits becomes the cause of increased costs, not only the fixed costs of expansion, but also of operating costs." Keirstead (110, p. 59).

"This sum (funds to establish a research department within a firm) would have to come from trading profits, and be spent in the hope a return would accrue from it at a later date." Carter and Williams (25, p. 48).

Table 12. Trends in industry research expenditure, U.S.A.^a

Industry	Research expenditure Increase 1956-1960 (percent)	Rank by proportion of revenue spent on research (1956)
Aircraft and parts	103	1
Professional and scientific instruments	79	5
Primary metals	49	8
Electrical equipment	62	2
Machinery	39	3
Stone, clay and glass	38	11
Petroleum products	35	6
Paper and allied products	34	12
Rubber products	30	10
Fabricated metal products	27	7
Food and kindred products	27	9
Textiles and apparel	25	13
Chemical and allied products	24	4
Other manufacturing	19	-
Non-manufacturing	35	-

^aSource: Holland (97, p. 60).

Table 13. Research in the U.S.A., 1941-1956^a

	1941	1946	1951	1956
Total research expenditure (millions current dollars)	900	1,780	3,360	9,000
Percent financed by:				
Government	41	51	59	59
Industry	57	47	39	38
Colleges and universities	2	2	2	3
Percent performed by:				
Government	22	26	21	16
Industry ^b	73	67	68	72
Colleges and universities	5	7	11	12

^aSource: Villard (227, p. 54).

^bOnly about 4 percent of research performed by industry was basic research (that is, unassociated with a specific product or process application).

When innovation is endogenous, it is possible to examine the influence of various human elements that might otherwise be ignored. There can be a shortage of key personnel such as engineers, scientists or Schumpeterian entrepreneurs. Further, capital is not just in tangible things, but may include knowledge and invention--the product of investment in research and personnel.

At the other pole (Ricardo, Malthus, Keynes, Jewkes), invention is

exogenous. It is something that occurs through insomnia to genius.*
 When invention is outside the system it becomes reasonable to ask what happens when the rate of investment (invention use) exceeds the rate of invention. The answer is simple. There will result a fall in the marginal efficiency of capital. And when the marginal efficiency of capital falls low enough (that is, to the interest rate), the inducement to invest will equal zero. The result is stagnation.**

This is not only a logical possibility but an eventuality of some significance so long as invention and innovation are the products of accident or inspiration rather than calculation and investment. The Canadian economy since 1952 is an interesting modern-day example of tendencies toward stagnation. The Canadian government is committed to

*This is not absurd. Probably considerable evidence could be found to support the hypothesis that ideas arise from environment and not from an effort to produce them. Alfred Russel Wallace who, independent of Charles Darwin, advanced a theory of natural selection (1859) is said to have had the basic ideas one evening as he lay on a cot reading Malthus' Essay on population. Archimedes is alleged to have discovered buoyancy in his bath and to have been so thrilled that he hastened home unclothed through the streets of Syracuse shouting "Eureka! I have found it." See (56, p. 168). Again the inventions that sparked the industrial revolution were made largely by artisans, although Watt's steam engine was the product of research and study (206). The anecdote connecting Newton and the apple is presumably apocryphal.

**This line of reasoning can lead to the conclusion that the standard of life may, over some range, be inversely proportional to the stock of capital.

"The post-war experiences of Great Britain and the United States are, indeed, actual examples of how an accumulation of wealth, so large that its marginal efficiency has fallen more rapidly than the rate of interest can fall in the face of prevailing institutional and psychological factors, can interfere, in conditions mainly of laissez-faire, with a reasonable level of employment and with the standard of life which the technical conditions of production are capable of furnishing." Keynes (113, p 219).

a policy of full employment. But the rate of unemployment except for seasonal fluctuations has risen steadily for nine years to a level (in March 1961) of 13.1 percent. Thus, it is not surprising that the term "stagnation" is applied by Armstrong (2).

Whether this tendency is attributable to the relatively low level of research is unknown. But it is significant that research expenditures in Canada are relatively smaller than in the U.S.A. or the U.K. (24). In terms of the percent of GNP, research expenditure in Canada is only about one-third that of the U.S.A., and between one-half and one-third that of the U.K.

One would expect, too, that small firms would less frequently undertake research than large firms. It is only when firms can anticipate a substantial share of the market that it pays to speculate in research.* Indeed there is very good evidence to indicate that relatively more research is done in large than in small firms. Table 14 provides evidence to this effect.

*Studies of returns from research are not easily or frequently made. Griliches (77) has estimated the social return from the development of hybrid corn at seven hundred percent or more. But if full benefits were appropriable by sellers of hybrid seed, firms with a relatively small share of the seed market (that is, competitive firms) would not be interested even in such a profitable venture, because all other sellers would share in the benefits but not in the costs.

Table 14. Concentration of research in large firms, U.S.A.^a

Number of employees	Percentage of firms undertaking research
Up to 100	8
100-499	22
500-999	42
1000-4999	60
5000 or more	94

^aSource: Villard (227).

D. Summary

In this chapter the definition of externalities has been examined. Because pecuniary externalities result in some redistribution of income among firms, a model of economic development stressing this aspect of economic development has been put forward.

Internal funds, as has often been noted, are relatively important primarily because they are cheaper and do not involve risk of loss of control. No doubt, for some purposes, the distribution of income among individuals and among firms is of little importance. But the income of firms provides, among other things, a source of investment funds. Thus, rents and pecuniary externalities appear to be of some considerable importance in economic development. The general tendency to ignore income distribution among firms is apparently but one aspect of the tendency to establish norms that relate only to the best of all possible

situations. When imperfections (for example, in the capital market) exist, a whole new range of norms becomes relevant, and a whole new range of questions must be asked.

V. EXTERNALITIES IN COMPARATIVE STATICS

A. Types of Market Failure in Comparative Statics

In the last chapter it was convenient to concentrate on pecuniary externalities because these are the easiest to quantify. But at least six other types of market failure are treated explicitly or implicitly in the development literature under the heading external economies and diseconomies (10, 13). Welfare implications of these types of market failure have already been considered in chapter four, but it remains to examine the manner in which such externalities influence parameters of development. Types of externalities most frequently mentioned are:

1. factor indivisibilities, 2. non-appropriabilities, 3. irreversibilities and technological advances, 4. non-convexity, 5. equilibrium difficulties, and 6. social goods. Brief definitions are in order before each of these is examined.

Factor indivisibility, or lumpiness, is involved whenever factor increments are discontinuous in a manner that prevents equating the factor price to the MVP in competitive situations or to the MRP in imperfectly competitive situations.

Non-appropriabilities are involved whenever costs of excluding the indirect beneficiaries of some action are greater than the revenue which could be derived from imposing a charge for the good or service indirectly provided.

Irreversibilities and technological advances are involved whenever it is impossible or inexpedient to return to a production function or consumption pattern that existed during some previous time period.

The set S is a non-convex set if between any two points in S there exists some point which does not belong to S.

Equilibrium difficulties are involved whenever some parameter of the system oscillates continuously or explosively about a fixed point or a trend.

Social goods are products or services (such as clean roadsides) which can only be produced by the combined effort of a large part of the population, or goods (such as park space) which are consumed in common by all members of the area.*

1. Factor indivisibilities

Factor indivisibilities may be significant for either firm or industry, or both, and may result in increasing or decreasing costs in either firm or industry, or both. When average revenue is falling and indivisibilities within the firm are significant in such a manner that average cost at the firm's level of output is falling even faster, then any outside influence which expands the market for the firm's goods will increase unit profits. In this case, an externality of at least short term duration exists. But if other firms are attracted by a high rate of profit and enter the industry, profits will be forced down (to the equilibrium level?) again. However, in the final situation as in the initial situation anything which enlarges the market causes unit profits to rise at least temporarily. Market failure is involved because activities which

* This loose sort of definition is open to objections as has been noted by Cole (39); a more precise definition is provided in what follows.

expand the market for the subject commodity are not sufficiently encouraged.*

If indivisibilities are relevant to the industry but not to the firm (for example, where overhead costs of an indivisible factor are equally shared by all firms as in the water rate from an irrigation dam) each firm may be producing in a region of increasing costs while total output in the industry may be subject to decreasing costs. For example, according to Hoselitz (101) the breadth of skills available in a city tends to eliminate bottlenecks caused by shortage of a particular type of labor.** In this situation any outside influence which increases the rate of profit and so encourages the entrance of firms will result in the lowering of average costs in the industry. The extra-normal profits*** may eventually be eliminated because the product price is depressed by additional output. But falling industry costs may remain. If so, market failure still exists as there is too little incentive for additional output in the industry. Negative rents are not paid! But this situation cannot arise under strictly perfect competition because it implies the existence of factors not privately owned; or of a factor-

*Increasing returns implies the marginal value product of the fixed factor is negative; consequently it is impossible to equate the factor price (which is positive) with its MVP. But the criterion $MVP = P$ derives from general equilibrium theory and is not necessarily relevant to imperfectly competitive situations. Lerner (122) is incorrect in treating the general equilibrium norms as absolute norms which are applicable in any situation.

**Also see Marshall (134, p. 265).

***There is nothing particularly desirable about normal profits. The term is used merely as a basis for comparison.

selling firm which is subject to decreasing cost. This would lead to integration and the elimination of the decreasing costs or to monopoly.*

Indivisibilities have received most careful consideration in an extensive literature dating from a paper by Hotelling (102). But the essential problems remain. Whenever it is necessary to make recommendations regarding whether a good should or should not be introduced or whether a given price should be lower or higher, or whether price discrimination should or should not be allowed, the general equilibrium analysis does not provide a very satisfactory method of solution. It is therefore necessary to use either consumers' and producers' surplus or the consumers' ordinal preference field.** One is about as difficult to obtain as the other.

There is, however, still considerable sympathy for the criterion which derives from Hotelling (102). An investment should be made if a perfectly discriminating monopolist could cover full costs.

Here it does not seem wise to give further consideration to such a well studied subject. But one thing is of importance. Indivisibility, if significant, appears to lead either to monopoly elements or to a partly

*Lumpiness, of factors, products or processes (though not the time-lumpiness of durability) is a familiar difficulty of welfare economics. It may involve failure to sustain a Pareto optimum because: 1. monopoly develops, or 2. profit-maximizing competitors produce an inappropriate quantity of goods, or 3. if the appropriate quantity of goods is produced, some producers sustain continued losses.

**The theory of producers' and consumers' surplus was developed to deal with the problems of finite changes (that is, just this problem), but it has had a most checkered history. Hicks has worked out the implications but Samuelson has virtually denied its usefulness. Samuelson prefers the consumers' ordinal preference field. A bibliography is provided in Little (128).

social good. One has but to examine the examples of indivisibilities generally given--ocean liners, automobile assembly yards, invention--to obtain evidence of the social nature of many indivisible goods or indivisible processes. The tendency toward monopoly is too familiar to require reiteration, and since consequences of monopoly are avoided in this thesis, it is convenient to eliminate the direct consideration of indivisibilities. The subject has been extensively studied; its important aspects in what follows may be considered under the heading social goods.

2. Non-appropriabilities

Assume now that all relevant final and intermediate goods may be produced at constant returns to scale by applying fixed technological methods to scarce raw materials. Consumers' tastes are independent. Non-zero prices must exist by the duality theorem, for every factor or product of significance. It is immediately observable that though tastes are independent, consumer welfare is not independent. Smith's lawn is a free good of value to his neighbors. His hammering on Sunday mornings is a free good of negative value.

There is also consumer-producer interdependence. Factory soot may dirty Smith's shirts or fumes may ruin his lawn* and architect's fees do not wholly reflect the satisfaction or dissatisfaction of those who observe the resultant buildings. One person's consumption of a mixture of driving and drinking increases everyone else's insurance rates.

*Sudbury, a nickel-mining city of 70,000 in Northern Ontario, has virtually no shrubbery and the surrounding countryside is denuded of former flora because of sulphur fumes from the International Nickel smelters.

Producer-producer interdependence may also exist. Bees collect free pollen and freely fertilize fruit crops. Crop spray does sometimes drift to, and damage neighborhood gardens. Poor cultural practices on farms and in suburbs encourage weeds (the seeds and pollen of which freely blow elsewhere). Road building sometimes causes harmful or helpful drainage or flooding.

All such difficulties characterize competitive as well as imperfectly competitive situations. And such situations may exist in statics or dynamics. Fundamentally, the difficulty involves costs of engendering appropriability or culpability* which are greater than the worth of the presently free good. Smith, for example, could build a wall and charge people for a peek at his lawn, but such a procedure would not likely involve a significant increase in his income.**

When non-appropriable or non-culpable products result in a disturbance of profit rates, do firms transfer their activities in such a manner that a normal rate of profit is returned? In general the non-appropriable product will be associated with some scarce factor (such as space) so that in a static sense changes in rents cause a return to normal profits. But this tends to conceal an important difficulty. Firms which generate such non-appropriable or non-culpable externalities will be too little or

*It is convenient to have a word for use in places when appropriability is not satisfactory because the good is of negative value. When blameworthiness or censure can be established the product will be called culpable. For example, the law in some places provides that owners of cars with noisy mufflers may be ticketed. Thus, there is provision for at least partial culpability.

**Ellis and Fellner (55) have gone so far as to attribute virtually all externalities that exist in competition to non-appropriability.

too much encouraged. When non-appropriable externalities exist, investment appears to remain too low. When non-culpability exists, investment in the externality-generating firm appears to be too high because the firm is generating a product of negative value to others.

In order to fit this situation into an equal profit analysis, it is necessary to consider these externalities as external costs or revenues to the firm generating them. Then if internal profit rates are equal everywhere, total profit rates (internal plus external profits) will, except by chance, be unequal for any two firms if one or both generates externalities. Ad valorem taxes or subsidies are therefore required in order to achieve the ideal output.

3. Appropriability, culpability and conditions of entry

In the following, it will be assumed that firm B initiates production in an economy composed of an agricultural segment (characterized by disguised unemployment and the absence of exchange) and a single monopolistic industry, C, producing only one product. *c* is exchanged for money, as is labor, the only factor of production. Firm B's choice-of-product decision will be considered.

Beneficial effects available to B due to C's presence are attributable to one or more of the following:

1. Products *c* and *b* are related in consumption.
2. The income elasticity of demand for *b* is other than zero.
3. *b*, including appropriable by-products of *b* production, is a factor or substitute for a factor in *c* production.

Is *b* likely to be chosen so that *c* and *b* are related in consumption,

and if so, would B prefer (C has no choice in the matter) the two products to be complements or substitutes? At first it seems that b will be chosen from among the c complements in consumption because such a choice involves an already large market.* That is, B's entrance would be easier (if b were chosen from among the c complements) because for any quantity of b, marketing expenditure would, ceteris paribus, be lower, or selling price higher or both.

Thus, if C were large and well established and B were small and without great liquid reserves, probably B would be content, at least initially, to choose a produce that complemented c. But if B were a powerful and growth-oriented firm, it might choose a product so as to maximize anticipated growth of sales, or maximize sales at some future date. This decision may not be compatible with choice of a product from among the c complements.

Other things being equal, B presumably wishes to choose a product for which demand will expand. If an increase in national income is anticipated, this amounts to a desire for maximum income elasticity. But if one dollar of extra income is spent on b, it cannot be spent elsewhere. B then has incentive to choose a product which reduces the income elasticity of demand of all other products.** Suppose national income is

*Evidently there will be a market for many goods in the area if c production generates income, because people have diverse tastes. Not all these goods will be c complements. But the market for c complements will, ceteris paribus, be larger than the market for goods which are c substitutes or are independent of c in consumption.

** Assume $\frac{dy}{dt}$ is not, in the knowledge of B, dependent on the choice between complementarity and substitutability.

expected to change by ΔY under constant prices. The change in expenditure on b may be larger than ΔY -- other goods are inferior; equal to ΔY -- the income elasticity of demand for other goods is zero; less than ΔY -- the income elasticity of demand for other goods is positive; negative -- b is an inferior good. Evidently this list is in order of b's preference. However, if production of b increases, causing a positive change in national income, and if c is an inferior good, c and b cannot be complements. Were they complements, increased consumption of b would increase the marginal rate of substitution of c for money and this would be associated with increased purchases of c.

It is convenient to define three terms: completing entry, neutral entry, and aggressive entry. Completing entry involves choice of a product complementary in consumption with the bundle of products already being produced in the area; aggressive entry involves choice of products intended to displace presently consumed products from the market; and neutral entry involves choice of products which are intended neither to be complementary with, nor to substitute for, but to add to, presently consumed products.*

a. Completing entry In the closed economy considered here, there are only two products exchanged, c and b. If b is chosen to complement c, then c also complements b by definition. However, it is known that when there are only two goods they must be substitutes.** Thus, b

*After this was written a somewhat similar set of definitions was discovered in Carter and Williams (25).

**See Hicks (92, p. 46).

can only enter in a completing manner when there exists some third good the ownership of which may be reduced. This third good can only be money since c and b include everything else. Completing entry therefore involves an upward shift to a steeper consumption function.* The new consumption function is higher because an addition to national income takes place. It is steeper because complementarity of goods increases the marginal propensity to consume.

b. Aggressive entry It may seem that C's activity would not have a beneficial influence on an aggressive entrant, since a substitution of c for money reduces the marginal rate of substitution of b for money. But this is true only in the short run. C's activity extended over some period has presumably resulted in an expanded market for product c. If b were a substitute (even a perfect substitute), it is possible that C's activity in developing the market would be more beneficial than its current competitive output would be harmful.** But as mentioned in chapter four it is impossible to quantify the importance of any firm's existence, so that use of terms such as $TRE_{A/B}$ would not be meaningful.

While C's activity may have generated a market which is of benefit to

*This is a somewhat succinct description because saving and investment have not been introduced. Alternatively, regard the complementarity of c and b as resulting in decreased demand for leisure or increased off-farm migration.

**Suppose C's activity has increased c demand at a given price from 10 to 100 units. If B produces a perfect substitute it would possibly get no more than one-half the clientele. But even at that, b sales will be 50 units while without C's activity they would only have been 10 units. There is reason (given some advances in technology) to suppose that B may do better than this, however. B may produce something the quality of which is superior to c or the price lower, and so displace c from the market altogether.

an aggressive entrant B, it is only by coincidence if B, on entering, produces a net positive revenue externality for C.* Thus, entering firms with their product flexibility have advantages over existent firms which are committed to given production functions and types of product.**

In general it seems that aggressive entry is associated with a beneficial effect on entering firms and a negative revenue externality to existing firms.*** There is an interesting parallel with motives given by firms for or against undertaking research. These include, according to Jewkes et al. (1955, p. 175):

1. Fear of being supplanted--if they do not engage in research.
2. Fear of loss of their own developed ideas to others may lead to failure to do research.
3. Hope of expansion of the market through the generation of ideas.

*B's investment will result in an increase in real income if less than full employment prevails, so that, if c is not an inferior good there is likely to be some tendency for benefit. C's revenue externality will depend on the income elasticity of demand for c and on the elasticity of substitution of b for c. However, by definition of aggressive entry, b is intended to displace c; consequently net revenue externality in C is likely to be negative.

**It has been suggested that aggressive entry may result in development without growth. Each entrant may offer a preferred commodity selling at the same price as some product presently consumed. Existing firms and their products will continually be displaced by new firms with new products. This is development because consumers obtain increasing satisfaction from their purchases. It is not growth because GNP does not change. (J. J. Hollenhorst, Instructor, Iowa State University. Ames, Iowa. Private communication. 1959.)

***According to Clark (37, p. 86): "The most basic security it (business) needs is protection against predatory tactics."

Competitive firms will be strongly motivated by 1., weakly motivated by 2., and strongly motivated by 3.*

c. Neutral entry Neutral entry involves production of a new product for which tendencies toward substitutability and toward complementarity with the presently consumed bundle just balance. Output of the new commodity involves income generating effects. And some of this extra income will be spent for the new commodity, some for the old commodities. On the other hand, some expenditure which formerly was channeled to old commodities now finds its way to the new commodity. But were the latter tendency to dominate, aggressive entry would result, and if the marginal propensity to consume increased, completing entry would result. Consequently, neutral entry involves moving to the right along the old consumption function. This, in the model economy described here, involves a shift of resources out of the non-commercial agricultural segment.

To summarize: aggressive entry involves development as new products replace old ones. It may not result in growth of total value of output. Completing entry involves a shift to a new steeper consumption function. Neutral entry involves an upward shift along the old consumption function. Consequently, neutral entry need not involve positive net revenue exter-

*The following examples are illustrative: 1. After introduction of the rotary kiln used in making Portland cement, the output of Portland cement increased (1880-1884 to 1910-1914) at an annual rate of 26.3 percent. Output of non-Portland cement increased from 1880-1884 to 1898-1902 at an annual rate of 5.9 percent, but from 1898-1902 to 1910-1914 decreased at an annual rate of 16.7 percent. 2. In 1893 the Bethlehem Iron Co. designed and built a 125 ton steam hammer that was rendered obsolete within three years by the hydraulic forging press. 3. By 1895 the Northrop loom, after ten year's use, had displaced practically all other looms. 4. The Curtiss screw making machine (1871) was rendered obsolete within five years by the automatic turret lathe. All examples are attributable to Strassmann (206).

nalities. Completing entry implies that total revenue externalities to the existent firm and total beneficial effects to the entering firm are positive, and aggressive entry implies that $TRE_{C/B} < 0$, but that the existence of C is beneficial to B. It seems clear that when entry is relatively easy there is a danger that entry will tend to be too aggressive.

d. Cost externalities and the choice of product Cost externalities, as well as those revenue externalities which operate through factor markets, are attributable to one or more of the following:

1. b (including appropriable by-products of b production) is a factor, or substitute for a factor, or complement in production with a factor, used in c production.
2. Income effects influence factor prices.

Some entering firms introduce new production functions rather than new products, while others use factors in the same proportion as existing processes to produce different products. Naturally entering firms use the least cost proportion of factors which the existent state of technology allows. Consequently, any changes in technology operate in favor of entering firms and to the relative disadvantage of existent firms. The introduction of a new production function is likely to cause a shift in factor prices and this tends to make all existent processes archaic. Existent firms can only counterattack by modernizing their facilities--updating their plants. This involves a cost to which there corresponds no increase in returns.*

*Returns are increased only in the sense that updating the plant results in higher net revenues than if the plant is not modernized.

Entry in a technologically advancing society is likely, therefore, to be associated with positive cost externalities among existent firms. These externalities are likely to be larger when entry is aggressive, than when entry is completing or neutral because aggressive entry involves similarity of products and this in turn is likely to be associated with similarity of factors.

In the two product economy under consideration, none of the three categories of entry above, could involve b (product of the entering firm) as a factor in c. But this is obviously a possibility and would involve a negative cost externality in C, for otherwise firm C would not purchase b. It is possible, too, that c will be a factor in b, and this might provide C with a positive revenue externality.

e. Entry and non-appropriabilities It would be possible to proceed with this type of analysis by introducing additional firms and specifying production functions. But that is not intended. The general type of interdependence of profits has been shown to result in a possible asymmetric transfer of costs or benefits when changes such as entry take place in imperfect competition. But it has been established in chapter three that an asymmetric flow of externalities results in failure of the marginal private cost-benefit analysis to correspond to marginal social cost and benefit. Consequently the pecuniary type of non-appropriabilities or non-culpabilities which are here discussed appear to lead to external economies and diseconomies in the traditional sense. That is (except when the revenue and cost influences of C on B are balanced by an equal and opposite flow from B to C) they lead to failure in achieving necessary

conditions for a Pareto optimum.

The argument can be put in more practical terms. Extremely free entry may result in too much aggressive entry and a consequent tendency toward an extremely high rate of product change and an unnecessarily wasteful rate of obsolescence of capital equipment. The more difficult is entry, (the more power existent firms have to resist aggressive entry) the more likely is entry to be neutral or completing. This may result in a technically stagnant society--in growth without technical development. Apparently either extreme is undesirable.

4. Irreversible externalities and technological advances

Ellis and Fellner (55) believed externalities to be typically of an irreversible nature, that is, they believed external economies to be primarily dynamic phenomena.* Irreversible externalities will be considered by using an example. The envelope of the average cost curve of either firm or industry, given enough time for a long-run adjustment, might look like the curve in Figure 10. Figure 10 is based on data provided in Table 15. This data was chosen because it exhibits an unusually large fall in real costs. No doubt the fall in real costs is partly attributable to technological improvements so that it is quite clear that the curve depicted in Figure 10 is not a supply curve. It is the envelope of a series of average cost curves each drawn at a different time for a different scale of output.

*It is possible that capital accumulation by individual firms is a nearly irreversible phenomenon. Large firms can compete with small firms but the reverse is not the case. See Hahn (81). Schumpeter, too, regards externalities as irreversible: "They depict historical processes in generalized form." (188, p. 995).

Figure 10. Real cost of cotton, U.K., 1865-1915
Ordinate: index of real costs (1914 = 100)
Abcissa: year

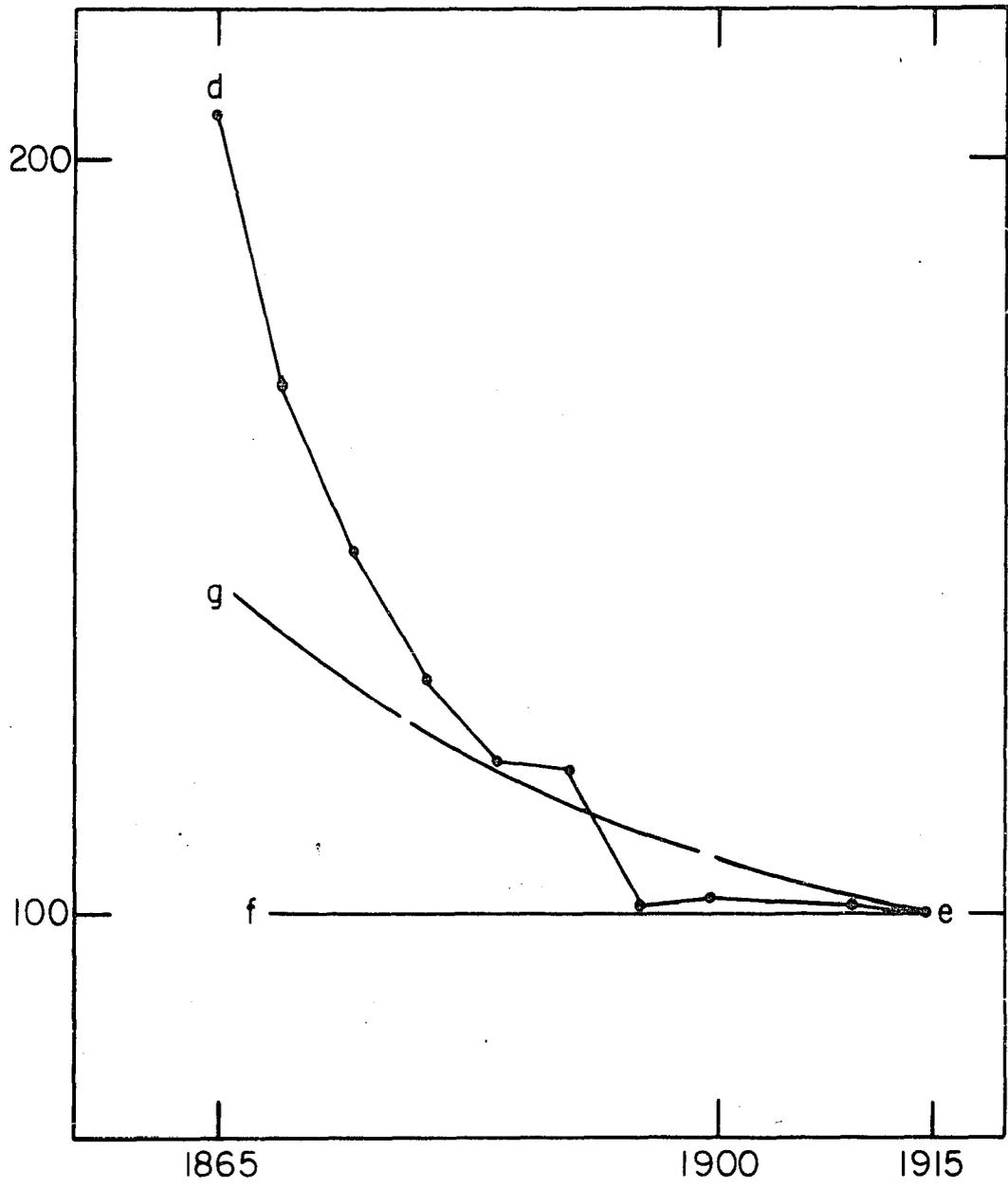


Table 15. Indexes of real cost and of output, cotton yarn in Manchester, 1854-1914^a

Year	Index no. active spindles	Index real cost, seven-year moving average
1854	14.4	181
1859	15.8	193
1865	18.1	206
1869	24.8	170
1874	36.5	148
1879	40.2	131
1884	48.6	120
1889	55.1	119
1899	73.7	101
1904	79.6	102
1909	88.7	101
1914	100.0	100

^aSource: Clark (36, p. 352).

The great fall in real costs in Figure 10, it might be argued, is attributable to economies of scale. But Kuznets (117), after presenting the similar data of Table 16, argues that the fall in real costs is attributable to improving technology over time and not to expansion in the scale of output.

The fall in costs of yarn exhibited in Table 15 and Table 16 was

Table 16. Cost of yarn in Britain, 1779-1882^a

Yarn 40 hanks to the pound			Yarn 100 hanks to the pound		
Year	Shillings	Pence	Year	Shillings	Pence
1779	14	0	1786	34	0
1784	8.76	11	1796	15	6
1799	4.42	2	1806	4	2
1812	1.00	0	1812	2	10
1830	0.56	6.75	1830	2	2.75
1860	0.52	6.25	1860	1	5
1882	0.28	3.375	1882	1	0.375

^aSource: Kuznets (117, p. 262).

associated with an expansion in output, but it seems unlikely the fall in costs was brought about exclusively by expansion of scale. Rather, it seems likely that expansion in scale was a contributing factor in technological developments. These technological developments brought about the fall in real costs. The technological changes may have been encouraged by expanding scale of output. But it seems likely, that were the scale to be reduced, the technological developments would not have to be given up. Again referring to Figure 10, average costs initially are d , costs fall as more output is sold, given time for adjustment to the new situation. Costs may fall to e , whereupon any diminution in quantity produced causes costs to move leftward along the different (hypothetical) path $e-f$ or $e-g$.

It is not possible that the situation exists in statics, for if costs could be made to fall without the elapse of time, then line segment $d-e$ would not be part of the envelope as every firm would have immediate

access to costs e-f or e-g. If technological advances are ignored, then this ratchet cost curve may be due to either an improvement in the quality of the labor force through training, or on-the-job education, or to the more exact adjustment of factor suppliers to the needs of the industry. Neither of these can be considered to be of modest dimensions.

It is not clear, in this situation, what the supply curve looks like because of imperfect competition and because there is no way of knowing if the firms concerned understand the workings of their own cost curves. Indeed, it is not even clear that producers will be unwilling to sustain a temporary loss at outputs as large as e in order to have access to line segment e-f or e-g. But it is possible to outline circumstances in which the low cost region e-f or e-g is never reached, and such circumstances generally must discourage growth.

It is also possible that the envelope of the cost curve increases to approach a high level e'-f', and fails to return to lower levels when output is decreased. This might happen for example when an industry becomes large enough to attract a union which wins certain irrevocable benefits such as coffee breaks, extra pay for overtime, and seniority promotion. Ordinarily such a cost-increasing situation would be regarded as basically different from the cost-reducing situation, because of the presence of rents on intramarginal units. But the present situation does not involve rents. Rents are reversible phenomena rising from scarcity. That is not the case here.

Again, the number of firms each having fixed costs may increase irreversibly as demand for the industry's product expands. Any decrease

in demand will involve a movement leftward along a new, higher average cost curve. Costs of production in the large number of cafeterias located beside former army barracks provides an example.

Perhaps the most important source of positive irreversible externalities involves technological developments. If Smith's dictum (that the division of labor is limited by the extent of the market) has any validity, costs in any one industry, given time for adjustment, may fall irreversibly. Improved machines, processes, and factor combinations are searched out because large scale output makes innovation worth while. And once found or invented new methods are not forgotten even when the industry shrinks to a fraction of its former size. Arndt (3) has hypothesized that the rate of invention is not exogenous; it increases with increased output. And Schumpeter (188, p. 228) believed innovation made subsequent innovation easier.

Evidence respecting the influence of changing technology on production costs is really quite convincing. Solow (199), in a study using American data for the period 1909-1949, concludes that gross output per man has doubled over the interval, with $87\frac{1}{2}$ percent of the increase attributable to technical change and the remaining $12\frac{1}{2}$ percent to increased use of capital.

Fabricant (58) has reported, for the period 1871-1951 that about 90 percent of the per capita increase in output was attributable to technical progress or productivity change. Some of the results of various studies are summarized in Table 17.

But, the rate of invention may not have kept up. Thus, while Solow's

Table 17. Shifts in productivity that are attributable to technological change

Source	Years	Change in productivity percent
Solow ^a	1909-1949	1.5 per year
Valavanis-Vail ^b	1869-1948	0.75 per year
Fabricant ^c	1889-1953	1.6-2.0 (labor) per year 0.7-1.2 (capital) per year
Fabricant ^d	1889-1953	90
Schmookler ^e	1904-1913 to 1929-1938	36.5

^aSource: Solow (199, p. 316), excludes agriculture.

^bValavanis-Vail (224, p. 217).

^cFabricant (58, p. 5).

^dFabricant (60).

^eSchmookler (183, p. 226).

index of technological change appears to increase in a manner that might best be approximated by a logarithmic function ($\Delta A/A = 0.015$ on the average) the same cannot be said for the number of patents.

It is quite possible, however, that there has been an improvement in the quality of patented ideas. But Kuznets (117) believes that technology is subject to a decreasing rate of improvement, and this is consistent with a relative decrease in the number of patents. A decline, after 1919-1921, in the number of patents relative to the number of workers, is indicated in Table 18.

Table 18. Number of patents and designs issued per 10,000 workers, U.S.A.^a

Years	Number
1899-1901	968.4
1919-1921	1094.0
1929-1931	837.2
1949-1951	781.9

^aAdapted from Schmookler (184, Table 4, p. 327).

Tables 15 through 18 and Figure 10 serve to prove what is common knowledge: technological innovation involves irreversible change. But this change does not take place at the same rate in all industries so that investors who concern themselves only with the short run, and only with their own small part of an industry, are not likely to be led to so invest that a maximum rate of technological expansion is realized. Only investors having monopoly in all alternatives, and having the same time horizon as the society will always be led to choose the course of action that maximizes technological progress.

a. Non-convexity The reason why so much space has been devoted to technological change is that this very central aspect of development may lead to market failure. It is possible, in fact it is even likely, that the cost-reducing consequences of technological change will result in non-convexities in the time path of production functions. (See Solow's index of technology which increases in a non-convex manner (199).) This

has a familiar and devastating effect on duality.* For present purposes it implies that even when the time path of costs in all industries is known in advance, profit maximizing firms may not be led to invest in appropriate enterprises.

If resources are allocated according to incremental net returns by myopic competitors, and if one industry is subject to increasing and another to decreasing returns, the former will receive no resources unless its incremental $P-MC$ is greater than $P-MC$ for the decreasing cost industry. But under increasing returns incremental $P-MC$ can be negative. In this case only firms with a large lump of resources will consider investment in the industry of increasing returns. But this static case has been extensively studied.

If there are non-convexities in the time path of costs in one industry, but not in an alternative industry, the difficulty resolves itself into a problem of planning horizon. No individual has a planning horizon as distant as the normative planning horizon of the society. Thus, some activities should perhaps be carried on (even at a short-term loss) by government intervention because only in this way will technology in that industry be developed to the point where the industry can make its

*Recent treatment of non-convexity by Rothenberg (175) appears, however, to indicate the difficulty has been exaggerated in application to competitive situations. When there are only a few firms, non-convexity will remain troublesome. Non-convexity in time paths of costs will also provide difficulty, no matter how many firms there are.

contribution.* The situation is analogous to the protection by tariffs of an infant industry. But in this case it is the infant technology that requires protection--and development.

5. Equilibrium difficulties

Scitovsky (192) has mentioned the possibility that firms inter-related through the market mechanism will be a source of waste to themselves and uncertainty to other investors while they are in a process of adjusting to each others' level of output. It is also possible that the result will be completely unstable or continuously oscillating. The hog-cycle (cobweb theorem) provides an example of the sort of difficulty that can arise.** When unintegrated interdependence exists, risks are greater and plans less efficient than appears necessary. Presumably failure of growth can result. This matter may be examined using a difference equation model.

Assume there are n industries. Of these, a fraction, a , are capital goods industries that sell produce to other capital goods industries and to consumer goods industries. The latter sell only into final consumption. There are $a \cdot n$ capital goods industries and $(1-a)n$ consumer goods industries.

A pecuniary externality exists in some capital goods industry A, when

*Tinbergen (213, p. 32), for example, claims in connection with social goods: "...the activities to which transportation facilities and power are complementary cannot themselves be accurately foreseen: total production of the region may still be a highly uncertain entity. This sometimes is the very reason why private investment in transportation and power is not forthcoming and why public investment is the only practical possibility."

**Buchanan (21) is incorrect in stating that there are no examples.

another capital goods industry (which sells to A) is induced to change its product price because of a shift in profits in a previous time period (call this backward linkage for A). Alternatively, pecuniary externalities exist in A, if some consumer goods industry or some capital goods industry shifts demand for A's product (forward linkage in A) because such industries experienced a shift in profits during a previous time period.

Assume constant returns to scale and fixed factor proportions. Then:

$$1. \Delta K_i(t) = mp\Delta K_j(t-1)$$

where $\Delta K_i(t)$ is the percent change in industry i's profits during period t, m is the elasticity of the price that affects i with respect to profits in industry j (factor price if i sells to j; product price times-1 if i buys from j), p is the proportion of industry j's output which is taken by industry i or (forward linkage for i) the proportion of i's output that is taken by j. $\Delta K_j(t-1)$ is the percent change in industry j's profits during period t-1. Further assume:

$$2. \Delta K_i(t) = mp\Delta C_j(t-1).$$

Now drop the i's and j's so that $\Delta K(t)$ and $\Delta K(t-1)$ are percent change in profits in each capital goods industry as well as capital goods industries in general and $C(t)$ is the percent change in profits in all or any consumer good industry. Let the elasticity of price with respect to profits be the same in all industries and a constant. Then:

$$3. \Delta K(t) = \alpha\Delta K(t-1) + \beta\Delta C(t-1).$$

But $\Delta C(t)$ is only subject to backward linkage so that: $\Delta C(t) = \gamma\Delta K(t-1)$

or:

$$4. \Delta C(t-1) = \gamma\Delta K(t-2)$$

Substitute equation 4 into 3 to obtain:

$$5a. \Delta K(t) = \alpha \Delta K(t-1) + \beta \sigma \Delta K(t-s), \text{ or:}$$

$$5b. 0 = -\Delta K(t) + \alpha \Delta K(t-1) + \beta \sigma \Delta K(t-2).$$

Now in order to specify $\alpha + \beta$ it is useful to examine equation 3 more carefully. If profit in period $t-1$ in all capital and consumer goods industries changed by some percentage, then:

$$6. \Delta K(t) = m(an-1)\frac{1}{an-1} \Delta K(t-1) + \frac{m(an-1)}{n-1} \Delta K(t-1) + mn(1-a)\frac{1}{n-1} \Delta C(t-1).$$

and simplifying:

$$7. \Delta K(t) = \frac{m(an + n-2)}{n-1} \Delta K(t-1) + \frac{mn(1-a)}{n-1} \Delta C(t-1)$$

In equation 6, m is the elasticity, $(an-1)$ is the number of capital goods industries affecting every other capital goods industry, and $\frac{1}{an-1}$ is the proportion of purchase from every other capital goods industry by any capital goods industry. The second element $m(an-1)\frac{1}{n-1}$ is the elasticity times the number of capital goods industries $(an-1)$ to which each capital goods industry sells, times $\frac{1}{n-1}$, the proportion of total sales of any capital goods industry that go to any other industry. The $C(t-1)$ coefficient is simple because only one way linkage is involved. It consists of the elasticity times the number of consumer goods industries, $n(1-a)$, times the proportion of output of each capital goods industry that goes to each other industry.

Now comparing equation 7 with equation 3:

$$8. \alpha = \frac{m(an + n-2)}{n-1} \text{ and:}$$

$$9. \beta = \frac{mn(1-a)}{n-1}$$

In a manner similar to the way in which equation 6 was derived, obtain:

$$10. \Delta C(t) = m(an) \frac{1}{an} \Delta K(t-1) \text{ and comparing equation 10 to equation 4:}$$

$$11. \gamma = m.$$

It is interesting to note that none of the three parameters (α, β, δ) is very sensitive to shifts in n . All respond monotonically to shifts in m .

Comparing equation 3 with equation 4 note that if $\alpha > \delta$ the profit expansion induced by an increase in $\Delta K(t-1)$ will be greater in the capital goods sector than in the consumer goods sector provided $\beta > 0$. A sufficient condition for $\alpha > \delta$ is $\frac{an + n-2}{n-1} > 1$ or $a > \frac{1}{n}$. That is, so long as there are two or more capital goods sectors, there will be a tendency for profit expansion in the capital goods industries to outstrip profit expansion in the consumer good industries.

The questions of primary importance are: is the system stable, and does the system oscillate. Equation 6 is a second order difference equation, the solution to which takes the form:

$$13. \Delta K(t) = C + a_1 x_1^t + a_2 x_2^t$$

where C , a_1 and a_2 are constants, x_1 and x_2 represent the solutions to 6. Then $\Delta K(t)$ is stable if (x_1) and $(x_2) < 1$. And $\Delta K(t)$ will be non-oscillating if x_1 and x_2 are real and not negative numbers.

Solving equation 5b yields:

$$14. x = \frac{\alpha + \sqrt{\alpha^2 + 4\beta\delta}}{2}$$

The system will be stable (the larger $|x|$ is less than one) if (simplifying the larger x value in equation 14):

$$15. 1 - \alpha > \beta \delta \text{ or if,}$$

$$16. \frac{n-1}{2n} > m - \frac{m}{2n}$$

Thus, for large n , the system will be stable if $m < \frac{1}{2}$. This is not a very difficult requirement to meet. The elasticity must generally be a small fraction and frequently must be zero. $K(t)$ will oscillate since x values are negative. m has been assumed ≥ 0 .

The model appears to suggest that inter-firm action and reaction will not lead to instability in the rate of profit. But the assumptions are fairly unrealistic. In particular the assumption of a single m value that applies to all firms seems unrealistic. The possibility remains (even without oscillations or instability) that as unintegrated interdependent firms adjust to new levels of output (in the absence of a perfect system of forward prices), there will be uncertainty and mal-planning among firms which buy from or sell to them.

6. Social goods

A product X is a social good if additional consumption X^i , by some individual i , fails to reduce the total remaining for others. Then:

$$17. X = X^i = X^j = \dots = X^n.$$

Social goods may be contrasted with private goods, (goods for which total consumption equals the sum of all individual's consumption). A private good Y may be defined by the equation:

$$18. Y = \sum_{i=1}^n (Y^i).$$

Samuelson (179, 182), Musgrave (143), and Bowen (18) have elaborated some of the welfare implications of this case in recent years. But much remains to be done; in particular the case has not been satisfactorily related to economic development.

Some examples will be useful. Ice cream is a private good. Total consumption equals the sum of consumption by individuals. Knowledge, however, is a social good. After consumption by $n-1$ individuals there still remains the original total available for the consumption of the n th individual.* Other examples of social goods include: national defense, resource conservation, law enforcement, public health, moderate as opposed to extreme business cycles, economic growth** and power development.***

Yet the definitions are polar and extreme so that it is convenient to think of every good as spotted somewhere along a continuum from pure social good to pure private good. When a book, for example, is borrowed from a library the amount of knowledge available to other borrowers is temporarily reduced; and when ice cream is bought, more retail outlets are

*The word "consumption" may seem inappropriate. Yet factors are used up in the production of such social goods as knowledge and national defense. This justifies use of the word.

**It is clear that a rapid rate of national economic growth is desired by many persons. One person's consumption of such a rate of growth does not reduce the amount remaining for others.

***Tinbergen (213, p. 6) speaking of power, water, and land reclamation projects notes: "Usually such investments cannot all be made by private individuals, since their yields spread through the community and do not readily take the form of income to the investor." For a definitional statement see Musgrave (143, p. 108).

encouraged so that ice cream may, after a while, be rather more than less available to other consumers.

Unfortunately, it is not clear simply from the nature of an object whether or not it is a social good. A highway, for example, is not noticeably consumed as one drives on it. It is there for others to use before, during, and after any individual's use. But during rush hours it may be congested and then one auto reduces the road available for another. A traffic jam may be well described by equation 18, though equation 17 is more appropriate at other times. Thus, roads may be close to the polar social good at 2 p.m. and on virtually the other end of the continuum by 5:30 p.m.

While social goods are generally of rather large dimension and are generally more or less indivisible like national defense, the important point is this: Smith's national defense is Brown's national defense. There is no way of dividing it up so that one person obtains more and another less. This property might be called non-consignability. It is different from non-appropriability, because where non-appropriability involves the absence of feasible rationing (because of absence of control over product flows), non-consignability involves the impossibility of rationing. Smith's consumption is Brown's consumption, so neither of them would save his ration book.

Knowledge is an interesting case. One person can have more and another less. And it would be rational of an individual to buy learning for himself (though he might not buy as much as the social welfare function indicates is desirable); but it would not be rational for him to buy

additional basic research. Basic research unlike applied research is not consignable. Smith's atomic physics is Brown's atomic physics whether he likes it or not.*

Highways, streets, and bridges have many social good characteristics. True, non-payers of tolls could be excluded if this were feasible so that the situation at first seems to involve non-appropriability. But as Hotelling (102) has pointed out in connection with his bridge example, benefits accrue to an extremely large and diverse group of non-users (who are also non-payers). Even if primary indirect users could be charged for use of the bridge (by increased cost of the transported item) it would not be possible to reach secondary and tertiary indirect users unless perfect price discrimination were achieved in setting tolls. Road and bridge effects are, in fact, so general that it appears appropriate to apply the term social good. But roads and bridges are not pure social goods like national defense; they are perhaps better described as partly social goods.

These examples are by no means exhaustive. Samuelson (182) offers the opinion that the propriety of any government activity directed toward a non-social good should be questioned. But he thinks most government activities probably are concerned with the provision of social goods.**

*Some large firms do basic research. These include firms in electronics, chemistry, and engineering. See chapter four. There is a high correlation between size of firm and amount expended on research. This suggests that there are degrees of consignability, and that the amount of consignability is a reflection of firm size.

**In underdeveloped countries there is increasing recognition of the importance of social goods. In nineteen underdeveloped countries during the eight year period 1950-51 to 1957-58, government investment increased at an annual rate of 11.7 percent while private investment increased at only 4.7 percent according to tables in World economic survey, 1959 (215).

It appears possible that there are social evils (social goods of negative value). If an educated populace is a social good, then a mis-educated populace is a social evil. Industrial effluent, city sewage, soil erosion, and malarial swamps are other candidates. In what follows social evils will be treated as social goods of negative sign.

Social overhead capital is closely related to social goods and has been extensively treated by economists concerned with development. It is alleged (see Higgins, 93) that social overhead capital must be provided by government because of the large indivisible nature of investment requirements and because of the historically low rates of return in such industries.

This approach, unfortunately, hides one important attribute of social goods. Namely, that market failure is involved and that government must provide such services or the area will do without them. The invisible hand does not work for social goods! The reason follows from the definition: private purchases of social goods, because of non-consignability, result in no greater contribution to the purchaser's welfare than to the welfare of anyone else. Then each person will avoid purchases in the hope that some other person or government will provide the social good.* Private purchases of social goods are like voluntarily paying taxes. The social results may be desirable but the private motives are absent.

In connection with social goods market failure can occur in four ways: 1. too little is produced; 2. too much is produced; 3. the

*This, in fact, makes the definition depend on expectations regarding government activity. Unfortunate as this may seem, it appears to conform to reality.

production is poorly allocated;* and, 4. costs are poorly allocated. It is possible here to examine whether failure of the first or second type is more likely, and to say something about failure of the last type.

In this section assume the existence of two goods, X (a social good) and Y (a private good). X and Y are produced and used under conditions of diminishing returns. There are two firms, A and B. Under what conditions will the ideal quantities of X and Y be produced?

Suppose some transformation function T-T and some MRS function for firm A as indicated in Figure 11, p. 126. Let A be at any position α on the MRS curve. The social good X, of quantity x, obtained by A is, by definition, also obtained by firm B. Then Figure 11 provides in curve y-y the amounts of commodity Y available for consumption in B at each given x consumption in A. This is obtained from Figure 11 by a vertical subtraction of MRS from T-T. The following property obtains on y-y; any position on y-y represents a combination of X and Y in firm A which is as good as, but not better than, any other position on y-y.

There is, in firm B some family of MRS curves and one of these must be tangent to y-y at α , or else it would be possible for firm B to produce more under a different combination of X and Y. The tangency condition is familiar enough, what is unfamiliar is the requirement that tangency occur on α . This is necessary because the quantity of social goods x must be the same for both firms.

These requirements define necessary but not sufficient conditions for a Pareto optimum. The criterion is $\sum MRS = MRT$. This is shown in

*Strotz (209) has made the interesting point that, within limits, public goods may be regarded as a means of redistributing income.

Figure 11. A Pareto optimum when there is one social and one private good

Figure 12. A Pareto optimum when there is one partly social and one private good

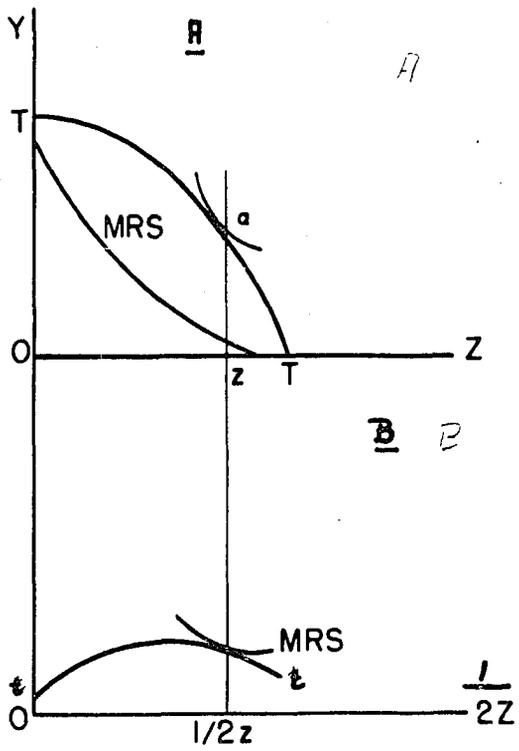
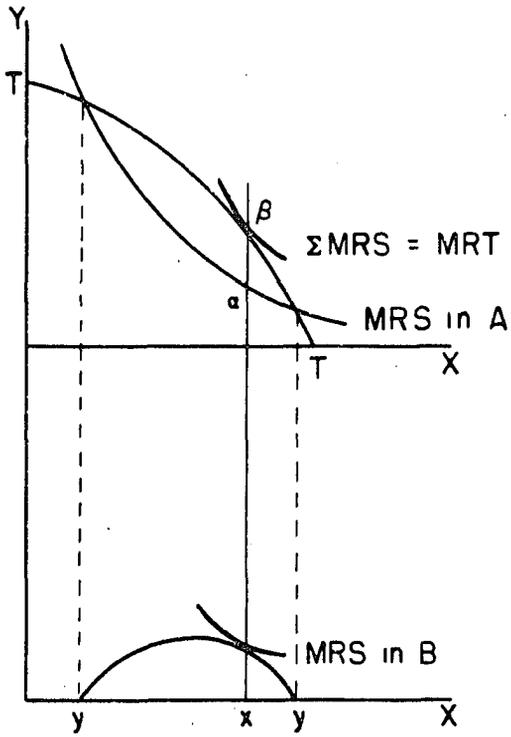


Figure 11 at the point β .

Bowen (18) has been able to demonstrate that a voting procedure combined with government activity may, under certain circumstances, be able to satisfy the $\sum MRS = MRT$ criterion.*

However, while difficulties and inadequacies of government finance and administration are related to development, it is not the purpose here to consider such problems. Market failure is always involved respecting social goods in the sense that the system of *laissez-faire* does not provide the desirable combination of goods. But failure of development may not follow if government acts in an appropriate manner. Further, government will presumably treat polar social goods as a first challenge because such goods will not be supplied without government activity.

Thus the interesting case, in western capitalism, is not pure social goods but goods which have a fairly large social content. These may be defined as goods for which the sum of individual consumption is greater than production, because, in effect, consumption by one person or firm provides satisfaction to some other persons or firms. The following inequality provides an alternative definition.

$$19. \quad Z < Z^i + Z^j \dots + Z^n < nZ.$$

It is possible that the provision by private industry of some considerable quantity of such a commodity will result in failure to recognize the partly social nature of such goods.

*These conditions involve, besides necessary information and that everyone votes, the condition that X be produced under constant or decreasing cost. The Arrow conditions are not fulfilled.

Consider the completely heuristic example below. There are two individuals, i and j , and two goods Z and Y . The transformation function is:

$$20. \quad Z + Y = 10.$$

Let U stand for an ordinal index of utility and, as usual, let the individual be denoted by a superscript.* Suppose, further, that the individuals have identical tastes, and that income distribution is equal.

Table 19 indicates a possible utility system.

Utility may be maximized using columns 5 and 9 (the marginal utility columns) and the transformation function. Ideally, $Y = 6$ (3 each), $Z = 4$ (2 each), utility = $27 \times 2 + 22\frac{1}{2} \times 2 = 99$. However, each individual is insensitive to how his own consumption affects others. Consequently, if left alone, each will consume one unit of Z and four units of Y so that total utility will be a suboptimum 98. Too little of the partly social good, Z , is purchased.

Alternatively i and j may be regarded as firms, Y and Z as factors and U as factor value product. Conclusions are the same.

Here i 's consumption is not j 's consumption as with a social good; rather, one half of i 's consumption is j 's consumption. The situation may be illustrated using Figure 12, p. 126.

Any position z on OZ occupied by individual i corresponds to a position $z/2$ occupied by individual j . Let the abscissa in Figure 12B be stretched so that any number of units is represented by a distance

*Thus $U^i Z^i$ is utility obtained by i due to own-purchase of commodity Z . $U^i Z^j$ is utility obtained by i due to j 's purchase of Z .

Table 19. A possible utility system

Quantity of Z purchased by both i and j	Z				Y			
	(2) $U^i Z^i$	(3) $U^i Z^j$	(4) U^i	(5) MU^i	(6) Quantity of Y purchased by both i and j	(7) $U^i Y^i$	(8) $U^i Y^j$	(9) MU^i
1	10	5	15	15	1	10	0	10
2	15	$7\frac{1}{2}$	$22\frac{1}{2}$	$7\frac{1}{2}$	2	19	0	9
3	20	10	30	$7\frac{1}{2}$	3	27	0	8
4	25	$12\frac{1}{2}$	$37\frac{1}{2}$	$7\frac{1}{2}$	4	34	0	7
5	30	15	45	$7\frac{1}{2}$	5	40	0	6

twice as great as in Figure 11. Let i be given some consumption level α . Proceed, as before, to find the necessary conditions for a Pareto optimum. $T-T$ minus MRS^i is given by $t-t$. It is necessary that some MRS^j be tangent to $t-t$ at α for otherwise some different combination of Y and Z which makes i no worse off, would increase j 's welfare (output).

The Pareto criterion involves as a necessary, but not sufficient condition that the sum of the two MRS curves depicted equal the MRT . But observe that while $MRS^i = \Delta Y/\Delta Z$, $MRS^j = \Delta Y/2\Delta Z$. The criterion therefore is $MRT = MRS^i + \frac{1}{2} (MRS^j)$.

Iteration may be used to demonstrate that in general, for any number n of individuals having like utility functions in which the partly social good Z consumed by individual i enters in some fractional manner (intensity) $1/\theta$ into the utility functions of all other individuals, a Pareto optimum requires:

$$21. \quad MRT = \left(1 + \frac{n-1}{\theta}\right) MRS.$$

Thus, in the case of the polar social good the criterion becomes $MRT = (1 + \frac{n-1}{1}) MRS = n(MRS)$, and in the case where intensity = $\frac{1}{n-1}$, $MRT = 2MRS$.

If partly social goods are treated as private goods each individual will equate MRS and MRT. The difference which results from using this criterion as opposed to 21 may be treated as due to an externality.

The utility analysis appropriate to consumers may be abandoned in favor of an externality analysis appropriate to firms. If Z is a partly social factor of intensity $1/\delta$ and there are n typical firms, and one of these finds it profitable to buy ten dollars worth of Z, there is generated a negative total cost externality in each other firm of $\$10/\delta$. This analysis may be extended at will.

For the moment it is important to examine whether partly social goods correspond to any real phenomena. Is there such a thing as partial consignability? Candidates are highways, bridges, education (as opposed to knowledge), applied research (as opposed to basic research), the railways, swamp drainage and soil conservation. In each case rational individuals would pay some charge based on their own consumption. Yet, in a sense, Jones' bridge is White's bridge, Jones' educated children elevate White's ignorant children, and swampbred mosquitoes bite everyone.

When one looks at the matter in this way the question of existence appears to be more pointedly applied to social than to partly social goods. Most cases of the former appear to exhibit partial consignability rather than absolute non-consignability.

When market failure results from the existence of social goods it,

therefore, appears reasonable to conclude: 1. without government intervention too little of the social or partly social good will be produced, and, 2. in the case of privately purchased partly social goods, benefits will fail to correspond to payment.

B. Summary

It is convenient to terminate this chapter with an anatomical summary. Externalities may arise in imperfect competition under two conditions.

1. Scarce goods or overly abundant goods, having a non-zero influence on some individual's welfare, may fail to exchange at a non-zero price.
2. The ideal configuration of goods fails to correspond to a positive, equilibrium rate of profit for each producer because some prices, though non-zero, are inappropriate, or because no appropriate set of prices exists*.

Such externalities arise due to six causes. 1. Factor, product, or production process indivisibilities, such as give rise to non-convexities, may lead profit maximizing producers away from the optimum position. 2. Non-appropriability of a given product results in the production of that product being too little encouraged. Non-culpability of a given product results in the production of that product being too

*These two modes might be consolidated. On the other hand Bator (10) provides (for static competition) five modes. These are: a. Failure by existence--no prices exist; b. Failure by incentive--negative profits accrue to some firm at the ideal output; c. Failure by signal--profits are not a maximum for some firm at the ideal output; d. Failure by structure--the system is not self-policing and monopoly results; e. Failure by enforcement--legal imperfections.

little discouraged. 3. Irreversible changes in cost structure, if unknown in advance, result in obvious if unavoidable difficulties. But if known in advance, they may be externally beneficial or harmful. This is because of other imperfections, such as capital rationing in monopoly. Alternatively the form of competition may make any one individual's action toward a cost goal pointless, as it does, for example, with respect to research under competitive conditions. Technological advances may involve known non-convexities in time, irreversibilities, or they may involve inter-temporal welfare considerations where inter-temporal over-compensation is impossible. 4. Production functions may be non-convex over some range of output or over some range of time. 5. Market interdependence in the absence of a complete system of forward prices is associated with insufficient information regarding the plans made by interdependent firms. This may lead to equilibrium difficulties. 6. Social goods imply the impossibility of any price system which would satisfy Pareto optimum conditions but may not result in failure of development if government acts in an appropriate manner.

VI. AN EMPIRICAL INVESTIGATION

A. The Externality Matrix

An externality matrix is a list of changes in net revenue in each sector that arises as a consequence of a one dollar change in the sales and therefore output of any other sector. These changes in net revenue derive from changes in demand under constant selling price (assumed) and a variable cost increment (to be empirically obtained). For example, if there is a one dollar expansion of demand for agricultural products when initially all receipts are disbursed, and if it is known from the agricultural cost function that $E = \frac{\% \text{ increase in costs}}{\% \text{ increase in output}} = 0.93$, then costs increase by \$0.93, revenue increases by \$1.00, and a \$0.07 pecuniary externality exists. This seven cents may be treated as an increment of profit or an increment of saving. This type of situation was called an external-internal economy by Robertson (166) and a pecuniary external economy by Viner (228).

In order to compute the pecuniary externality matrix, it is necessary to know what changes in intermediate demand arise as a consequence of a one dollar increase in final demand for the product of each sector. This is the information provided by an input-output study. But in the ordinary input-output model $MC = AC$. (Intermediate use plus final demand = total output for the average dollar and the marginal dollar.) Moreover total gross output in each sector may be interpreted as equal to total gross inputs, so there is no residual saving--no pecuniary externality. It is necessary to alter these assumptions.

If each sector is assumed to have some MPS, there will be a

corresponding sector multiplier. Each sector multiplier conceptually is similar to the Keynesian multiplier; the primary difference being that Keynes imputes all savings to consumers and so is able to obtain a single multiplier, whereas in this computation, savings of each sector are not imputed to any other sector. The matrix of multipliers obtained is closely related to the matrix multiplier of the Leontief system. But the elements will all be slightly different because $MC \neq AC$ in the present model; that is, incremental disbursements do not equal average disbursements. Thus, on the average, one dollar's exogeneous expenditure by consumers on commodity x is associated with some level of demand for x as a factor. Factor use plus final use equals total output. But at the margin, a one dollar change in exogeneous expenditure may be associated with changes in factor demand and changes in total output that are not indicated by average factor requirements.

The primary use of this matrix multiplier in the present calculation is in obtaining the externality matrix. The multiplier provides inputs required from each sector which result from a one dollar increase in demand (exogeneous expenditure) for the product of any other sector. But there is also available a system of coefficients relating incremental costs to incremental demand. And it is possible to use these coefficients to adjust the elements of the matrix multiplier. In this way, an incremental cost matrix (the elements of which are the extra costs in any sector that arise as a consequence of a one dollar increase in demand for the product of any other sector) can be obtained. Alternatively, it is possible to obtain a matrix, the elements of which are the extra savings

(profits) that accrue in any sector as a consequence of a one dollar increase in demand for the products of any sector. The latter is the pecuniary externality matrix. It is a description of the extent and location of some of the potential investment funds in an expanding economy.

Consider an example. Extra output in some sector may be worth one dollar and extra costs may be \$0.93 so that savings of seven percent of increments to sales accrue in the sector. Do these savings correspond in any way to extra capital required in order to produce the extra output? Suppose capital worth \$100 will produce \$14.25 worth of product per year for ten years. Then \$100 worth of capital will produce (ignoring time discounts) \$142.50 worth of product. Or seven cents worth of capital, employed for its ten year life, would produce a product of \$1.00. Thus, ignoring indivisibilities of capital and time, a one dollar increase in demand gives rise to seven cents profit which is just sufficient (when added to the capital of the sector) to enable output to be expanded by one dollar's worth.

This simple example illustrates one use of the pecuniary externality matrix. If some sectors, by this calculation, have an accumulation of pecuniary externalities that are more than sufficient to finance additional investment, the capital market will be called upon to redirect investment funds. Failure to satisfactorily do so is likely to be associated with failure to achieve maximum economic growth.

1. Data

a. Data requirements The computations require data of several kinds. First production functions or cost functions for each sector are required. Ideally, these would specify the amount by which each factor must be increased in order to expand the product of the industry by one dollar's worth. For example, production functions of the form $O = aB^E$ (where O is output, B is an index of inputs, a and E are coefficients) might be fitted for each industry. This function is homogeneous of degree E , that is, expanding B by one percent will expand O by E percent. Under constant prices E obtained in this way estimates: (the percent change in costs) / (the percent change in output).

It should be noted that this is not the information provided by a Leontief input-output matrix. (Factor requirements for production of a marginal unit are not, ordinarily equal to average factor requirements as is generally assumed in the input-output system.)

The second data requirement involves a transactions matrix. These two sets of data are sufficient for calculation of a matrix multiplier and an externality matrix in the event that constancy of prices may be assumed.

Further investigation (comparison of externalities with capital requirements) involves finding, for each sector, the capital-output coefficients and the durability of capital.

Unfortunately, even such complete data as this involves using assumptions respecting factor supplies that are, in the short run, quite extreme. It is necessary to assume that factor prices are constant. If

output in any industry expands it must follow that all or at least some factors are present in supplies greater than those presently utilized, or alternatively that all changes are incremental.

In the long run, elastic factor supplies are probably more realistic than in the short run because new methods of using previously useless materials are discovered. (For example, Taconite may be enriched to replace higher grade Mesabi ores.) But in the long run, production functions are likely to change a good deal and this makes use of a transaction matrix risky. In the short run, factor supply elasticities appear to imply unused capacity, and if this is the case, production functions may not appropriately describe factor requirements. This is an important difficulty as Furtado (68) has pointed out in another connection.

b. Available data There are available a relatively large number of cost studies.* These studies generally indicate either the proportionate increase in costs as output is expanded by one unit or provide an equation relating costs and outputs, or inputs and outputs.

Transactions matrixes are available. Here the input-output study completed for Canada, 1949 (22) will be used. Capital-output coefficients may be derived from data published by Hood (99). Unfortunately, information on capital longevity is scarce and of poor quality, but some data is available in Hood (99).

Most of the cost studies previously completed must be reworked to provide the information needed. Frequently equations of the form desired (linear in logarithms) are found to fit the data tolerably well, but

*Most of the cost studies used here are derived from U.S.A. data but some use British, French, or other information.

almost all the studies examined used equations linear in natural data. Unfortunately, the latter type does not provide a unique value of the elasticity relating costs and output so the equations must be refitted in logarithms. Such a procedure may result in lower R^2 values. This is not particularly disconcerting in practice, as R^2 does not often drop very much and the procedure is superior to calculating elasticities at mean values in natural data.

c. Deficiencies of data It is unlikely that mn elasticities (one for each factor in every sector) would be available; a rather more reasonable goal is to seek n elasticities--one for each sector--and assume that factor substitution does not take place. This assumption is likely to bias costs toward the high side, but it may be set against the assumption of constant prices which biases costs toward the low side.

Again most studies relate to firms and not industries (though studies based on cross-sectional data do not necessarily relate to any one firm). Such cross-sectional studies are probably as descriptive of the industry as they are of the firm; though the cross-sectional data is usually used for studies of cost structure of firms. Using cross-sectional data to estimate cost curves for the industry leads to some important difficulties:

1. Factors may be unique to individual firms on the envelope. (The envelope is not a valid LAC curve.)
2. The industry may confront factor shortages not felt when any single firm expands so that the assumption of constant prices is more extreme when dealing with the industry.

3. Costs to the industry depend on how extra business is distributed among firms.

4. Costs to the industry also depend on the amount and location of excess capacity which, in turn, reflects error, anticipated expansion in demand (which tends to make plants too large), anticipated technological advances (which tend to make plants too small), and cyclical phenomena.

5. Cost to the industry reflects location of increments in demand relative to location of excess capacity.

6. Costs in the industry reflect the amount of time allowed for adjustment of firm sizes and the amount of pressure put on firms by competition.

7. The industry may, given time, generate service subsidiaries or a trained labor force or research teams (and a more rapid development of technology) or political power which would not be available to expanding firms. That is, technological externalities may exist. But these difficulties are no greater than difficulties involved in obtaining the LAC curve in the first place.

The problem of estimating LAC curves has received a good deal of attention. There are two primary methods: Use of cross-sectional contemporaneous data, and use of data from a single firm which has grown or decreased in size. The latter procedure is subject to all the exigencies of time-series analysis. Technology changes, products and factors change, prices change and demand conditions change. In particular costs must be intertemporally allocated. Yet use of cross-sectional data yields a strange hybrid. Do cost curves fitted to cross-sectional data describe

how costs change as some one firm expands? Do they describe how costs in the industry change as the whole industry expands? Evidently they are not a very accurate description of either. There are a number of other difficulties in estimating LAC curves. Some of these involve:

1. Failure of some firms to operate efficiently (that is, on the LAC curve).
2. Prices change during the period of observation (in time series) or between firms (in cross section) because of locational advantages or marketing advantages. Marketing advantages tend to systematically interfere in fitting cost functions because large firms typically have more bargaining power. All price changes necessitate the use of cost indexes, but since factor substitution takes place, cost indexes are likely to be biased upward except during the base period.
3. Observations generally are in terms of totals for a year, quarter or month, but when cost functions are not linear and there is a seasonal variation in output, this introduces a bias so that choice of different periods leads to different conclusions. Moreover, there may be seasonal variation in both costs and output so that results show spurious relations.
4. Capital inputs are conceptually very difficult to deal with. For example, if capital is measured by stock prices it is not independent of profits, and efficiency (198).
5. Accounting methods vary from one firm to another so that some items may be allocated to costs in one firm but elsewhere in other firms.
6. Large firms apparently tend systematically to use more capital

intensive methods than small firms (67). This implies that errors in factor cost allocation (because of accounting techniques or inappropriate capital values) may lead to spurious economies or diseconomies of scale.

7. Firms produce different products and most firms produce several; therefore, it is difficult to compare costs or production functions in different firms. Is detergent soap? Do jets and propeller-driven planes provide the same product?

8. If selling and transportation are included as costs, then geographic dispersion of the market will tend to limit scale advantages of manufacturing in any one plant (67).

9. How can the economies of multi-plant firms be compared with the economies achieved by large plants?

10. Technology continually changes as do prices, market size, et cetera, yet LAC is a static concept. Therefore trying to find LAC through observing actual cost structures of plants built at different times is likely to prove misleading. Wiles' (231) concepts of partial and total adaptation appear more meaningful.

But it will not do to let these objections inundate the study. There is a good deal of evidence to support the hypothesis of scale advantages. Compare the productivity of U.S.A. and other economies (155). Note the failure of steel rolling in Chile (167), and the opinion of business people, engineers and economists (231, 106, 82, 94). Examine evidence that economies are provided not only in the scale of plant but in having several plants under the management of one firm (67).

As pointed out by Robinson (167) in connection with Bain's (7) work

the advantages of scale are far more ramifying than might be first supposed. Even if the most efficient scale of plant could produce only ten, twenty, or thirty percent of the U.S.A. requirements, the size of the markets implies competition between firms so that:

"firms and plants can and do specialize more narrowly and concentrate their efforts on a more limited range of products. At the same time the addition of an efficient unit of production to the market required less growth of the market to justify it, risks were to that extent reduced, and confidence in necessary minimum of expansion more readily created" (167, p. xvii).

The assumption of increasing productivity with larger scale is more likely to be realized as the size of the subject nation decreases. In Canada, increasing productivity should be the rule to judge from Robinson (167, p xviii):

"It is not going too far, perhaps, to say that it seemed to be our general impression that most of the major industrial economies of scale could be achieved by a relatively high income nation of fifty million: ..."

2. The productivity vector

What is required for every industry is a coefficient of elasticity which relates the percent change in inputs to the percent change in outputs. That is, for every industry it is desirable to have an $E = \frac{di}{do} \cdot \frac{o}{i}$: where i = input and o = output. This is, in fact, the information which is provided by a cost function of the form:

1. $O = a(i)^E$, or by a Cobb-Douglas function of the form:
2. $O = aL^bC^c$, under constant price. (Here $E = b + c$.)

One would expect, if the LAC or the cost under total adaptation is

L-shaped,* that a curve fitted to the logarithm of i (on the ordinate) and the logarithm of o (on the abscissa) would be a convex (from below) curve having a slope at large outputs approaching one. (Marginal cost is constant.) But for most of the scatter diagrams this tendency does not appear. A straight line on double logarithm paper fits very well indeed. Its slope (with cost on the ordinate) is generally less than 1.0. (There are economies of scale.) The same is generally true for diagrams that have not been presented here.

When a single economies-of-scale coefficient adequately describes the differences in efficiency at various levels of output of the firm, the same coefficient may be used to describe differences in efficiency at various levels of output of the industry. But if used this way, it is necessary to assume that increments or decrements in volume are distributed among firms so that the relative volume done by each does not change. It is further necessary to assume there are no inter-firm technological externalities, and that the total cost curve satisfactorily describes costs in each firm.

Because of the relatively high cost of graph drawing, empirical work is not presented in full. The following tables and graphs represent only summaries and conclusions. Discussion is kept to a minimum. Moreover, the material on which these tables and charts are based is of highly variable quality so that it has been thought necessary, in many cases, to indicate a subjective evaluation of the original data. In some cases tests

*See Salter (177), Wiles (231), Johnston (106). Also see Schuman and Alpert (185) for an expression of engineers' opinion. In a study by Eiteman and Guthrie (53), 366 replies to a questionnaire sent to businessmen were almost 2:1 (203:113) in favor of an average cost curve that fell throughout the range of output.

of significance are not useful because the available data represent averages. Still, best estimates of E are given by a regression estimator.

3. Summary of previous studies

Table 20 provides a summary of cost studies published elsewhere. In all cases, the data have been refitted using logarithms. In almost all cases $E < 1.0$ and in several cases the hypothesis: $E = 1$ is rejected. Generally the number of observations is too small to adequately test this hypothesis. No attempt has been made to test whether or not residuals are randomly distributed. Again an E less than one in Table 20 indicates that average costs fall as output expands.

Haldi (82, pp. 36-50) has considered evidence on increasing returns in various manufacturing processes. He uses the equation $C = aX^b$ where C = cost, X = capacity, a and b are coefficients obtained by the method of least squares. It is evident that if $b > 1$ there is decreasing returns to scale; if $b < 1$ there is increasing returns to scale. The smaller is b , the more important are scale advantages (or the less important are scale disadvantages). Table 21 provides a summary of results.

In part A of Table 21, scale factors are given for total installation costs of various types of equipment. Information on number of observations, range in size of equipment, and significance of the coefficients has been suppressed. In part B scale factors for construction costs of total plants are presented; again information is suppressed. Part C provides scale factors for operating costs. Data on operating costs is not as complete, and appears less accurate than data in parts A and B.

Table 20. Estimates of E obtained from refitting secondary data using the equation $\text{cost} = a (\text{output})^E$

Industry	E	Standard error	Description of data Source
1-Bus passenger transport	0.7138*	0.0315	Johnston (106, p. 80)
2-Gas production	0.9734	0.0542	Gribbin (76, p. 206)
3-Gas production	0.9046	0.0794	Verhulst (226, p. 295)
4-Gas production	0.9029	0.1093	Verhulst (226, p. 295)
5-Steel production	0.7825*	0.0762	Wylie & Ezekiel (232)
6-Steel production	0.7247*	0.0237	Yntema (237, p. 36)
7-Cement production	0.8058*	0.0222	Wiles (231, p. 227)
8-Banking	0.8757*	0.0129	Wiles (231, p. 240)
9-Industrial assurance	0.9604	0.0172	Wiles (231, p. 237)
10-Industrial assurance	0.9285*	0.0199	Wiles (231, p. 237)
11-Building societies	0.9345*	0.0125	Johnston (106, p. 104)
12-Steam railways	0.8842*	0.0342	Wiles (231, p. 239)
13-Leather belts	0.9588*	0.0092	Dean (44, chart 5)
14-Coal mining	0.9782	0.0129	Johnston (106, p. 101)
15-Crude petroleum	1.1080	0.0627	Wiles (231, p. 236)
16-Coal mining	0.9023*	0.0078	Wiles (231, p. 237)
17-Coal mining	0.8300*	0.0261	Wiles (231, p. 237)
18-Coal mining	0.9654	0.0258	Wiles (231, p. 239)
19-Automobiles	0.9101*	0.0181	Wiles (231, p. 234)
20-Newsprint	0.8228*	0.0830	Wiles (231, p. 229)
21-Fir lumber	0.9520	0.0461	Wiles (231, p. 239)
22-Clay products	0.9520	0.0338	Rautenstrauch (163, p. 335)
23-Cigar production	0.8816*	0.0161	Rautenstrauch (163, p. 324)
24-Food products	0.9010*	0.0322	Rautenstrauch (163, p. 320)
25-Restaurants	0.8359	0.1391	Rautenstrauch (163, p. 320)
26-Baking	0.9848	0.0656	Wiles (231, p. 235)
27-Butcher shops	0.7056	0.1179	Wiles (231, p. 230)
28-Butcher shops	0.7231	0.6616	Wiles (231, p. 230)
29-Wholesale trade	0.8179*	0.0299	Wiles (231, p. 234)
30-Wholesale trade	0.8674*	0.0199	Wiles (231, p. 234)
31-Department stores	0.9813	0.0129	Wiles (231, p. 231)
32-Department stores	1.0371	0.0177	Wiles (231, p. 231)
33-Clothing retail	0.9803	0.0230	Rautenstrauch (163, p. 335)
34-Tire & rubber production	0.9291*	0.0184	Rautenstrauch (163, p. 340)
35-Factory construction	0.6607*	0.0856	Markham (133, p. 52)
36-Electrical products	1.0533*	0.0152	Rautenstrauch (163, p. 320)
37-Food processing	0.9384	0.0361	Johnston (106, p. 95)

*Hypothesis that $E = 1$ is rejected at 5% level.

Table 20. (Continued)

Quality	Description of data			
	Type of observation ^a	N	Place	Date
Very high	Single firm; 4 week time series	33	U.K.	1949-1952
High	Cross sectional	20	U.K.	
High	Cross sectional	10	France	1945
High	Cross sectional	15	France	1945
High	U.S. steel, time series	11	U.S.A.	1929-1939
High	U.S. steel, time series	12	U.S.A.	1927-1938
Good	Cross sectional data	u 14	U.S.A.	1929
Good	Cross sectional o = deposits	d 7	U.S.A.	1939
Good	Cross sectional	8	U.K.	1912-1917
Good	Cross sectional	8	U.K.	1937-1940
Good	Cross sectional averages	15	U.K.	1953
Good	Cross sectional averages	u 9	U.S.A.	1945
Very high	Time series	44	U.S.A.	1935-1938
Good	Cross sectional averages	7	U.K.	1950
Fair	Cross sectional averages	u 12	U.S.A.	1929
Fair	Cross sectional averages (613) ^b	u 10	U.K.	1925
Fair	Cross sectional averages (653)	7	U.K.	1923
Fair	Cross sectional averages (68)	4	U.S.A.	1917
Poor	Costs assumed equal to price	13	U.S.A.	1953
Fair	Cross sectional averages (33)	6	U.S.A.	1913
Fair	Cross sectional (28 and 49)	6	U.S.A.	1918
Very poor	Break-even chart, time series data	15	U.S.A.	1849-1929
Very poor	Break-even chart, time series data	13	U.S.A.	1928-1934
Poor	Time series	18	U.S.A.	1919-1933
Fair	Branches of one company	13	U.S.A.	
Good	Cross sectional	u 14	U.S.A.	1922-1925
Poor	Cross sectional unknown numbers	5	U.K.	1936
Poor	Cross sectional unknown numbers	d 5	Holland	1932
Poor	Cross sectional unknown numbers	d 7	U.S.A.	1929
Poor	Cross sectional unknown numbers	d 8	U.S.A.	1939
Poor	Cross sectional unknown numbers	d 4	U.S.A.	1935-1937
Poor	Cross sectional unknown numbers	u 4	U.K.	1935-1937
Poor	Time series	11	U.S.A.	1904-1933
Poor	Time series	10	U.S.A.	1925-1934
Very poor	Time series	8	U.S.A.	1925-1939
Poor	Time series	30	U.S.A.	1899-1936
Very good	Time series	37	?	1950-1951

^aWhen only the lower limit of the final class size is given, it is necessary to exclude that class from the study. This is indicated by a u if average costs go up, d if they go down.

^bWhen cross sectional averages are used the number of items in the class is given in brackets if known.

Table 21. Scale factors for various pieces of industrial equipment and plants^a using the equation $C = a(X)^b$, Cost = $a(\text{capacity})^b$

	b value
A. Installed equipment	
Agitated vessels	0.47
Autoclaves	0.26-0.46 ^b
Compressors	0.54-0.87
Condensers	0.51-0.54
Construction and mining machinery	0.60
Continuous thickeners	0.60
Cross country pipelines	0.67
Demineralized water systems	0.90
Electric motors	0.36-0.82
Evaporators	0.54-0.80
Furnaces	0.43-0.85
Gas holders, gas producers	0.43-0.60
Heat exchangers	0.54-0.60
Kraft paper mill boilers	0.69
Liquid filters	0.43-0.60
Liquid pumps	0.33-0.60
Refrigeration units	0.30-0.78
Tanks	0.45-0.73
Towers	0.31-1.00
Average (all operations)	0.59
B. Plant construction costs	
Non-petroleum chemicals; butadiene except butylenes; high purity oxygen; synthetic rubber (Buna S) and T.N.T.	1.01-1.39
Metals: aluminum extrusions, aluminum sulphate from bauxite	1.00-4.2

^aSource: A - adapted from Haldi (82, Table 1, pp. 36-38); B - adapted from Haldi (82, Table 2, pp. 45-46; C - adapted from Haldi (82, Table 3, p. 50).

^bWhere more than one "b" value is given, information on one or more different types of equipment has been suppressed.

Table 21. (Continued)

	b value
The remaining 43 types of plants have "b" values below 1.0. Average of total is 0.76.	
C. Plant operating costs	
Liquor evaporation in paper mills	0.60
Electrolytic evaporator for NaOH	0.53
Low temperature dehydration	0.66
Petroleum refining	0.50-0.73
Spin bath evaporation for rayon mills	0.68
Pig iron	0.50
Steel ingots	0.37
Finished steel	0.50
Tonnage oxygen	0.66
Average	0.57

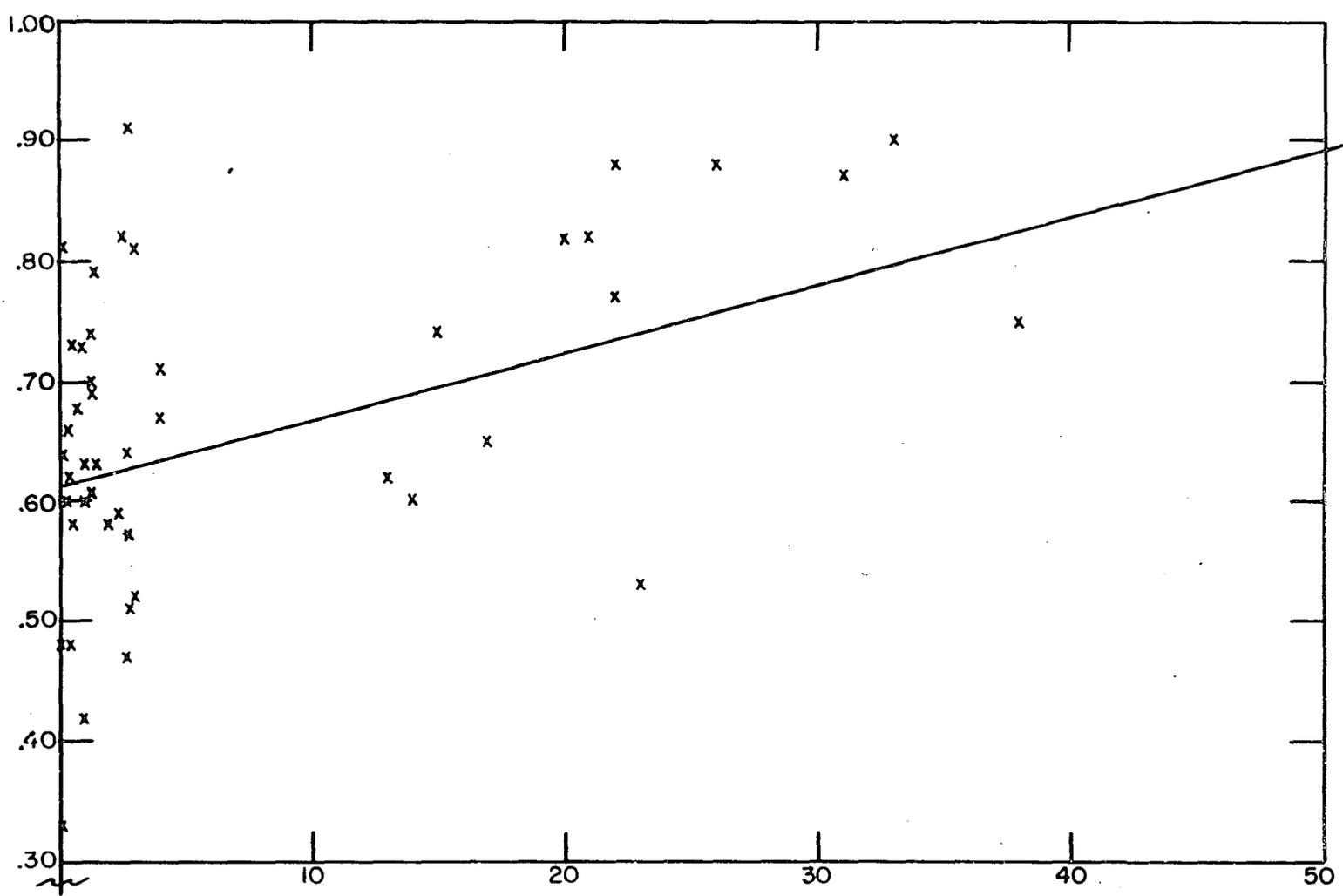
Cross-sectional scatter diagrams were prepared from Haldi's data. No hypotheses were suggested by the scatter diagrams relating size of equipment installed and b values or by the data on operating costs. But there does appear to be a tendency (see Figure 13) for scale factors to approach the value 1.0 as plant size become larger. That is, the larger the plant, the more likely it is that plant construction will be subject to neither economies nor diseconomies of scale. A regression equation was fitted by least squares to this data with b value as dependent variable and mean construction cost in millions of dollars as independent variable. German data included by Haldi were excluded in order to increase homogeneity. The result is:

Figure 13. Economies of scale in plant construction

Ordinate: 'b' value in equation $\text{cost} = a(\text{scale})^b$

Abscissa: average plant scale used in the equation above in millions of dollars

Source: Haldi (82, pp. 45, 46)



$$3. \quad b = 0.6123 + 0.005597C \\ \quad \quad \quad (0.001089)$$

$n = 45$, $t = 5.137$, $R^2 = 0.3749$. The standard error of the regression coefficient is in brackets.

An engineering study of costs done by Schuman and Alpert (185) for a water pumping plant, using the equation $\text{Cost} = k (\text{size})^b$ is of interest. Table 22 provides an extract.

Table 22. Values of b for costs of water pumping equipment

Flow g.p.m.	b
1	0.3
10	0.3
100	0.4
1,000	0.7
10,000	0.9
100,000	1.0
1,000,000	

This data was drawn up by design engineers. It has two important advantages over Haldi's data. First, output changes by one million fold, and second, this is the sort of ex ante cost information that management has to deal with.* It may be wrong; it cannot be irrelevant. However, Haldi's data is superior for most uses because it reflects ex post phenomena.

*Apparently engineering use of 0.6 factor rule is quite common. The formula is as follows: $\frac{C_1}{C_2} = \left(\frac{X_1}{X_2}\right)^{0.6}$ where C_1 is the cost of a piece of equipment of size X_1 , and C_2 is the cost of a piece of equipment of size X_2 . ($X_2 > X_1$).

There are a number of studies or bits of information which for one reason or another did not warrant fitting an equation by least squares. These have been plotted on double logarithm paper, lines have been fitted visually, and slopes evaluated. The following graphs (Figure 11 to Figure 31) resulted. Slopes and sources together with other information is provided on the facing page. These self-explanatory graphs will not be discussed further.

A further source of information is related to pecuniary externalities. It is the rate of change of profit with increased scale. This unfortunately has only indirect bearing as large firms are presumably able to charge higher prices for their products and pay lower prices for their factors. Nevertheless, studies appear to indicate that higher profits accrue to large firms. Table 23 summarizes material prepared by Crum (41). Unfortunately, the data is old and refers to an unusual period.

It should be noted that when all firms are divided into two groups: those exhibiting profits, and those exhibiting losses, the results are quite different. Profit rates among those firms exhibiting a profit tend to decrease as firm size increases.

In general, industries exhibiting rapid technological change presumably have higher values of E than others.* Indexes of output and

*Assume: $T = g(t, q)$ where T = an index of technological contribution to output, t = time, q = output.

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial q} \frac{dq}{dt} \cdot \frac{dT}{dt}, \text{ thus, is partly a reflection of } \frac{dT}{dq} .$$

Figure 14.
Iron ore mining U.S.A.,
1909
No. of observations = ?
Mesabi open E = 0.51
Mesabi underground
E = 0.82
Old range E = 0.85
Wiles (231, p. 227)

Figure 17.
Crude petroleum
46 small, 42 medium,
10 large companies
E = 0.78 (1914); 0.61
(1915); 0.73 (1916);
0.67 (1917); 0.71
1918 and 1919
Wiles (231, p. 241)

Figure 20.
Milk dealers W. Va.,
1933
No. producers = ?
E = 0.94
Wiles (231, p. 228)

Figure 23.
Wheat flour milling
U.S.A., 1913-1918
17 small,
14 large companies
E = 0.97
Wiles (231, p. 241)

Figure 15.
Copper mining U.S.A.,
1918
(a) Lake Co's. E = 0.59
(b) Porphyry Co's.
E = 0.82
(c) Others E = 0.85
Wiles (231, p. 241)

Figure 18.
Oil refineries U.S.A.,
1939
Engineer's estimates of
Costs independent of
location E = 0.72
Wiles (231, p. 232)

Figure 21.
Creameries U.S.A.,
1920; Canada, 1933
? creameries in U.S.A.
78 creameries in Canada
E = 0.73 (U.S.A.)
E = 0.80 (Canada)
Wiles (231, p. 233)

Figure 24.^a
Iron U.S.A., 1909
21 Bessemer pig iron
(lowest cost)
13 Bessemer (2nd high-
est cost)
15 Basic pig iron (2nd
lowest cost)
22 open hearths (high-
est cost)
E = 0.69-0.80^d
Wiles (231, p. 228)

Figure 16.
Bituminous coal U.S.A.
1917
68 producers
E = 0.95
Wiles (231, p. 239)

Figure 19.
Milk evaporation U.S.A.
1918
28 small
9 large companies
E = 0.94
Wiles (231, p. 241)

Figure 22.
Fish canning U.S.A.,
1916 and 1917
17 small plants 1916
40 small plants 1917
52 large plants 1916
41 large plants 1917
E = 0.93
Wiles (231, p. 229)

Figure 25.
Farm and industrial
machinery U.S.A.,
1924-1934
1 producer
E = 0.70-0.99
Rautenstrauch (163,
P. 337)

^aThe d superscript indicates average costs in an excluded observation are below previous average costs.

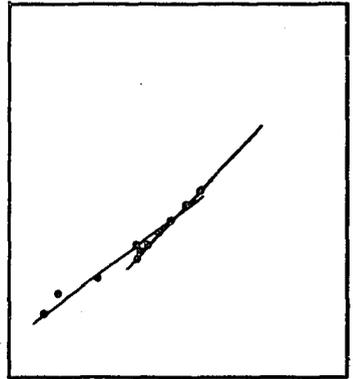
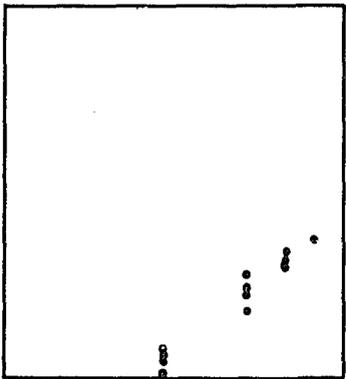
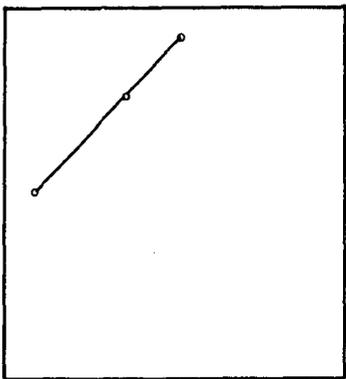
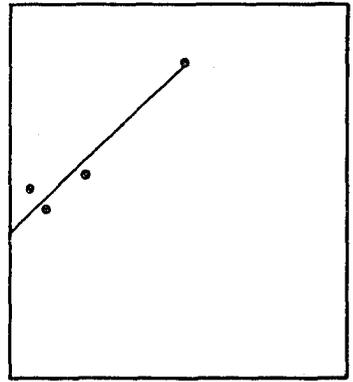
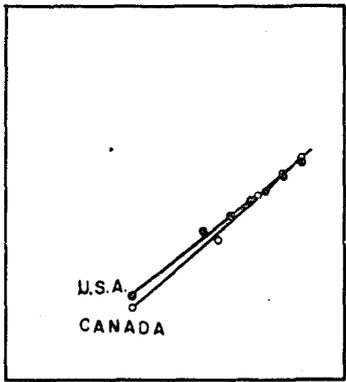
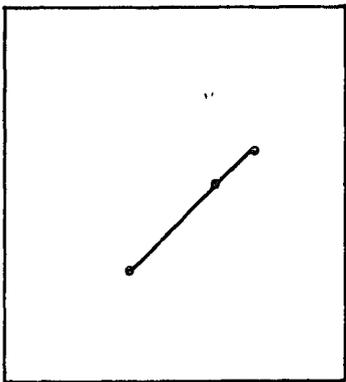
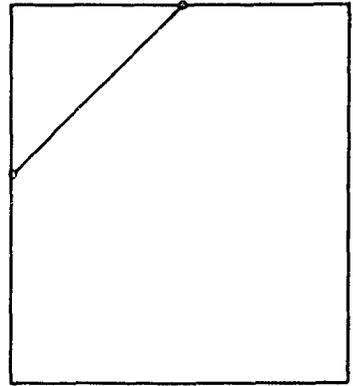
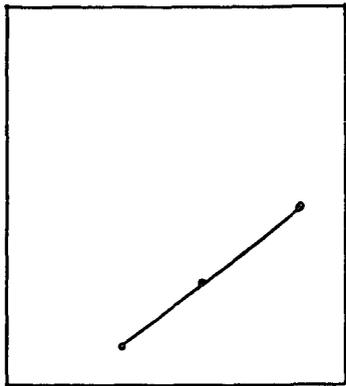
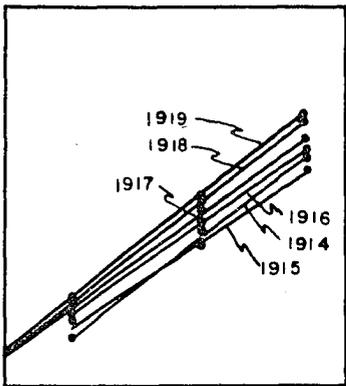
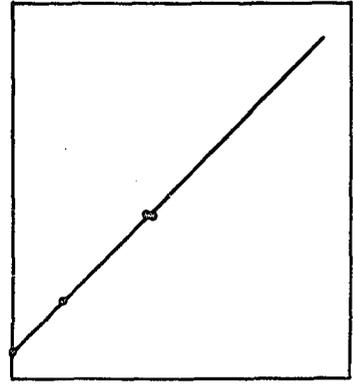
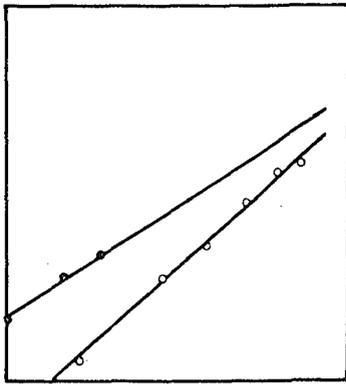
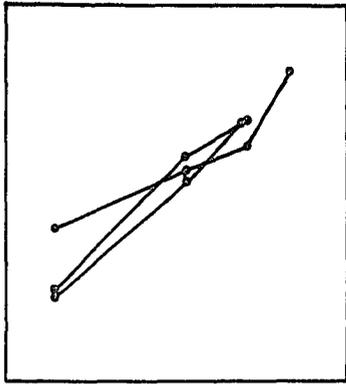


Figure 26.
Construction costs rayon
plants U.S.A., 1925-1935
8 observations
E = 0.75
Markham (133, p. 52)

Figure 27.
Shoe retailing U.S.A.,
1937
53 observations
E = 0.78
Dean and James (45)

Figure 28.
Retailing U.K., 1931-
1937
E = 0.83
Plant and Fowler (162,
Table 8)

Figure 29.
Electrical products
U.S.A., 1899-1936
1 Producer
E = 0.96-1.20
Rautenstrauch (163,
p. 315)

Figure 30.
Fertilizer U.S.A.,
1941-1942
No. producers = ?
E = 0.99 (mixed
fertilizer)
E = 0.91 (bulk super-
phosphate)
Wiles (231, p. 236)

Figure 31.
Beet sugar U.S.A.,
1917-1918
56 units
E = 0.81
Wiles (231, p. 229)

Figure 32.
Operating costs
railways U.K., 1928-
1937
E = 0.31
Broster (20)

Figure 33.^a
Co-op grocery stores
U.K., 1930
Number = ?
E = 0.78
Wiles (231, p. 230)

^aThe d superscript indicates average costs in an excluded observation are below previous average costs.

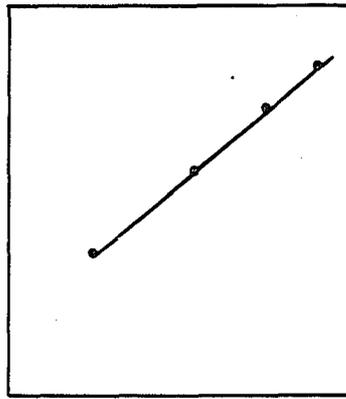
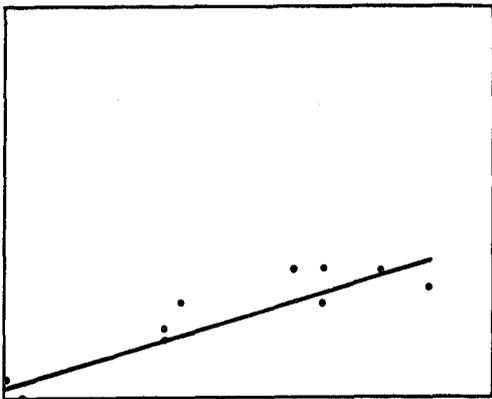
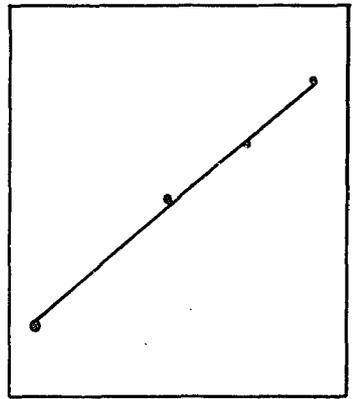
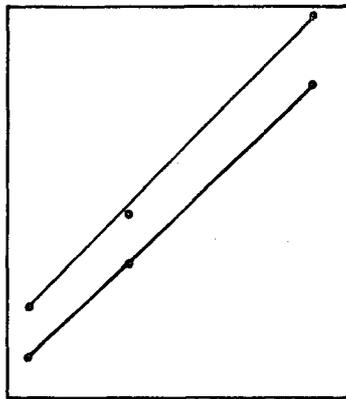
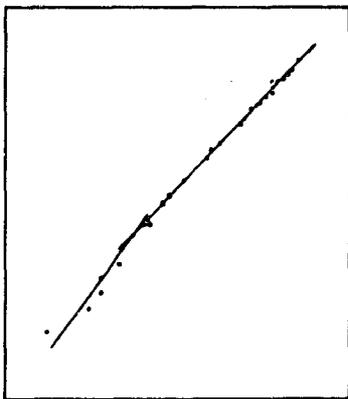
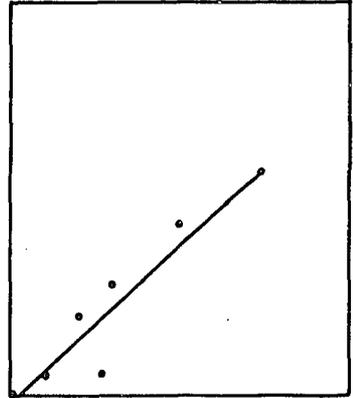
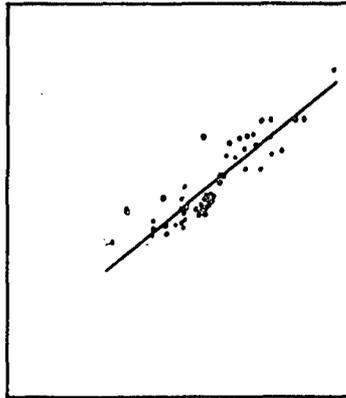
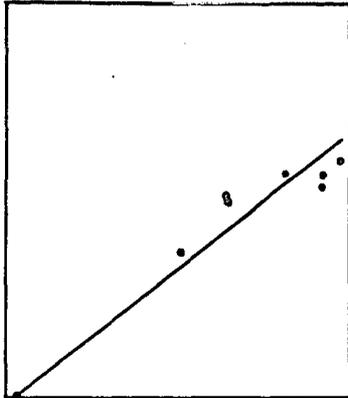


Table 23. Net profits as a percent of equity, unweighted averages, U.S.A., 1931-1936^a

Type of enterprise	Corporation size interval in thousands of dollars assets ^b								
	0	50	100	250	500	1,000	5,000	10,000	50,000
All corporate industry	-16.34	- 4.73	-2.64	-1.40	-0.80	-0.03	0.38	1.45	2.78
All manufacturing	-17.24	- 5.21	-2.36	-0.67	0.60	1.56	2.54	2.48	3.88
Metal manufacturing	-19.17	- 5.85	-3.19	-1.65	-0.54	3.05	0.89	0.56	n.a.
Food group	-11.08	- 2.71	-0.79	2.08	3.22	3.44	5.48	n.a.	n.a.
Chemicals	-15.48	- 1.48	1.37	3.68	5.00	6.13	8.46	5.22	2.46
Textiles	-23.86	- 7.64	-3.74	-0.73	0.01	0.38	n.a.	n.a.	n.a.
Printing	-13.84	- 3.49	-0.33	2.37	3.06	4.77	8.33	n.a.	n.a.
Paper	-11.67	- 1.64	1.42	2.88	3.36	2.30	n.a.	n.a.	n.a.
Forest products	-20.08	-11.61	-7.14	-5.40	-3.83	n.a.	n.a.	n.a.	n.a.
Stone, clay, glass	-17.43	- 6.67	-4.99	-3.37	-1.06	-1.01	1.15	n.a.	n.a.
Liquors	3.83	4.07	5.96	6.34	7.87	n.a.	n.a.	n.a.	n.a.
Tobacco	-14.58	- 3.04	0.48	-1.69	-1.07	n.a.	n.a.	n.a.	n.a.
Leather	-22.80	- 7.43	-2.44	-1.89	-1.57	n.a.	n.a.	n.a.	n.a.
Rubber	n.a.	- 2.85	-1.16	3.23	n.a.	n.a.	n.a.	n.a.	n.a.
Misc. manufacturing	-21.29	- 7.34	-2.97	-1.00	-0.56	1.18	n.a.	n.a.	n.a.

^aSource; Adapted from Crum (41, Tables 2 - 26). Total assets as given by balance sheet on tax returns. Rate of return = ratio of profit or loss after taxes to estimate average equity.

^bLower limit of the class interval is given. Note that intervals are not of equal size.

technological change for the period 1939-1950 have been published for 53 industries by the U.S.A. Department of Labor (220). Multicollinearity interferes with attempts to obtain $\frac{dT}{dq}$ for any one industry. But a cross sectional approach yields the equation:

$$4. \Delta(\text{Productivity index}) = K + 0.41974 \Delta(\text{Output index}).$$

$$t = 4.4912, n = 53.$$

Consequently an array of industries by the historic index of technological change should approximately correspond to an array of industries according to values of E .

Two roughly correspondent arrays of industries by order of historic technological change are provided in Tables 24 and 25.

4. Economies of scale vector

The cost and production functions, and indexes of technological change presented above are of uneven quality. Moreover, comparison of the sectors in the available input-output model with the cost information above indicates that for some sectors, there is very little information. These difficulties stand as barriers to the general use of an index number in computing E for each sector. Approximations and rough estimates have been used where there was not enough information for an accurate estimate, or where estimates of E for single industries were thought to be in error.

Details regarding the classification (numbers refer to the standard industrial classification, 1948 (23)) and the estimation of E for each sector are provided in Table 29 of the Appendix. Estimates of E involve judgment rather than calculations from the data. In general estimates tend to be conservative (that is, tend toward the value 1.0) in order to

Table 24. Indexes of output per man-hour in 27 industries, U.S.A., 1950^a

Rank ^b	Industry	Index number	Rank ^b	Industry	Index number
1923=100					
1	Rayon and allied products	1,217	15	Pulp and paper mills	184
2	Ice cream	341	16	Paint and varnishes	183
3	Electricity	335	17	Non-ferrous metals	180
4	Rubber tires and tubes	312	18	Hosiery	172
5	Petroleum refining	290	19	Footwear	168
6	Tobacco products	289	20	Bit. coal	167
7	Iron and steel	273	21	Leather	164
8	Confectionary	256	22	Cane sugar refining	158
9	Glass products	238	23	Clay cons. products	154
10	Cement	234	24	Coke	131
11	Woolen and worsted	214	25	Flour, etc.	130
12	Cotton goods	203	26	Meat packing	125
13	Canning and preserving	200	27	Bread, etc.	117
14	Fertilizers	198			

^aSource: Salter (177, p. 164).

^bRanked by size of index number, from largest to smallest.

Table 25. Indexes of productivity in U.S.A. industries, 1950^a

Rank ^b	Industry	Index number	Rank ^b	Industry	Index number
1939=100					
1.	Electric light and power	1,764	18.	Telephone	307
2.	Manufactured gas	1,176	19.	Natural gas	296
3.	Rubber products	878	20.	Misc. manufacturing	292
4.	Tobacco manufacturing	620	21.	Fab. metals	291
5.	Transportation cost	608	22.	Primary metals	284
6.	Oil and gas	501	23.	Telegraph	263
7.	Chemicals	435	24.	Non-electric machines	251
8.	Printing and publishing	432	25.	Apparel	246
9.	Stone, clay, glass	412	26.	Farming (gross income)	244
10.	Railroad	390	27.	Foods	241
11.	Non metals	390	28.	Beverages	238
12.	Local transit	372	29.	Bit. coal	230
13.	Paper	359	30.	Furniture	208
14.	Petroleum and coal	338	31.	Leather production	198
15.	Electric machines	338	32.	Farming (net income)	184
16.	Textiles	325	33.	Lumber products	177
17.	Metals	317	34.	Anthracite coal	147

^aSource: U.S. Department of Labor (220).

^bRanked by size of index number from largest to smallest.

provide an opposite bias to that involved in assuming constance of prices. The E values estimated in Table 29 of the Appendix are presented in tabular form in Table 26.

Table 26. The E vector^a

Sector	E value	Sector	E value
1 Agriculture	1.00	22 Furniture	1.00
2 Forestry	0.95	23 Other wood	1.00
3 Fishing and hunting	1.00	24 Paper	0.95
4 Metal mining	0.90	25 Printing	0.80
5 Coal and petroleum	0.85	26 Iron and steel	0.80
6 Non-metal mining	1.00	27 Agricultural implements	0.90
7 Meat products	1.00	28 Iron n.e.s.	0.90
8 Dairy products	0.98	29 Transport equipment	0.93
9 Fish processing	0.95	30 Jewelery	1.00
10 Fruit and vegetables	0.90	31 Non-Fe. metal n.e.s.	0.90
11 Grain mill	0.97	32 Elect. apparatus	1.00
12 Bakery products	0.98	33 Non-metallic minerals	0.95
13 Carbonated bev.	1.00	34 Products of oil	0.95
14 Alcoholic bev.	1.00	35 Chemicals	0.90
15 Conf. and sugar	0.89	36 Misc. manufacturing	1.00
16 Misc. foods	0.90	37 Construction	0.85
17 Tobacco	0.88	38 Transportation, trade	0.80
18 Rubber	0.93	39 Communication	0.95
19 Leather	0.96	40 Power, gas and water	0.80
20 Textiles	0.80	41 Finance	0.95
21 Clothing and furs	1.00	42 Service	0.95

^aSource: Table 29 of the Appendix.

5. Summary of calculations

Using the E vector and a transactions matrix, it is possible in the manner already described, to compute a pecuniary externality matrix. This has been done. The results are presented in Table 30 of the Appendix. The procedure is as follows: Obtain a set of equations in the usual manner of input-output analysis.

$$5. [x] + [Y] = [X]. \text{ That is,}$$

$$[\text{Interindustry use}] + [\text{direct consumption}] = [\text{total output.}]$$

Then, obtain an a matrix, the elements of which are $a_{ij} = \frac{x_{ij}}{X_j}$ and write

$$6. [a][X] = [Y].$$

Equation 6 is thought inaccurate when applied at the margin due to scale economies. Next, modify the a_{ij} values so they express estimated factor requirements at the margin. That is, $\bar{a}_{ij} = \frac{x_{ij}}{X_j} (E_j)$. Invert the resultant $[\bar{a}]$ matrix.

$$7. [\bar{a}]^{-1} [Y] = [X].$$

Here $[\bar{a}]^{-1}$ is the matrix multiplier. Premultiply the $[\bar{a}]^{-1}$ by a vector obtained by subtracting each value of E from one.

$$8. [I - E][\bar{a}]^{-1} = [T].$$

Here T is the pecuniary externality matrix. T_{ij} is the extra profits or savings or undisbursed funds available in industry i which arise as a consequence of a one dollar expansion in industry j.

An example of the use to which this matrix may be put is provided by Table 27 and the subsequent discussion. Unfortunately Table 27 provides a contrast with the E vector and the externality matrix, rather than a test of the accuracy of the externality matrix. No method of

Total 27. Investment required per dollar's worth of output^a

Sector	Net capital ^b		Total (3)	Output ^c (4)	Capital output (5)	Longevity ^d		Weighted ^e longevity (8)	Investment per dollar output ^f (9)
	Const. (1)	Eq. (2)				Const. (6)	Eq. (7)		
1	560.5	1351.3	1911.8	1601	1.19	40	13	20.9	0.057
2	91.3	58.1	149.4	316	0.47	21	9	16.3	0.029
3	148.3(E)	67.3	215.6	81	2.66	-	6	?	?
4									
5	486.1(E)	295.5	781.6(E)	665	1.18	30(E)	16	24.7	0.048
6									
7									
8									
9	99.9(E)	126.7	226.6	247	0.92	50	18	32.10	0.028
10									
11									
12									
13									
14	128.4(E)	122.8	251.2	304	0.83	50	14	32.4	0.026
15									
16									
17									
18	45.8(E)	43.8	89.6	185	0.48	50	16	33.4	0.014
19									

^aSource: Calculated from tables in Hood (99, Appendix to chapter 6).

^bCapital is divided into two categories which correspond to buildings (construction) and equipment and machinery (equipment).

^cOutput is in terms of gross domestic product in 1949.

^dLife expectancy (years).

^eWeights are values of construction capital and equipment capital.

^fColumn 5 divided by column 8.

Total 27. (Continued)

Sector	Net capital ^c			Output ^e (4)	Capital output (5)	Longevity ^b		Weighted ^d longevity (8)	Investment per dollar output ^f (9)
	Const. (1)	Eqt. (2)	Total (3)			Const. (6)	Eqt. (7)		
20	159.4(E)	152.4	311.8	239	1.30	50	21	26.6	0.049
21	49.7(E)	47.6	97.3	290	0.34	50	16	33.4	0.010
22	77.0(E)	22.9	99.9	72	1.39	35	18	31.1	0.045
23	118.3	70.0	188.3	245	0.77	35	18	29.7	0.026
24	374.7	273.8	648.5	407	1.59	50	21	37.8	0.042
25	62.1(E)	59.4	121.5	189	0.64	50	17	33.9	0.019
26									
27	289.0(E)	276.3	565.3	619	0.91	50	16	33.4	0.027
28									
29 ^g	78.7(E)	75.2	153.9	454	0.34	50	19(E)	34.9	0.009
30									
31	107.0(E)	102.3	209.3	305	0.69	50	20	33.8	0.020
32									
33	86.1(E)	109.1	195.2	197	0.99	50	21	33.8	0.029
34									
35	134.3(E)	106.4	240.7	199	1.21	50	20	36.7	0.033
36	23.2(E)	22.2	45.4	82	0.56	50	15	32.9	0.017
37	59.6	205.6	265.2	1090	0.24	25	9	12.6	0.019
38									
39	3835.8	1630.3	5466.1	3353	1.63	50	16	39.9	0.041
40	1505.8	438.5	1944.3	277	7.02	55	30	49.4	0.142
41	304.1	30.6	334.7	1183	0.28	50	15	46.8	0.006
42	371.2	218.8	590.0	2298	0.26	50	13	36.1	0.007
Subtotal	9196.3	5906.9	15103.2						
Error	749.6	790.9	1540.5						
Total econ.	9945.9	6697.8	16643.7	14885	1.12			35.2	0.032

^gThe method of evaluating capital does not seem appropriate in this case.

testing the validity of the pecuniary externality matrix has been developed.

6. Requirements of investment funds

If Table 30 of the Appendix provides a description of the manner in which potential investment funds accrue as the Canadian economy grows, is there some measure of whether or not such funds accrue in an appropriate manner? The calculations in Table 27 have been undertaken in an attempt to provide such a criterion. The data, especially that on capital longevity, must be regarded as of extremely poor quality. Nevertheless such detailed information as is provided in Table 27 is not available in many countries. It is interesting to compare Table 30 of the Appendix (the externality matrix) with column 9 of Table 27.

The absolute level of the figures in column 9 must be regarded as approximate. Capital longevity figures are not very accurate and the level of capital values depends on the deflators used. But the relative values of these figures are presumably much more satisfactory. Probably it does take about twice as much capital to produce one dollar's worth of agricultural products as it takes to produce one dollar's worth of fishery products. Again observe that column 9 of this table abstracts from time considerations.

If additional output must be obtained immediately, column 5 is the relevant column. Columns 5 and 9 may be compared with the pecuniary externality matrix $[T]$ (Table 30 of the Appendix). Naturally, it is necessary to simplify $[T]$ somewhat in order to make this comparison. The method of rank correlation has been used to compare the pecuniary

externality matrix with columns 5 and 9 of Table 27.

Unfortunately the interpretation of this test is not unambiguous. If the correlation is low it may be because of the crude nature of the data, or it may be because profits accrue in an inappropriate manner. The coefficient of rank correlation is unlikely to be very meaningful for this reason. If the coefficient is high it means both that the data is tolerably accurate and that pecuniary externalities accrue in an appropriate manner.

Rank correlation between the pecuniary externality matrix (appropriately collapsed into a vector by assuming a one dollar expansion in every sector) and column 5 Table 27 is only 0.007, $p = 0.488$. Rank correlation between column 9 of Table 27 (capital output ratio where capital is adjusted for longevity) is 0.802, $p = 0.198$.

The coefficients of rank correlation are not large enough to lead to rejection of the hypothesis that there fails to be a correspondence between the accruals of pecuniary externalities and the need for investment funds. However, the hypothesis that pecuniary externalities accrue to those industries which most need investment funds cannot be accepted. There may be some tendency in this direction (as is indicated by the relatively low probability of obtaining a rank correlation of 0.802 by chance alone) but the model here used is not adequate for testing this hypothesis.

When the capital market is imperfect, economic development may fail to be a maximum because pecuniary externalities accrue in an inappropriate manner.

B. Summary

In this chapter an attempt is made to quantify pecuniary externalities in the Canadian economy. To the author's knowledge there has been no previous attempt to measure the importance of externalities. This is partly because the influences of an outside action on any one firm are extremely complex. The direct effect of firm A's action on firm B's profits tells only part of the story. Firm A's action stimulates action on the part of other firms, C,D,E, and their reactions in turn have secondary effects on Firm B. A's action affects B in both a direct and an indirect manner. In this chapter an input-output model of the economy is used as a framework for studying the complex nature of action and reaction.

Using an estimate of the cost function in each sector, it is possible to modify the input-output matrixes to obtain a matrix multiplier that is consistent with economies or diseconomies of scale. But as this involves a fairly expensive matrix inversion, and may not greatly alter the Leontief input-output matrix, it could be avoided for many purposes.

The cost functions may be used, under the assumption of constant prices, in combination with the matrix multiplier to compute the amount of undisbursed funds accruing in each sector as a consequence of a one dollar expansion in output in any other sector. A table of such values has been computed and is presented in the Appendix (Table 30).

Such a table may be used to study the question whether, in a growing economy, funds accrue in industries which most need them. By way of example, a brief study is made (see Table 27) of accrual of undisbursed

funds in the Canadian economy. An attempt is made to see if such funds accrue in those sectors which most need capital investment. Although no definite conclusions can be reached it seems likely that, there is little tendency toward correspondence between sources of investment funds (in pecuniary externalities) and funds required for capital expansion.

VII. SUMMARY AND CONCLUSIONS

A. Purpose

This is a study of the manner in which external economies and diseconomies influence economic development. External economies and diseconomies comprise a rather old subject, but unfortunately, there are still many ambiguities and confusing passages in the voluminous literature on the subject. Moreover, there has been comparatively little attention devoted to the manner in which external economies and diseconomies influence economic growth and development. It is not clear, for example, whether the presence of external economies acts to promote or to retard the rate of economic development. In fact, it is not even clear that the terms external economy and external diseconomy can be meaningfully defined. What is by now clear, is that economists writing about economic development impart a different meaning to these words than that which was spelled out by Alfred Marshall (134) when he first defined them.

This study is therefore a part of a broader question: does the invisible hand work to promote economic development in modern mixed economies containing government and monopoly elements: Dorfman, Samuelson and Solow (48) have put forward a dynamic invisible hand formulation. It appears that a correspondence between the results of the freely operating price system and a Pareto welfare criterion exists, provided that time rates of change of all money prices are perfectly known to all marketers. This requirement is never met in practice. Moreover, it is not clear that a dynamic Pareto optimum can be satisfactorily defined. Still

the practical question is: does the invisible hand provide an approximate model of reality?

Practical limitations have forced the all-encompassing problems into the background in this study. More limited and specific questions, such as the following, have been considered. What market imperfections interfere with duality? What types of market imperfections may be called external economies? How can external economies be defined? And, should external diseconomies always be avoided?

B. Definitions

In this thesis the terms external economy and external diseconomy are collapsed into the single term externality.* This is a convenience that simplifies exposition. Moreover, in situations that involve imperfect competition it is not altogether clear how economies can be separated from diseconomies. For example, were a firm to undertake additional expenditure in order to obtain advantages from a market enlarged by a second firm, it may be difficult to determine whether an economy or a diseconomy is involved. This dilemma can be circumvented by using the word externality.

There are three difficulties which must be overcome before a meaningful definition of externalities is available. The first problem is one of relating externalities to welfare norms when these norms (for example, the ideal rate of capital formation) are unknown. The second

*This term was first used by Bator (9).

problem is one of obtaining a quantitatively meaningful definition (in terms of money values) even though external influences are not exchanged in the market place, and when, under non-constant returns to scale, external influences cannot be evaluated using productivity theory. This difficulty in evaluating externalities is called the problem of imputation. The third problem involves obtaining a definition which has value for policy decisions. When the business world is imperfect (that is, not perfectly competitive) it is not valid to make policy recommendations which are oriented toward competitive norms. This is called the problem of the second best. Thus, although an external influence obviously interferes with perfectly competitive norms, it may be undesirable in an imperfect world, to recommend the removal of such an external influence.

There is a temptation to define externalities as a deviation from the MPC equals MSC criterion. But such a definition is not useful because the meaning of MSC is by no means clear. In fact use of the term MSC presupposes an ability to measure externalities.*

Here externalities are defined in two ways. If socially relevant information is ignored by decision makers because their incentives are not identical with those of society in general, an externality may be said to exist. This definition is rather relaxed and imprecise. A more exact definition and the one most frequently used in this study is as follows. When some firm A takes some action that influences B in such a

* $MSC = MPC + \text{externalities}$, where externalities are appropriately defined. Obviously, this equation does not define externalities unless MSC is known or can be obtained, from some other source.

way that B's profits are increased or reduced, but when A receives or pays no compensation for this action, an externality may be said to exist. The ramifications of this definition received extensive treatment in chapter 4. The definition is used as a tool of positive economics. This, unfortunately, is not in the tradition of Marshall and Pigou who were able to relate external economies and diseconomies to norms of public welfare. But until more satisfactory intertemporal welfare models are available, it seems wise to take a positive rather than a normative approach. Positive rather than normative usage also avoids problems of the second best, but unfortunately this means that very little may be said about the policy implications of externalities. The problem of imputation is avoided, as far as possible by considering only incremental changes. Thus it is unnecessary (as well as impossible) to evaluate a total external influence. It is not possible to say anything about how much A's presence is worth to B -- but it still may be possible to say something worthwhile about how much it is worth to B if A increases its output by one percent.

A number of other terms have been defined in this study. Concepts such as economic growth, economic development, invention, and innovation were defined and considered in context.

C. Historical Development

The classical economists apparently treated most or all industrial pursuits as subject to falling average costs. Only agriculture was subject to increasing costs. Marshall had a more refined view. Some firms were subject to increasing, others to decreasing costs, moreover, some

whole industries were subject to increasing, and others to decreasing costs. This was because of such phenomena as falling industrial demand schedules, general progress of the environment, or scarce skilled labor, and technological progress. The last two, when generated in one firm, might be used by another. Thus, to Marshall (134), external economies and diseconomies were partly a means of treating the problem of imperfect competition.

Pigou (159) attempted to relate external economies and diseconomies to welfare economics. His statements, while intuitively plausible, lacked theoretic rigor, so that Allyn Young (236), Robertson (166), Clapham (35), and Knight (116) were able to find fault with his models. However, there were by this time sufficient examples of externalities that the subject remained of interest. One particularly intriguing example involved financing a subway. The subway would be profitable if the financing firm owned contiguous real estate because such real estate would appreciate in value due to presence of the subway. But the subway would be unprofitable if the financing firm did not own the contiguous real estate. Moreover, the financing company might not be able to purchase the appropriate real estate without a prohibitive inflation of real estate values. It was thought that appreciation in values of contiguous real estate would, so far as the whole society was concerned, be negated by depreciation in values of non-contiguous real estate. But the present study appears to indicate that where the subway generates growth rather than just movement, there is no necessity for a depreciation of any non-contiguous real estate values. However, subject matters such as this

received comparatively little attention in Marshall's time because economists were working out the welfare implications of perfect competition.

Economists at this time were handicapped by a lack of a theory of imperfect competition. If there were economies of scale it seemed that individual firms must expand until either monopoly resulted or the economies of scale were completely utilized. About this time, following the pioneer effort of Douglas and Cobb, a fairly large number of empirical studies purported to show that increasing returns to scale was not a particularly important phenomenon. Rather, approximately constant returns to scale appeared to be the rule. These empirical findings discouraged the study of economies of scale whether these be internal or external to the individual firm.

Welfare economists continued to investigate the consequences of increasing returns but were unable to overcome the problem of income distribution. Under increasing returns, if factors are paid their MVP, firms will disburse more in factor payments than they receive for the product.

Income distribution is a similar problem (though not as pressing) when there is decreasing returns to scale. Unfortunately, contemporary welfare theory still is unable to tell us very much about normative income distribution.

Because most of the study of income distribution was done by welfare economists, the orientation of such studies was toward income distribution among ultimate consumers. Implications of income distribution among intermediate manufacturing firms was frequently ignored. The means to

this end was simply the assumption of a perfect capital market. Only in comparatively recent times have the implications of this assumption been seriously investigated. It is, for example, now clear that a large part of the capital expansion of most firms is internally financed. Similarly, technological development upon which economic growth so greatly depends, is largely a product of research and development work undertaken by large corporations and financed out of internal profits.

Moreover, it has now become clear that earlier empirical studies which concluded that most industries were subject to approximately constant returns to scale, were not the last word. It appears that most firms are subject to gradually falling costs throughout a great part if not all of their feasible output range. It also seems likely that many industries are subject to increasing returns to scale, but that the limited size of the market prevents such industries from expanding into the lowest-cost output range.

D. Types of Market Failure

By market failure is meant the failure of the mixed economy to produce results in correspondence with an arbitrary set of conventional welfare norms. But, economists concerned with economic development have defined external economies and diseconomies in such a way as to relate these concepts to profits within the firm and not to welfare norms. The above definition of externalities is in the more recent tradition: externalities are related to profits. But it seems wise to follow Marshall, and direct most of the study towards problems of scale; that is, the relation of A's changes in scale to B's profits.

Economies of scale may be internal or external. They may not result in market failure. But there are various types of situations which relate to externalities, and which do exhibit tendencies toward market failure. It is appropriate to consider these one by one.

1. Factor indivisibilities

Factor indivisibilities may give rise to non-constant returns to scale. The indivisible factor may be privately owned or publicly owned. Especially in the latter case there arises the problem of pricing. It may not be possible to set price equal to MVP or MRP. The same price may not be appropriate on both sides of the market. This is a rather old problem that has been extensively treated elsewhere.

2. Non-convexity

It may happen that technological developments give rise to non-convexity in the time path of cost functions. An example may be useful. An economy which makes use of blacksmiths for a comparatively long period of time may result in a generation of blacksmiths and blacksmiths' sons who are extremely inventive and therefore who develop new technological methods of forging iron, stamping metals and machining hard substances. These inventions may be of such great importance that costs fall a good deal. But costs may fall even farther when those who become skilled and knowledgeable in the new machines in turn invent better machines and improve the old machines. But an alternative procedure may make use of blacksmiths for such a short period of time (for example, because of the opportunity to borrow technology from a neighboring economy) that

blacksmiths' skills and innovating ability are not developed. The latter economy may continue to borrow technology and never develop its own. As a consequence of this, if it has different factor endowments and different technological problems, its costs may forever remain high relative to what they might be, paradoxically because of its very anxiety to obtain the latest technological equipment. The matter is extremely complex, and has received little study. It is possible that an economy must remain in a given technological situation, using given (perhaps archaic) production functions for a comparatively long period of time before the technology-creating process becomes self-sustaining. There may be a take-off into self-sustaining technology as well as a take-off into self-sustaining growth.

3. Non-appropriabilities

When activity of one firm creates an economy in a second firm, (as when the bees from an apiary fertilize an orchardist's fruit blossoms), and when the economy creating activity is not rewarded by the firm which receives it, a non-appropriability is said to exist. However, when such activity is harmful, (without retribution) the activity is said to be non-culpable. (For example, air pollution increases the corrosion rate of exposed metals.) In either case duality fails, and the market system apparently does not provide the ideal allocation of resources.

4. Irreversibilities

A great part of economic theory is based upon the assumption that changes are not irreversible. But, in fact, soils do erode and certain

economic institutions (old age pensions and children's allowances, for example) appear to be politically irrevocable. In such situations the process of *tâtonnement* does not work. Intervention by government may be required! Government, for example, may have to prevent individual persons from following policies that erode soils or reduce the reproductive ability of forests. Such people may be acting rationally given their time horizon, but the time horizon of society is longer than that of any individual, so that it is not appropriate to allow individuals to commit society to an irreversible course of action. If this is allowed, market failure may result and the rate of economic developments may be less than the possible rate.

5. Equilibrium difficulties

The hog-cycle is an example of market failure due to interdependent anticipations. It is evident that such situations involve interdependence of profits. There is presumably no reason to assume that when equilibrium difficulties confront an economy, profits will accrue to those firms which are particularly in need of funds for expansion. It seems, in fact, as likely that windfall gains and losses will occur in a more or less random fashion. This is not serious where the capital market is perfect. Elsewhere it may result in failure of economic development.

6. Social goods

When consumption by any one individual of a given commodity does not reduce the amount of that commodity remaining for others to consume, the commodity may be termed a social good. Perhaps the best example is

national defense. But there are other similar situations--road side parks, clearing malarial swamps, highways and roads, good health, and economic development. By the very nature of a social good, individual purchases are inappropriate. Private police protection as well as private defense against an attacking country is absurd. Similarly most individuals working alone cannot drain a malarial swamp, bring about economic development, or reduce the hazards of communicable diseases.

But there are all grades of social goods from the purely social, through the partly social to those having very small social contents and finally to purely private goods. And whether or not a good is a social good depends partly on whether or not individuals would rationally purchase such a good. Individuals can do something about communicable diseases -- they may avoid other people. Individuals also may do something about national defense -- they may volunteer for dangerous military service at low rates of pay. But in general, these things must be forced upon individuals who realize that their own action comprises but a tiny fraction of the resources needed for accomplishment of the goal. Thus it is, that rumors of government activity in any area which has hitherto been a subject for private individual responsibility tends to discourage such private responsibility. Individuals expect the subject good to become a social good. But such expectations reduce private purchases so that, if the good is really needed by the society, government finds after a while that it must provide such services or they will not be provided. The good becomes a social good solely because it is expected to become a social good.

Health insurance may be a case in point; individuals can purchase health insurance, but they are not likely to do so if they believe the government will soon provide a health insurance scheme. And, the fact that individuals fail to look after themselves in this manner may force the government to provide a health scheme.

All these difficulties may involve market failure of a type that leads to sub-optimum economic development. It is reasonable to suppose that at times each type of market failure acts to reduce the rate of economic growth.

E. The Question of Entry

Entry represents what is perhaps one of the more important types of externalities. Establishing firms may, of course, provide either substitute or complementary goods. They may also use the same factors as, or factors that are complementary in some production process with, those used by the affected firm. It is possible to define three types of entry: neutral, completing, and aggressive. 1. Neutral entry involves establishment of a new firm, B, in an area in such a way that firm A's profits are not positively or negatively affected. B's entry does not cause an externality in A. 2. Completing entry involves establishment in an area of some firm, B, which produces a good which firm A may use as a factor, or which is complementary with firm A's product. Therefore, completing entry involves a positive externality in firm A. 3. Aggressive entry involves establishment of some firm, B, in an area serviced by firm A in such a way that A's business (profit) is reduced by B's entry. For example, if firm B produces the same or a competing product,

there will be a negative externality in firm A. These cases are all possible although they are perhaps not all equally likely to happen. Neutral entry would seem rather unlikely and completing entry must generally involve factor suppliers. In the modern oligopolistic situation aggressive entry appears to be the general case.

It is possible that economic development might be promoted in some situations by a device intended to protect existent firms from the predatory tactics of entrants. The fact is that western capitalism provides more protection for the developer of a technical device than for the developer of a market; more protection for an invention than for a product. Indeed, market size and power or monopoly is one of the few effective defenses against predatory tactics. There is no reason to assume that the right amount of protection is offered to either inventors or developers. Perhaps inventors obtain too much protection and developers too little.

F. Quantification of Pecuniary Externalities

An attempt was made to provide some measurement of externalities in the Canadian economy. The Canadian economy has been characterized in recent years by a tendency toward stagnation and a high level of unemployment, as well as by a vast increase in some industries in the flow of investment funds from other countries. It is therefore thought that a study of the generation of pecuniary externalities (loosely, profits caused by the activity of other firms) would be of some use in examining the reasons for these two tendencies.

The effect of any one sector of the economy on any other sector is,

of course, extremely complex and to attempt to quantify this effect and to measure it in terms of profits caused cannot be undertaken with a simple model. It is not possible to examine only firms A and B and to say that if A increases its output by one percent then B's profits will be changed by x percent. Rather, A's action will affect all the other industries in the economy; they will in turn react, and all these reactions will influence B's profits. In this study a modified input-output model was used. The input-output coefficients were modified by estimates for each sector of the cost function that was applicable to that sector.

Unfortunately, while the data are relatively good, they are not of sufficient accuracy to make estimates of pecuniary externalities very precise. Perhaps the weakest link in the data is the estimates of cost functions. Despite the enormous amount of work that has been done in estimation of cost functions there is still great doubt respecting the shape of the cost functions for any firm and any industry.

In this study, estimates of cost functions for the sector were obtained by estimating the elasticity of costs with respect to output. That is, a single coefficient for each sector was obtained; this coefficient represents the percent change in costs divided by the percent change in output. Inter-firm data were used in obtaining these elasticities. It should be noticed that such cross-sectional data are frequently used as the basis for estimates of LAC curves within firms. But such cross-sectional data are just as valid a source of information respecting the cost function for the whole industry. It is necessary to

assume that increments in output are distributed among firms in proportion to output of firms in the previous time period, that is, firms having i percent of the total output in the initial time period obtain i percent of the increment in output. Constant prices are assumed.

Proceeding in this manner the modified inverse is a matrix multiplier having properties analogous to the Keynesian multiplier. This multiplier indicates the change in demand for the product of each sector that derives from a change of one dollar in the exogenous expenditure on any other sector. It is possible to combine this multiplier with the cost function for the industry in order to obtain estimates of pecuniary externalities. For example, if sector A expands by one dollar's worth and this gives rise to a derived demand for the products of sector B of \$0.45, and if we know that one percent increments in the output of sector B may be had for an additional expenditure of only 0.8 percent, there will arise in sector B a pecuniary externality of \$0.09. (That is, $1 - 0.8 = 0.2 \times \$0.45$ equals \$0.09.)

This procedure has been followed and the pecuniary externality matrix presented in Table 30 of the Appendix is the result.

Conclusions that follow directly from the multiplier or externality matrix are as follows. 1. Making allowance for a cost function in each sector results in a matrix multiplier with some elements in common and some elements smaller than the Leontief matrix. That is, elements exhibit more dispersion than the Leontief matrix; they are in general smaller. They are sufficiently similar to the Leontief interdependence matrix that for many purposes the two are interchangeable.

2. A \$1.00 expansion of every sector of the Canadian economy would produce undisbursed funds of 25 to 34 cents in textiles, printing and publishing, primary iron and steel, construction, electric power, gas and water utilities. But a similar change leads to less than one cent undisbursed funds in agriculture, fisheries, forestry, mining, meat products, carbonated beverages, alcoholic beverages, tobacco, clothing, furniture, wood products, jewelery and silverware, electric appliances and supplies, and miscellaneous manufacturing industries. The latter industries are, in general, those industries subject to roughly constant returns to scale.

3. Undisbursed funds in sector i that result from a \$1.00 expansion in output in sector j are generally of rather small magnitude. Only about 4 percent involve sums larger than one cent. But a large number (about 23 percent) involve sums larger than one tenth of a cent.

Comparison of externalities created by expansion of all sectors with funds needed for investment to facilitate that expansion is useful. Undisbursed funds resultant from an expansion of \$1.00 in every sector of the Canadian economy range from zero dollars in agriculture and fisheries to \$1.13 in the transportation sector. Thus, even though the transportation industry is relatively capital intensive (Capital output ratio = 1.63) it appears that this sector could easily be self-financing. In Table 28 pecuniary externalities are presented for each sector. Again, these are the undisbursed funds which arise in each sector following a \$42.00 expansion (one dollar for each sector) in the Canadian economy. Capital output ratios are also presented in Table 28 for comparison.

Table 28. Pecuniary externalities and capital output ratios following one dollar expansion in every sector of the Canadian economy

Sector	Externality ^a (dollars)	Capital ^b Output
	(1)	(2)
1. Agriculture	0.000	1.19
2. Forestry	0.090	0.47
3. Fisheries	0.000	2.66
4. Metal mining	0.153	
5. Coal mining	0.202	1.18
6. Non-metal mining	0.000	
7. Meat products	0.000	
8. Dairy products	0.021	
9. Fish processing	0.051	0.92
10. Fruit and vegetable preparation	0.220	
11. Grain mill products	0.014	
12. Bakery products	0.020	
13. Carbonated beverages	0.000	
14. Alcoholic beverages	0.000	0.83
15. Confectionery and sugar	0.039	
16. Miscellaneous foods	0.123	
17. Tobacco and products	0.000	
18. Rubber products	0.082	0.48
19. Leather products	0.041	
20. Textile products	0.341	1.30
21. Clothing	0.000	0.34
22. Furniture	0.000	1.39
23. Wood products	0.000	0.77
24. Paper	0.088	1.59
25. Printing, publishing	0.252	0.64
26. Iron and steel	0.295	
27. Agricultural implements	0.101	0.91
28. Iron and steel n.e.s.	0.238	
29. Transportation	0.112	0.34
30. Jewellery	0.000	
31. Non-ferrous metal	0.122	0.69
32. Electrical apparatus	0.000	
33. Non-metallic minerals	0.010	
34. Products of petroleum and coal	0.100	0.99
35. Chemical and allied products	0.159	1.21
36. Miscellaneous manufacturing	0.000	0.56
37. Construction	0.302	0.24
38. Transportation, storage, trade	1.130	
39. Communication	0.067	1.63
40. Electric power, gas, water	0.293	7.02
41. Finance	0.104	0.28
42. Service	0.089	0.26

^aSource: from column 43, Table 30 of the Appendix.

^bSource: from column 5, Table 27.

The rank correlation between the pecuniary externalities in the first column and the capital output ratios in the second column is 0.802. A rank correlation this high occurs with probability 0.198, or about one time in every five. The hypothesis that pecuniary externalities fail to accrue in an appropriate manner can therefore not be rejected.

G. Suggestions for Further Research

The quantitative section of this study suffers from use of imprecise data and rather poor estimates, especially of cost functions. The situation is, in fact, something of a paradox, for in a modern capitalist country where data is abundant the capital market is relatively good. Thus, if it were possible in such an economy to demonstrate that pecuniary externalities did not accrue in an appropriate manner, the results might be largely irrelevant because of the excellence of the capital market. In an underdeveloped economy the capital market is likely to be most imperfect and failure of development may accompany inappropriate profit accruals. At the same time, in an underdeveloped economy, the sources of data are likely to be so imperfect that no one can find out very much about pecuniary externalities. Thus, results may only be obtained where there is little point in obtaining them. The whole subject is one rather well suited to investigation in a relatively small, relatively underdeveloped economy such as is Canada's. It is to be hoped that the matrix of pecuniary externalities developed in this study can be checked against data on changing profits in Canadian industries.

The cost structure of industries is worth investigating for its own sake. But unfortunately, production costs in firms still get most of the

attention.

Much more study should be made of the income of firms and industries. It is not really possible to take shelter behind the assumed perfection of the capital market. For even a perfect capital market would not allot funds to an elixir manufacturer, if the preparation proved non-appropriable. The general tendency to ignore income distribution among firms may be a carry-over from the failure of welfare economics to satisfactorily treat this problem in consumption. If so, the tendency is both unfortunate and based on a false premise, for income among firms is very intimately a production problem and only remotely a welfare problem.

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IX. APPENDIX

Table 29. Estimation of the economies of scale vector

Description of relevant information	Coefficient	Classification of industry and source of data
1. Agriculture		
		000 to 079
Iowa crop farms	1.06	Heady (84, p. 359)
Iowa hog farms	1.03	"
Iowa dairy farms	1.22	"
Iowa general farms	1.06	"
Iowa large farms	1.03	"
Iowa small farms	1.23	"
Southern Iowa	0.90	"
Central Iowa	0.91	"
Iowa crop production	1.23	"
Iowa livestock production	0.90	"
United States 1.82, 1.18, 1.15, 0.27		Table 2
Rank = 25/34 (gross income) and 31/34 (net income)		Table 25
E = 1.00		
2. Forestry		
		080 to 089
Fir lumber production	0.9520	Eq. 21, Table 20
Lumber products	Rank = 32/34	Table 25
E = 0.95		
3. Fishing, hunting and trapping		
		091 to 097
No information		
E = 1.00		
4. Metal mining, smelting and refining		
		101-119, 345
Iron ore mining	0.51-0.85	Figure 12
Copper mining	0.59-0.84	Figure 13
Pig iron (operating costs)	0.50	Table 21
Agitated vessels (installation)...	0.47	Table 21
Furnaces (installed)	0.43-0.85	Table 21
Metals	Rank = 17/34	Table 25
Iron and steel	Rank = 7/27	Table 24
E = 0.90		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
5. Coal mining, crude petroleum and natural gas 121-126		
Coal mining U.K.	0.9782	Eq. 14, Table 20
Coal mining U.K.	0.9023	Eq. 16, Table 20
Coal mining U.K.	0.8300	Eq. 17, Table 20
Coal mining U.S.A.	0.9654	Eq. 18, Table 20
Coal mining U.S.A.	0.95	Figure 14
Crude petroleum....	0.78, 0.61, 0.73, 0.67, 0.71	Figure 15
Pipe lines	0.67	Halci (82, p. 36)
Bituminous mining.....	Rank = 29/34 anthracite	
	Rank = 34/34	Table 25
Oil and gas.....	Rank = 6/34	Table 25
Petroleum and coal.....	Rank = 14/34	Table 25
Bituminous.....	Rank = 20/27	Table 24
E = 0.85		
6. Non-metal mining, quarrying and prospecting 131-133, 139-179		
See coal mining data in sector 5.		
See metal mining data under sector 4.		
Non-metal mining.....	Rank = 11/24	Table 25
E = 1.00		
7. Meat products 200		
Refrigeration (installed).....	0.68	Halci (82, p. 37)
Meat packing.....	Rank = 26/27	Table 24
E = 1.00		
8. Dairy products 201-209		
Ice cream	Rank = 2/27	Table 24
Food products.....	0.9010	Eq. 24, Table 20
Food processing.....	0.9384	Eq. 37, Table 20
Heat exchangers.....	0.54-0.60	Table 21
Liquid filters (installed).....	0.43-0.60	Table 21
Liquid pumps (installed).....	0.33-0.60	Table 21
Refrigeration units (installed)...	0.68	Table 21
Tanks.....	0.45-0.73	Table 21
Milk evaporation.....	0.94	Figure 17
Milk dealers.....	0.94	Figure 18
Creameries.....	0.73-0.80	Figure 19
E = 0.98		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
9. Fish processing		210
Fish canning	0.93	Figure 20
Refrigeration units (installed)...	0.68	Haldi (82, p. 27)
E = 0.95		
10. Fruit and vegetable preparation.		212
Food products	0.9010	Eq. 24, Table 20
Food processing.....	0.9384	Eq. 37, Table 20
Low temp. dehydration(Fruit juices	0.66	Haldi (82, p. 50)
Foods	Rank = 27/34	Table 25
Canning and preserving.....	Rank = 13/27	Table 24
E = 0.90		
11. Grain mill products		213-216
Wheat flour milling.....	0.97	Figure 21
Soybean extraction plants.....		
(construction).....	0.70	Haldi (82, p. 46)
Flour, etc.	Rank = 25/27	Table 24
E = 0.97		
12. Bakery products		218-219
Baking	E = 0.9843	Eq. 26, Table 20
Bread, etc.	Rank = 27/27	Table 24
E = 0.98		
13. Carbonated beverages		220
Beverages	Rank = 28/34	Table 25
Demineralized water systems.....	0.47	Haldi (82, p. 37)
Pumps (installation)	0.47	" (82, p. 36)
Agitated vessels (installation).....	0.47	" (82, p. 36)
Refrigeration units (installation)...	0.68	" (82, p. 37)
E = 1.00		
14. Alcoholic beverages		221-224
See under sector 13.		
Condensers (installation).....	0.51-0.54	Haldi (82, p. 36)
E = 1.00		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
15. Confectionery and sugar refining		225, 227
Sugar beet refining.....	0.8907	Wiles (231, p. 228)
Sugar beet refining.....	0.81	Figure 29
Confectionery.....	Rank = 8/27	Table 24
Cane sugar refining.....	Rank = 22/27	Table 24
E = 0.89		
16. Misc. food preparations		228
See under sector 10.		
E = 0.90		
17. Tobacco and tobacco products		230
Cigar production.....	E = 0.8816	Eq. 23, Table 20
Tobacco manufacturing.....	Rank = 4/34	Table 25
Tobacco products.....	Rank = 23/34	Table 25
E = 0.88		
18. Rubber products		236, 239
Tire and rubber production....	E = 0.9291	Eq. 34, Table 20
Rubber tires and tubes.....	Rank = 4/27	Table 24
Rubber products.....	Rank = 3/34	Table 25
E = 0.93		
19. Leather products		241-249
Leather belts	0.9588	Eq. 13, Table 20
Footwear.....	Rank = 19/27	Table 24
Leather.....	Rank = 21/27	Table 24
Leather products	Rank = 31/34	Table 25
E = 0.96		
20. Textile products except clothing		251-269
Spin bath (rayon).....	E = 0.68	Haldi (82, p. 50)
Textiles.....	Rank = 16/34	Table 25
Cotton goods.....	Rank = 12/27	Table 24
Rayon and allied products.....	Rank = 1/27	Table 24
E = 0.80		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
21. Clothing (textiles and fur)		270-279
Apparel	Rank = 25/34	Table 25
Hosiery	Rank = 18/27	Table 24
Woolens and worsteds.....	Rank = 11/27	Table 24
E = 1.00		
22. Furniture		286
Furniture	Rank = 29/34	Table 25
See under sector 23		
E = 1.00		
23. Wood products except furniture		281-285, 287-289
Fir lumber	E = 0.9520	Eq. 21, Table 20
E = 1.00		
24. Paper products		292-299
Paper mills	E = 0.80	Haldi (82, p. 46)
Newsprint	E = 0.8228	Eq. 20, Table 20
Kraft paper mill boilers.....	E = 0.69	Haldi (82, p. 37)
Black liquor (sulphate operating)....	E = 0.60	Haldi (82, p. 50)
Liquid filters (installed)....	E = 0.54	Haldi (82, p. 37)
Liquid pumps (installed).....	E = 0.47	Haldi (82, p. 37)
Paper	Rank = 13/34	Table 25
Pulp and paper mills	Rank = 15/27	Table 24
E = 0.95		
25. Printing, publishing, etc.		301-309
Printing and publishing	Rank = 8/34	Table 25
E = 0.80		
26. Primary iron and steel		325
Steel production	0.7825	Eq. 5, Table 20
Steel production	0.7247	Eq. 6, Table 20
Iron production	0.69-0.80	Figure 22
Pig iron (operating)	E = 0.50	Haldi (82, p. 50)
Steel ingots (operating)	E = 0.37	" (82, p. 50)
Finished steel (operating) ...	E = 0.50	" (82, p. 50)
E = 0.80		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
27. Agricultural implements		325
Farm and industrial machinery ... E = 0.70-0.99 E = 0.90		Figure 23
28. Iron and steel products n.e.s.		312-324, 326-329
Stainless plate E = 0.63 Construction and mining machinery E = 0.60 E = 0.90		Haldi (82, p. 38) Haldi (82, p. 36)
29. Transportation equipment		330-339
Automobiles E = 0.9101 Tire and rubber production E = 0.9291 Transportation equipment E = 5/34 E = 0.93		Eq. 19, Table 20 Eq. 34, Table 20 Table 25
30. Jewelery and silverware		343-346
E = 1.00		
31. Non-ferrous metal products n.e.s.		341, 342, 347, 349
Construction costs metal plants . E = 1.01 Copper condensers E = 0.54 See items under sector 28. E = 0.90		Haldi (82, p. 45) Haldi (82, p. 37)
32. Electrical apparatus and supplies		351-359
Electrical products E = 1.0533 Electric machines Rank = 15/34 Electric motors (installed) (small motors)..... E = 0.36 (large motors)..... E = 0.82 E = 1.00		Eq. 36, Table 20 Table 25 Haldi (82, p. 36) Haldi (82, p. 36)

Table 29. (Continued)

Description of relevant information	coefficient	Classification of industry and source of data
33. Non-metallic mineral products 137, 361-369		
Clay products	E = 0.9520	Eq. 22, Table 20
Cement plants (construction).....	E = 0.77	Haldi (82, p. 45)
Cement	Rank = 10/27	Table 24
Clay products	Rank = 23/27	Table 24
Glass products	Rank = 9/27	Table 24
Stone, clay, glass	Rank = 9/34	Table 25
E = 0.95		
34. Products of petroleum and coal 373-379		
Oil refining	E = 0.72	Figure 16
Petroleum refining (operating).....	E = 0.50	Haldi (82, p. 50)
Coke plant (construction)	E = 0.60	
Petroleum and petro-chemicals ..		
(construction).....	E = 0.68	Haldi (82, p. 45)
Natural gas	Rank = 19/34	Table 25
Petroleum refining	Rank = 5/27	Table 24
Coke	Rank = 24/27	Table 24
E = 0.95		
35. Chemical and allied products 380-389		
Chemical plants (construction)	E = 0.69	Haldi (82, p. 45)
Oxygen plants (operating)	E = 0.66	" (82, p. 50)
Electrolytic evaporator (installed) E = 0.53		" " "
Sulphur from H ₂ S (installed).....	E = 0.73	" " "
Gas production	E = 0.9046	Eq. 3, Table 20
Gas production	E = 0.9029	Eq. 4, Table 20
Gas production	E = 0.9734	Eq. 2, Table 20
Storage tanks (installed)	E = 0.57	Haldi (82, p. 38)
Chemicals	Rank = 7/34	Table 25
Fertilizers	Rank = 14/27	Table 24
E = 0.90		
36. Misc. manufacturing 391-399		
Misc. manufacturing	Rank = 20/34	Table 25
Non-electric machines	Rank = 24/34	Table 25
E = 1.00		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
37. Construction		404-439
Factory construction	E = 0.6607	Eq. 35, Table 20
Construction of plants	E = 0.6850	
(Estimated at mean value)		Eq. 1, Chapter 6
Construction and mining machinery...	E = 0.60	Haldi (82, p. 36)
Construction costs rayon plants	E = 0.75	Figure 24
E = 0.85		
38. Transportation, storage and trade		501-527, 701-799
Steam railways	E = 0.8842	Eq. 12, Table 20
Railway operating costs	E = 0.31	Figure 30
Bus transport	E = 0.7138	Eq. 1, Table 20
Wholesale trade	E = 0.8179,	
	E = 0.8674	Eq. 29, 30, Table 20
Department stores	E = 0.9813,	Eq. 33, Table 20
	1.0371	
Clothing stores	E = 0.9803	Eq. 33, Table 20
Local transit	Rank = 12/34	Table 25
Railways	Rank = 10/34	Table 25
E = 0.80		
39. Communications		543-549, 914
Telegraph	Rank = 23/34	Table 25
Telephone	Rank = 18/34	Table 25
E = 0.95		
40. Electrical power, gas and water utilities		
Gas production	E = 0.9734	Eq. 2, Table 20
Gas production	E = 0.9046	Eq. 3, Table 20
Gas production	E = 0.9029	Eq. 4, Table 20
Electricity	Rank = 3/27	Table 24
Electric light and power	Rank = 1/34	Table 25
Pipelines	E = 0.67	Haldi (82, p. 45)
E = 0.80		

Table 29. (Continued)

Description of relevant information	Coefficient	Classification of industry and source of data
41. Finance, insurance and real estate		802-809
Banking	E = 0.8757	Eq. 8, Table 20
Industrial assurance	E = 0.9604	Eq. 9, Table 20
Industrial assurance	E = 0.9285	Eq. 10, Table 20
Building societies (service)	E = 0.60	(Estimated from Monthly Review 61, p. 5)
E = 0.95		
42. Service industries		901-911, 916-949
Industrial assurance	E = 0.9604	Eq. 9, Table 20
Industrial assurance	E = 0.9285	Eq. 10, Table 20
Building societies	E = 0.9345	Eq. 11, Table 20
E = 0.95		

Table 30. The pecuniary externality matrix

	1	2	3	4	5
1.	0.000000	0.000000	0.000000	0.000000	0.000000
2.	0.000128	0.050054	0.000152	0.000224	0.000045
3.	0.000000	0.000000	0.000000	0.000000	0.000000
4.	0.000067	0.000118	0.000044	0.100089	0.000043
5.	0.001024	0.000640	0.000825	0.000892	0.150306
6.	0.000000	0.000000	0.000000	0.000000	0.000000
7.	0.000000	0.000000	0.000000	0.000000	0.000000
8.	0.000003	0.000000	0.000000	0.000001	0.000000
9.	0.000033	0.000001	--	0.000001	--
10.	0.000001	--	--	0.000001	--
11.	0.002445	0.000030	0.000002	0.000005	0.000001
12.	--	--	--	--	--
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000056	0.000001	0.000001	0.000002	0.000001
16.	0.000067	0.000002	0.000004	0.000004	0.000001
17.	--	--	--	--	--
18.	0.000585	0.000059	0.000060	0.000163	0.000172
19.	0.000019	0.000001	0.000006	0.000002	0.000001
20.	0.001478	0.000820	0.006280	0.000306	0.000102
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.000304	0.000053	0.000082	0.000107	0.000067
25.	0.000420	0.000122	0.000183	0.000349	0.000636
26.	0.000713	0.000970	0.000426	0.000706	0.000466
27.	0.000204	0.000003	--	0.000001	--
28.	0.001939	0.003224	0.000734	0.001913	0.001063
29.	0.000840	0.000977	0.001766	0.001228	0.001257
30.	--	--	--	--	--
31.	0.000127	0.000351	0.000070	0.000145	0.000083
32.	--	--	--	--	--
33.	0.000090	0.000041	0.000153	0.000218	0.000042
34.	0.002552	0.001699	0.002161	0.000787	0.000624
35.	0.002048	0.000142	0.000416	0.002049	0.000561
36.	--	--	--	--	--
37.	0.004933	0.002494	0.000077	0.002258	0.002552
38.	0.018873	0.007275	0.009676	0.009841	0.004987
39.	0.000113	0.000041	0.000062	0.000140	0.000083
40.	0.000954	0.000290	0.000476	0.008764	0.004264
41.	0.002043	0.000326	0.000786	0.000684	0.000581
42.	0.000369	0.000117	0.000158	0.000522	0.000295

Table 30. (Continued)

	6	7	8	9	10
1.	0.000000	0.000000	--	--	--
2.	0.000381	0.000176	0.000264	0.000231	0.000486
3.	0.000000	0.000000	--	--	--
4.	0.000485	0.000092	0.000080	0.000106	0.000177
5.	0.002598	0.000972	0.001161	0.001056	0.000741
6.	0.000000	0.000000	--	--	--
7.	0.000000	0.000000	--	--	--
8.	--	0.000003	0.020004	0.000001	0.000091
9.	0.000001	0.000158	0.000021	0.050000	0.000011
10.	0.000001	0.000001	0.000024	0.000001	0.100006
11.	0.000004	0.001533	0.001508	0.000005	0.000498
12.	--	--	--	--	0.000001
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000001	0.000089	0.000695	0.000001	0.005277
16.	0.000003	0.000167	0.000170	0.000004	0.000897
17.	--	--	--	--	--
18.	0.000240	0.000461	0.000474	0.000286	0.000167
19.	0.000003	0.000014	0.000014	0.000005	0.000007
20.	0.000226	0.001046	0.001032	0.002837	0.000529
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.000721	0.000450	0.000837	0.000346	0.001183
25.	0.000719	0.000794	0.000902	0.000977	0.001463
26.	0.000942	0.000946	0.000888	0.001600	0.003047
27.	--	0.000128	0.000126	0.000001	0.000037
28.	0.001687	0.002827	0.002566	0.005478	0.011179
29.	0.003668	0.000976	0.001039	0.001383	0.001071
30.	--	--	--	--	--
31.	0.000167	0.000160	0.000151	0.000152	0.000226
32.	--	--	--	--	--
33.	0.000076	0.000160	0.000501	0.000338	0.001929
34.	0.001366	0.001981	0.002162	0.001864	0.001109
35.	0.000939	0.003071	0.001554	0.000325	0.000734
36.	--	--	--	--	--
37.	0.003816	0.004723	0.004547	0.002483	0.003138
38.	0.013780	0.040927	0.029100	0.028381	0.028517
39.	0.000231	0.000319	0.000357	0.000686	0.000273
40.	0.003366	0.001400	0.001744	0.001269	0.001759
41.	0.000993	0.001843	0.001871	0.001391	0.001534
42.	0.000524	0.000656	0.000651	0.000876	0.001456

Table 30. (Continued)

	11	12	13	14	15
1.	--	--	--	--	--
2.	0.000478	0.000646	0.000481	0.000582	0.000370
3.	--	--	--	--	--
4.	0.000091	0.000072	0.000104	0.000066	0.000099
5.	0.000961	0.000899	0.000953	0.000636	0.000592
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	0.000037	0.000290	0.000038	0.000005	0.000282
9.	0.000410	0.000070	0.000003	0.000003	0.000037
10.	0.103098	0.015539	0.000245	0.000483	0.000688
11.	0.030929	0.004662	0.000074	0.000145	0.000206
12.	0.000001	0.020001	0.000001	0.000001	--
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000329	0.005540	0.013304	0.000662	0.011050
16.	0.000797	0.002761	0.006480	0.009159	0.001894
17.	--	--	--	--	--
18.	0.000309	0.000301	0.000680	0.000186	0.000100
19.	0.000013	0.000007	0.000004	0.000004	0.000004
20.	0.003070	0.000866	0.000553	0.000281	0.001448
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.001987	0.002940	0.001549	0.002467	0.001718
25.	0.001841	0.001782	0.002018	0.001171	0.001053
26.	0.000639	0.000454	0.001089	0.000707	0.000252
27.	0.000076	0.000027	0.000004	0.000007	0.000013
28.	0.001555	0.001007	0.035700	0.002237	0.000604
29.	0.001099	0.000964	0.000618	0.000671	0.000442
30.	--	--	--	--	--
31.	0.000186	0.000162	0.000211	0.000118	0.000344
32.	--	--	--	--	--
33.	0.000145	0.000146	0.000123	0.000697	0.000048
34.	0.001483	0.001587	0.002160	0.000983	0.000641
35.	0.003038	0.001478	0.001656	0.000482	0.000582
36.	--	--	--	--	--
37.	0.004929	0.003714	0.006518	0.001867	0.001723
38.	0.073617	0.037605	0.019357	0.014280	0.011928
39.	0.000512	0.000651	0.000689	0.000357	0.000296
40.	0.002222	0.002748	0.001638	0.001992	0.000990
41.	0.001795	0.001795	0.001917	0.001470	0.000992
42.	0.001596	0.001630	0.002833	0.001617	0.000763

Table 30. (Continued)

	16	17	18	19	20
1.	--	--	--	--	--
2.	0.000337	0.000540	0.000159	0.000421	0.000158
3.	--	--	--	--	--
4.	0.000048	0.000234	0.000136	0.000061	0.000048
5.	0.000420	0.000637	0.000452	0.000338	0.000320
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	0.000032	0.000003	0.000002	0.000001	--
9.	0.000010	0.000012	0.000003	0.000060	0.000001
10.	0.000173	0.000003	0.000002	0.000001	0.000001
11.	0.000677	0.000834	0.000010	0.000141	0.000011
12.	0.000001	0.000001	0.000001	--	--
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.001761	0.000022	0.00005	0.000064	0.000003
16.	0.100065	0.000027	0.000017	0.000014	0.000081
17.	--	--	--	--	--
18.	0.000131	0.000275	0.070056	0.000551	0.000123
19.	0.000005	0.000009	0.000134	0.040005	0.000051
20.	0.000356	0.000663	0.023359	0.002483	0.200109
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.001510	0.002085	0.000577	0.000772	0.000500
25.	0.001299	0.001288	0.001124	0.000972	0.000413
26.	0.000449	0.000496	0.000513	0.000710	0.000250
27.	0.000023	0.000070	0.000001	0.000012	0.000001
28.	0.001354	0.001288	0.001415	0.002350	0.000616
29.	0.000451	0.000707	0.000689	0.000553	0.000422
30.	--	--	--	--	--
31.	0.000094	0.000861	0.000128	0.000105	0.000106
32.	--	--	--	--	--
33.	0.000288	0.000070	0.000067	0.000101	0.000050
34.	0.000732	0.001352	0.000861	0.000560	0.000228
35.	0.000788	0.001321	0.004996	0.000676	0.001614
36.	--	--	--	--	--
37.	0.001265	0.000483	0.002098	0.001808	0.000672
38.	0.022710	0.020612	0.013996	0.020808	0.008560
39.	0.000422	0.000161	0.000699	0.000503	0.000224
40.	0.001268	0.001251	0.002070	0.001335	0.001617
41.	0.001429	0.001838	0.002411	0.001284	0.000739
42.	0.001127	0.002253	0.001367	0.000799	0.000313

Table 30. (Continued)

	21	22	23	24	25
1.	--	--	--	--	--
2.	0.000141	0.001423	0.015341	0.010157	0.001624
3.	--	--	--	--	--
4.	0.000037	0.000391	0.000095	0.000221	0.000062
5.	0.000242	0.000397	0.000606	0.001151	0.000449
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	0.000001	0.000001	0.000001	0.000001	0.000001
9.	0.000001	0.000002	0.000001	0.000001	0.000001
10.	0.000001	0.000001	0.000001	0.000001	0.000001
11.	0.000014	0.000012	0.000065	0.000047	0.000010
12.	--	--	--	--	--
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000002	0.000003	0.000003	0.000003	0.000002
16.	0.000026	0.000014	0.000038	0.000042	0.000009
17.	--	--	--	--	--
18.	0.000112	0.000316	0.000087	0.000088	0.000066
19.	0.000023	0.000203	0.000003	0.000036	0.000007
20.	0.060356	0.017933	0.000718	0.001901	0.000365
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.000500	0.000430	0.000217	0.050126	0.007914
25.	0.000922	0.001014	0.000616	0.000762	0.200462
26.	0.000187	0.001113	0.000864	0.000725	0.000240
27.	0.000001	0.000001	0.000006	0.000004	0.000001
28.	0.000422	0.003872	0.002608	0.001782	0.000536
29.	0.000336	0.000638	0.001084	0.001217	0.000409
30.	--	--	--	--	--
31.	0.000075	0.001422	0.000224	0.000534	0.000136
32.	--	--	--	--	--
33.	0.000036	0.000058	0.000064	0.000181	0.000060
34.	0.000218	0.000674	0.001225	0.001221	0.000558
35.	0.000754	0.001894	0.000876	0.001365	0.001571
36.	--	--	--	--	--
37.	0.001549	0.002483	0.003433	0.003066	0.002082
38.	0.009573	0.020686	0.025414	0.016032	0.011176
39.	0.000602	0.000727	0.000477	0.000491	0.000846
40.	0.001167	0.001955	0.001575	0.004866	0.001867
41.	0.000950	0.001400	0.001152	0.001135	0.001639
42.	0.000495	0.000949	0.000640	0.000650	0.000904

Table 30. (Continued)

	26	27	28	29	30
1.	--	--	--	--	--
2.	0.000010	0.000302	0.000210	0.000198	0.000258
3.	--	--	--	--	--
4.	0.002321	0.000477	0.001156	0.000601	0.015657
5.	0.002028	0.000509	0.000627	0.000481	0.000440
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	--	--	0.000001	0.000001	0.000001
9.	--	0.000001	0.000001	0.000001	0.000001
10.	0.000001	0.000001	0.000001	0.000001	0.000002
11.	0.000002	0.000004	0.000003	0.000003	0.000004
12.	--	--	--	--	0.000001
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000001	0.000001	0.000002	0.000001	0.000002
16.	0.000002	0.000003	0.000003	0.000004	0.000003
17.	--	--	--	--	--
18.	0.000123	0.002282	0.000130	0.001350	0.000062
19.	0.000002	0.000109	0.000003	0.000033	0.000003
20.	0.000615	0.001597	0.000172	0.001890	0.000164
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.000162	0.000223	0.000285	0.000249	0.000655
25.	0.000411	0.000476	0.000684	0.000489	0.000978
26.	0.200478	0.022340	0.025580	0.016397	0.000755
27.	--	0.100000	--	--	0.000001
28.	0.001057	0.016773	0.100607	0.006786	0.000779
29.	0.001001	0.002144	0.002723	0.070397	0.000581
30.	--	--	--	--	--
31.	0.000150	0.000411	0.000870	0.001494	0.000954
32.	--	--	--	--	--
33.	0.000199	0.000104	0.000156	0.000352	0.000077
34.	0.004274	0.000962	0.001167	0.000857	0.000594
35.	0.000469	0.000944	0.000500	0.001018	0.000866
36.	--	--	--	--	--
37.	0.003511	0.002139	0.002620	0.001398	0.002582
38.	0.016847	0.018414	0.014797	0.013763	0.022590
39.	0.000170	0.000208	0.000437	0.000267	0.000838
40.	0.006925	0.002039	0.002528	0.001645	0.002605
41.	0.000674	0.000716	0.001377	0.000845	0.001634
42.	0.000406	0.000471	0.000902	0.000576	0.001176

Table 30. (Continued)

	31	32	33	34	35
1.	--	--	--	--	--
2.	0.000180	0.000264	0.000327	0.000077	0.000398
3.	--	--	--	--	--
4.	0.023560	0.002784	0.000133	0.000046	0.000630
5.	0.000608	0.000373	0.001975	0.017677	0.000933
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	0.000001	0.000001	0.000001	0.000001	0.000011
9.	0.000001	0.000001	0.000001	--	0.000049
10.	0.000001	0.000001	0.000001	0.000001	0.000002
11.	0.000003	0.000004	0.000009	0.000002	0.000131
12.	--	--	--	--	0.000001
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000001	0.000002	0.000004	0.000001	0.000063
16.	0.000003	0.000005	0.000124	0.000002	0.000122
17.	--	--	--	--	--
18.	0.000088	0.000290	0.000185	0.000264	0.000128
19.	0.000002	0.000004	0.000003	0.000002	0.000004
20.	0.000176	0.001635	0.001003	0.001581	0.000576
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.000226	0.000493	0.001086	0.000227	0.001207
25.	0.001204	0.000820	0.000723	0.000699	0.001284
26.	0.001104	0.004430	0.001737	0.000358	0.000891
27.	--	0.000001	0.000001	--	0.000010
28.	0.001640	0.003099	0.001665	0.000873	0.001866
29.	0.001112	0.000689	0.001493	0.000675	0.000865
30.	--	--	--	--	--
31.	0.100165	0.006757	0.000199	0.000099	0.000309
32.	--	--	--	--	--
33.	0.000136	0.000274	0.050058	0.000044	0.000483
34.	0.000870	0.000654	0.001973	0.050183	0.001836
35.	0.000672	0.001367	0.001092	0.000460	0.100336
36.	--	--	--	--	--
37.	0.004287	0.002222	0.002565	0.002466	0.002929
38.	0.029364	0.015890	0.018689	0.018521	0.019474
39.	0.000268	0.000526	0.000238	0.000248	0.000506
40.	0.003800	0.001887	0.005160	0.001909	0.003367
41.	0.000958	0.001194	0.001279	0.001693	0.001619
42.	0.000785	0.000952	0.000632	0.000625	0.001486

Table 30. (Continued)

	36	37	38	39	40
1.	--	--	--	--	--
2.	0.000392	0.001549	0.000206	0.000149	0.000295
3.	--	--	--	--	--
4.	0.000773	0.000711	0.000086	0.000152	0.000407
5.	0.000537	0.000680	0.001133	0.000410	0.002929
6.	--	--	--	--	--
7.	--	--	--	--	--
8.	0.000012	0.000001	0.000001	0.000001	--
9.	0.000002	0.000002	0.000001	0.000001	0.000001
10.	0.000002	0.000002	0.000002	0.000002	0.000001
11.	0.000008	0.000013	0.000004	0.000003	0.000004
12.	0.000001	0.000001	0.000001	0.000001	--
13.	--	--	--	--	--
14.	--	--	--	--	--
15.	0.000004	0.000003	0.000002	0.000002	0.000002
16.	0.000006	0.000013	0.000003	0.000003	0.000004
17.	--	--	--	--	--
18.	0.000195	0.000195	0.000215	0.000253	0.000064
19.	0.000182	0.000004	0.000010	0.000003	0.000001
20.	0.001756	0.000037	0.000126	0.000032	0.000011
21.	--	--	--	--	--
22.	--	--	--	--	--
23.	--	--	--	--	--
24.	0.001107	0.001064	0.000495	0.000308	0.000224
25.	0.001174	0.000865	0.001739	0.003438	0.000530
26.	0.000647	0.004367	0.000531	0.000469	0.000536
27.	0.000001	0.000001	0.000001	0.000001	--
28.	0.000785	0.007195	0.000901	0.000678	0.001043
29.	0.000465	0.001041	0.001524	0.000828	0.000350
30.	--	--	--	--	--
31.	0.000431	0.002380	0.000247	0.000455	0.000427
32.	--	--	--	--	--
33.	0.000078	0.001838	0.000156	0.000137	0.000149
34.	0.000965	0.000947	0.000637	0.000814	0.001100
35.	0.001913	0.002594	0.000328	0.000301	0.001755
36.	--	--	--	--	--
37.	0.002230	0.152258	0.006567	0.007129	0.008527
38.	0.012513	0.033541	0.202946	0.017793	0.008900
39.	0.000761	0.000310	0.000329	0.050103	0.000102
40.	0.001226	0.001800	0.001298	0.001217	0.200413
41.	0.001806	0.001339	0.001546	0.001254	0.000365
42.	0.001248	0.001126	0.001347	0.001704	0.000327

Table 30. (Continued)

	41	42	$\sum_{i=1}^{42}$
1.	--	--	--
2.	0.000247	0.000300	0.090363
3.	--	--	--
4.	0.000115	0.000144	0.152719
5.	0.000695	0.000589	0.201892
6.	--	--	--
7.	--	--	--
8.	0.000001	0.000036	0.020868
9.	0.000001	0.000016	0.050921
10.	0.000002	0.000064	0.220360
11.	0.000005	0.000053	0.044123
12.	0.000001	0.000024	0.020039
13.	--	--	--
14.	--	--	--
15.	0.000003	0.000065	0.039036
16.	0.000004	0.000054	0.123101
17.	--	--	--
18.	0.000043	0.000095	0.062010
19.	0.000002	0.000013	0.040960
20.	0.000153	0.000693	0.341334
21.	--	--	--
22.	--	--	--
23.	--	--	--
24.	0.000267	0.000752	0.088472
25.	0.002658	0.010134	0.251658
26.	0.000681	0.000788	0.295481
27.	0.000001	0.000012	0.100777
28.	0.000682	0.001870	0.238255
29.	0.000478	0.001309	0.112180
30.	--	--	--
31.	0.000362	0.000288	0.122336
32.	--	--	--
33.	0.000260	0.000203	0.010386
34.	0.000436	0.000605	0.099662
35.	0.000436	0.001035	0.159016
36.	--	--	--
37.	0.020985	0.008875	0.302001
38.	0.007875	0.140020	1.129648
39.	0.000410	0.000734	0.066517
40.	0.001878	0.003556	0.293090
41.	0.050900	0.001260	0.104452
42.	0.001889	0.050295	0.089507