BULK WAVE CHARACTERIZATION OF LAMINATED COMPOSITES

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INTRODUCTION

Composite materials are currently seeing wider use in the aerospace and automobile industries. Composites offer many advantages over conventional materials, such as a greater strength to weight ratio and the ability to engineer their mechanical properties to a specific task. The major problems associated with composites are cost and reliability. Like virtually all engineering materials, composites can have flaws which may compromise their strength and reliability. The ability to detect these flaws in a reliable, cost effective fashion is significantly essential in the utilization of composite materials in critical structural areas. Currently, nondestructive evaluation using ultrasonic wave amplitude analysis, is most often used to inspect materials for flaws. This method can detect gross macroscopic flaws such as delamination or cracks, but more subtle flaws in the individual layers of a composite such as incomplete cure or low fiber volume ratio, cannot be found using conventional inspection techniques. Full stiffness modulus reconstruction, using acoustic wave velocities, is an alternative way to nondestructively determine the exact mechanical properties of a given composite part. Much research has been done in the area of modulus reconstruction of single layered composites [1-3]. The objective of this paper is to develop schemes for modeling multi-layered composites commonly seen in practice. Two basic methods of modeling composites are presented here; the layered method and the averaged method. The layer method treats each ply as a separate material. The averaged method consists of taking all the layers and averaging their material properties together. This paper will look at the differences between these two methods and will show how the relationship between the wavelength and the ply thickness determines which theory will apply.

Laminate Models

In this work, two alternative models for wave propagation in composite laminates were investigated: 1) Layer Method - ply by ply treatment and 2) Average Method - composite treated as a homogeneous medium. Both models were studied for two typical composite lay-ups, cross-ply $(0°/90°)$ and $\pi/4 \, (0°/\pm 45°/90°)$. Of particular importance in this study was determining the effect of frequency on wave propagation in laminated composites.

Model 1 - Layer Method

With this approach, one must track the progress of each wave through the laminate, taking into account mode conversion at each interface. The time of flight through the sample is obtained by examining each layer in the composite and adding the results together. It should be pointed out that certain waves, though theoretically possible, will not be generated in practice due to amplitude consideration.

Since water supports only longitudinal waves, the only particle motion at the boundary between the water and composite is in the *y-z* plane, hence only waves with a component of particle motion in the *y-z* plane can be generated. Thus, two waves will be generated in the 0° and 90° layers but three waves will be excited in the 45° layers. This means that, with mode conversion, twelve direct ray paths are possible in a $\pi/4$ laminate, i.e.

For the cross ply sample, there are four possible ray paths (since only two waves will be excited in each layer), i.e.

This information allows us to determine the transit times through the sample for any experimental geometry. Results are presented in Figures 1 and 2 for $\pi/4$ and cross ply samples as a function of angle of incidence.

Table 1	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6
0° ply	V_{p2}	V_{p2}	V_{p2}	V_{p3}	V_{p3}	V_{p3}
45° ply	V_{p1}	V_{p2}	V_{p3}	V_{p1}	V_{p2}	V_{p3}
90° ply	V_{p1}	V_{p1}	V_{p1}	V_{p1}	V_{p1}	V_{p1}
	Path 7	Path 8	Path 9	Path 10	Path 11	Path 12
0° ply	V_{p2}	V_{p2}	V_{p2}	V_{p3}	V_{p3}	V_{p3}
45° ply	V_{p1}	V_{p2}	V_{p3}	V_{p1}	V_{p2}	V_{p3}
90° ply	V_{p2}	V_{p2}	V_{p2}	V_{p2}	V_{p2}	V_{p2}

Table 2	Path 1	Path 2	Path 3	Path 4
0° ply	V_{p2}	V_{p2}	V_{p3}	V_{p3}
90° ply	V_{p1}	V_{p2}	V_{p1}	V_{p2}
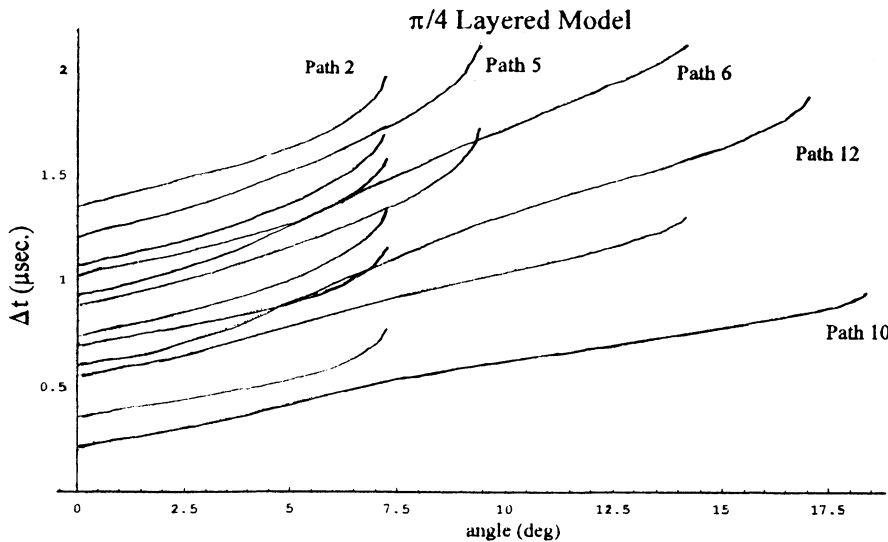
Figure 1 $\pi/4$ Layup - transit time vs angle of incidence (theory).

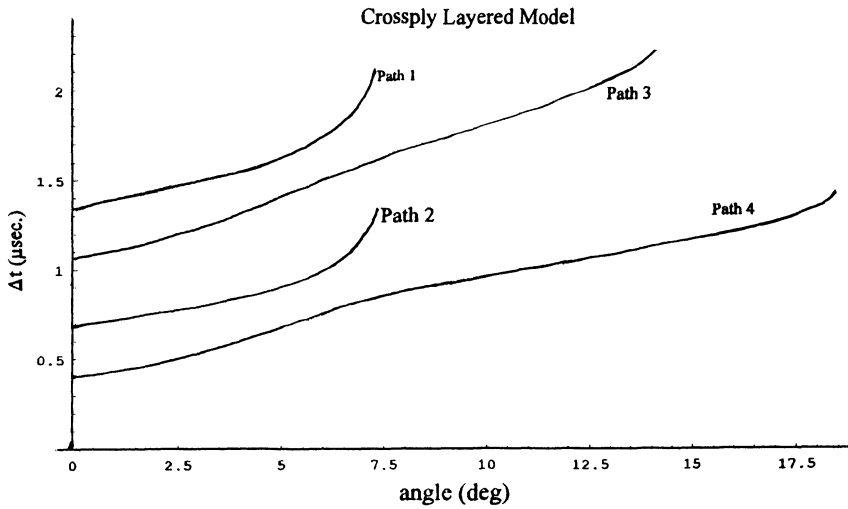**Crossply Layered Model**

Figure 2 Crossply layup - transit time vs angle of incidence (theory).

Model 2 - The Averaging Method

   In this method the composite is viewed as a single medium. The stiffness matrix components of the individual layers will be averaged together to obtain an overall new stiffness matrix. This procedure requires that each layer be given a weighting factor based on the thickness of that layer relative to the total length. This can be expressed in equation form as follows:

$$C_{ij}^{avg} = C_{ij}^1 \frac{d_1}{d_{total}} + C_{ij}^2 \frac{d_2}{d_{total}} + \dots + C_{ij}^n \frac{d_n}{d_{total}} \tag{1}$$

Here, based on polarization considerations, only two waves are predicted in each layup. Predicted transit times are presented in Figures 3 and 4.
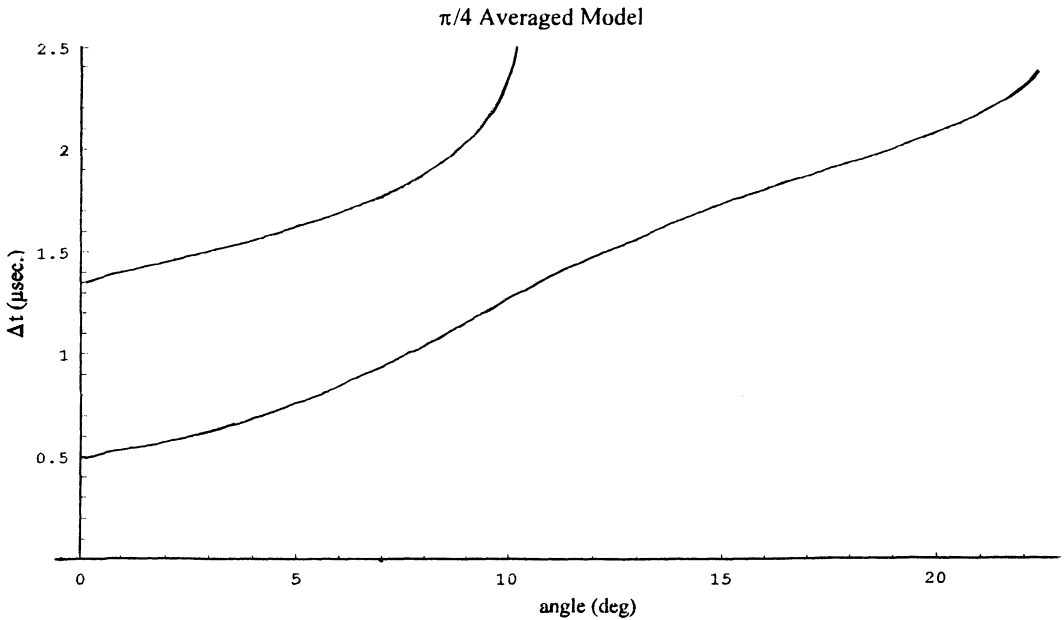

π/4 Averaged Model

Figure 3   π/4 Layup - transit time vs angle of incidence (theory).
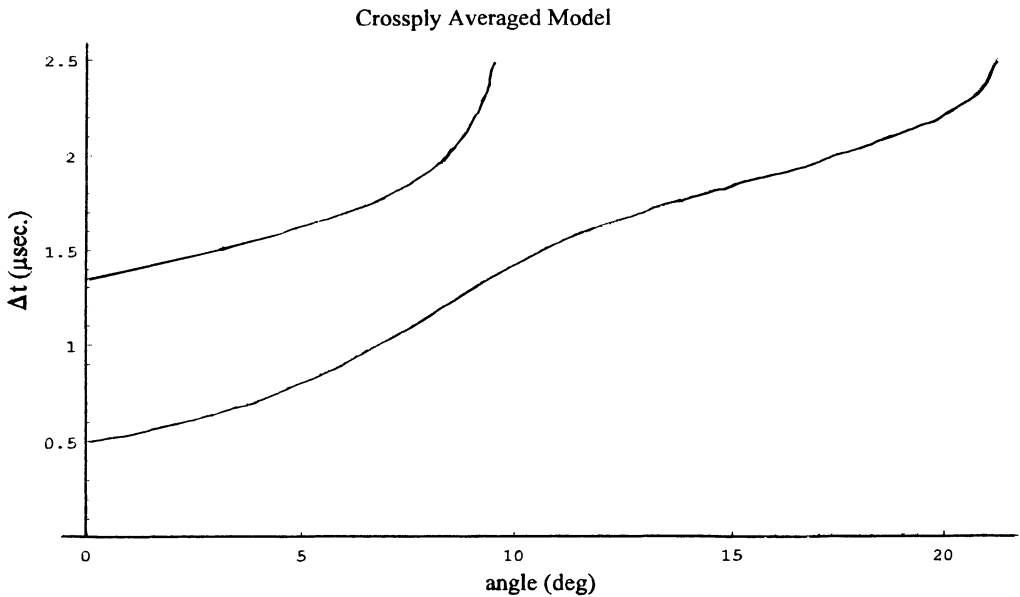
Figure 4   Cross ply layup - transit time vs angle of incidence (theory).

Model Verification

The finite difference method consists of developing algebraic approximation of a partial differential equation. In this work, the system that is being solved is the equation of motion for an inhomogeneous anisotropic material. It is true that within each layer the material is homogeneous, but by making the formulation account for inhomogeneous properties, the boundaries conditions between the layers do not need to be examined explicitly. This will greatly simplify the computer analysis program and will give more realistic results. The governing equation of motion now must be modified to allow spatially varying material properties, i.e.

$$\rho \ddot{u}_i = C_{ijkl}\, u_{k,lj} + \left(C_{ijkl}\right)_{,j} u_{k,l} \tag{2}$$

Here, we will concentrate our effort on a two dimensional model and restrict our attention to cross ply composites. Applying central difference equations to equation 2 allows us to calculate the particle displacements for each time step.

The next step in developing a finite difference model is to lay out the grid over the area of interest. The models derived in the previous sections calculate the change in the time of flight due to the presence of the sample. A uniform grid with distance steps that are determined at the time the program is run is laid over the area. The thickness of each ply is 2 mm and the width of the transducer pulse (plane wave) is around 10 mm. The axis of the finite difference grid is aligned with the transducers and not the sample. This was done to simplify the boundary conditions. This alignment means that the stiffness matrix components, that are needed for finite difference scheme must be rotated by angle θ about the x axis. The standard tensor tranformation relations are used to change the coordinate systems. Each grid point is assigned stiffness matrix components and densities consistant with the material properties at the grid point's loctation.

Next, we must deal with the boundary conditions. The top boundary is where the wave is introduced. The control region is pulsed to simulate transducer motion. The outer regions are stress free. After the pulse is finished the entire boundary is set to zero. The other three boundaries are free boundaries. This causes a problem when trying to calculate the value of the

w and *v* at the edge grid points. Points outside the grid are needed for these calculations. To generate, these points, a second order polynomial curve fit, using the edge point and the next two interior points was used to calculate points outside the grid. This procedure will insure that waves will be allowed to pass out of the model without being impeded and causing a reflection.

The last point of interest of the model is stability. Alterman and Loewenthal (4) calculated the stability condition of the isotropic wave equation with open boundary conditions as:

$$\frac{\Delta x}{\Delta t} \le \sqrt{V_{pl}^2 + V_{ps}^2}$$ (3)

where: Δt = time step Δx = spacial step V_{pl} = longitudinal phase velocity V_{ps} = shear phase velocity

However, there is no counterpart for anisotropic media. Here, we use step sizes of .00025 μ sec in time and .025 mm in space. This insures that the fastest traveling wave (longitudinal wave, parallel to fiber direction) will not traverse more than a single pixel in a single time step. Since in this case we are propagating waves through the composite at oblique incidence (not in the reinforcement direction), no wave propagates more than a third of a pixel in any given time step. No stability problems were observed.

RESULTS AND DISCUSSION

Typical results from the finite difference calculation are shown in Figures 5 - 6. At high frequencies (as illustrated for 5mHz in Figure 5), four distinct waves were observed in the cross ply response as predicted using the layer model. The predicted values for the behavior of the transit time as a function of incidence angle agrees well with the theory for all four propagation paths.
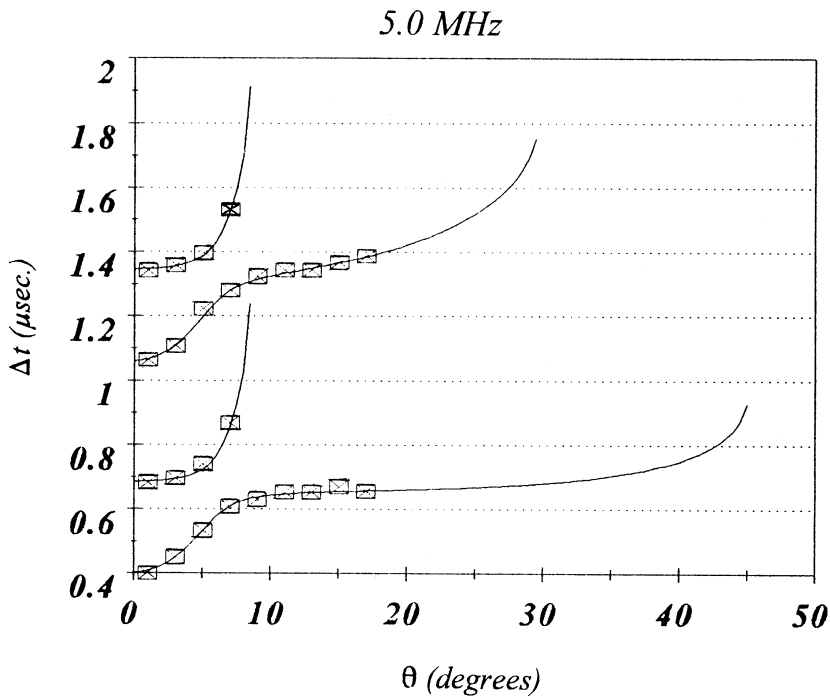
5.0 MHz

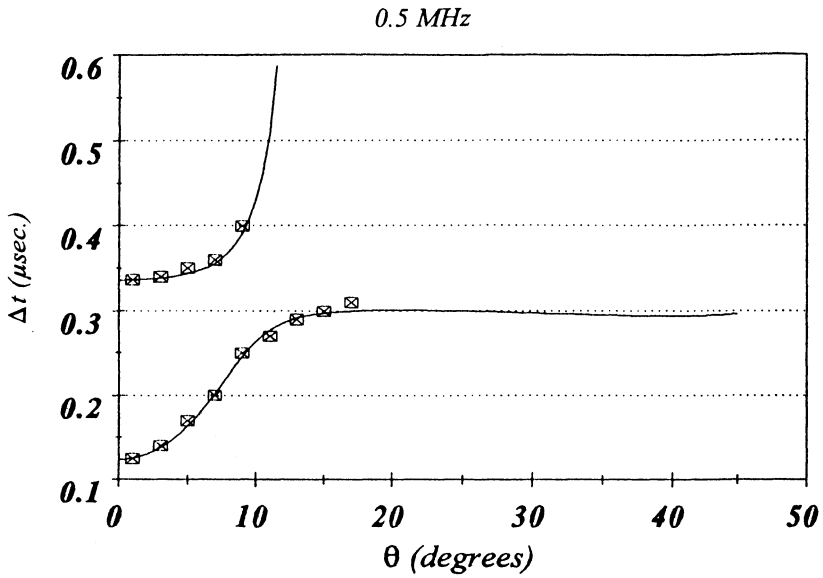Figure 5 Finite difference results (high frequency).

Figure 6 Finite difference results (low frequency).

At lower frequencies (as shown in Figure 6) only two waves could be discerned. This is in agreement with the predictions from the average model. The angular variation expected from the average model was also found to agree well with the finite difference simulation. If we fix the angle of incidence, we obtain results similar to those of Figure 7 for an incidence angle of 2.5°. Above 1.5 MHz four distinct arrival times may be found in the simulated wave form. Below this frequency, the amplitudes of two of the waves are undetectable.
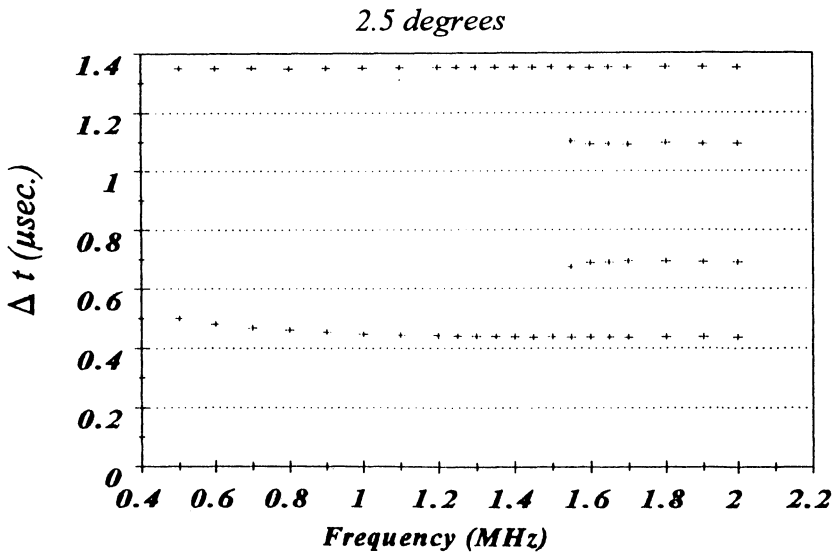
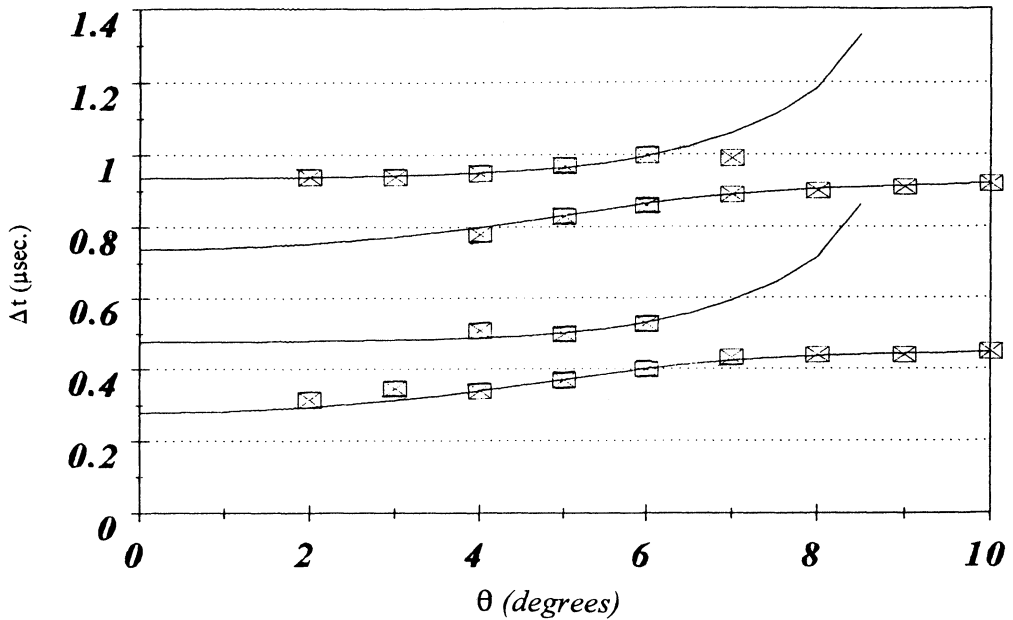Figure 7 Effect of frequency on acoustic response.

Figure 8 Experimental results.

Experimentally, the major challenge one encounters lies in separating out several closely spaced echoes. To do this we employ the analytic, signal magnitude approach of Gamel [5]. This procedure yields a signal proportional to the energy in the acoustic wave and allows us to recover all four echoes in the cross ply sample.

From these results, we observe two distinct specimen responses at high frequency and low frequency.

If we postulate that the transition between high and low frequency behavior depends on the ratio of the acoustic wavelength to the refracted ray path in a given ply, we obtain the following expression for the critical transition frequency:

$$F_{cr} = \frac{\sqrt{V_p^2 - \sin^2(\theta)\dfrac{V_p^4}{V_{water}^2}}}{d} \tag{4}$$

In Figure 9 these critical frequency values are compared with the extinction frequencies observed (finite difference) for the quasilongitudinal wave in the $0°$ and $90°$ plies of the cross-ply laminate. Again, good agreement is observed.
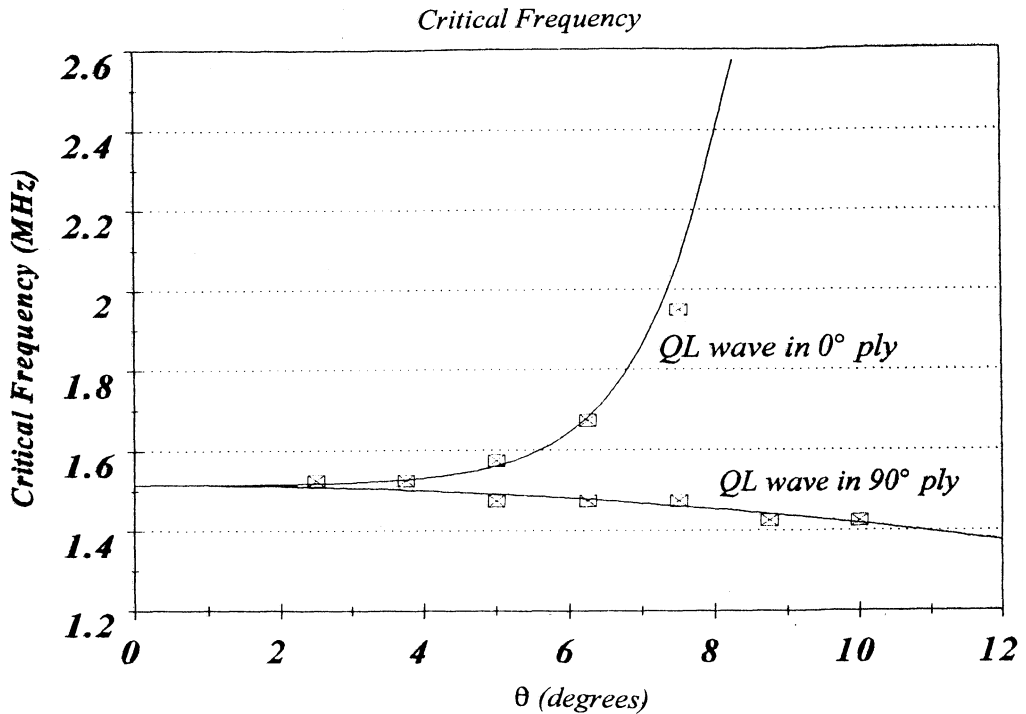
Figure 9 Critical frequency results.

CONCLUSIONS

In this work, acoustic wave propagation in laminated composites has been studied. Two models for the behavior of the composites have been presented: 1) a layered model which treats each ply as a distinct entity and 2) An average model which treats the composite as a continuum. Results were compared with both finite difference simulations and laboratory experiments for several different cases. At high frequencies, the layer model was found to work best. At lower frequencies, the average model produced the best results. Based on the observations we developed an expression to predict the transition between the two regimes based on the ratio of the acoustic wavelength to the refracted path length in each ply. This expression was found to agree quite well with observations.

REFERENCES

1. L. Moore, "An Ultrasonic Method of Determining the Effects of Processing on Carbon-Carbon Composites," Masters Thesis, University of Oklahoma, 1992.
2. Z. Chen, "A New Method to Ultrasonically Measure Elastic Moduli in Composites," Master's Thesis, University of Oklahoma, 1978.
3. R. Kline, Nondestructive Characterization of Composite Media, Technomic Publishing Co., Lancaster, 1992.
4. A. Ilan and D. Loewenthal, :Instability of Finite Difference Schemes Due to Boundary Conditions in Elastic Media." Geophysical Prospecting, Vol. 24, 1976.
5. P. Gamel, "Analogue Implementation of Analytic Signal Magnitude for Pulse Echo Systems," Ultrasonics, 19 (1981) pp. 219-283.