BOUNDARY INTEGRAL AND FINITE ELEMENT SIMULATION OF ELECTROMAGNETIC NDE PHENOMENA

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INTRODUCTION

Finite element (FE) studies of energy/material interactions associated with the nondestructive evaluation (NDE) of materials have not only yielded useful information concerning the physics of new NDE phenomena [1] but also provided "test-beds" for the simulation of NDE situations too difficult to replicate in a laboratory environment [2]. FE code has been developed for the analysis of those NDE processes governed by elliptic [3], parabolic [4] and hyperbolic [5] partial differential equation (PDE) types taking advantage of axisymmetry wherever possible in order to conserve computer capacity. In those situations requiring fine spatial and/or temporal discretization, it has been found that the FE code makes excessive demands on even the best computer resources. Examples of this situation include the finite element modeling of the remote field effect in large diameter pipelines [6] and the simulation of ultrasonic wave propagation through large structures [7].

The boundary element (BE) method, on the other hand, is ideally suited to infinite domain problems and generally requires a smaller set of equations to be solved, particularly when a solution is needed at only discrete points on the boundary of the material, as if often the case in NDE studies.

Rizzo [8] describes the common theoretical background of both FE and BE methods. Zienkiewicz et al [9], Li et al [10], and Brebbia and Georgiou [11] discuss the application of both methods to structural mechanics problems and also show how FE and BE discretized regions can be coupled to give a hybrid solution procedure.

This research is concerned with the relative merits and limitations of FE and BE methods as applied to the simulation of electromagnetic NDE phenomena. To illustrate the procedure, a comparative study is described in this paper of the direct current potential drop (DCPD) technique as used in fatigue crack monitoring.

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POTENTIAL DROP METHOD

The DCPD method has been accepted in fracture mechanics as one of the most accurate and efficient methods for monitoring crack growth [12]. It has been applied to monitor fatigue, stress corrosion and creep cracks. It is also useful for measuring velocities of fast running cleavage cracks and evaluating the extent of crack closure in fatigue crack propagation studies [12,13].

In the DCPD method (Fig. 1), a constant current is passed through a cracked specimen, and the electric potential drop across the crack is monitored. As the crack increases, the uncracked cross-sectional area of the test piece decreases, the current path resistance increases and thus the potential increases. In practice for a particular geometry, calibration curves are given in the form of U/U_0 v/s A/W where U_0 is the reference potential drop across the initial cracked specimen (Fig. 2), U is the potential drop as the crack length increases, and A/W is the crack length to width ratio. These calibration curves have been determined experimentally [16-18], analytically (Johnson's formula) [14,15], and numerically [12,13], and they are accepted for use in fracture mechanics.

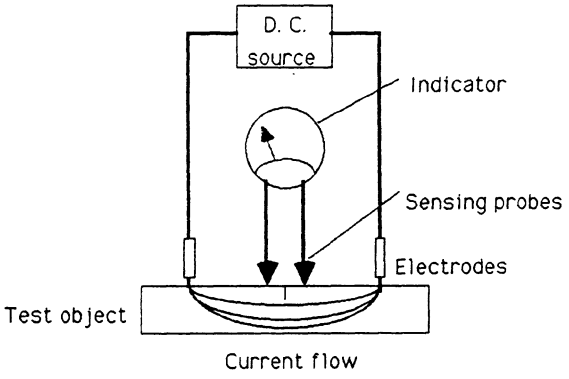

Fig. 1. DCPD experimental setup.

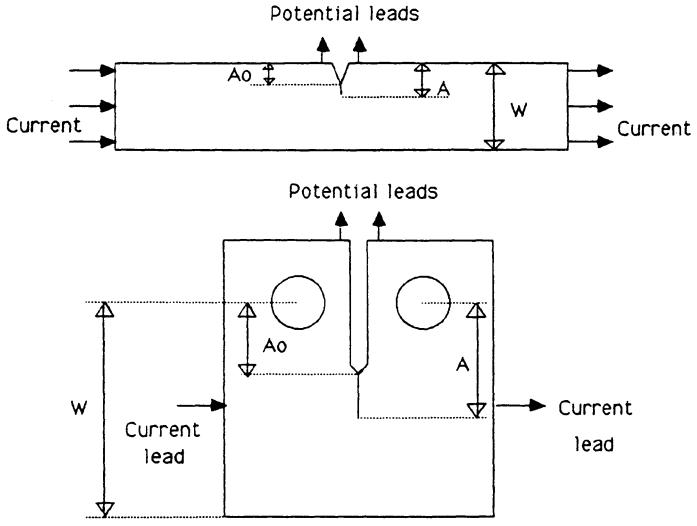

Fig. 2. SEN and CT geometry.

The underlying elliptic PDE describing the DCPD technique is Laplace's equation

$$\nabla^2 V = 0 \tag{1}$$

where V is the steady state voltage in the specimen geometry, with a constant current I in the plane of the geometry. The next section describes briefly the procedure used in FE and BE analysis to solve Laplace's equation.

FINITE ELEMENT FORMULATION

Laplace's equation is solved by the FE method through a variational formulation. The functional for Eq. (1) becomes [12,13,20]

$$F = \int_v \frac{1}{2}\sigma \left(\left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 \right) dv - \int_S V_s i_s \, ds \tag{2}$$

where v is the volume enclosed, S is the surface that bounds the volume v, V_s and i_s are the voltage and current specified at the boundary. By taking the first variation of F and setting it to zero, that is

$$\delta F = 0 \tag{3}$$

the potential V is obtained by solving the matrix equation

$$[S][V] = [Q] \tag{4}$$

where $[S]$ is the conductivity matrix and $[Q]$ is the forcing vector due to the boundary conditions. The FE meshes for the two geometries are shown in Figs. 3a and 3b.

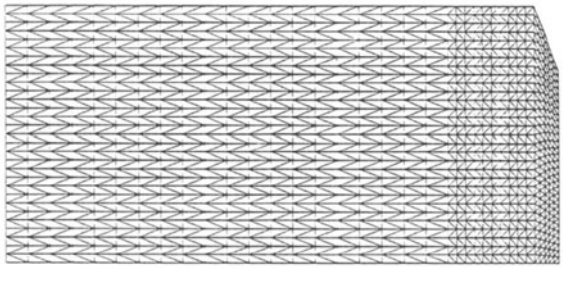

Fig. 3a. SEN finite element mesh.

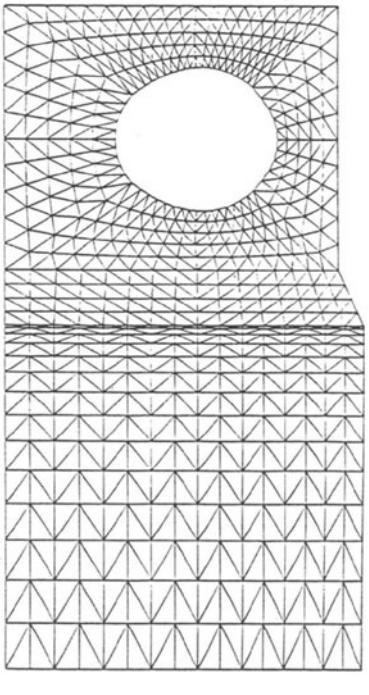

Fig. 3b. CT finite element mesh.

BOUNDARY ELEMENT FORMULATION

The boundary integral equation corresponding to Eq. (1) can be written as

$$c(\xi)V(\xi) = \int_{ds}\left[F(x,\xi)\frac{\partial V}{\partial n}(x) - \frac{\partial F}{\partial n}(x,\xi)V(x)\right]ds(x) \tag{5}$$

where ds is the boundary, F denotes a fundamental solution to the elliptic pde, and the coefficient $c(\xi)$ is the interior angle 2π or π [21]. The fundamental solution is the Green's function, which is

$$F(x,\xi) = -\frac{1}{2\pi\sigma}ln(r) \tag{6}$$

where r is the vector from ξ to x. This integral equation is solved by standard numerical techniques for the potential V on the boundary.

RESULTS

The geometries considered for comparison are the single edge notched (SEN) specimen and the compact tension (CT) specimen (Fig. 2). Fig. 4 compares the FE and BE results, and Johnson's formula for the SEN specimen. Calibration curves for the CT test piece are shown in Fig. 5. The FE and BE results indicate less than 10 percent deviation from Johnson's formula. The deviation is more pronounced for the CT specimen as the input current is applied only at a point. The memory requirements and the execution times are outlined in Table I. The results in Table I are rough estimates as an initial study.

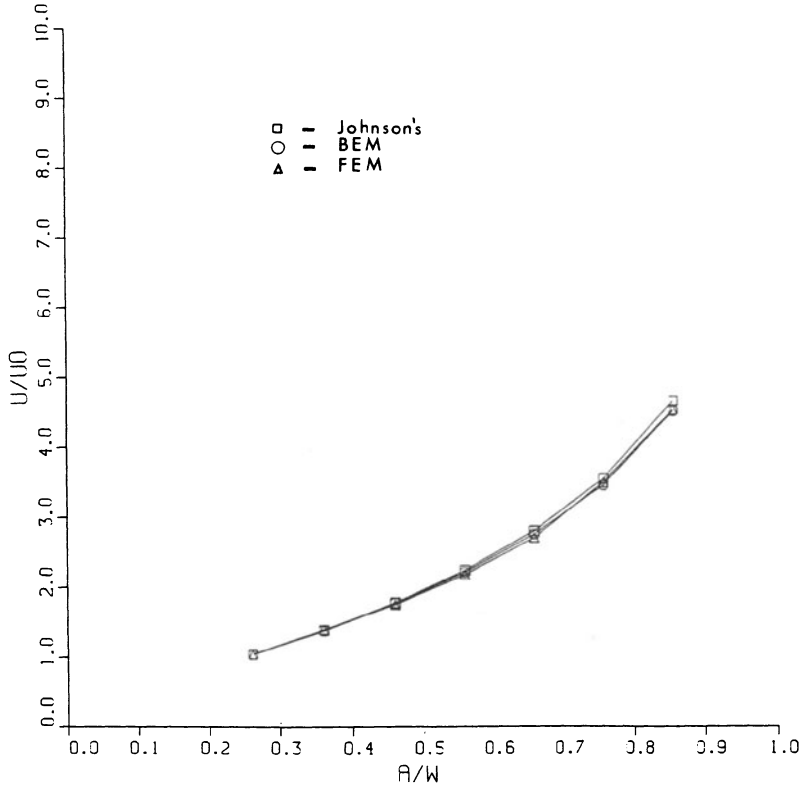

Fig. 4. Calibration curves for the SEN specimen.

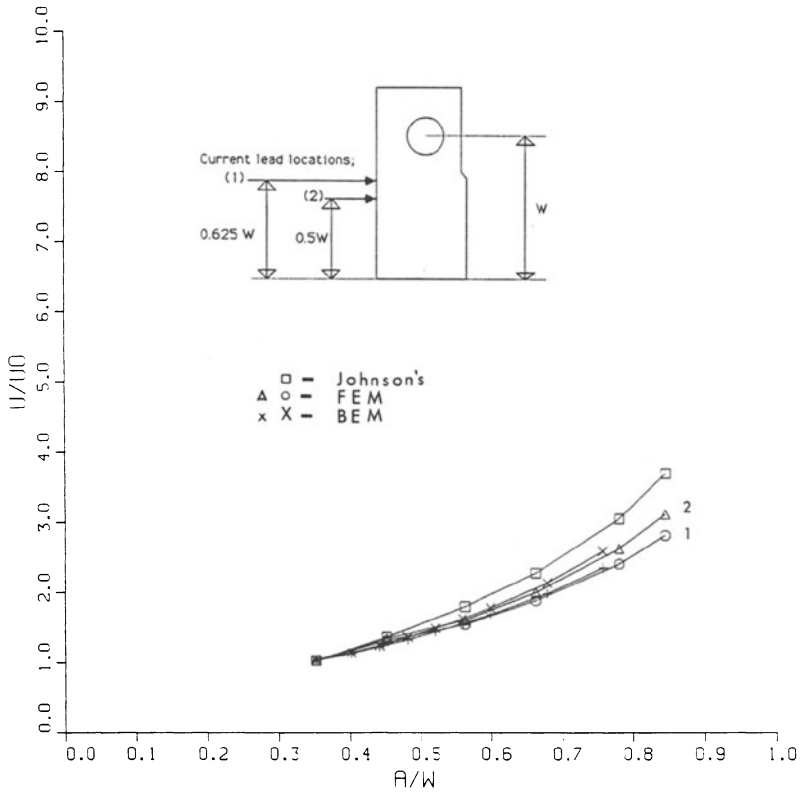

Fig. 5. Calibration curves for the CT specimen.

Table I. Comparison of computer resources for FE and BE.

	SINGLE EDGE NOTCH		COMPACT TENSION	
	FE	BE	FE	BE
Number of Nodes	1116	38	972	68
Number of Elements	2100	19	1720	34
Bandwidth	33	39	54	69
CPU Time (seconds)	3.05	0.566	4.06	1.07
Total Memory	46K	3K	53K	9K

Figs. 6a and 6b are the equi-potential contours obtained by post-processing of the FE method results. These contour plots help to visualize the effect of the current input and the behavior of the voltage contours as the crack propagates in the material. (The current contours, if plotted, would be orthogonal to the voltage

contours). The equi-potential contours were used to derive the optimum condition for the current input and voltage measurement positions [18]. To generate the calibration curves, the voltage is monitored at only one point near the crack edge, as the crack length increases. Thus, the BEM is more convenient in the generation of the calibration curves. The computer resources in Table I are self explanatory and indicate the tradeoffs required for the two numerical techniques.

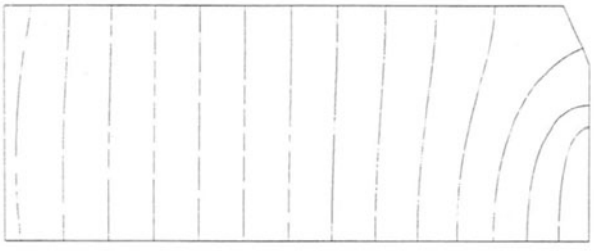

Fig. 6a. Equipotential contours for the SEN specimen using FE.

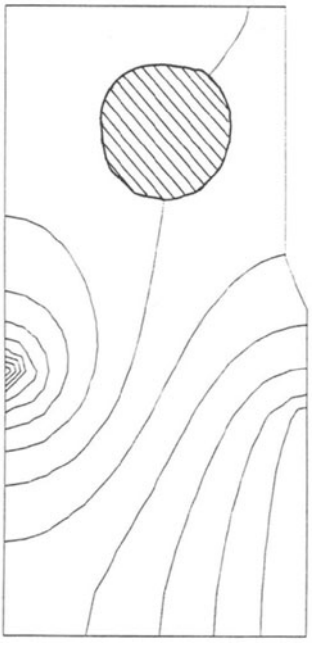

Fig. 6b. Equipotential contours for the CT specimen using FE.

CONCLUSIONS

Despite the preliminary nature of this study, a number of tentative observations can be drawn from the BE/FE results.

1. Where specific output data is needed at discrete points on a test geometry (as is often the case in NDT), the BE approach requires much less computational effort.
2. Where a complete qualitative and quantitative picture of the energy/material interaction physics is required, then the FE technique is most efficient.

These comments apply to the solution for Laplace's equation for the DCPD testing of isotropic materials described in this paper. A more detailed study is already underway to extend the comparison to ACPD and eddy current NDT methods.

With regard to the DCPD method for measuring fatigue crack growth, it is interesting to note that for the compact tension specimen, Johnson's formula appears to be unduly pessimistic in its estimation of crack length.

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REFERENCES

1. W. Lord, Y. S. Sun, S. S. Udpa and S. Nath, "A Finite Element Study of the Remote Field Eddy Current Phenomenon," IEEE Transactions on Magnetics, Vol. 24, No. 1, pp. 435-438, Jan., 1988.
2. N. Ida, H. Hoshikawa and W. Lord, "Finite Element Prediction of Differential EC Probe Signals from Fe_3O_4 Deposits in PWR Steam Generators," NDT International, Vol. 18, No. 6, pp. 331-338, Dec., 1985.
3. W. Lord and J. H. Hwang, "Defect Characterization from Magnetic Leakage Fields," British Journal of Nondestructive Testing, Vol. 19, No. 1, pp. 14-18, Jan., 1977.
4. B. Allen, N. Ida and W. Lord, "Finite Element Modeling of Pulsed Eddy Current NDT Phenomena," IEEE Transactions on Magnetics, Vol. 21, No. 6, pp. 2165-2168, Nov., 1985.
5. R. Ludwig and W. Lord, "A Finite Element Study of Ultrasonic Wave Propagation in an Aluminum Block," Materials Evaluation, Vol. 46, pp. 108-113, Jan., 1988.
6. S. Nath, M. S. thesis, Colorado State University, 1988.
7. R. Ludwig and W. Lord, "A Finite Element Formulation for the Study of Ultrasonic NDT Systems," IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, Vol. 35, No. 6, pp. 809-820, Nov., 1988.
8. F. J. Rizzo, "The Finite and Boundary Element Methods: One Review of Their Foundations," Advanced Boundary Element Methods, IUTAM Symposium, San Antonio, Texas, 1987.
9. O. C. Zienkiewicz et al, "The Coupling of the Finite Element Method and the Boundary Solution Procedure," International Journal of Numerical Methods in Engineering, Vol. 11, pp. 357-375, 1976.
10. H. B. Li et al, "A New Method for the Coupling of Finite Elements and Boundary Element Discretized Subdomains of Elastic Bodies," Computer Methods in Applied Mechanics and Engineering, Vol. 54, pp. 161-185, 1986.
11. C. A. Brebbia and P. Georgiou, "Combination of Boundary and Finite Elements in Electrostatics," Applied Math Modelling, Vol. 3, pp. 212-220, June, 1979.
12. R. O. Ritchie and K. J. Bathe, "On the Calibration of the Electrical Potential Technique for Monitoring Crack Growth Using Finite Element Methods," International Journal of Fracture, Vol. 15, No. 1, pp. 45-55, Feb., 1979.
13. G. H. Aronson and R. O. Ritchie, "Optimization of the Electrical Potential Technique for Crack Growth Monitoring in Compact Test Pieces Using Finite Element Analysis," Journal of Testing and Evaluation, Vol. 7, No. 4, pp. 208-215, July, 1979.
14. H. H. Johnson, "Calibrating the Electric Potential Method for Studying Slow Crack Growth," Material Research and Standards, Vol. 5, No. 9, pp. 442-445, Sept., 1965.

15. Che-Yu Li and R. P. Wei, "Calibrating the Electrical Potential Method for Studying Slow Crack Growth," Journal of Testing and Evaluation, Vol. 6, No. 8, pp. 392-394, Aug., 1966.

16. K. H. Schwalbe and D. Hellman, "Application of the Electrical Potential Method to Crack Length Measurements Using Johnson's Formula," Journal of Testing and Evaluation, Vol. 9, No. 3, pp. 218-221, May, 1981.

17. G. Clark and J. F. Knott, "Measurement of Fracture Cracks in Notched Specimens by Means of Theoretical Electrical Potential Calibrations," Journal of Mechanics and Physics of Solids, Vol. 23, No. 4/5, pp. 265-276, Aug./Oct., 1975.

18. R. O. Ritchie et al, "Crack Growth Monitoring: Optimization of the Electrical Potential Method Using an Analog Method," International Journal of Fracture Mechanics, Vol. 7, No. 4, pp. 462-467, Dec., 1971.

19. M. E. Orazem and W. Ruch,"An Improved Analysis of the Potential Drop Method for Measuring Crack Lengths in Compact Tension Specimens," International Journal of Fracture, Vol. 31, pp. 245-252, 1986.

20. K. J. Bathe and E. L. Wilson, Numerical Methods in Finite Element Analysis, Prentice-Hall, Englewood, New Jersey, 1976.

21. T. Rudolphi and V. Peterson, "A Modular Program for Poisson's Equation with Linear Boundary and Domain Elements," Advanced Engineering Software, Vol. 9, No. 1, pp. 7-15, 1987.